# **Generative Modelling for HEP**

Deep-faking a high-energy physics detector

Quarks online workshop "Advanced Computing in Particle Physics" June 8-9, 2021

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#### This X does not exist



#### This Person Does Not Exist

The site that started it all, with the name that says it all. Created using a style-based generative adversarial network (StyleGAN), this website had the tech community buzzing with excitement and intrigue and inspired many more sites.

Created by Phillip Wang.



#### This Cat Does Not Exist

These purr-fect GAN-made cats will freshen your feeline-gs and make you wish you could reach through your screen and cuddle them. Once in a while the cats have visual deformities due to imperfections in the model – beware, they can cause nightmares.

Created by Ryan Hoover.



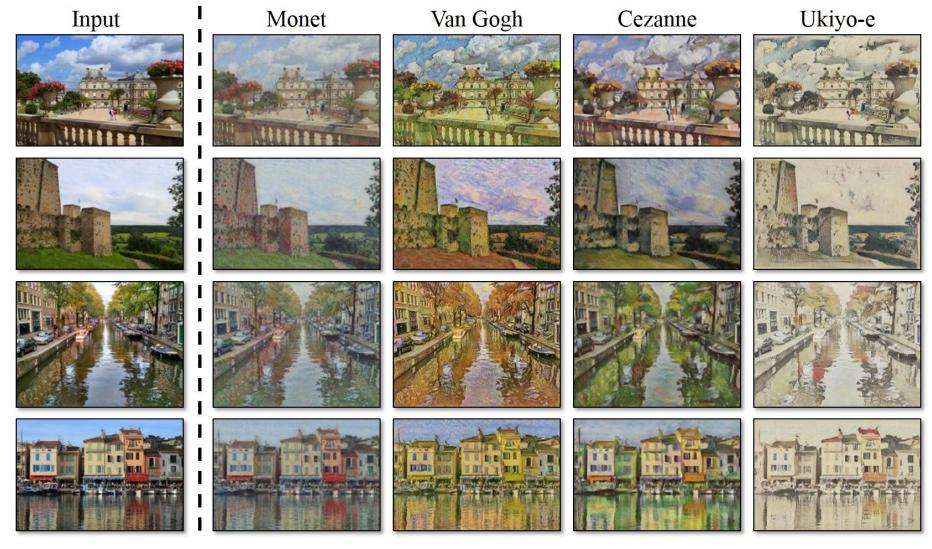
#### This Rental Does Not Exist

Why bother trying to look for the perfect home when you can create one instead? Just find a listing you like, buy some land, build it, and then enjoy the rest of your life.

Created by Christopher Schmidt.

https://thisxdoesnotexist.com/

### Style transfer



https://junyanz.github.io/CycleGAN/

### Generative models progress

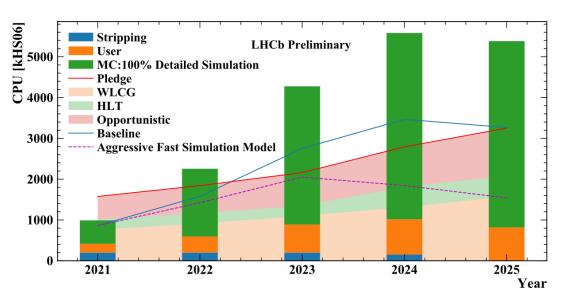


https://twitter.com/goodfellow\_ian/status/1084973596236144640

#### Deep generative models for fast detector simulation

- What if, instead of generating images, we train these models to generate detector responses?
- Generating a response as fast as a single forward pass through the network
  - Should be orders of magnitude faster compared to e.g. detailed Geant4 simulation

#### LHCb-FIGURE-2019-018



Estimated CPU usage for LHCb

#### Outline

- Generative modelling
- Deep learning models for generative modelling
  - GANs
  - VAEs
- ► HEP applications

# Generative modelling

### Problem setup

- ► Training data a set of objects, e.g.:
  - Photos of animals / people faces / rooms / whatever
  - Text
  - Audio of speech / music / whatever
  - Signals from a high energy physics experiment detector
- ► Goal: build a model to sample similar data

Set of objects:  $\{x_i \mid i = 1, ... N\}$ 

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I.e.  $\{x_i\}$  are i.i.d. sampled from p(x)

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  - Photos of animals / people faces / rooms / whatever
  - Text
  - Audio of speech / music / whatever
  - Signals from a high energy physics experiment detector
- Goal: build a model to sample similar data
  - Learn the population distribution to sample more objects
     from it
    - (may be done implicitly, i.e. when we can't evaluate the probability density, yet can sample from it)

Set of objects:

 $\{x_i \mid i = 1, ... N\}$ 

Population PDF:

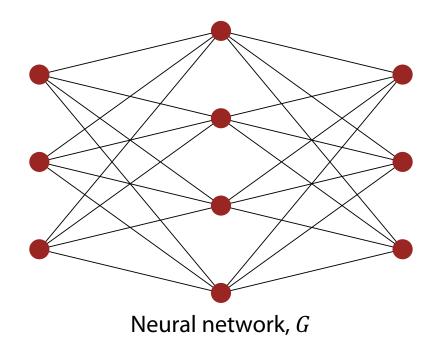
p(x)

I.e.  $\{x_i\}$  are i.i.d. sampled from p(x)

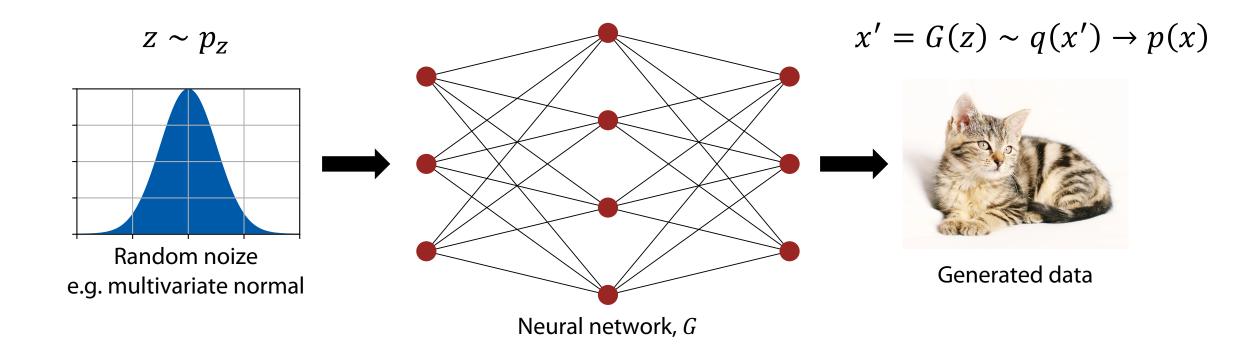
Learn  $q(x) \sim p(x)$  to sample x' from q(x)

# Generative Adversarial Networks (GANs)

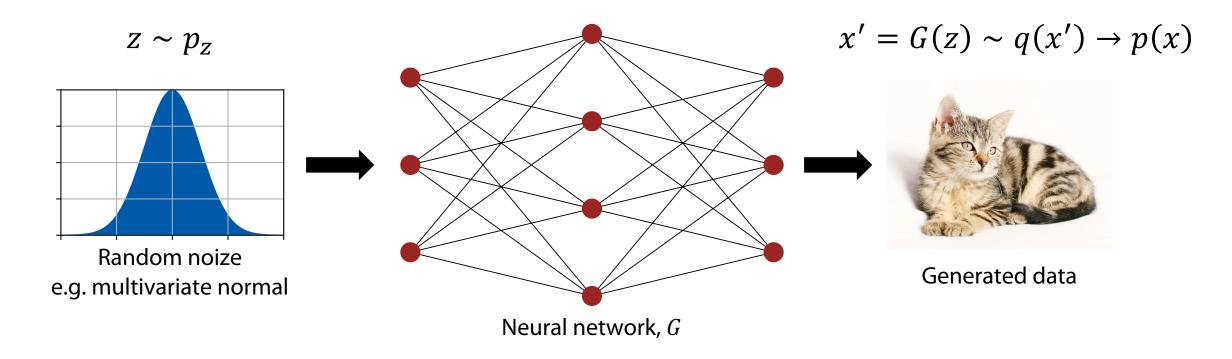
### How can a neural network generate data?



