Aspects of Superfluid and Quantum Dark Matter

Mark Hertzberg, Tufts University

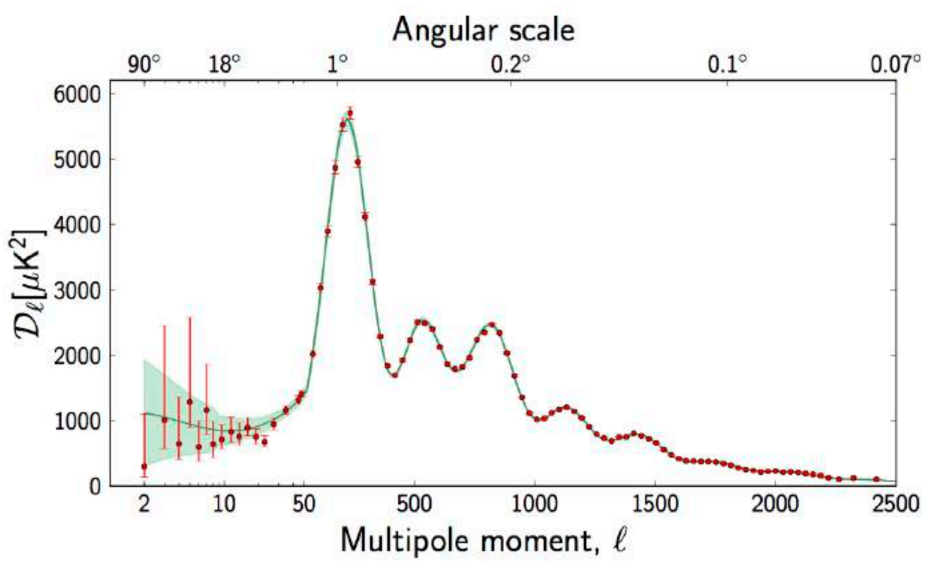
Quarks 2021, June 24

Part 1: Aspects of Superfluid Dark Matter

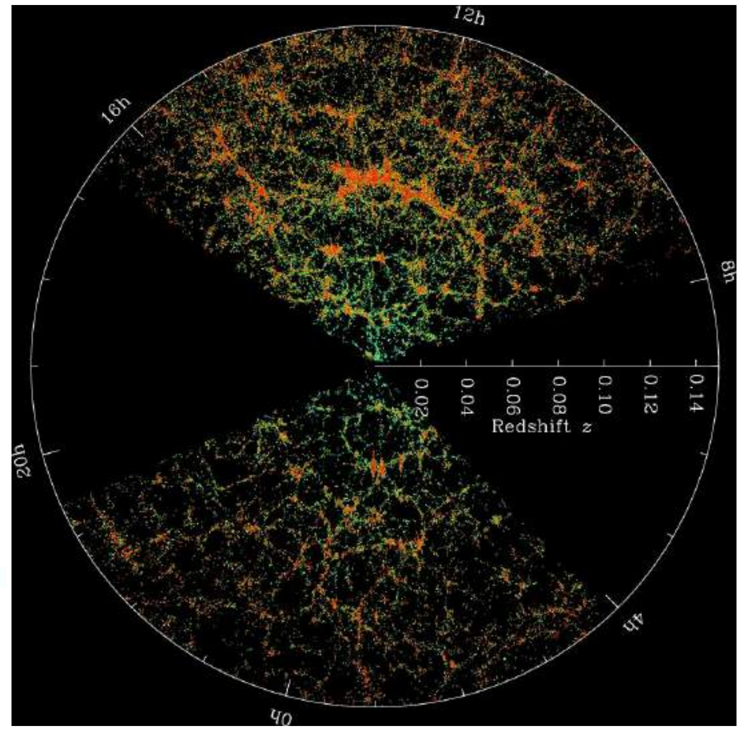
Based on work with:

Jacob Litterer  Neil Shah 

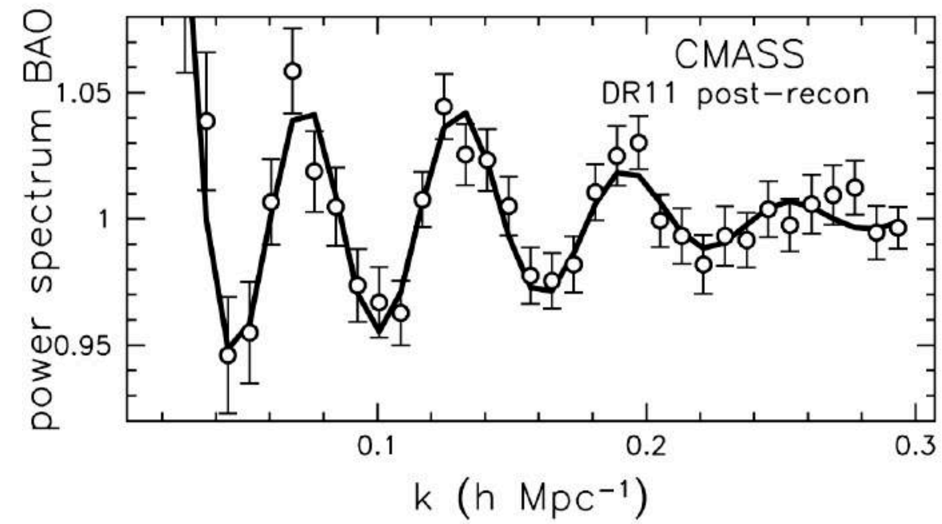
Tremendous Success of CDM on Large Scales



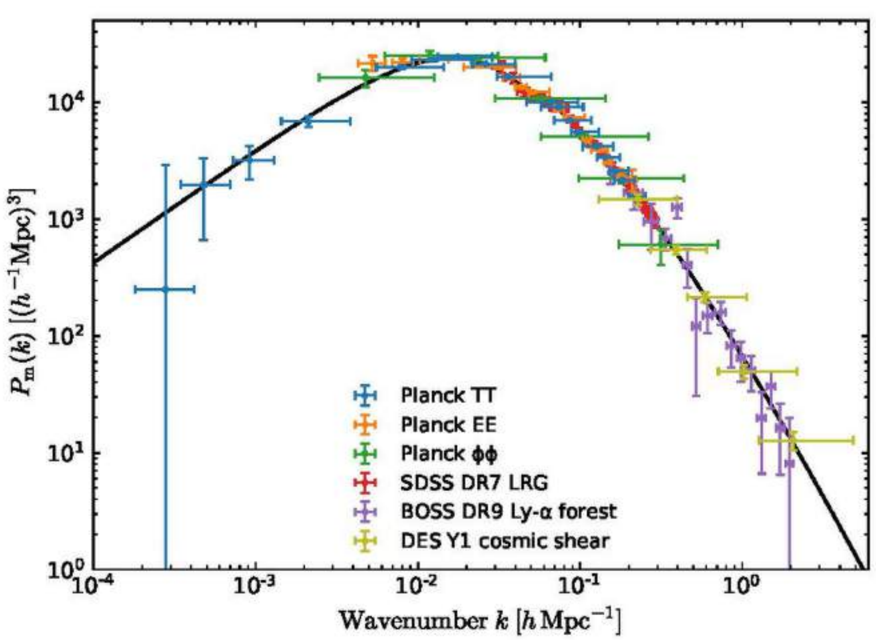
CMB (Planck)



Large Scale Structure (SDSS)

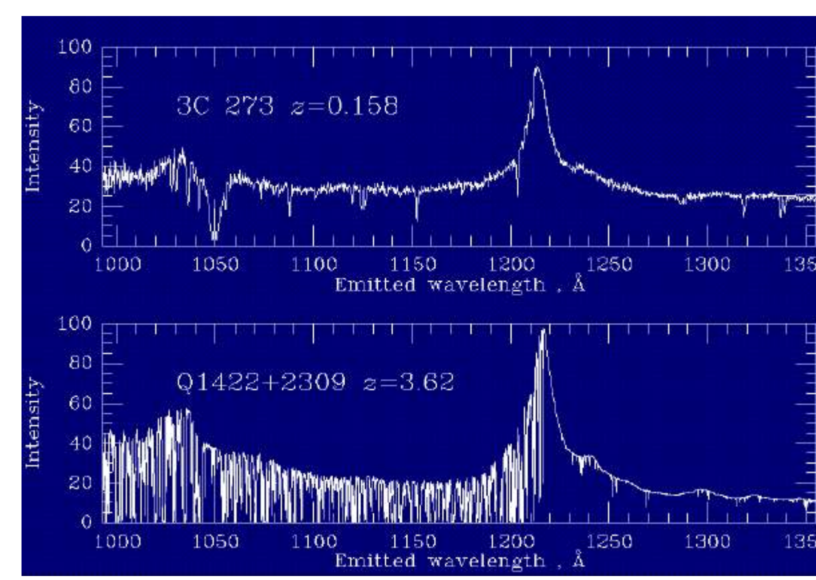


BAO (BOSS)



Concordance (Planck)

Galaxy Clustering (Hubble)

