

Instability of rotating Bose stars



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XXI INTERNATIONAL SEMINAR
ON HIGH-ENERGY PHYSICS

Workshop “Dark Matter”

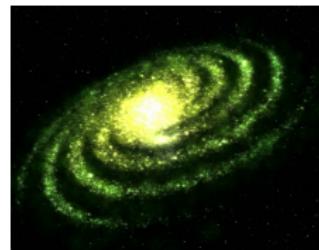
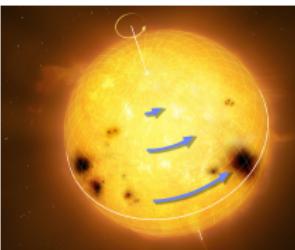
online, 24/06/2021

A. Dmitriev, DL, A. Panin, E. Pushnaya, I. Tkachev, [arXiv:2104.00962](https://arxiv.org/abs/2104.00962)

Message to take home

We expect that....

every compact object can rotate

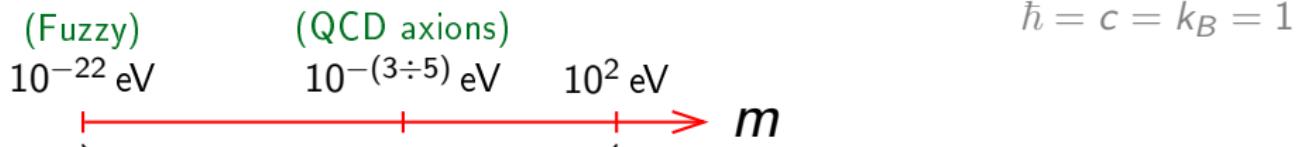


But Bose stars cannot!

if self-interaction is zero or negative: $\lambda = 0$ or $\lambda < 0$

although they are stable at zero spin...

Light bosonic (ALP) dark matter



- Large occupation numbers!

$$m \ll 10^2 \text{ eV} \Rightarrow f \sim \frac{\rho/m}{(mv)^3} \gg 1$$

\Rightarrow classical field $\phi(t, x)$:

$$\square\phi + m^2\phi + \lambda_4\phi^3/6 + \dots = 0$$

self-interaction

- Nonrelativistic approximation: $v \ll 1$

Gross–Pitaevsky–Poisson system

$$i\partial_t \psi = -\Delta \psi / 2m + mU\psi + \underline{\lambda_4 |\psi|^2 \psi / 8m^2}$$

$$\Delta U = 4\pi G m |\psi|^2$$



$$\phi = \text{Re } \psi(t, x) e^{-imt}$$

$U(t, x)$

\Rightarrow rich wave phenomena

$$\psi(t, x)$$

- (Pseudo)scalars with small mass m
- Can form cold dark matter
via vacuum realignment

*Wise, Wilczek '83
Abbott, Sikivie '83*

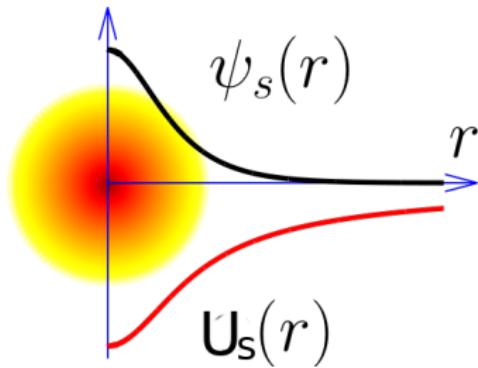
QCD axions

- Solve strong CP problem
Peccei, Quinn '77
- CDM: $m \sim 10^{-5}$ eV
Klaer, Moore '17
 $m \gtrsim 10^{-3}$ eV
Gorghetto, Hardy, Villadoro '20
- $\lambda_4 < 0$
attraction!

String axions

- Consequence of string theory
- Any mass m
Arvanitaki et al '10
- Fuzzy DM: $m \sim 10^{-22}$ eV
e.g. Hu, Barkana, Gruzinov '00
- λ_4 is small and naturally < 0
attraction!