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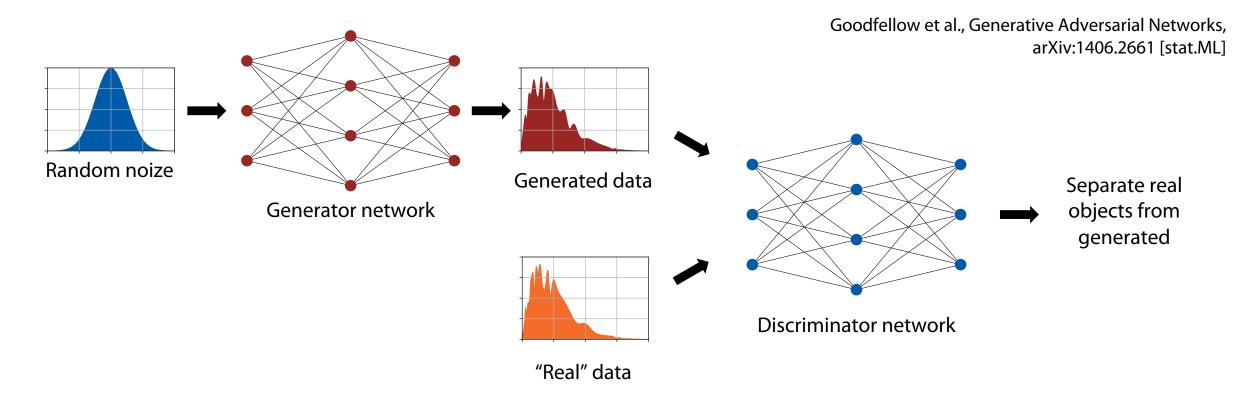


► This makes the generated object being a differentiable function of the network parameters

### How to train such a generator?

- Generated object is a differentiable function of the network parameters
- Need a differentiable measure of similarity between the sets of generated objects and real ones
  - Can optimize with gradient descent
- How to find such a measure?

### Adversarial approach



 Measure of similarity: how well can another neural network (discriminator) tell the generated objects apart from the real ones

### Training the networks

**Algorithm 1** Minibatch stochastic gradient descent training of generative adversarial nets. The number of steps to apply to the discriminator, k, is a hyperparameter. We used k=1, the least expensive option, in our experiments.

for number of training iterations do

#### for k steps do

- Sample minibatch of m noise samples  $\{z^{(1)}, \ldots, z^{(m)}\}$  from noise prior  $p_g(z)$ . Sample minibatch of m examples  $\{x^{(1)}, \ldots, x^{(m)}\}$  from data generating distribution  $p_{\mathrm{data}}(oldsymbol{x}).$
- Update the discriminator by ascending its stochastic gradient:

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[ \log D\left(\boldsymbol{x}^{(i)}\right) + \log\left(1 - D\left(G\left(\boldsymbol{z}^{(i)}\right)\right)\right) \right].$$

#### end for

discriminator

steps

• Sample minibatch of 
$$m$$
 noise samples  $\{\boldsymbol{z}^{(1)},\dots,\boldsymbol{z}^{(m)}\}$  from noise prior  $p_g(\boldsymbol{z})$ .
• Update the generator by descending its stochastic gradient: 
$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^m \log\left(1 - D\left(G\left(\boldsymbol{z}^{(i)}\right)\right)\right).$$

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#### end for

The gradient-based updates can use any standard gradient-based learning rule. We used momentum in our experiments.

https://arxiv.org/abs/1406.2661

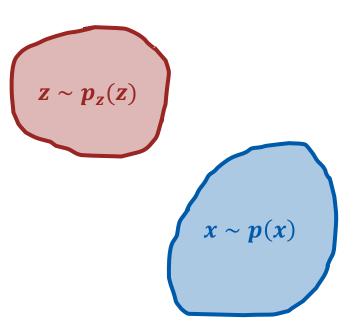
## Variational Autoencoders (VAE)



<u>arXiv:1401.4082</u>

arXiv:1312.6114

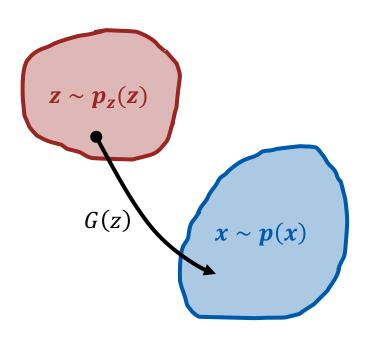
Simarly to GANs, we want to find a transformation from a known distribution  $p_z(z)$  to the data distribution p(x), using only samples from p(x)



arXiv:1401.4082 arXiv:1312.6114

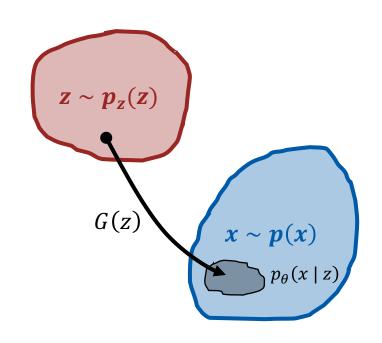
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▶ In GANs, this transformation is deterministic (x' = G(z))



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- ▶ In GANs, this transformation is deterministic (x' = G(z))
- In VAEs, it is stochastic, modelled with parametric distribution  $p_{\theta}(x \mid z)$ 
  - E.g.:  $p_{\theta}(x \mid z) \equiv p(x; \lambda = G_{\theta}(z)),$
  - i.e. a neural network  $G_{\theta}(z)$  maps (**decodes**) latent codes z to parameters  $\lambda$  of some distribution  $p(x; \lambda)$

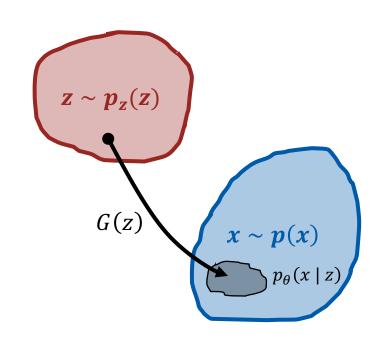


arXiv:1401.4082

arXiv:1312.6114

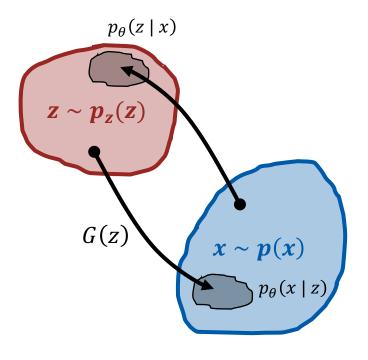
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- So, our approximation to the target distribution is:

$$p_{\theta}(x) = \int p_{\theta}(x, z) dz = \int p_{\theta}(x \mid z) p_{z}(z) dz = \mathbb{E}_{z \sim p_{z}} p_{\theta}(x \mid z)$$



arXiv:1401.4082 arXiv:1312.6114

- ▶ Assume we know the inverse transformation  $p_{\theta}(z|x)$ 
  - (though, this is typically intractable)