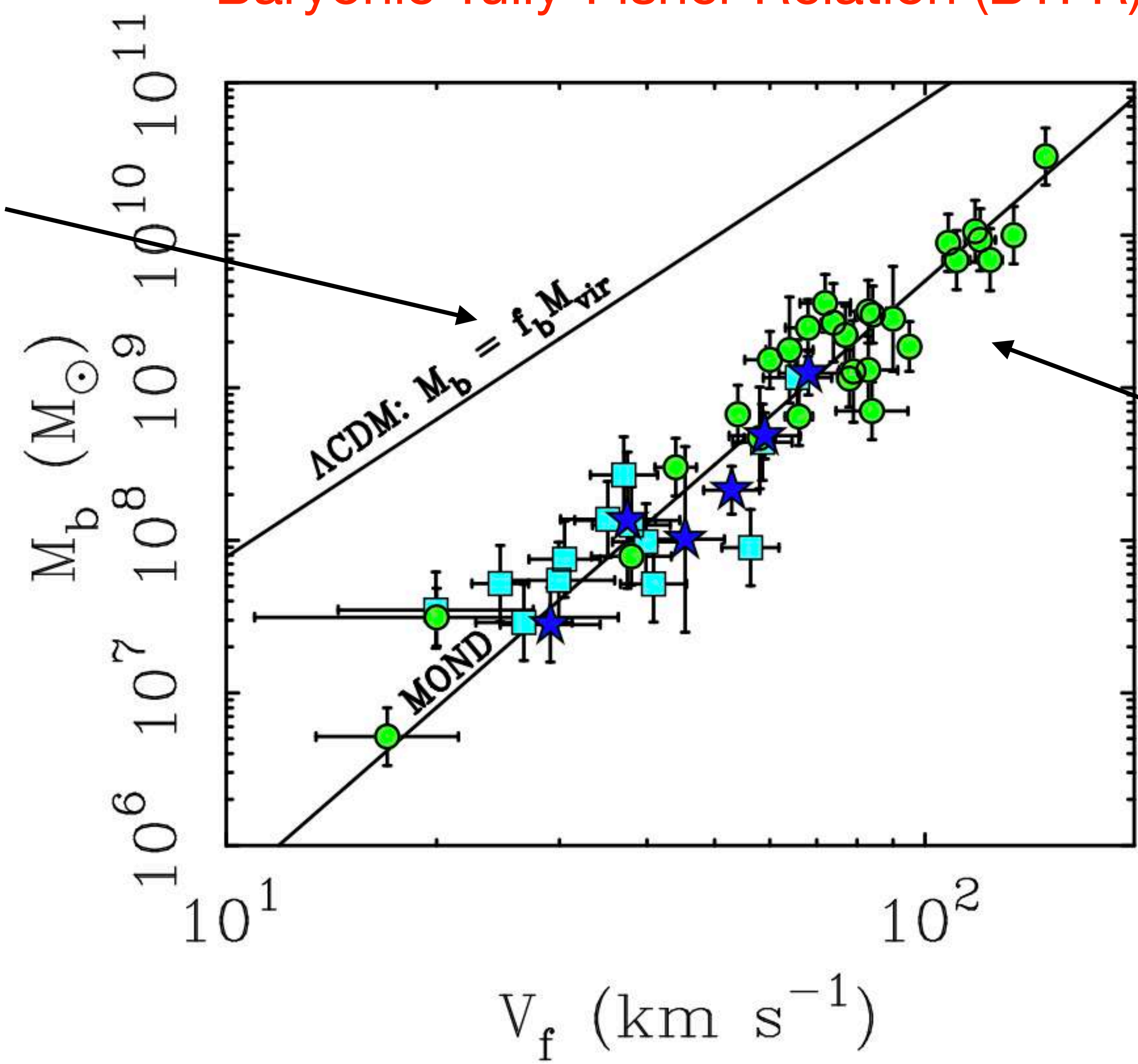


Lyman Alpha Forest (Keck)

Possible Difficulties with CDM on Galactic Scales?

Baryonic Tully-Fisher Relation (BTFR)

$M_b \propto v_f^3?$



$\boxed{M_b \propto v_f^4}$

(from McGaugh 2011)

Modify Gravity on Galactic Scales (MOND)?

$$a \propto \frac{M_{enc}}{R^2}$$

If instead: $$\frac{v^2}{R} = a \propto \sqrt{\frac{M_{enc}}{R^2}} \implies \boxed{M_b \propto v_f^4}$$

(Milgrom)

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(Milgrom)

Implementing this is very difficult:

> The unique, causal, Lorentz invariant, theory of massless spin 2 particles, at large distances, is general relativity

(Feynman, Weinberg, Deser,…)

However, one can add new degrees of freedom. In particular, new scalars could mediate a new long range (peculiar) interaction

Simplistic Attempt

$$X \equiv \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$$

$$S = -\int d^4x \sqrt{-g} \left[F(X, \varphi) \; - \tilde{\beta}\, \varphi\, T_{\mathrm{B}} + \frac{\mathcal{R}}{16\pi G} + \mathcal{L}_{SM} \right]$$

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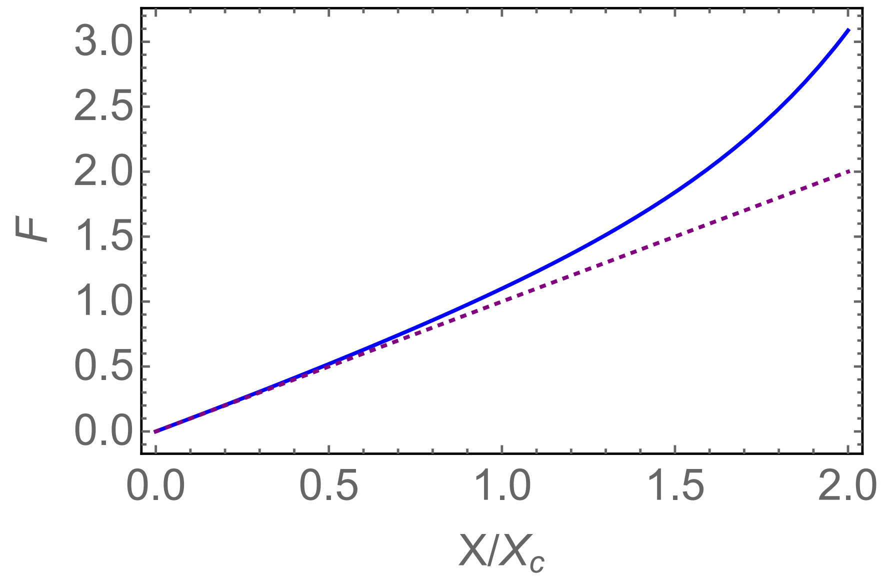
Introduce function *F* with 2 different asymptotic regimes:



Low densities/large scales $F = X$ (canonical)

(3/2 scaling)

High densities/galactic scales $\boxed{F = \tilde{\alpha}\, X \sqrt{|X|}}$ Mediates a MOND-like force

Example: $F = X\,(1 + \tilde{\alpha}^4\, X^2)^{1/4}$

Large φ, can stay within regime of EFT

Simplistic Attempt

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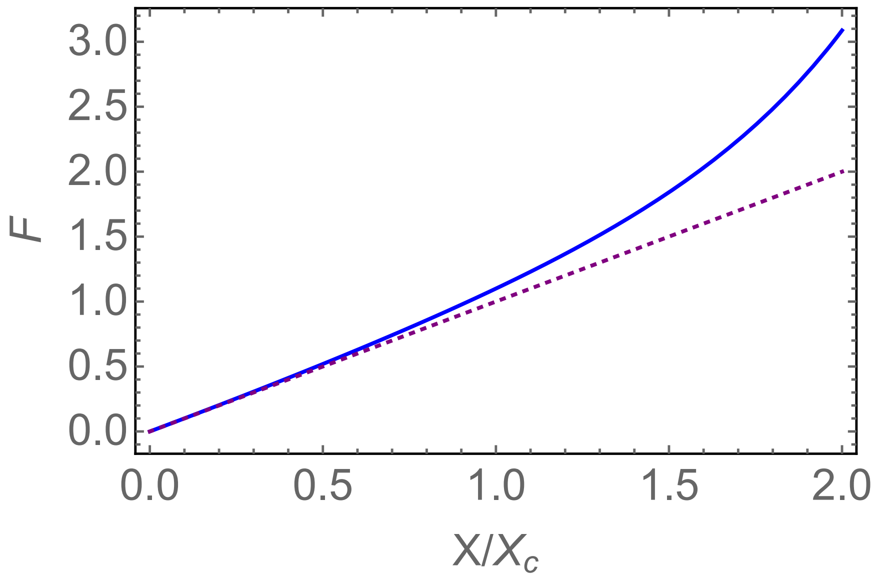

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$$-\frac{3\tilde{\alpha}}{2^{3/2}} \nabla \cdot (\nabla\varphi |\nabla\varphi|) = \tilde{\beta}\, T_B \qquad \mathbf{a} \propto -\mathrm{sign}(\tilde{\alpha}) \sqrt{\frac{M_{enc}}{R^2}}\, \hat{r}$$

Simplistic Attempt

Two Problems

$$F = \tilde{\alpha}\, X \sqrt{|X|}$$

Theoretical: High energy perturbations on top of the MONDian solution are superluminal (related details later)