Mass is small, self-interaction is naturally attractive!



Solitonic solution

- $\psi = \psi_s(r) e^{-i\omega_s t}$
↑
Ground state of U
- $U = U_s(r)$ — potential of ψ_s
- $\omega_s < 0$ — energy level
- $M_s = M_s(\omega_s)$ — parameter

Ruffini, Bonazolla '69; Tkachev '86

All particles on the same level

⇒ gravitationally bound **Bose–Einstein Condensate**

Properties of Bose stars

① Form in popular DM models via BE condensation

Fuzzy DM ($m \sim 10^{-22}$ eV)

In (dwarf) galaxies
during structure formation

$$M_{\text{halo}} \sim 5 \cdot 10^9 M_{\odot}$$

$$M_s \sim 10^8 M_{\odot}$$

Schive, Chiueh, Broadhurst '14

Veltmaat, Niemeyer '16

QCD axions ($m \sim 10^{-(3 \div 5)}$ eV)

In axion miniclusters

at RD/MD

$$M_{\text{halo}} \sim 10^{-13} M_{\odot}$$

$$M_s \sim 10^{-15} M_{\odot}$$

DL, Panin, Tkachev '18

Eggemeier, Niemeyer '19

② Stop growing beyond certain mass

Schive, Liao, Woo, Wong, Chiueh '14

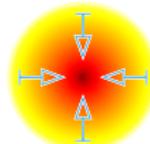
Eggemeier, Niemeyer '19; Chen, Du, Lentz, Marsh, Niemeyer '20

③ Collapse as Bosenova

Attractive self-interaction: $\lambda_4 < 0$

Large mass: $M > M_{cr}$

} \Rightarrow squeeze-in!



Chavanis '11; DL, Panin, Tkachev '17

Tkachev '87; DL, Panin, Tkachev '20

④ Radio-explode at $M > M'_{cr}$

But this may not happen due to Bosenova

Prevent Bosenova — add rotation?

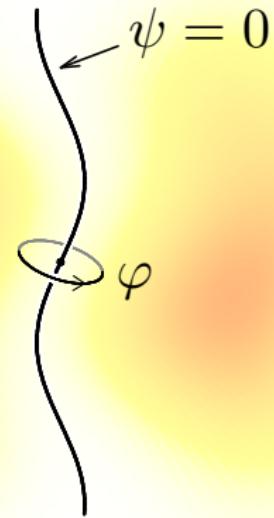
Can the Bose stars rotate?

BEC velocity is irrotational:

$$\mathbf{v} = \frac{\text{Im}(\psi^* \nabla \psi)}{m|\psi|^2} = \nabla \arg \psi / m$$



$\text{rot } \mathbf{v} = 0$



Rotation \leftrightarrow vortices!

$$\oint \mathbf{v} \cdot d\mathbf{x} = I$$

\Leftarrow

$$\psi \propto r^l e^{il\varphi}$$

/ rotation quanta

This costs energy!

Rotating Bose stars

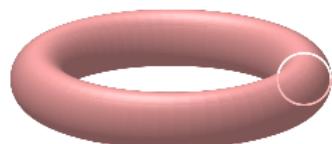
Axially-symmetric Ansatz:

$$\psi = \psi_s(r, z) e^{-i\omega_s t + il\varphi}$$

$$U = U_s(r, z)$$

$$\psi_s \propto r^l \text{ as } r \rightarrow 0 \quad \Rightarrow$$

vortex at $r = 0$

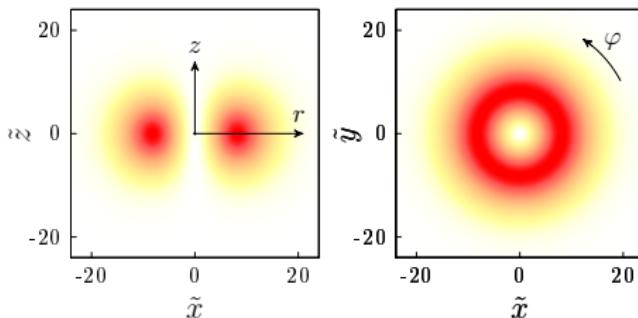


donut form!