arXiv:1401.4082

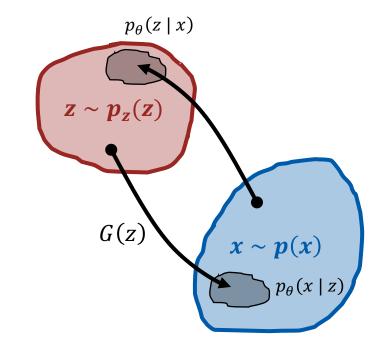
arXiv:1312.6114

- ▶ Assume we know the inverse transformation  $p_{\theta}(z|x)$ 
  - (though, this is typically intractable)
- ► Then, we could efficiently train our model by maximizing the log-likelihood:

$$\log p_{\theta}(x) = \mathbb{E}_{z \sim p_{\theta}(z|x)} \log \left[ p_{\theta}(x) \frac{p_{\theta}(z|x)}{p_{\theta}(z|x)} \right]$$

$$= \mathbb{E}_{z \sim p_{\theta}(z|x)} [\log p_{\theta}(x, z) - \log p_{\theta}(z|x)]$$

$$= \mathbb{E}_{z \sim p_{\theta}(z|x)} \log p_{\theta}(x, z) + \mathcal{H}(p_{\theta}(z|x))$$



So, for the log-likelihood we're sampling not all z values, but only those corresponding to this particular x

Maximizing this encourages placing high probability mass on many z values that could've generated x

e.g., 
$$\mathcal{N}\left(z;(\mu,\sigma)=D_{\phi}(x)\right)$$

▶ In practice,  $p_{\theta}(z|x)$  is not known, so we approximate it with some  $q_{\phi}(z|x)$ :

$$[\log p_{\theta}(x)]_{\text{approx.},\phi} = \mathbb{E}_{z \sim q_{\phi}(z|x)} \log p_{\theta}(x,z) + \mathcal{H}\left(q_{\phi}(z|x)\right)$$

- One can prove, that this approximate log-likelihood is a **lower bound** to the true log-likelihood  $\log p_{\theta}(x)$
- ightharpoonup Maximizing it wrt  $\theta$  and  $\phi$  will also maximize the true log-likelihood
  - Will lead to the true optimum if the family  $q_{\phi}(z|x)$  is rich enough to include  $p_{\theta}(z|x)$  for any  $\theta$
- ▶ It's easy to derive this alternative form, which is simpler to optimize:

$$[\log p_{\theta}(x)]_{\text{approx.},\phi} = \mathbb{E}_{z \sim q_{\phi}(z|x)} \log p_{\theta}(x|z) - D_{KL}(q_{\phi}(z|x) || p(z)) \to \max_{\theta,\phi}$$

# Applications in HEP

### Deep learning for fast simulation in HEP

Quite a developing field!

(not pretending to be able to cover all applications)

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K. Matchev, P. Shyamsundar, Uncertainties associated with GAN-generated datasets in high energy physics, arXiv:2002.06307 [hep-ph]

#### Where it all started: LAGAN

de Oliveira, L., Paganini, M. & Nachman, B., arXiv:1701.05927

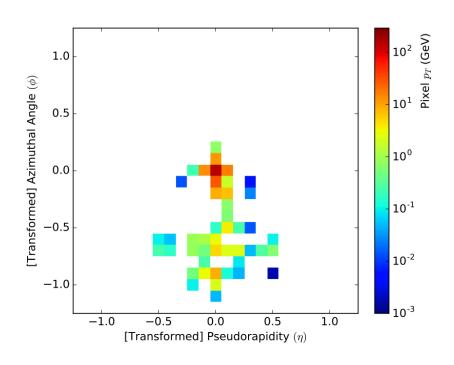


Figure 1: A typical jet image.

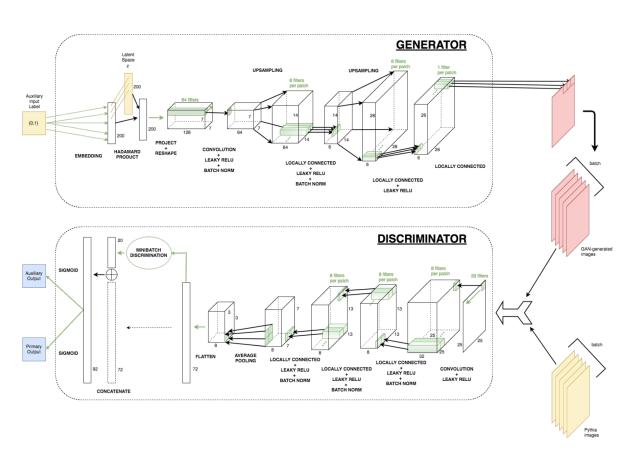


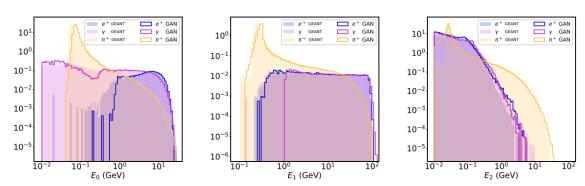
Figure 4: LAGAN architecture

Demonstrated the ability to generate realistic jet images

### CaloGAN (3D calorimeter)

L., Paganini, de Oliveira, M. & Nachman, B., arXiv:1705.02355

- ▶ Up to  $\mathcal{O}(10^3)$  time improvement on CPU
- ▶ Up to  $\mathcal{O}(10^5)$  on GPU



Some physically-motivated variables for validation (not seen at training time)

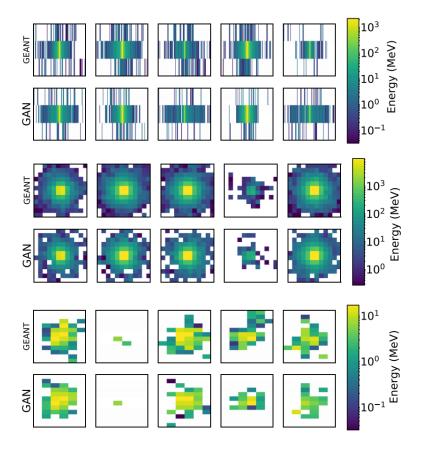
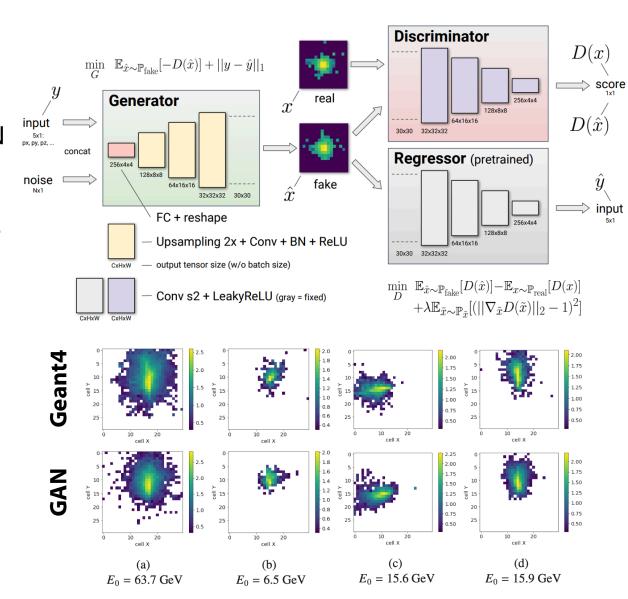
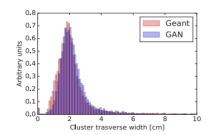


FIG. 2. Five randomly selected  $\gamma$  showers per calorimeter layer from GEANT4 (top rows) and their five nearest neighbors (by Euclidean distance) from a set of CaloGAN candidates.