Simplistic Attempt

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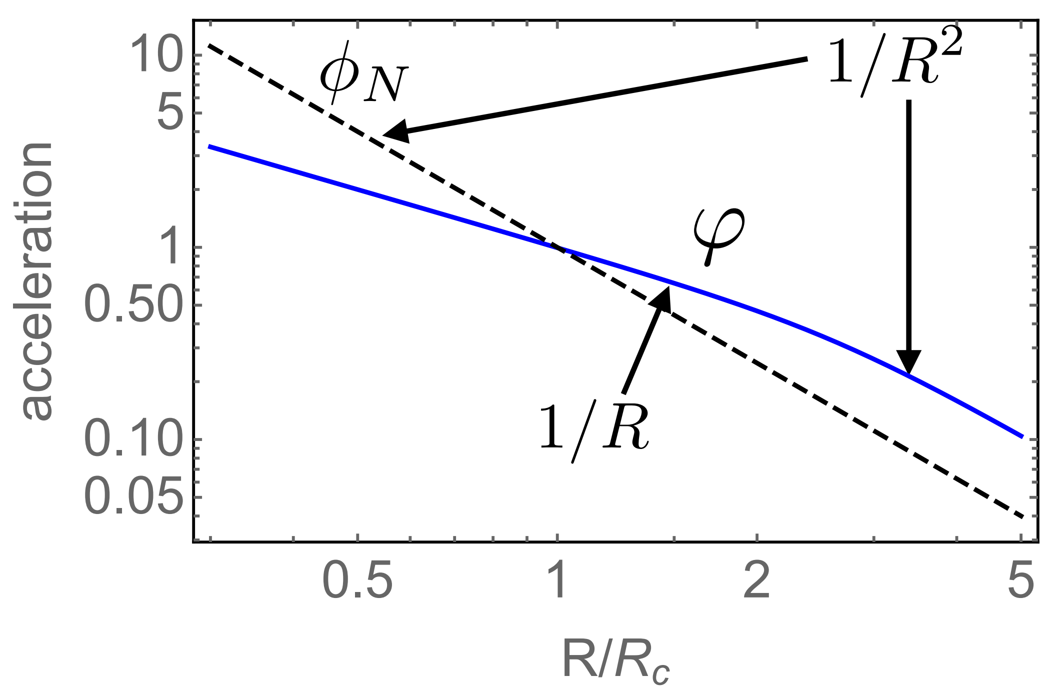
$$F = \tilde{\alpha}\, X \sqrt{|X|}$$

Theoretical: High energy perturbations on top of the MONDian solution are superluminal (related details later)

$$F = X$$

Phenomenological: Although the scalar becomes canonical at large scales, it introduces another $1/r^2$ force. So it is difficult to consistently obtain the desired galactic and large scale behaviors



Sophisticated Attempt - SuperFluid Dark Matter (SFDM)

Clever idea: Use Spontaneous Symmetry Breaking

Complex Scalar Dark Matter

$U(1)$ Symmetry

Φ

$$X = g^{\mu\nu}\partial_\mu \Phi \partial_\nu \Phi^*$$

Berezhiani, Khoury 2015

Sophisticated Attempt - SuperFluid Dark Matter (SFDM)

Clever idea: Use Spontaneous Symmetry Breaking

Complex Scalar Dark Matter Φ $X = g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi^*$

$U(1)$ Symmetry

Example:

$$\boxed{F = \frac{1}{2}\left(X + m^2|\Phi|^2\right) + \frac{\Lambda^4}{6(\Lambda_c^2 + |\Phi|^2)^6}\left(X + m^2|\Phi|^2\right)^3}$$

Reproduces CDM on large scales

(Quantum, from particle point of view, Classical, from field point of view)

Allows for phase transition to superfluid at galactic densities

$$\Phi = \rho\, e^{i(\theta + mt)}$$

Goldstone θ can act as long-ranged force mediator

Berezhiani, Khoury 2015

Sophisticated Attempt - SuperFluid Dark Matter (SFDM)

$$\Phi = \rho\, e^{i(\theta + mt)}$$

Slowly varying phase θ and modulus ρ around superfluid condensate

$$X + m^2|\Phi|^2 = (\nabla\rho)^2 - 2\,m\,\rho^2\,Y \qquad \text{with} \qquad Y \equiv \dot{\theta} - m\,\phi_N - \frac{(\nabla\theta)^2}{2m}$$

At tree-level, can integrate out heavy modulus (Higgs mode)

$$\rho^2 = \Lambda\sqrt{2m|Y|}$$

(3/2 scaling)

Find low energy effective action for Goldstone is

$$\boxed{F_{\text{eff}} = -\frac{2\Lambda(2m)^{3/2}}{3} Y\sqrt{|Y|}}$$

By coupling to baryons, can mediate MOND-like force — reproduce BTFR, and CDM on large scales

Berezhiani, Khoury 2015

Analysis of High Energy Perturbations ε_j

Decompose into components $\Phi = (\phi_1 + i\,\phi_2)/\sqrt{2}$

Expand around superfluid $\phi_j = \phi_j^b + \varepsilon_j \qquad (j = 1, 2) \qquad \left(F' \equiv \frac{\partial F}{\partial X}\right)$

Linear equation of motion for high energy perturbations

$$\sum_{j=1}^{2} \left[F' \eta^{\mu\nu} \delta^{ij} + F'' \partial^\mu \phi_i^b \partial^\nu \phi_j^b\right] \partial_\mu \partial_\nu \varepsilon_j = 0$$

Diagonalize to obtain Higgs normal mode perturbations and associated effective metric

$$\psi = \partial^\mu \phi_1^b \partial_\mu \varepsilon_1 + \partial^\mu \phi_2^b \partial_\mu \varepsilon_2$$

$$G_\phi^{\mu\nu} \partial_\mu \partial_\nu \psi = 0$$

$$\boxed{G_\phi^{\mu\nu} = F' g^{\mu\nu} + F''(\partial^\mu \phi_1^b \partial^\nu \phi_1^b + \partial^\mu \phi_2^b \partial^\nu \phi_2^b)}$$

MH, J. Litterer, N. Shah 2105.02241

Causal Propagation?

Obtain eigenvalues of effective metric $G_\phi^{\mu\nu} = F' g^{\mu\nu} + F''(\partial^\mu \phi_1^b \partial^\nu \phi_1^b + \partial^\mu \phi_2^b \partial^\nu \phi_2^b)$

Conditions for hyperbolicity

$$(A) \quad A \equiv F' > 0$$

$$(B) \quad B \equiv F' + 2XF'' > 0$$

(Aharanov, Komar, Susskind; Wald; Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi; Bruneton;…)

Condition for subluminality

$$(C) \quad C \equiv -F'' \geq 0$$

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Condition for subluminality

$$(C) \quad C \equiv -F'' \geq 0$$

Evaluate in SFDM model $A > 0$ $\quad B = \dfrac{4\, m^3\, \Lambda^4\, Y}{\rho^8}$ $\quad C = \dfrac{2\, m\, \Lambda^4\, Y}{\rho^{10}}$

MOND regime $$Y \approx -\frac{(\nabla\theta)^2}{2m} \implies \boxed{B < 0} \text{ and } C < 0$$

(ghost-like)

MH, J. Litterer, N. Shah 2105.02241

Causal Propagation? - General Analysis

Obtain eigenvalues of effective metric $$G_\phi^{\mu\nu} = F' g^{\mu\nu} + F''(\partial^\mu \phi_1^b \partial^\nu \phi_1^b + \partial^\mu \phi_2^b \partial^\nu \phi_2^b)$$

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Condition for subluminality $$(C) \quad C \equiv -F'' \geq 0$$

Most general form $$F = (X + m^2|\Phi|^2) \sum_{n=0} g_n \frac{\Lambda^{2n}\left(X + m^2|\Phi|^2\right)^n}{(\Lambda_c^2 + |\Phi|^2)^{3n}}$$

We proved that when g_n allow MOND regime $(\tilde{\alpha} > 0)$ $\Longrightarrow$ $\boxed{B < 0}$ (ghost-like) and $C < 0$

MH, J. Litterer, N. Shah 2105.02241

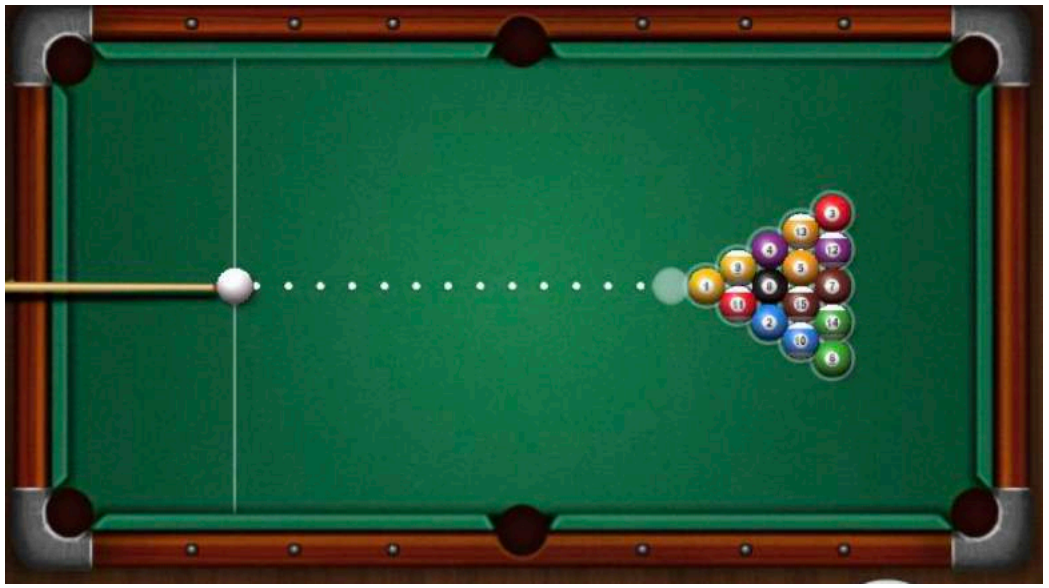
Part 2: Aspects of Quantum Dark Matter

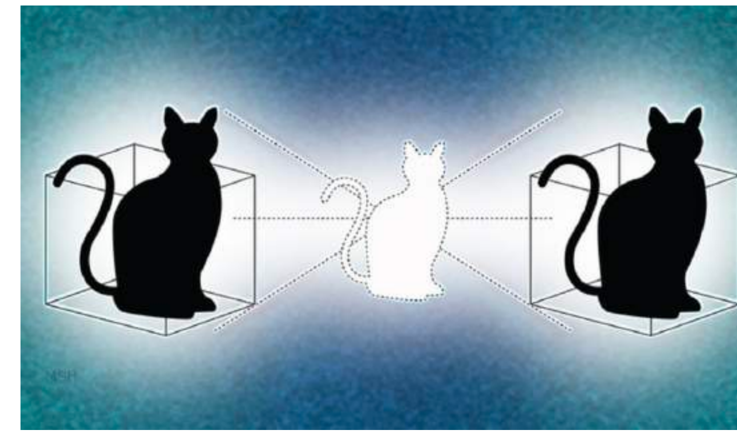
Based on work with:

Itamar Allali

Quantum: Any aspects of light DM not approximated by classical field theory?