BEC of particles with angular momenta l

$$L_{s,z} = M_s l / m$$

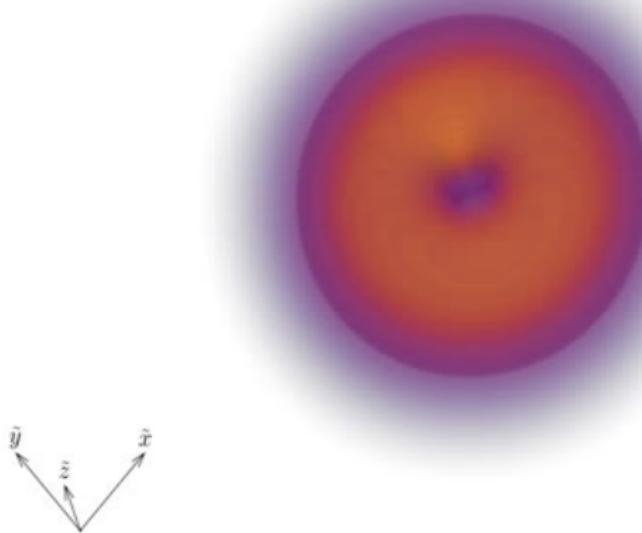
Solve equations numerically at $l = 1$, $\lambda_4 = 0$: (project to $\hat{R}_{\pi/2}\psi = e^{i\pi/2}\psi$)



Two-parametric family: l , M_s

Evolve the $l = 1$ star numerically ($\lambda_4 = 0$)

$$t = 0: \quad \psi = \psi_s + \delta\psi_{\text{small}}$$



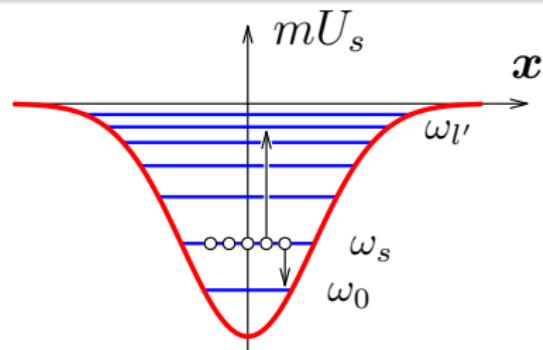
arXiv: 2104.00962

They are unstable!

Instability mechanism: $I \neq 0$

- ① Fix $U_s(r, z)$, set $\lambda_4 = 0$
 - ② Send: $\begin{cases} dN_0 \text{ particles} \rightarrow I_0 = 0 \text{ state} \\ dN' \text{ particles} \rightarrow I' \gg 1 \text{ state} \end{cases}$
 - ③ Angular momentum conserves:
- $$dN'I' = I(dN_0 + dN') \Rightarrow \boxed{dN' \ll dN_0}$$
- ④ But energy decreases!

$$\Delta E \approx dN_0(\omega_0 - \omega_s) < 0$$



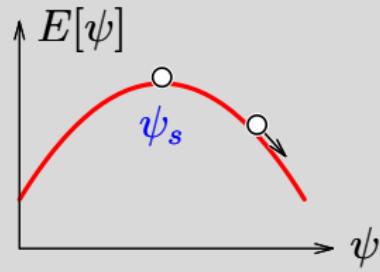
No-Go theorem: dynamical U_s & $\lambda_4 \leq 0$

$$\psi = \psi_s + \sqrt{dN_0} \Psi_0(x) + \sqrt{dN'} \Psi_{I'}(x)$$

\Downarrow

$$\boxed{E[\psi] < E[\psi_s] \leftarrow I \neq 0 \text{ & } \lambda_4 \leq 0}$$

$$\delta\psi \propto e^{\mu t}$$



Instability modes — numerically

- Linearize equations w.r.t. perturbations:

$$U = U_s + \delta U(r, z, t) e^{i\Delta l \varphi} + \delta U^*(r, z, t) e^{-i\Delta l \varphi}$$
$$\psi = \psi_s e^{il\varphi} + \underbrace{\delta \psi(r, z, t) e^{i(l+\Delta l)\varphi} + \delta \bar{\psi}^*(r, z, t) e^{i(l-\Delta l)\varphi}}_{\text{coupled in linear equations}}$$

pairwise: $(l, l) \rightarrow (l + \Delta l, l - \Delta l)$

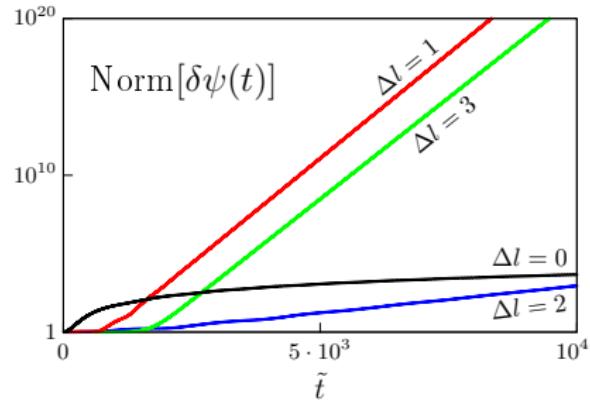
- Time-evolve: $\delta U, \delta \psi, \delta \bar{\psi} \propto e^{\mu t}$

- Dominant instability: $\max\{\mu(\Delta l)\}$

At $l = 2$:

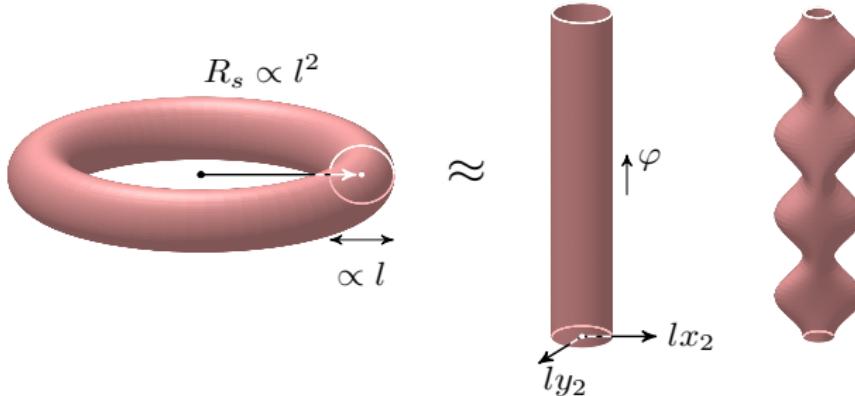
dominant mode $\Delta l = 1 \rightarrow$

\Rightarrow exponent $\mu(\Delta l = 1)$



Fast-rotating stars — analytically ($I \gg 1$)

$$\underbrace{p_{r,z}^2}_{kinetic} \sim \underbrace{\frac{GM_s}{R_s}}_{U_s} \sim \underbrace{\frac{l^2}{R_s^2}}_{centrifugal} \Rightarrow \left\{ \begin{array}{l} R_s \propto l^2 \\ \Delta r, \Delta z \sim p_{r,z}^{-1} \propto l \end{array} \right.$$



$$\left. \begin{array}{l} \psi_s \approx l^{-2} \psi_2(l r_2) \leftarrow \text{non-rotating 2D star} \\ \delta\psi = \delta\psi_2(l r_2) e^{\mu t + i(l + \Delta l)\varphi} \leftarrow \text{perturbation} \end{array} \right\}$$