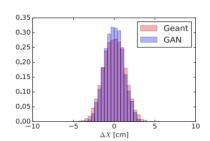
#### Fast Calorimeter Simulation: the LHCb case

- ► arXiv:1812.01319
  - Uses WGAN-GP modification of GAN (arXiv:1704.00028)
- 0.07 ms per sample (GPU)
- 4.9 ms per sample(CPU)

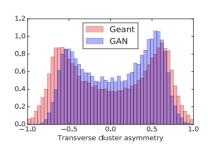




(a) The transverse width of real and generated clusters



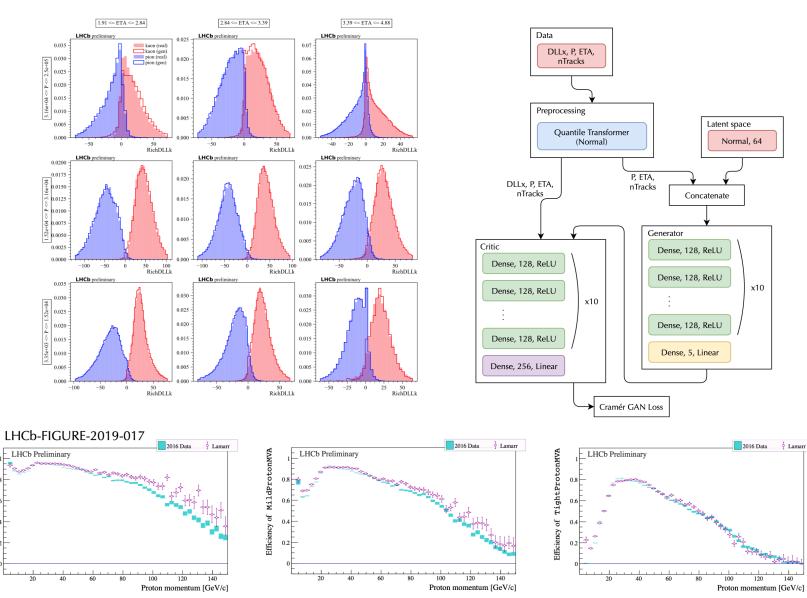
(c)  $\Delta X$  between cluster center of mass and the true particle coordinate



(e) The transverse asymmetry of real and generated clusters

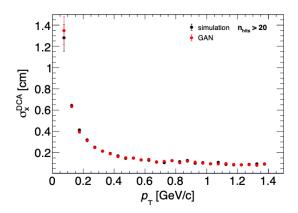
#### Data-driven simulation of LHCb Cherenkov detectors

- ► <u>arXiv:1905.11825</u>
- Cramer-GAN (arXiv:1705.10743)
- Trained on real data
  - Utilized sPlot for background subtraction

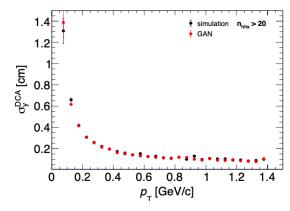


### Time projection chamber fastsim at MPD (NICA)

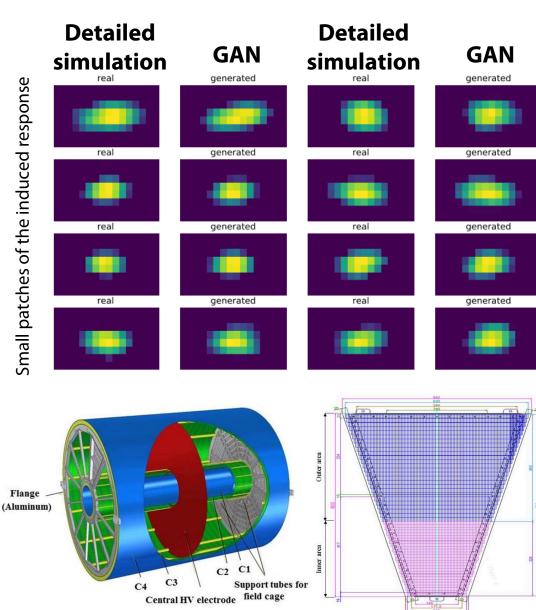
- arXiv:2012.04595
- Tracking characteristics are spot-on
- $\triangleright$   $\mathcal{O}(10)$  speed-up factor



(a) Distance of closest approach resolution along *x* 



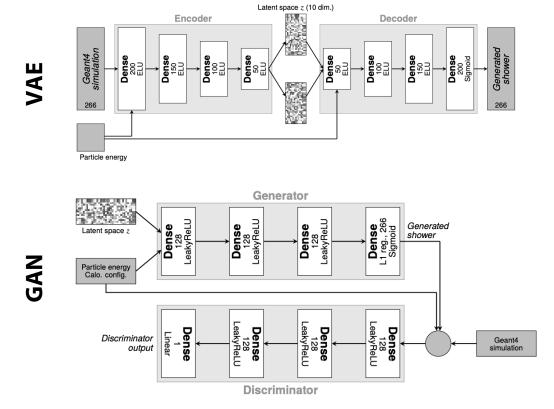
(b) Distance of closest approach resolution along *y* 

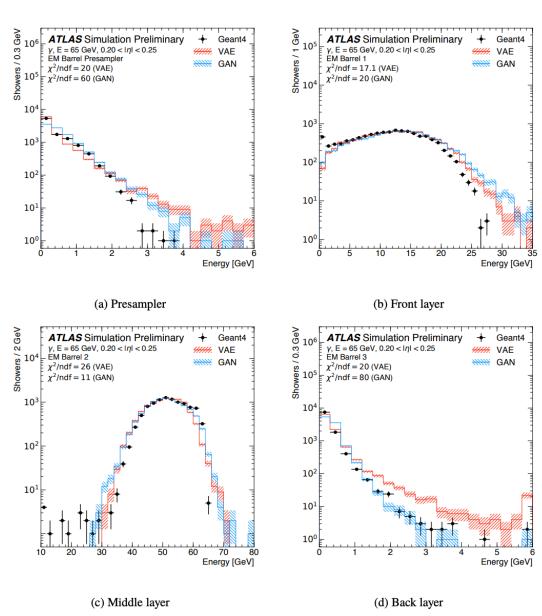


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#### Fast shower simulation in ATLAS

- ► ATL-SOFT-PUB-2018-001
- ▶ 3d calorimeter simulation
- Tested both GAN and VAE





### There's many more...

Check out this list if interested:
<a href="https://github.com/iml-wg/HEPML-LivingReview">https://github.com/iml-wg/HEPML-LivingReview</a>