Non-linear dynamics can launch states into Schrodinger cat-like states





Schrodinger Cat Billiards

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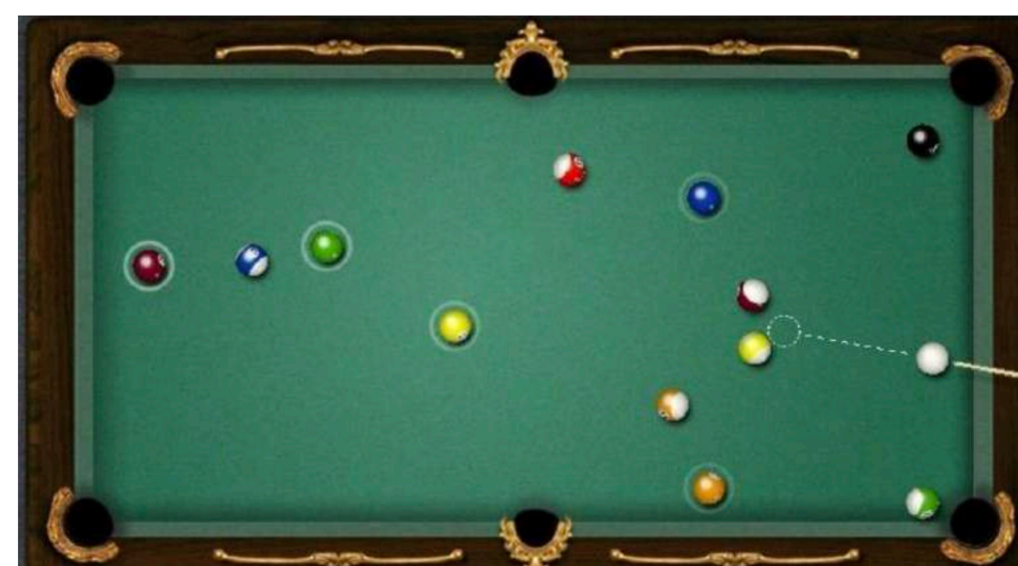

Albrecht, Phillips

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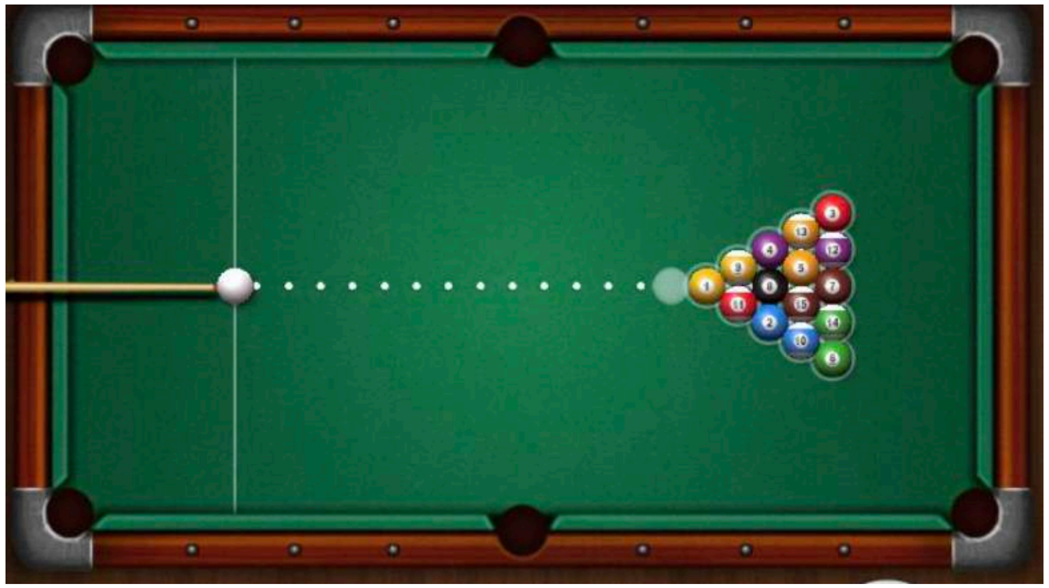

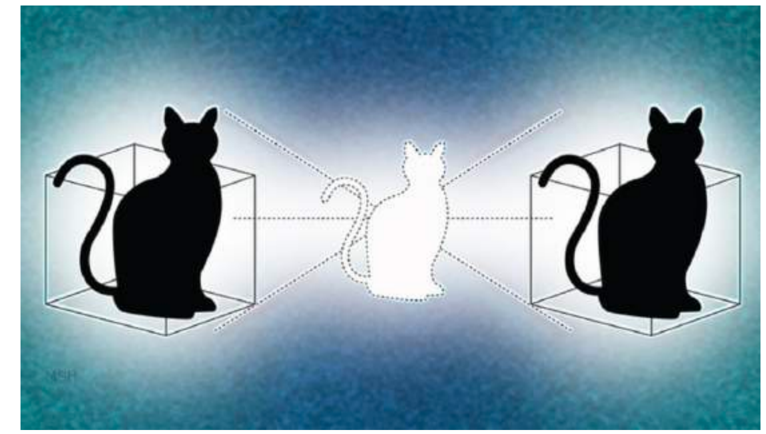

Quantumness destroyed due to DECOHERENCE

Schrodinger Cat Billiards

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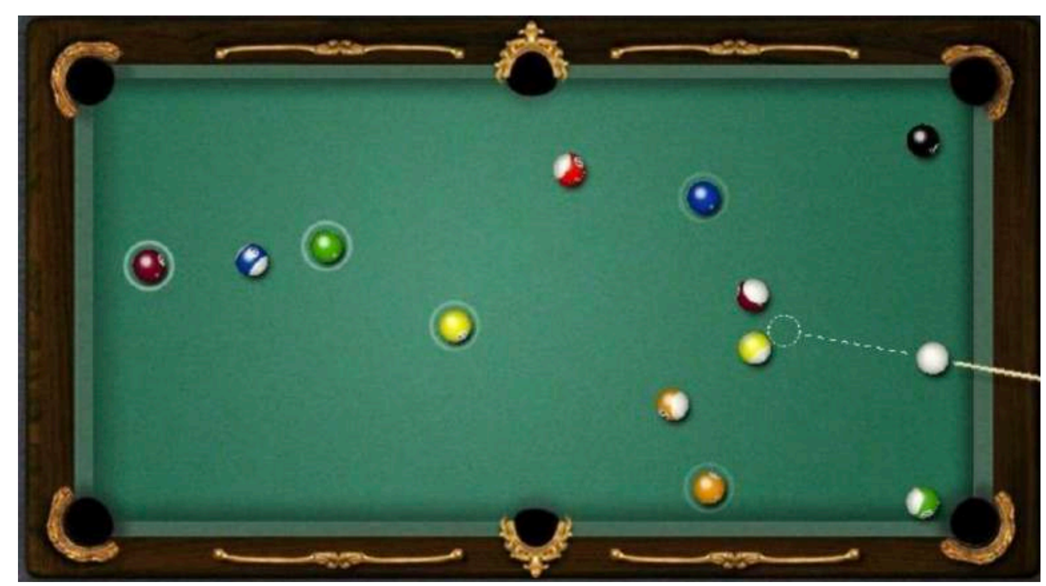

