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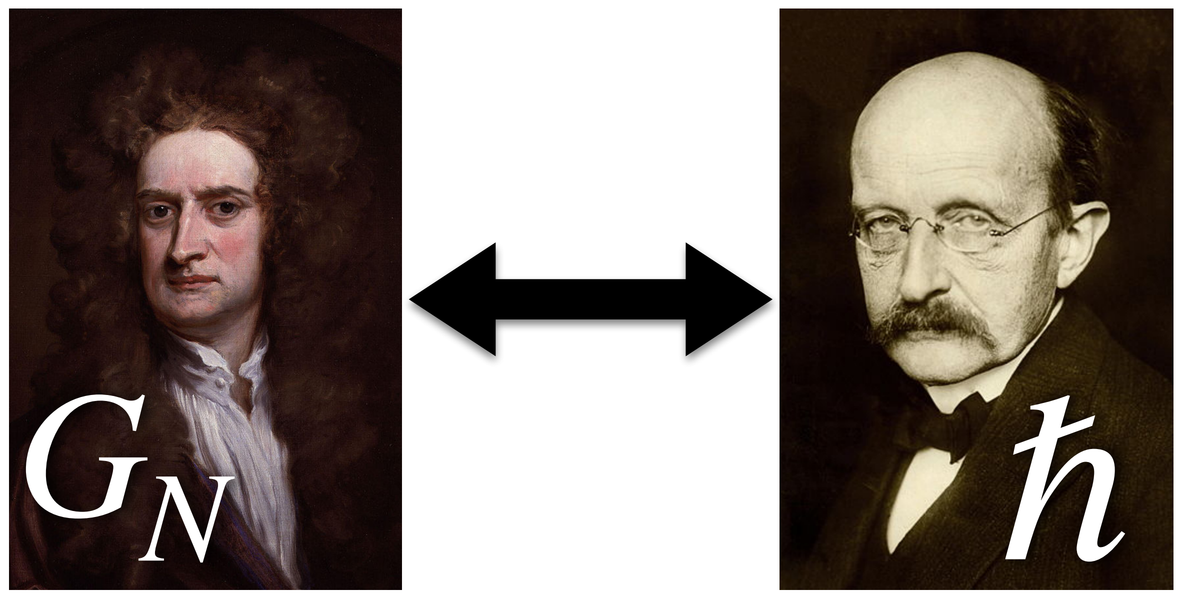
(dedicated to A.D. Sakharov's centennial)

Global Dynamics for



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Losing the trace to find dynamical Newton or Planck constants

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Message to take home

- There are *local, general-covariant field theories* where the dynamical degrees of freedom are *global,* e.g. given by integrals over the whole Cauchy hypersurface. This is a field theory cover-up of the usual “high school” mechanics.

- These global degrees of freedom can mimic free parameters or “fundamental constants”.

- Λ, G_N and even $\hbar$ can be such global degrees of freedom. Now they are subject to quantum fluctuations!

- Uncertainty relations for global degrees of freedom (e.g. Λ) can be the way to avoid classical “end of time” singularities of GR.

- Λ, G_N and even $\hbar$ as global degrees of freedom can be *frozen* axions for a confined Yang-Mills / QCD.

- The origin of the values of these global degrees of freedom should be in quantum cosmology - they are remnants of the BIG BANG - ideal “Landscape” for poor people

What a Strange Theory!

$$S = \frac{1}{8\pi G_N} \int d^4x \sqrt{-g} \left(-\frac{R}{2} + \Lambda \right)$$