#### Generative models / density estimation

#### GANs:

- Learning Particle Physics by Example: Location-Aware Generative Adversarial Networks for Physics Synthesis [DOI]
- Accelerating Science with Generative Adversarial Networks: An Application to 3D Particle Showers in Multilayer Calorimeters [DOI]
- CaloGAN: Simulating 3D high energy particle showers in multilayer electromagnetic calorimeters with generative adversarial networks [DOI]
- Image-based model parameter optimization using Model-Assisted Generative Adversarial Networks [D0I]
- How to GAN Event Subtraction [DOI]
- Particle Generative Adversarial Networks for full-event simulation at the LHC and their application to pileup description [DOI]
- How to GAN away Detector Effects [DOI]
- 3D convolutional GAN for fast simulation
- Fast simulation of muons produced at the SHIP experiment using Generative Adversarial Networks [DOI]
- Lund jet images from generative and cycle-consistent adversarial networks [DOI]
- How to GAN LHC Events [DOI]
- Machine Learning Templates for QCD Factorization in the Search for Physics Beyond the Standard Model [DOI]
- DijetGAN: A Generative-Adversarial Network Approach for the Simulation of QCD Dijet Events at the LHC [DOI]
- LHC analysis-specific datasets with Generative Adversarial Networks
- Generative Models for Fast Calorimeter Simulation.LHCb case [DOI]
- Deep generative models for fast shower simulation in ATLAS
- Regressive and generative neural networks for scalar field theory [DOI]
- Three dimensional Generative Adversarial Networks for fast simulation
- Generative models for fast simulation
- Unfolding with Generative Adversarial Networks
- Fast and Accurate Simulation of Particle Detectors Using Generative Adversarial Networks [DOI]
- Generating and refining particle detector simulations using the Wasserstein distance in adversarial networks [DOI]
- Generative models for fast cluster simulations in the TPC for the ALICE experiment
   RICH 2018 [DOI]
- GANs for generating EFT models [DOI]
- Precise simulation of electromagnetic calorimeter showers using a Wasserstein Generative Adversarial Network [DOI]
- Reducing Autocorrelation Times in Lattice Simulations with Generative Adversarial Networks [DOI]
- Tips and Tricks for Training GANs with Physics Constraints
- Controlling Physical Attributes in GAN-Accelerated Simulation of Electromagnetic Calorimeters [DOI]
- Next Generation Generative Neural Networks for HEP
- Calorimetry with Deep Learning: Particle Classification, Energy Regression, and Simulation for High-Energy Physics
- Calorimetry with Deep Learning: Particle Simulation and Reconstruction for Collider Physics [DOI]
- Getting High: High Fidelity Simulation of High Granularity Calorimeters with High Speed
- Al-based Monte Carlo event generator for electron-proton scattering
- DCTRGAN: Improving the Precision of Generative Models with Reweighting [DOI]
- GANplifying Event Samples
- Graph Generative Adversarial Networks for Sparse Data Generation in High Energy Physics
- Simulating the Time Projection Chamber responses at the MPD detector using Generative Adversarial Networks
- Explainable machine learning of the underlying physics of high-energy particle collisions
   A Data-driven Event Generator for Hadron Colliders using Wasserstein Generative
- Reduced Precision Strategies for Deep Learning: A High Energy Physics Generative Adversarial Network Use Case [DOI]
- Validation of Deep Convolutional Generative Adversarial Networks for High Energy Physics Calorimeter Simulations
- Compressing PDF sets using generative adversarial networks
- Physics Validation of Novel Convolutional 2D Architectures for Speeding Up High Energy Physics Simulations

#### Autoencoders

- Deep Learning as a Parton Shower
- Deep generative models for fast shower simulation in ATLAS
- Variational Autoencoders for Anomalous Jet Tagging
- Variational Autoencoders for Jet Simulation
- Foundations of a Fast, Data-Driven, Machine-Learned Simulator
- Decoding Photons: Physics in the Latent Space of a BIB-AE Generative Network
- Bump Hunting in Latent Space
- {End-to-end Sinkhorn Autoencoder with Noise Generator
- Graph Generative Models for Fast Detector Simulations in High Energy Physics
- DeepRICH: Learning Deeply Cherenkov Detectors [DOI]

#### Normalizing flows

- Flow-based generative models for Markov chain Monte Carlo in lattice field theory [DOI]
- Equivariant flow-based sampling for lattice gauge theory [DOI]
- · Flows for simultaneous manifold learning and density estimation
- Exploring phase space with Neural Importance Sampling [DOI]
- Event Generation with Normalizing Flows [DOI]
- i-flow: High-Dimensional Integration and Sampling with Normalizing Flows [DOI]
- Anomaly Detection with Density Estimation [DOI]
- Data-driven Estimation of Background Distribution through Neural Autoregressive Flows
- SARM: Sparse Autoregressive Model for Scalable Generation of Sparse Images in Particle Physics [DOI]
- Measuring QCD Splittings with Invertible Networks
- Efficient sampling of constrained high-dimensional theoretical spaces with machine learning

#### Physics-inspired

- JUNIPR: a Framework for Unsupervised Machine Learning in Particle Physics
- Binary JUNIPR: an interpretable probabilistic model for discrimination [DOI]
- Exploring the Possibility of a Recovery of Physics Process Properties from a Neural Network Model [DOI]
- Explainable machine learning of the underlying physics of high-energy particle collisions
- Symmetry meets Al

#### Mixture Models

- Data Augmentation at the LHC through Analysis-specific Fast Simulation with Deep Learning
- Mixture Density Network Estimation of Continuous Variable Maximum Likelihood Using Discrete Training Samples

#### Phase space generation

- Efficient Monte Carlo Integration Using Boosted Decision
- Exploring phase space with Neural Importance Sampling [DOI]
- Event Generation with Normalizing Flows [DOI]
- i-flow: High-Dimensional Integration and Sampling with Normalizing Flows [DOI]
- Neural Network-Based Approach to Phase Space Integration [DOI]
- VegasFlow: accelerating Monte Carlo simulation across multiple hardware platforms [DOI]
- A Neural Resampler for Monte Carlo Reweighting with Preserved Uncertainties [DOI]
- Improved Neural Network Monte Carlo Simulation [DOI]
- Phase Space Sampling and Inference from Weighted Events with Autoregressive Flows fDOI1
- How to GAN Event Unweighting

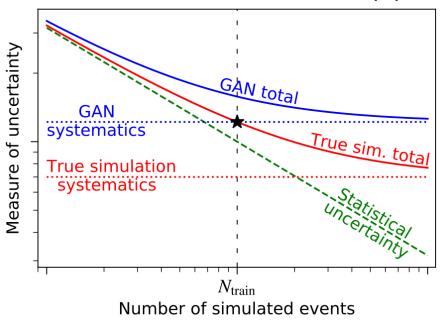
#### Gaussian processes

- Modeling Smooth Backgrounds and Generic Localized Signals with Gaussian Processes
- Accelerating the BSM interpretation of LHC data with machine learning [DOI]
- \$\textsf{Xsec}\$: the cross-section evaluation code [DOI]
- Al-optimized detector design for the future Electron-Ion Collider: the dual-radiator RICH case [D0I]

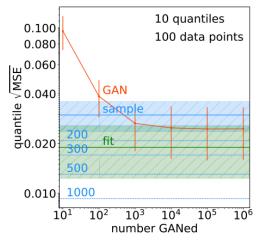
### Note on systematic uncertainties

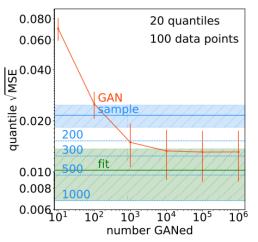
- Trained on a finite sample, generative models introduce additional systematics
- However, a generative model may contain more statistical power than the original training dataset — due to interpolation

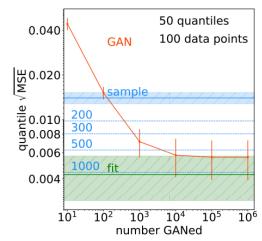
#### arXiv:2002.06307 [hep-ph]



#### <u>arXiv:2008.06545 [hep-ph]</u>







### Summary

- Deep generative models are an exciting and very quickly developing field of machine learning
- They promise to be good candidates for fast simulation models in HEP
- GANs and VAEs briefly covered in this talk, did not get to other models, e.g. based on Normalizing Flows or Mixture models
- Performance evaluation of a trained model is always tricky

# Thank you!





Artem Maevskiy

# Backup

# GAN



## Let's put it in formulas

Noise samples:

$$z_i \sim p_z(z)$$

where  $p_z$  is some simple PDF we can sample from, e.g.  $\mathcal{N}(0, \mathbb{I})$ .