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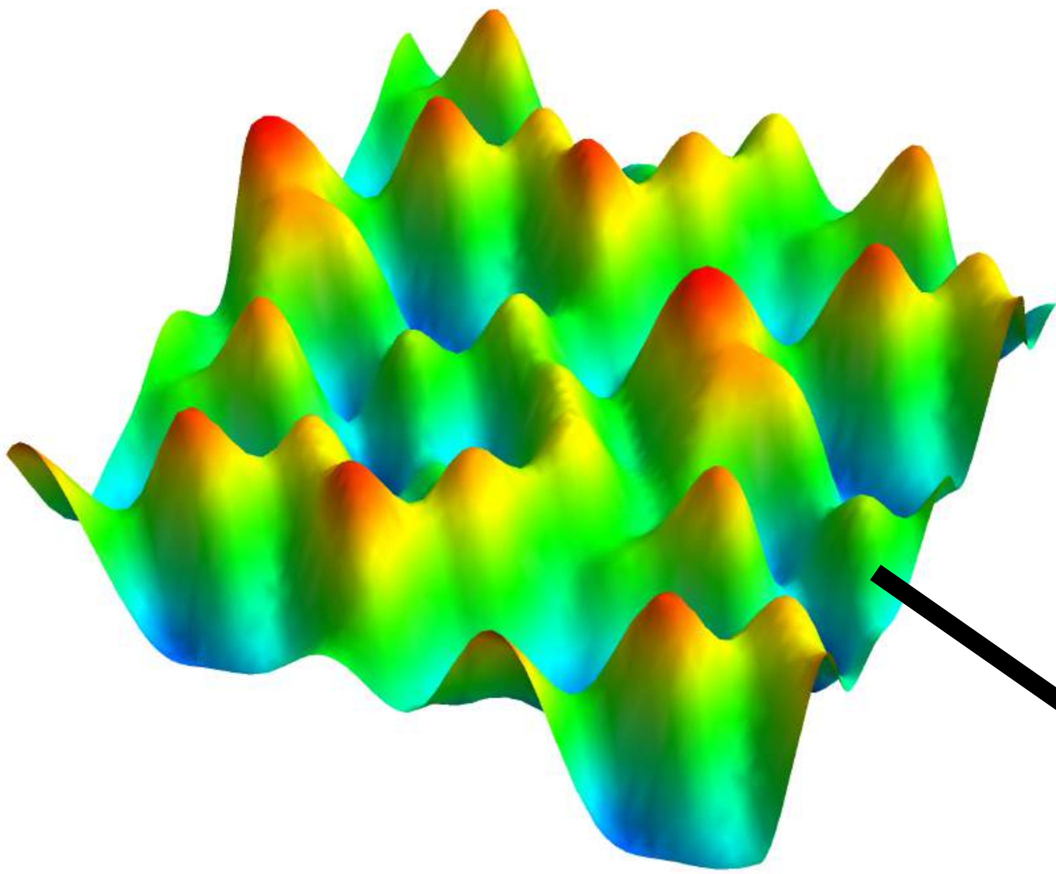
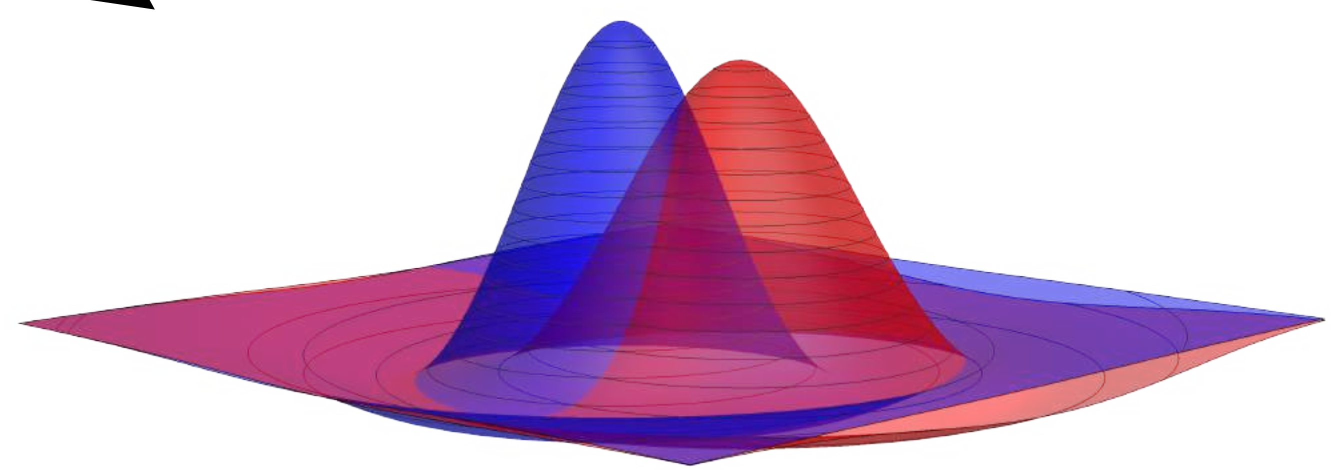


Dark Matter Schrodinger Cat (Axions)



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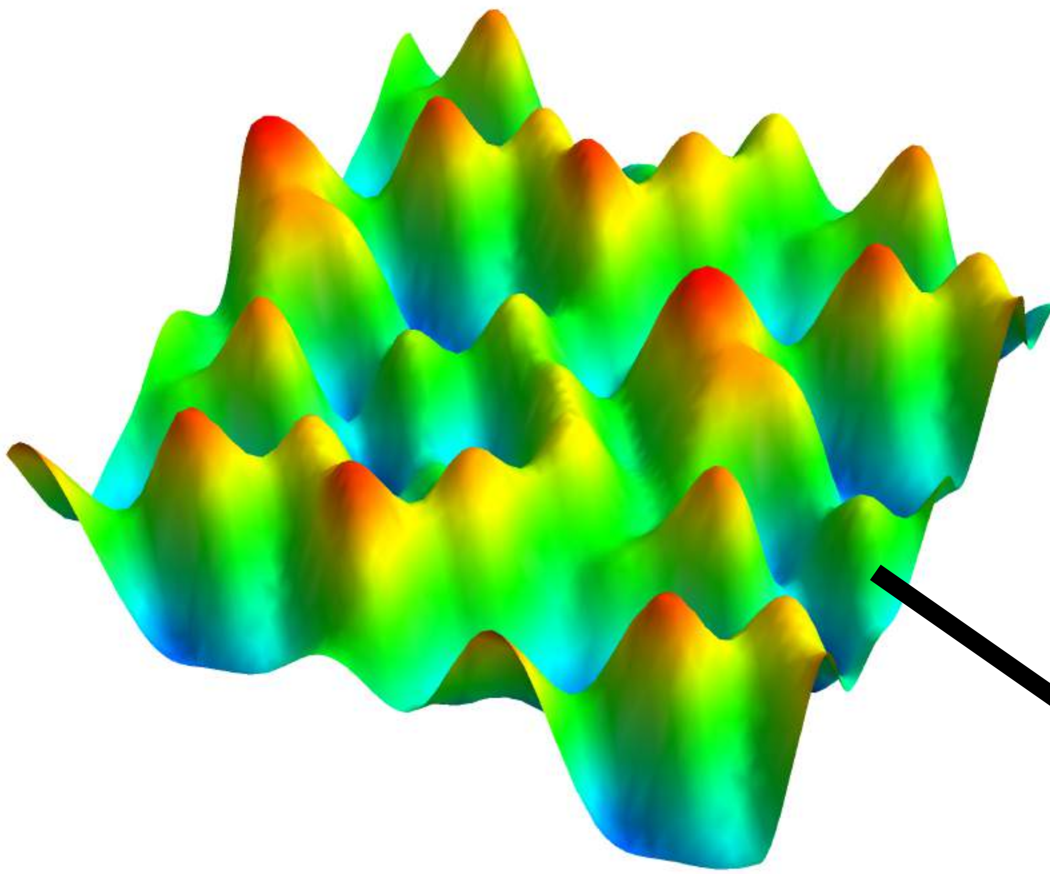

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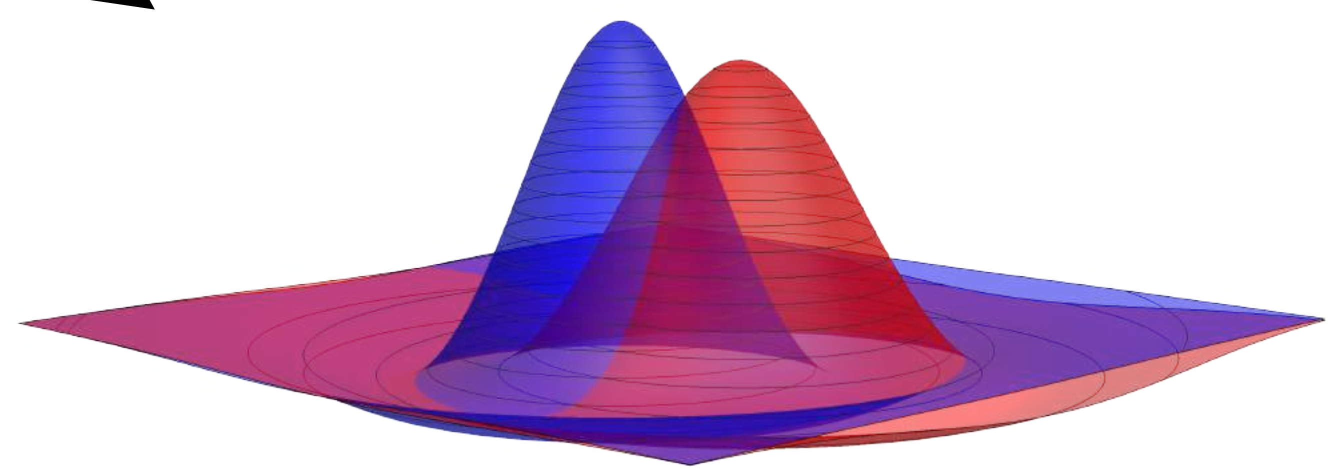


Quantumness destroyed due to DECOHERENCE???

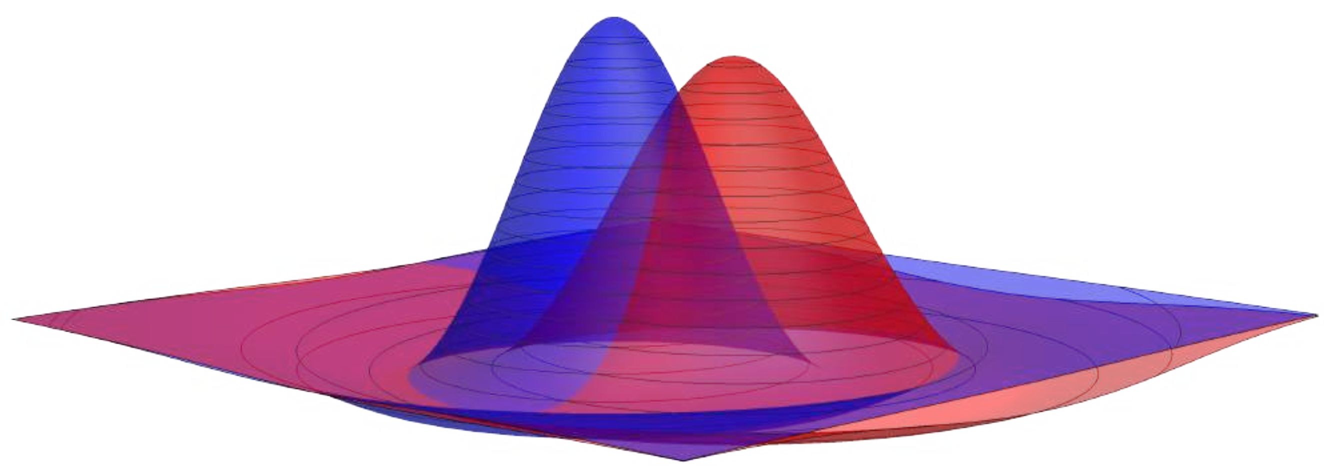
Less clear because dark matter has tiny (non-gravitational) interactions

Could Dark Matter Schrodinger Cats Survive?