$$\Lambda = 1.7 \times 10^{-66}\,\mathrm{eV}^2$$

$$G_N = \frac{1}{M_{pl}^2} = 0.7 \times 10^{-56}\,\mathrm{eV}^{-2}$$

$$G_N \Lambda \sim 10^{-122}$$

The Enigma of Gravity

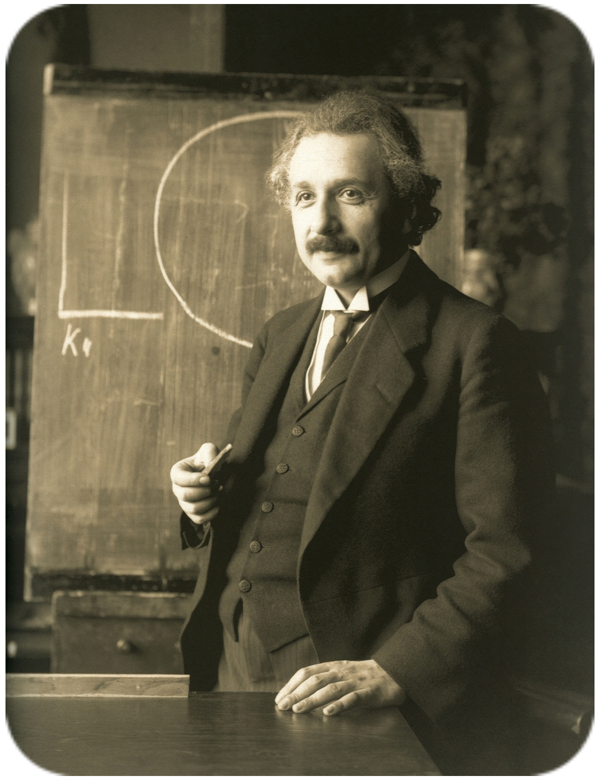
Which mechanism is behind Λ?

$$G_N \Lambda \sim 10^{-122}$$

Which mechanism is behind G_N?

Λ as global dynamical degree of freedom

first way to lose the trace



Traceless Einstein Equations?

Einstein: Gravitationsfelder im Aufbau der materiellen Elementarteilchen 349

In: *Königlich Preußische Akademie der Wissenschaften* (Berlin). *Sitzungsberichte* (1919): 349–356.

Spielen Gravitationsfelder im Aufbau der materiellen Elementarteilchen eine wesentliche Rolle?

Von A. Einstein.

Weder die Newtonsche noch die relativistische Gravitationstheorie hat bisher der Theorie von der Konstitution der Materie einen Fortschritt gebracht. Demgegenüber soll im folgenden gezeigt werden, daß Anhaltspunkte für die Auffassung vorhanden sind, daß die die Bausteine der Atome bildenden elektrischen Elementargebilde durch Gravitationskräfte zusammengehalten werden.

§ 1. Mängel der gegenwärtigen Auffassung.

Die Theoretiker haben sich viel bemüht, eine Theorie zu ersinnen, welche von dem Gleichgewicht der das Elektron konstituierenden Elektrizität Rechenschaft gibt. Insbesondere G. Mie hat dieser Frage tiefgehende Unternehmungen gewidmet. Seine Theorie, welche bei den Fachgenossen vielfach Zustimmung gefunden hat, beruht im wesentlichen darauf, daß außer den Energietermen der Maxwell-Lorentzschen Theorie des elektromagnetischen Feldes von den Komponenten des elektrodyna-

$$\cancel{G_{\mu\nu} - T_{\mu\nu} = 0}$$

$$G_{\mu\nu} - T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} (G - T) = 0$$

Bianchi identity + energy-momentum conservation

$$\nabla_\mu G^{\mu\nu} = 0 \quad + \quad \nabla_\mu T^{\mu\nu} = 0 \quad \Rightarrow \quad \partial_\mu (G - T) = 0$$

$$G_{\mu\nu} - T_{\mu\nu} - \Lambda g_{\mu\nu} = 0$$

Λ is merely an integration constant!

Decoupling vacuum energy from spacetime curvature

$$G_{\mu\nu} - T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} (G - T) = 0$$

invariant under *vacuum shifts* of energy-momentum tensor

$$T_{\mu\nu} \rightarrow T_{\mu\nu} + \Lambda g_{\mu\nu}$$

known under the name “unimodular” gravity

One can obtain Λ as an integration constant by making a constraint in the action

$$\sqrt{-g} = f(x)$$

i.e. partially fixing a gauge *before* variation

where $f(x)$ is *arbitrary* unspecified / external non-dynamical function which is often taken $f(x) = 1$

$$\Downarrow$$

Let's make $f(x)$ internal / dynamical function, which would still be irrelevant, but save general covariance!