Generated samples:

$$x_i' = G_{\theta}(z_i)$$

where  $G_{\theta}$  is the generator network with parameters  $\theta$ .

▶ Discriminator network (with parameters  $\phi$ ):

$$D_{\phi}(x)$$

returns the probability for x being a real sample rather than a generated one

► Measure of similarity between the generated and real samples:

Probability the sample was generated

$$L_G = \max_{\phi} \mathbb{E}_{x \sim p(x)} \left[ \log D_{\phi}(x) \right] + \mathbb{E}_{z \sim p_z(z)} \left[ \log \left( 1 - D_{\phi} \left( G_{\theta}(z) \right) \right) \right] \xrightarrow{\text{was general}} \mathbb{E}_{x \sim p(x)} \left[ \log D_{\phi}(x) \right] + \mathbb{E}_{z \sim p_z(z)} \left[ \log \left( 1 - D_{\phi} \left( G_{\theta}(z) \right) \right) \right] \xrightarrow{\text{was general}} \mathbb{E}_{x \sim p(x)} \left[ \log D_{\phi}(x) \right] + \mathbb{E}_{z \sim p_z(z)} \left[ \log \left( 1 - D_{\phi} \left( G_{\theta}(z) \right) \right) \right] \xrightarrow{\text{was general}} \mathbb{E}_{x \sim p(x)} \left[ \log D_{\phi}(x) \right] + \mathbb{E}_{z \sim p_z(z)} \left[ \log \left( 1 - D_{\phi} \left( G_{\theta}(z) \right) \right) \right] \xrightarrow{\text{was general}} \mathbb{E}_{x \sim p(x)} \left[ \log D_{\phi}(x) \right] + \mathbb{E}_{z \sim p_z(z)} \left[ \log \left( 1 - D_{\phi} \left( G_{\theta}(z) \right) \right) \right] \xrightarrow{\text{was general}} \mathbb{E}_{x \sim p(x)} \left[ \log D_{\phi}(x) \right] + \mathbb{E}_{z \sim p_z(z)} \left[ \log \left( 1 - D_{\phi} \left( G_{\theta}(z) \right) \right) \right] \xrightarrow{\text{was general}} \mathbb{E}_{x \sim p(x)} \left[ \log D_{\phi}(x) \right] + \mathbb{E}_{z \sim p_z(z)} \left[ \log \left( 1 - D_{\phi} \left( G_{\theta}(z) \right) \right) \right] \xrightarrow{\text{was general}} \mathbb{E}_{x \sim p(x)} \left[ \log D_{\phi}(x) \right] + \mathbb{E}_{z \sim p_z(z)} \left[ \log \left( 1 - D_{\phi} \left( G_{\theta}(z) \right) \right) \right] \xrightarrow{\text{was general}} \mathbb{E}_{x \sim p(x)} \left[ \log D_{\phi}(x) \right] + \mathbb{E}_{z \sim p_z(z)} \left[ \log \left( 1 - D_{\phi} \left( G_{\theta}(z) \right) \right) \right] \xrightarrow{\text{was general}} \mathbb{E}_{x \sim p(x)} \left[ \log D_{\phi}(x) \right] + \mathbb{E}_{z \sim p_z(z)} \left[ \log \left( 1 - D_{\phi} \left( G_{\theta}(z) \right) \right) \right] \xrightarrow{\text{was general}} \mathbb{E}_{x \sim p(x)} \left[ \log D_{\phi}(x) \right] + \mathbb{E}_{z \sim p_z(z)} \left[ \log \left( 1 - D_{\phi} \left( G_{\theta}(z) \right) \right) \right] \xrightarrow{\text{was general}} \mathbb{E}_{x \sim p(x)} \left[ \log D_{\phi}(x) \right] + \mathbb{E}_{z \sim p_z(z)} \left[ \log D_{\phi}(x) \right] + \mathbb{E}_{z \sim p_z(z)} \left[ \log D_{\phi}(x) \right]$$

# Training the networks

**Algorithm 1** Minibatch stochastic gradient descent training of generative adversarial nets. The number of steps to apply to the discriminator, k, is a hyperparameter. We used k = 1, the least expensive option, in our experiments.

for number of training iterations do

for k steps do

- Sample minibatch of m noise samples  $\{z^{(1)}, \ldots, z^{(m)}\}$  from noise prior  $p_g(z)$ .
- Sample minibatch of m examples  $\{x^{(1)}, \ldots, x^{(m)}\}$  from data generating distribution  $p_{\text{data}}(x)$ .
- Update the discriminator by ascending its stochastic gradient:

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^{m} \left[ \log D\left( \boldsymbol{x}^{(i)} \right) + \log \left( 1 - D\left( G\left( \boldsymbol{z}^{(i)} \right) \right) \right) \right].$$

#### end for

- Sample minibatch of m noise samples  $\{z^{(1)}, \ldots, z^{(m)}\}$  from noise prior  $p_g(z)$ .
- Update the generator by descending its stochastic gradient:

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^{m} \log \left( 1 - D\left(G\left(\boldsymbol{z}^{(i)}\right)\right) \right).$$

#### end for

The gradient-based updates can use any standard gradient-based learning rule. We used momentum in our experiments.

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# Problems with GANs

## The optimal discriminator solution

▶ If we re-write the loss using  $p_{\text{gen},\theta}(x)$  – the distribution of  $x' = G_{\theta}(z)$ , and expand the expectations as integrals:

$$L_G = \max_{\phi} \int_{x} \left[ p(x) \log \left( D_{\phi}(x) \right) + p_{\text{gen},\theta}(x) \log \left( 1 - D_{\phi}(x) \right) \right] dx$$

• it's easy to show that  $\max_{\phi}$  is obtained at  $\phi^*(\theta)$  with:

$$D_{\phi^*(\theta)}(x) = \frac{p(x)}{p(x) + p_{\text{gen},\theta}(x)}$$

So the objective becomes:

$$L_G = \mathbb{E}_{x \sim p(x)} \left[ \log \frac{p(x)}{p(x) + p_{\text{gen},\theta}(x)} \right] + \mathbb{E}_{x \sim p_{\text{gen},\theta}(x)} \left[ \log \frac{p_{\text{gen},\theta}(x)}{p(x) + p_{\text{gen},\theta}(x)} \right]$$
$$= -\log 4 + JSD(p \mid\mid p_{\text{gen},\theta})$$

Artem Maevskiy, NRU HSE

Jensen–Shannon divergence

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# Vanishing gradients

▶ In case p and  $p_{\text{gen},\theta}$  have non-overlapping support:

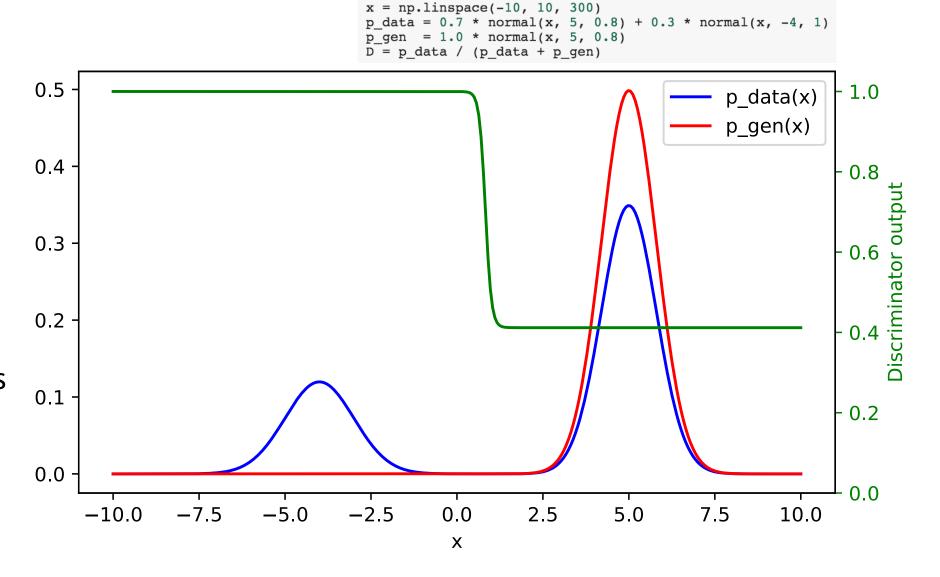
$$L_G = \mathbb{E}_{x \sim p(x)} \left[ \log \frac{p(x)}{p(x) + p_{\text{gen},\theta}(x)} \right] + \mathbb{E}_{x \sim p_{\text{gen},\theta}(x)} \left[ \log \frac{p_{\text{gen},\theta}(x)}{p(x) + p_{\text{gen},\theta}(x)} \right]$$

$$= \mathbb{E}_{x \sim p(x)} \left[ \log \frac{p(x)}{p(x)} \right] + \mathbb{E}_{x \sim p_{\text{gen},\theta}(x)} \left[ \log \frac{p_{\text{gen},\theta}(x)}{p_{\text{gen},\theta}(x)} \right] = 0 = const$$

No meaningful gradient, can't learn

# Mode collapse

- Assume at some point the generator has learned one of the modes
- No meaningful gradients to drive the solution towards covering the other modes



# Wasserstein GAN



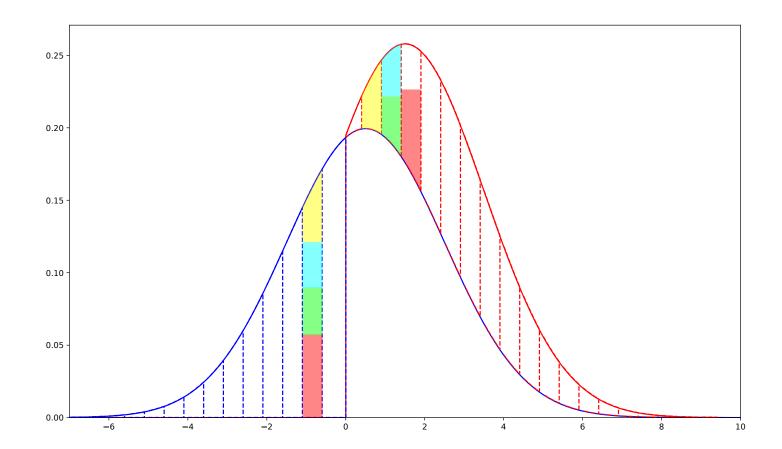
#### Alternative distance measure

- ➤ The problems with GANs are mainly due to Jensen–Shannon divergence providing problematic gradients
- ► What if we try to find some other measure of distance between real and generated distributions that doesn't have these problems?

#### Wasserstein distance

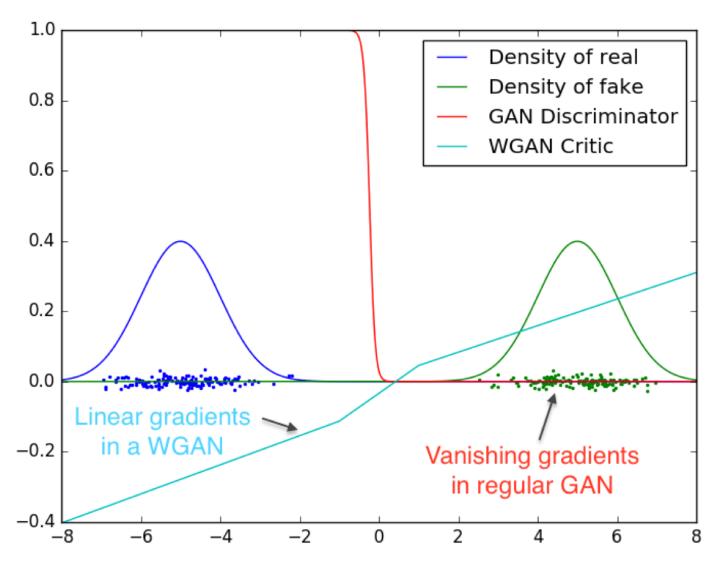
Also called "Earth mover's distance" (EMD)

- ► Distributions P(x) and Q(x) are viewed as describing the amounts of "dirt" at point x
- We want to convert one distribution into the other by moving around some amounts of dirt



- ▶ The cost of moving an amount m from  $x_1$  to  $x_2$  is  $m \times ||x_2 x_1||$
- ► EMD(P, Q) = minimum total cost of converting P into Q

# Why is it better?



#### Formal definition

► Say, we have a moving plan  $\gamma(x_1, x_2) \ge 0$ :

$$\gamma(x_1, x_2)dx_1dx_2$$
 – how much dirt we're moving from  $[x_1, x_1 + dx_1]$  to  $[x_2, x_2 + dx_2]$ 

▶ Then, the cost of moving from  $[x_1, x_1 + dx_1]$  to  $[x_2, x_2 + dx_2]$  is:

$$||x_2 - x_1|| \cdot \gamma(x_1, x_2) dx_1 dx_2$$

and the total cost is:

$$C = \int_{x_1, x_2} \|x_2 - x_1\| \cdot \gamma(x_1, x_2) dx_1 dx_2 = \mathbb{E}_{x_1, x_2 \sim \gamma(x_1, x_2)} \|x_2 - x_1\|$$

▶ Since we want to convert *P* to *Q*, the plan has to satisfy:

$$\int_{x_1} \gamma(x_1, x_2) dx_1 = Q(x_2), \qquad \int_{x_2} \gamma(x_1, x_2) dx_2 = P(x_1)$$

Interpreting  $\gamma$  as a PDF

#### Formal definition

Let  $\pi$  be the set of all plans that convert P to Q, i.e.:

$$\pi = \left\{ \gamma: \quad \gamma \ge 0, \quad \int_{x_1} \gamma(x_1, x_2) dx_1 = Q(x_2), \quad \int_{x_2} \gamma(x_1, x_2) dx_2 = P(x_1) \right\}$$

▶ Then, the Wasserstein distance between *P* and *Q* is:

$$\mathrm{EMD}(P,Q) = \inf_{\gamma \in \pi} \mathbb{E}_{x_1, x_2 \sim \gamma} \|x_2 - x_1\|$$

**Optimization over all transport plans – not too friendly** 

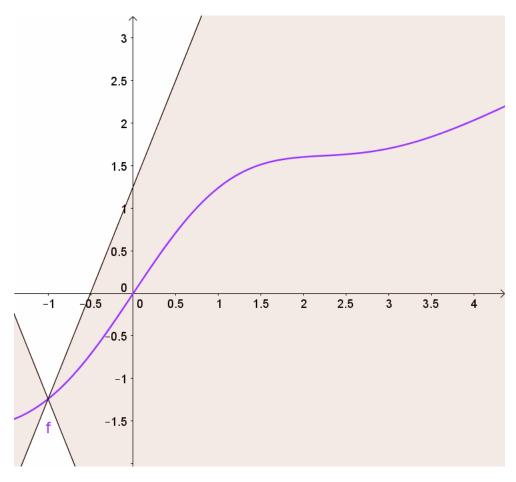
Dual form (Kantorovich-Rubinstein duality):

$$\mathrm{EMD}(P,Q) = \sup_{\|f\|_L \leq 1} \left[ \mathbb{E}_{x \sim P} f(x) - \mathbb{E}_{x \sim Q} f(x) \right]$$
 Optimization over Lipschitz-1 continuous functions acting in  $\mathcal{X} \to \mathbb{R}$ 