Solve with
shooting method

$\Rightarrow \mu$ and Δl at $l \gg 1$

Lifetimes of rotating Bose stars ($\lambda_4 = 0$)

$$\tau^{-1} \equiv \mu = \tilde{\mu} m^3 G^2 M_s^2$$

↑
numerically

$I \gg 1$ analytics ($\alpha_I \sim 2$)

$$\Delta I \approx [0.944 \cdot I / \sqrt{\alpha_I}] \propto I$$

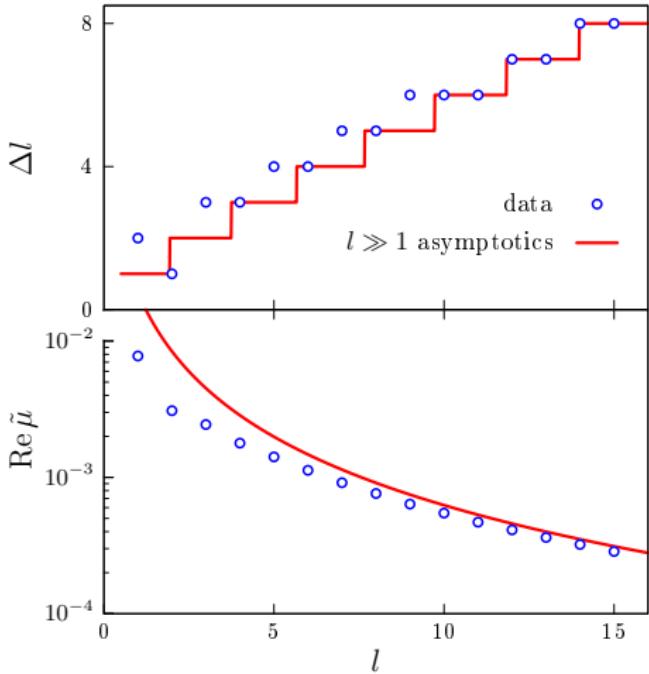
$$\tilde{\mu} \approx 2.22 \cdot 10^{-2} \alpha_I / I^2 \propto I^{-2}$$

Fuzzy DM: $m = 10^{-22}$ eV

$$\tau \simeq 10^{10} \text{ yr} \cdot I^2 \left(\frac{M_s}{4 \cdot 10^7 M_\odot} \right)^{-2}$$

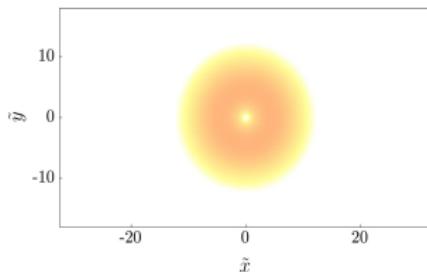
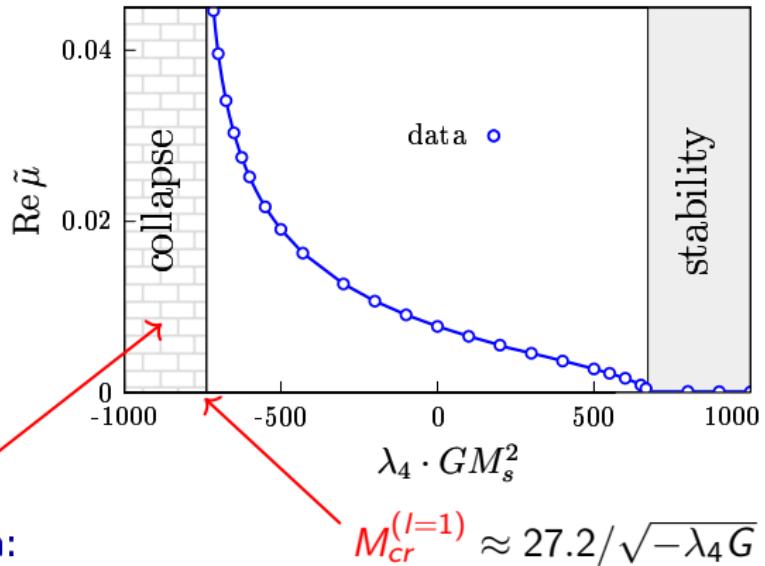
But: $\tau \omega_s \approx 1.7(\alpha_I + 1) \sim O(1)$

⇒ Rotating BSs are not quasi-stationary states (do not exist)!