Henneaux, Teitelboim(Bunster)(1989)

Theory of *all* (a)dS

$$S_{dS}\left[g, W, \Lambda\right] = \int d^4x \sqrt{-g}\,\Lambda\left[\nabla_\mu W^\mu - 1\right]$$

$$T_{\mu\nu} = \Lambda g_{\mu\nu}$$

Henneaux, Teitelboim(Bunster)(1989)

- Gauge degeneracy $W^\mu \to W^\mu + \epsilon^\mu$ where $\nabla_\mu \epsilon^\mu = 0$
- Fake violation of the Lorenz-symmetry, $\langle W^\mu \rangle \neq 0$
- DE / CC energy density as a Lagrange multiplier to make $\sqrt{-g} = \partial_\mu\left(\sqrt{-g}\,W^\mu\right)$
- Similarly to the Ostrogradsky Hamiltonian, the system is *linear* in canonical momentum π, i.e. in energy density of DE, Λ

$$\pi = \frac{\delta L}{\delta \dot{W}^t} = \sqrt{-g}\,\Lambda \qquad \text{However,} \qquad \partial_\mu \Lambda = 0$$

- Scale invariance, as there is no fixed scale in the action

Henneaux–Teitelboim “unimodular” gravity (1989)

$$S_{dS}\left[g, W, \Lambda\right] = \int d^4x\sqrt{-g}\,\Lambda\left[\nabla_\mu W^\mu - 1\right]$$

cf. with $S = \int dt\left(p\dot{q} - H\right)$, mind constraint $\partial_i\Lambda = 0$, use it in the action following Faddeev-Jackiw

$$S = \int dt\,\Lambda\,\partial_t\int d^3\mathbf{x}\sqrt{-g}\,W^t + \ldots$$

global degree of freedom canonically conjugated to the CC

$$\mathcal{T}(t) = \int d^3\mathbf{x}\sqrt{-g}\,W^t(t,\mathbf{x})$$

gauge invariance $W^\mu \to W^\mu + \epsilon^\mu$ generates global shift-symmetry $\mathcal{T} \to \mathcal{T} + c$

$$\dot{\mathcal{T}} = \int_\Sigma d^3\mathbf{x}\,\partial_t\left(\sqrt{-g}\,W^t(t,\mathbf{x})\right) = \int_\Sigma d^3\mathbf{x}\left[\sqrt{-g} - \partial_i\cancel{\left(\sqrt{-g}\,W^i(t,\mathbf{x})\right)}\right]$$

invariant - four volume of space time $\mathcal{T}\left(t_2\right) - \mathcal{T}\left(t_1\right) = \int_{t_1}^{t_2} dt\int d^3\mathbf{x}\sqrt{-g}$

Four-volume of spacetime is canonically conjugated to the cosmological constant

Heisenberg uncertainty relation

$$\delta\Lambda \times \delta \int_{\Omega} d^4x \sqrt{-g} \geq 4\pi \ell_{Pl}^2$$

Energy-Time Uncertainty Relation

$$\delta\varepsilon_\lambda \times \delta\mathscr{V} \geq \frac{\hbar}{2}$$

$$\varepsilon_\lambda = \frac{\Lambda}{8\pi G_N}$$

for collapsing radiation dominated closed universe

The total four volume $\mathscr{V}_{tot} = \frac{3}{4}\pi^3 a_m^4$ at least $\delta\mathscr{V} \leq \mathscr{V}_{tot}$ ⇒ $\varepsilon_\Lambda \geq \frac{\hbar}{2\mathscr{V}_{tot}}$

Global Solution to Global Problem!

actually $\delta\varepsilon_\lambda \simeq \varepsilon_r$ close to final singularity,
dS avoids "end of time"!

W^μ is a rather unusual field,
is there something more familiar,
e.g. a gauge potential / connection A_μ
(e.g. for SU(N))?