# Lipschitz continuity

- f is Lipschitz-k continuous if
- ▶ there exists a constant  $k \ge 0$ , such that for all  $x_1$  and  $x_2$ :

$$|f(x_1) - f(x_2)| \le k \cdot ||x_1 - x_2||$$



img from <a href="https://en.wikipedia.org/wiki/Lipschitz">https://en.wikipedia.org/wiki/Lipschitz</a> continuity

## [intuition behind the dual form]

disclaimer: not a strict mathematical derivation

$$EMD(P,Q) = \inf_{\gamma \in \pi} \mathbb{E}_{x_1, x_2 \sim \gamma} ||x_1 - x_2||$$

Let's add the following term to this expression:

$$+\inf_{\gamma}\sup_{f}\mathbb{E}_{x_1,x_2\sim\gamma}\left[\mathbb{E}_{s\sim P}f(s)-\mathbb{E}_{t\sim Q}f(t)-(f(x_1)-f(x_2))\right]$$

f(x) — real-valued function

These cancel out when  $\gamma \in \pi$ otherwise supremum over f(x) goes to  $+\infty$ 

Therefore, we can remove the  $\gamma \in \pi$  condition from the whole expression:

$$=\inf_{\gamma}\sup_{f}\mathbb{E}_{x_1,x_2\sim\gamma}\left[||x_1-x_2||+\mathbb{E}_{s\sim P}f(s)-\mathbb{E}_{t\sim Q}f(t)-(f(x_1)-f(x_2))\right]$$
 Infimum and supremum operations can be swapped under certain conditions

(satisfied here — see <a href="https://vincentherrmann.github.io/blog/wasserstein/">https://vincentherrmann.github.io/blog/wasserstein/</a> for more detailed info)

### [intuition behind the dual form]

disclaimer: not a strict mathematical derivation

$$= \sup_{f} \inf_{\gamma} \left[ \mathbb{E}_{s \sim P} f(s) - \mathbb{E}_{t \sim Q} f(t) + \mathbb{E}_{x_1, x_2 \sim \gamma} \left[ ||x_1 - x_2|| - (f(x_1) - f(x_2)) \right] \right]$$

Consider the following case:  $|f(a) - f(b)| \le ||a - b||$ ,  $\forall a, b \ne a$ 

We'll denote it as:  $||f||_L \leq 1$ 

For such case this term is 0

Otherwise the whole expression is -∞

Therefore we can finally rewrite the whole thing as:

$$EMD(P,Q) = \sup_{||f||_{L} \le 1} \left[ \mathbb{E}_{x \sim P} f(x) - \mathbb{E}_{x \sim Q} f(x) \right]$$

### **WGAN**

$$EMD(P,Q) = \sup_{\|f\|_{L} \le 1} \left[ \mathbb{E}_{x \sim P} f(x) - \mathbb{E}_{x \sim Q} f(x) \right]$$

- ► The function can be expressed as a neural net discriminator ('critic' in the original paper)
- The expectations can be estimated as sample mean

- Lipschitz-1 continuity can be replaced with Lipschitz-k continuity
  - In such case we'll estimate  $k \times \text{EMD}(P, Q)$
  - Can be achieved by clipping the weights of the critic:  $w \rightarrow \text{clip}(w, -c, c)$  with some constant c

We wouldn't know what k is, but it doesn't matter: all we want is to **minimize** the EMD!

#### **WGAN**

12: end while

**Algorithm 1** WGAN, our proposed algorithm. All experiments in the paper used the default values  $\alpha = 0.00005$ , c = 0.01, m = 64,  $n_{\text{critic}} = 5$ .

```
Require: : \alpha, the learning rate. c, the clipping parameter. m, the batch size.
     n_{\text{critic}}, the number of iterations of the critic per generator iteration.
Require: : w_0, initial critic parameters. \theta_0, initial generator's parameters.
 1: while \theta has not converged do
          for t = 0, ..., n_{\text{critic}} do
                Sample \{x^{(i)}\}_{i=1}^m \sim \mathbb{P}_r a batch from the real data.
 3:
                Sample \{z^{(i)}\}_{i=1}^m \sim p(z) a batch of prior samples.
               g_w \leftarrow \nabla_w \left[ \frac{1}{m} \sum_{i=1}^m f_w(x^{(i)}) - \frac{1}{m} \sum_{i=1}^m f_w(g_\theta(z^{(i)})) \right]
 5:
               w \leftarrow w + \alpha \cdot \text{RMSProp}(w, q_w)
 6:
               w \leftarrow \text{clip}(w, -c, c)
 7:
          end for
 8:
          Sample \{z^{(i)}\}_{i=1}^m \sim p(z) a batch of prior samples.
 9:
          g_{\theta} \leftarrow -\nabla_{\theta} \frac{1}{m} \sum_{i=1}^{m} f_{w}(g_{\theta}(z^{(i)}))
          \theta \leftarrow \theta - \alpha \cdot \text{RMSProp}(\theta, q_{\theta})
11:
```

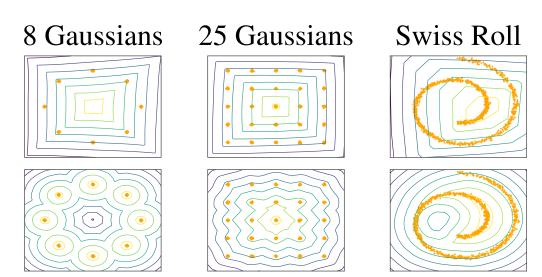
#### **WGAN-GP**

- Weight clipping makes the critic less expressive and the training harder to converge
- ▶ Optimal f should satisfy  $\|\nabla f\| = 1$  almost everywhere under P and Q
- ▶ Also:  $||f||_L \le 1 \iff ||\nabla f|| \le 1$
- Can replace weight clipping with a gradient penalty term:

$$GP = \lambda \mathbb{E}_{\widetilde{x} \sim \mathbb{P}_{\widetilde{x}}} [(\|\nabla_{\widetilde{x}} f(\widetilde{x})\| - 1)^2]$$

or alternatively ('one-sided' penalty):

$$GP = \lambda \mathbb{E}_{\tilde{x} \sim \mathbb{P}_{\tilde{x}}} [\max(0, \|\nabla_{\tilde{x}} f(\tilde{x})\| - 1)^2]$$



$$\mathbb{P}_{\tilde{x}}: \begin{bmatrix} \tilde{x} = \alpha x_1 + (1 - \alpha) x_2 \\ \alpha \sim \text{Uniform}(0, 1) \\ x_1 \sim P \\ x_2 \sim Q \end{bmatrix}$$

https://arxiv.org/abs/1704.00028

#### Sidenote

- There's an argument that the (true) Wasserstein distance might not be ideal for generative modelling
  - being a function of L2 norm of the difference vector (e.g. per-pixel difference between images)

$$EMD(P,Q) = \inf_{\gamma \in \pi} \mathbb{E}_{x_1, x_2 \sim \gamma} ||x_2 - x_1||$$

- ► A curious reading:
  - J. Stanczuk et. al. Wasserstein GANs Work Because They Fail (to Approximate the Wasserstein Distance), <a href="https://arxiv.org/abs/2103.01678">https://arxiv.org/abs/2103.01678</a>