Entanglement from Gravitational Scattering

$|\mathrm{DM}_1\rangle + |\mathrm{DM}_2\rangle$

Probe particle $|\psi\rangle$

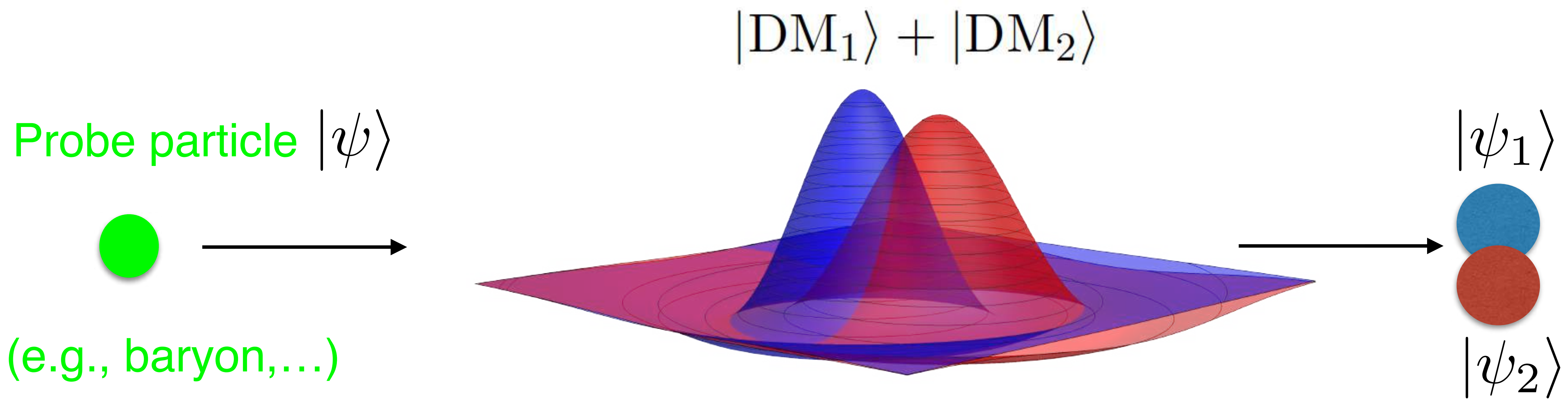


(e.g., baryon,…)

$$|\Psi_{\mathrm{ini}}\rangle = \left(|\mathrm{DM}_1\rangle + |\mathrm{DM}_2\rangle\right)|\psi\rangle$$

Product State

Entanglement from Gravitational Scattering



$$|\Psi_{\rm ini}\rangle = \left(|\mathrm{DM}_1\rangle + |\mathrm{DM}_2\rangle\right)|\psi\rangle \qquad |\Psi_{\rm fin}\rangle = |\mathrm{DM}_1\rangle\,|\psi_1\rangle + |\mathrm{DM}_2\rangle\,|\psi_2\rangle$$

Product State

Entangled State

Trace Out Probe Particle

$\hat{\rho} \equiv |\Psi\rangle \langle\Psi|$ Full Density Matrix

$\hat{\rho}_{\mathrm{red}} = \mathrm{Tr}_{|\psi\rangle}[\hat{\rho}]$ Reduced Density Matrix

$$= |\mathrm{DM}_1\rangle \langle\mathrm{DM}_1| + \langle\psi_2|\psi_1\rangle |\mathrm{DM}_1\rangle \langle\mathrm{DM}_2| + \langle\psi_1|\psi_2\rangle |\mathrm{DM}_2\rangle \langle\mathrm{DM}_1| + |\mathrm{DM}_2\rangle \langle\mathrm{DM}_2|$$

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Full Density Matrix

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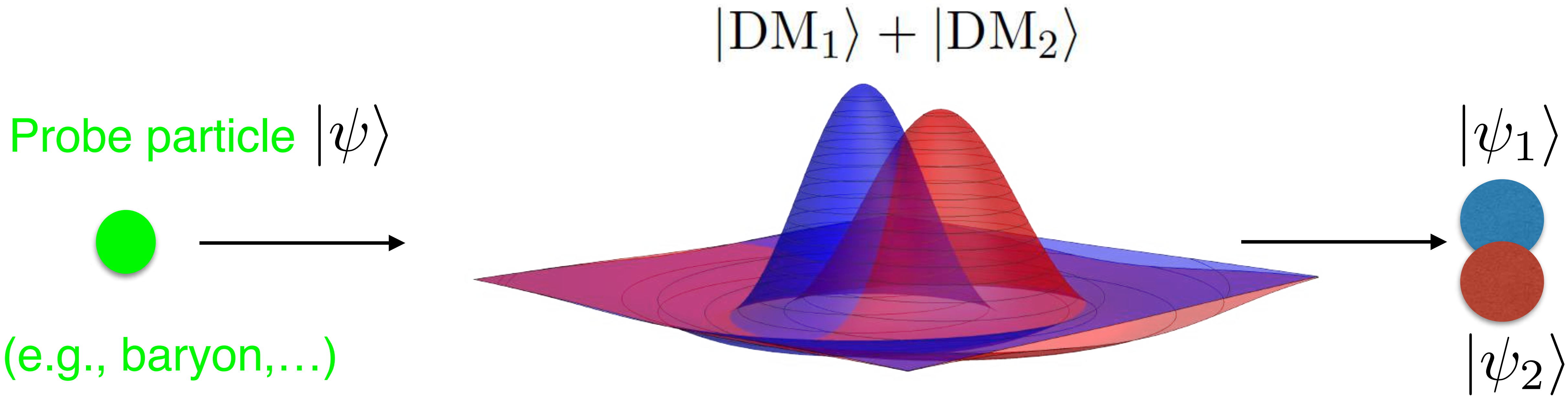
Off diagonal elements;
controlling true quantum effects

Overlap of Probe Particle States



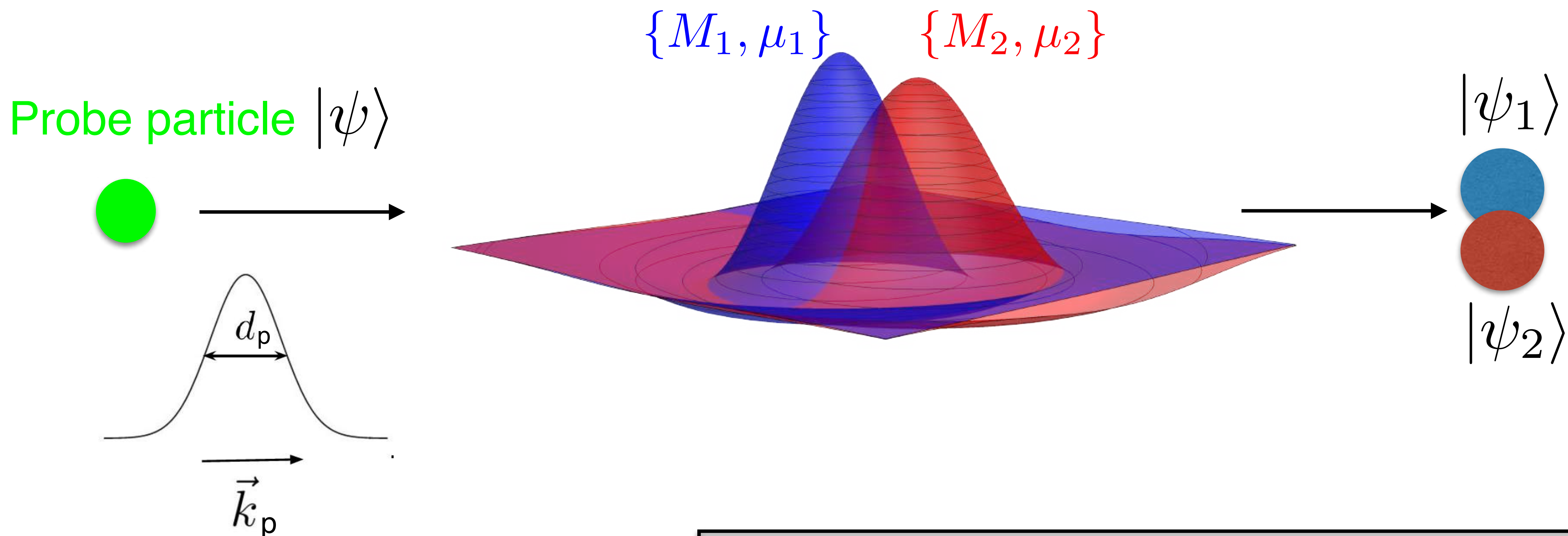
$$|\psi_i\rangle = |\mathrm{uns}\rangle + |\mathrm{sc}_i\rangle$$

$$|\langle\psi_1|\psi_2\rangle|^2 = 1 + 2\Big(\langle sc_1|sc_2\rangle_{\mathrm{R}} - \frac{1}{2}\Big\{\langle sc_1|sc_1\rangle + \langle sc_2|sc_2\rangle\Big\}\Big) + \Big(\langle sc_1|uns\rangle_{\mathrm{I}} + \langle uns|sc_2\rangle_{\mathrm{I}}\Big)^2 + O(G^3)$$

Allali, MH 2005.12287 (JCAP)

Result for Overlap of Probe Particle States

We evolve a Gaussian wave packet using perturbation theory



$$|\langle\psi_1|\psi_2\rangle|^2 = 1 - 2\Delta$$

$$\boxed{\Delta_0 = \frac{2G^2m^4}{\hbar^4k^2d^2}\left[\frac{M_1^2}{\mu_1^2}\chi_{11} + \frac{M_2^2}{\mu_2^2}\chi_{22} - 2\frac{M_1M_2}{\mu_1\mu_2}\chi_{12}\right]}$$