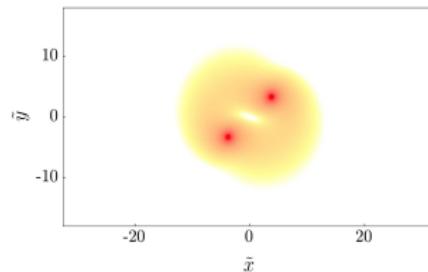


Adding self-interaction: $\lambda_4 \neq 0$

$I = 1$ Bose star with $M_s = \text{const}$

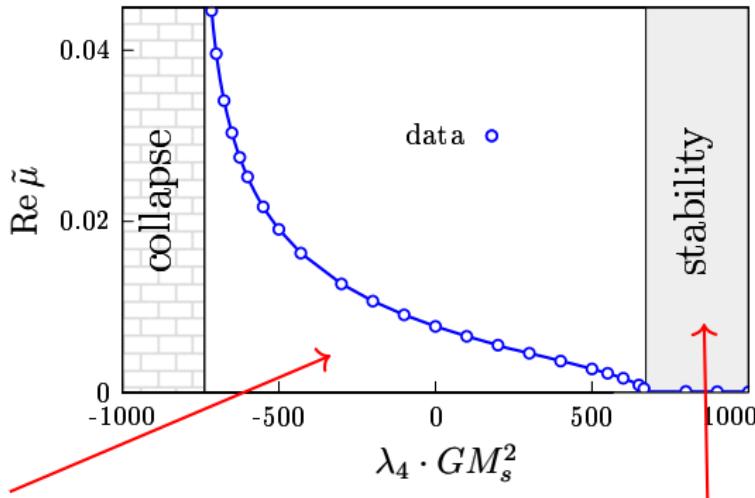


time



Adding self-interaction: $\lambda_4 \neq 0$

$|l| = 1$ Bose star with $M_s = \text{const}$



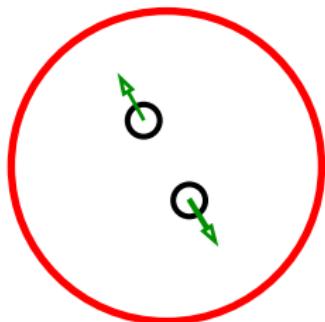
Rotational instability

Stability:

- ① $\lambda_4 > 0$
- ② $|l| = \pm 1$
- ③ $M_s \gtrsim 25.9/\sqrt{G\lambda_4}$

Dominating repulsion

$(\lambda_4 > 0 \leftrightarrow$ laboratory BEC)



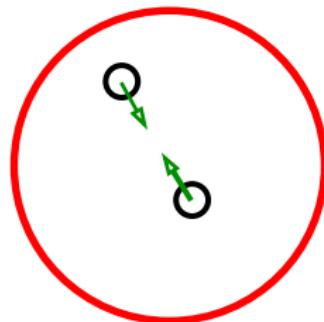
Vortices repulse

- from each other
- from star boundaries

Stable at $I = 1$

Dominating attraction

$(\lambda_4 < 0$ or gravity)



Vortices attract

- to each other
- to star boundaries

Unstable

Conclusions

- All rotating Bose stars are unstable at $\lambda_4 \leq 0$.
 - Similar: instability of relativistic rotating boson stars
Sanchis-Gual, Di Giovanni, Zilhão, Herdeiro, Cerdá-Durán, Font, Radu '19
 - Similar: tidal disruption of orbiting Bose stars
Du, Schwabe, Niemeyer, Bürger '18

- They decay fast: $\tau \sim \omega_s^{-1}$ (do not exist)

— Rotation does not stabilize against Bosenova!

cf. Davidson, Schwetz '16; Hertzberg, Schiappacasse '18

+

Angular momentum can be measured!

- Coalescence \Rightarrow gravity waves
- Bose stars \Rightarrow spin-zero BH \Rightarrow coalescence \Rightarrow gravity waves

can explain LIGO hints

LIGO & Virgo '19

- $I = 1$ Bose star is stable at $\lambda_4 > 0$ & large M_s
Siemonsen, East '20
- Fast-rotating stars can be described analytically



Thanks for attention!