Allali, MH 2005.12287 (JCAP)

Decoherence Rate from N-Probe Particles

Off diagonal element of density matrix

$$\prod_{n=1}^{N} |\langle \psi_1 | \psi_2 \rangle|_n = \prod_{n=1}^{N} (1 - \Delta_b) \sim e^{-\sum_{n=1}^{N} \Delta_b}$$

Decoherence rate

$$\Gamma_{\rm dec} = n\, v \int d^2b\, \Delta_b$$

$$\boxed{\Gamma_{\rm dec} = \frac{4\pi G^2 m^4 n\, v}{\hbar^4 k^2} \left[\frac{M_1^2}{\mu_1^2} \chi_{11} + \frac{M_2^2}{\mu_2^2} \chi_{22} - 2 \frac{M_1 M_2}{\mu_1 \mu_2} \chi_{12} \right]}$$

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Application to Light Diffuse DM (axions)

Diffuse scalars (axions)

$$\mu_i \sim \frac{1}{\lambda_{dB,a}} \sim \frac{2\pi}{m_a v_{vir}} \qquad M_i \sim \frac{4\pi}{3}\rho_{DM}\lambda_{dB,a}^3$$

Probe: Diffuse baryons

$$k_p \sim m_p\, v_{vir}$$

Decoherence Rate

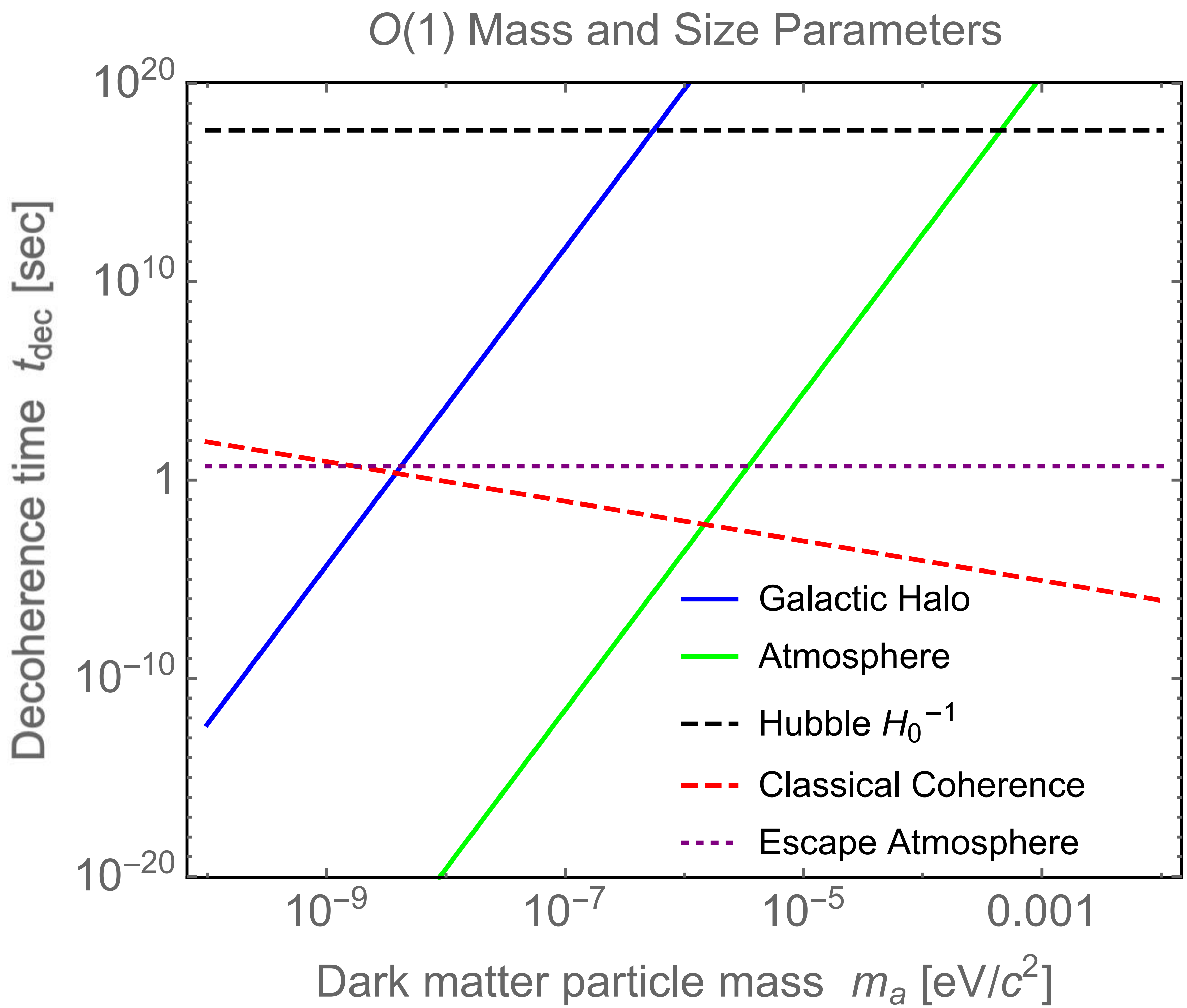
$$\boxed{\Gamma_{dec} \sim \frac{G^2\, m_p\, \rho_b\, \rho_{DM}^2}{m_a^8\, v_{vir}^9}}$$

Decoherence Time

$$t_{dec} = 1/\Gamma_{dec} \propto m_a^8$$

Allali, MH 2005.12287 (JCAP)

Application to Diffuse DM (Axions)



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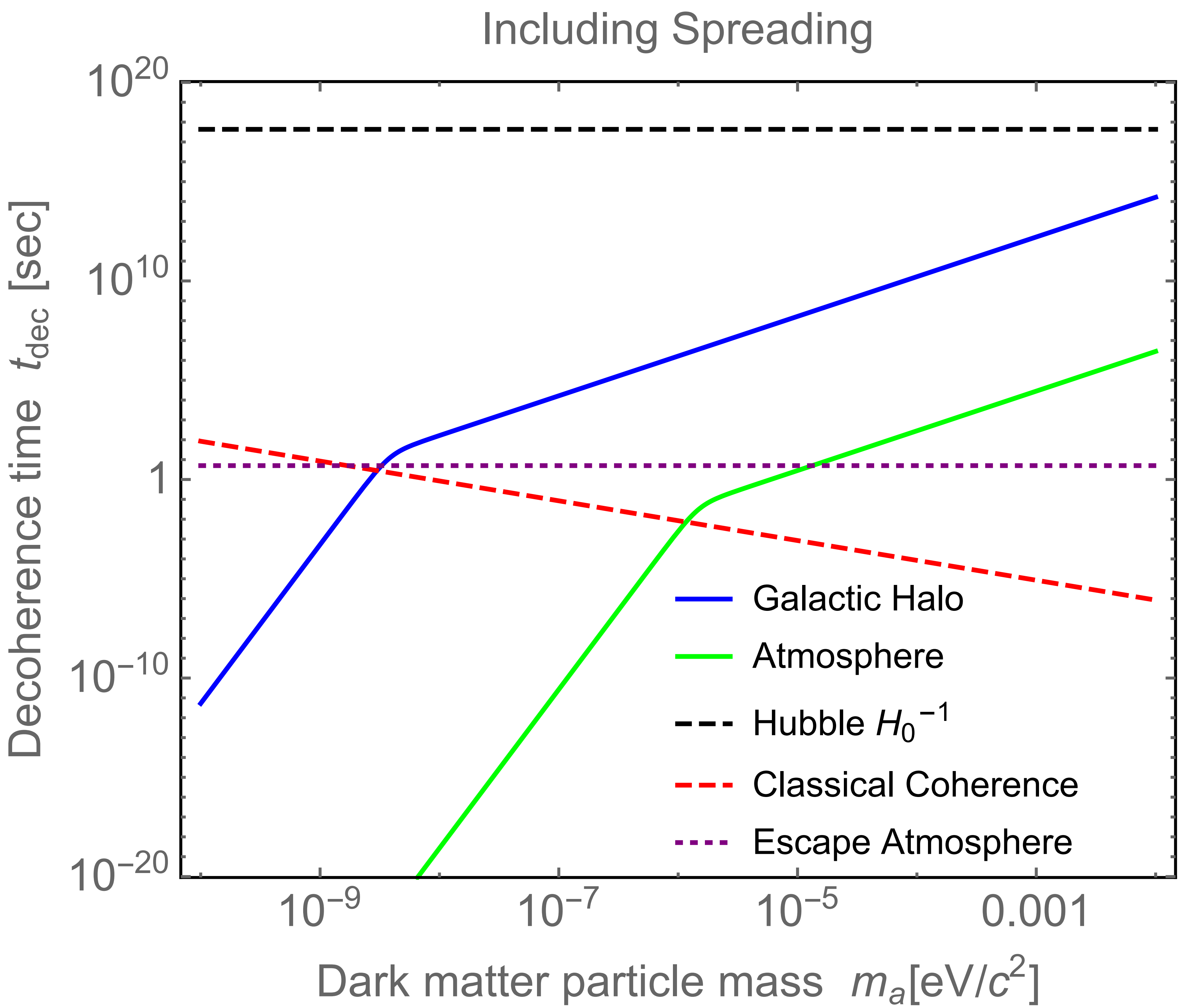


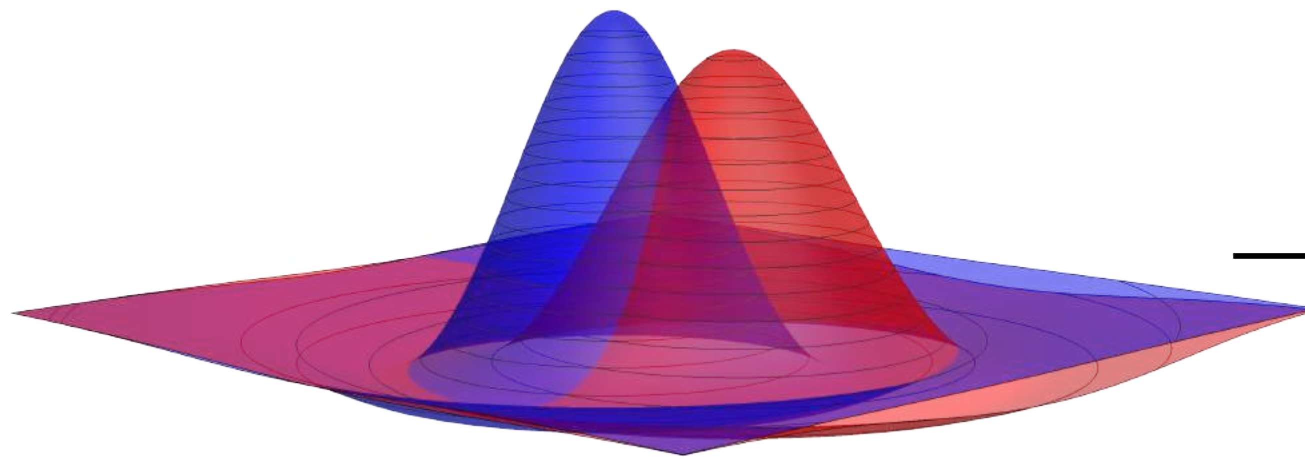
Allali, MH 2005.12287 (JCAP)

Application to Boson Stars

Boson star

Probe particle $|\psi\rangle$

(e.g., baryon,…)



$|\psi_1\rangle$

$|\psi_2\rangle$

Boson stars are much denser

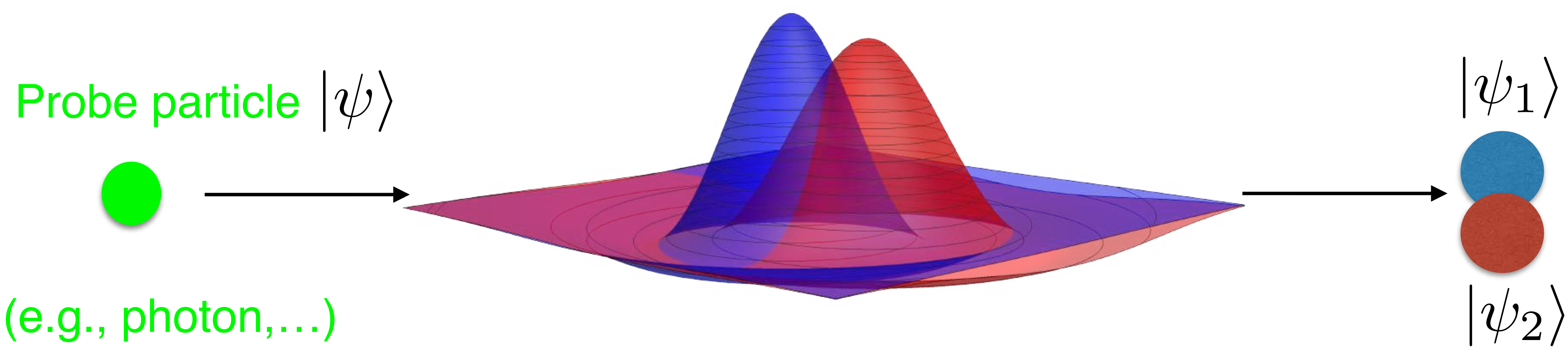
Decoherence Rate

$$\boxed{\Gamma_{\rm dec} \gtrsim \frac{\hbar^2 m_p \rho_p}{v_p m_a^4} \sim 10^{21}\,\mathrm{sec}^{-1}\left(\frac{1\,\mathrm{eV}}{m_a\, c^2}\right)^4}$$

Extremely rapid decoherence —> Very classical

Allali, MH 2005.12287 (JCAP)

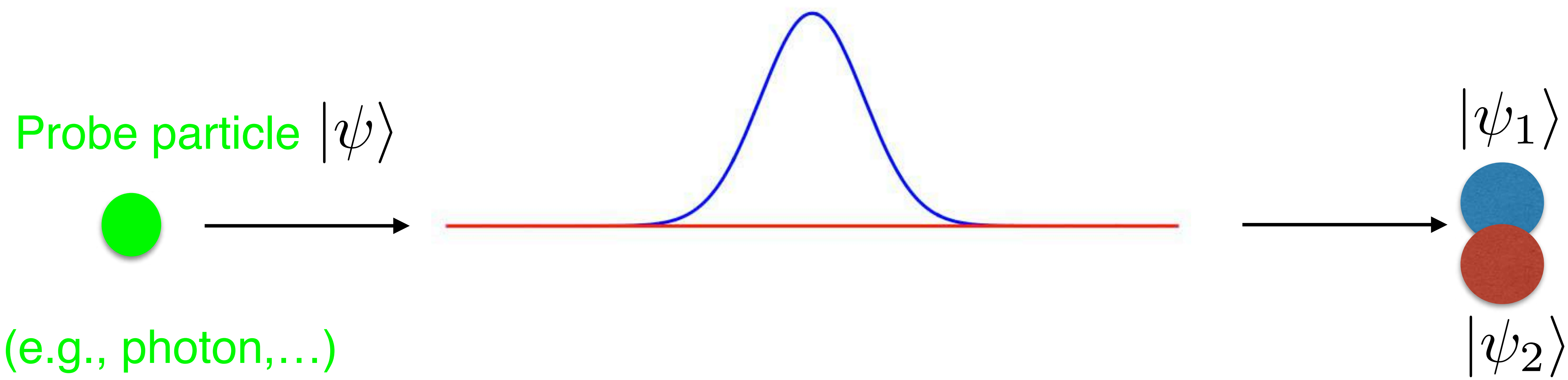
General Relativistic Extension



Rigorous quantum gravity calculation — General Relativity is a well behaved effective theory

Allali, MH 2012.12903 (PRD), 2103.15892 (PRL to appear)

General Relativistic Extension



Rigorous quantum gravity calculation — General Relativity is a well behaved effective theory

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General Relativistic Extension

Starting from QFT, can derive RSE for 1-particle sub-space: (ignoring spin)

$$(i\partial_t - \sqrt{-\nabla^2 + m^2})\psi(\mathbf{x},t) = \left(\Phi(\mathbf{x},t)\sqrt{-\nabla^2 + m^2} - \frac{\Psi(\mathbf{x},t)\nabla^2}{\sqrt{-\nabla^2 + m^2}}\right)\psi(\mathbf{x},t)$$

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Decoherence Rate
for superposition of
different phases

$$\boxed{\Gamma_{dec} \propto \exp\left[-\frac{m_a^2 E_p^2}{\mu^2 k^2}\right] \sim \exp\left[-\frac{1}{v_a^2 v_p^2}\right]}$$

Exponentially suppressed for non-relativistic DM or probes

So the phase is rather robust against decoherence - may be relevant to direct detection

$$|\mathrm{DM}\rangle \sim \sum_i c_i |\cos(\omega t - \mathbf{k}_a \cdot \mathbf{x} + \varphi_i)\rangle$$

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General Relativistic Extension

Starting from QFT, can derive RSE for 1-particle sub-space: (ignoring spin)

$$(i\partial_t - \sqrt{-\nabla^2 + m^2})\psi(\mathbf{x}, t) = \left(\Phi(\mathbf{x}, t)\sqrt{-\nabla^2 + m^2} - \frac{\Psi(\mathbf{x}, t)\nabla^2}{\sqrt{-\nabla^2 + m^2}}\right)\psi(\mathbf{x}, t)$$

Decoherence Rate
for superposition of
different phases

$$\boxed{\Gamma_{dec} \propto \exp\left[-\frac{m_a^2 E_p^2}{\mu^2 k^2}\right] \sim \exp\left[-\frac{1}{v_a^2 v_p^2}\right]}$$

Exponentially suppressed for non-relativistic DM or probes

So the phase is rather robust against decoherence - may be relevant to direct detection

$$|\mathrm{DM}\rangle \sim \sum_i c_i |\cos(\omega t - \mathbf{k}_a \cdot \mathbf{x} + \varphi_i)\rangle$$

(Although may decohere near black hole horizons)

Allali, MH 2012.12903 (PRD), 2103.15892 (PRL to appear)

Conclusions (Part 1: Superfluid DM)

Superfluid Dark Matter is a novel way (only known?) to obtain success of CDM on large scales and success of MOND on galactic scales

We studied a general class of models, and proved that high energy perturbations always violate hyperbolicity — ghost like behavior — in MOND regime

Intermediate regions can exhibit forms of superluminality.
There are problems in related models too.

Open question: is there ANY other model free of these theoretical problems?
Alternatively, can one rigorously show that CDM reproduces BTFR, etc?

MH, J. Litterer, N. Shah 2105.02241

Conclusions (Part 2: Quantum DM)

Macroscopic quantum states (Schrodinger cats) of light scalar dark matter might exist, and would be potentially robust against decoherence

We studied the decoherence of such states due to gravitationally scattering from probe particles; a rigorous quantum gravity result

We found that superpositions of spatial profiles decohere rapidly for very light DM (axions), and boson stars decohere extremely rapidly

We found that superpositions of phases live exponentially long, may launch detectors into superpositions. While relativistic states (near black holes) decohere quickly

Allali, MH 2005.12287 (JCAP), 2012.12903 (PRD), 2103.15892 (PRL to appear)