Use Chern-Simons Current C^{μ} instead of W^{μ} !

Axionic Cosmological Constant

$$S\left[g, A, \Lambda\right] = \int d^4x\sqrt{-g}\left[-\frac{1}{2}R\left(g\right) + \Lambda\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta} - 1\right)\right]$$

Hammer, Jiroušek, Vikman arXiv:2001.03169

Chern-Simons Current

$$C^{\alpha} = \mathrm{Tr}\frac{\varepsilon^{\alpha\beta\gamma\delta}}{\sqrt{-g}}\left(\boldsymbol{F}_{\beta\gamma}\boldsymbol{A}_{\delta} - \frac{2}{3}ig\boldsymbol{A}_{\beta}\boldsymbol{A}_{\gamma}\boldsymbol{A}_{\delta}\right)$$

composite vector variable, yet C^t does not depend on $\partial_t A_\mu$!

$$\nabla_{\alpha}C^{\alpha} = F_{\alpha\beta}\widetilde{F}^{\alpha\beta} \qquad \widetilde{F}^{\alpha\beta} = \frac{1}{2}\cdot\frac{\epsilon^{\alpha\beta\mu\nu}}{\sqrt{-h}}\cdot F_{\mu\nu}$$

gauge transformations $\quad \boldsymbol{A}_{\mu} \to U\boldsymbol{A}_{\mu}U^{-1} + \frac{i}{g}\left(\partial_{\mu}U\right)U^{-1}$

introduce the shifts $\quad C^{\mu} \to C^{\mu} + \epsilon^{\mu} \qquad \nabla_{\mu}\epsilon^{\mu} = 0$

Axionic Cosmological Constant, Comment I

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Cosmological Constant and Fundamental Length

In usual formulations of general relativity, the cosmological constant Λ appears as an inelegant ambiguity in the fundamental action principle. With a slight reformulation, Λ appears as an unavoidable Lagrange multiplier, belonging to a constraint. The constraint expresses the existence of a fundamental element of space–time hypervolume at every point. The fundamental scale of length in atomic physics provides such a hypervolume element. In this sense, the presence in relativity of an undetermined cosmological length is a direct consequence of the existence of a fundamental atomic length.

Maybe this fundamental scale is a confinement scale of a Yang-Mills theory/QCD?

$$S\left[g,A,\Lambda\right]=\int d^4x\sqrt{-g}\left[-\frac{1}{2}R\left(g\right)+\Lambda\left(F_{\alpha\beta}\widetilde{F}^{\alpha\beta}-1\right)\right]$$

Hammer, Jiroušek, Vikman arXiv:2001.03169

cf. Zhitnisky talk at this workshop

Frozen Axion for Λ

Canonically normalised θ instead of Λ

$$S\left[g, A, \theta\right] = \int d^4x\sqrt{-g}\left(-\frac{R}{2\varkappa} + \frac{1}{2}\left(\partial\theta\right)^2 + \frac{\theta}{f_\Lambda}F_{\alpha\beta}F^{\star\alpha\beta} - V_\lambda\left(\theta\right) \quad - \frac{1}{4\mathbf{g}^2}F_{\alpha\beta}F^{\alpha\beta}\right)$$

formal limit / “confinement” $\mathbf{g} \rightarrow \infty$

$$\varkappa = 8\pi G_N$$

New vacuum energy density $V_\lambda\left(\theta\right)$

G_N as global dynamical degree of freedom

Second way to lose the trace

Side product: $\hbar$ as global dynamical degree of freedom

1=1 instead of 0=0!

$$\frac{G_{\mu\nu}}{G} = \frac{T_{\mu\nu}}{T}$$

Jiroušek, Shimada, Vikman, Yamaguchi (2020)

Bianchi identity
+energy-momentum conservation
+non-degeneracy of $T_{\mu\nu}$ (say it contains a small Λ)

$$\partial_\mu \log G/T = 0 \quad \Rightarrow \quad G = 8\pi \overline{G}_N T$$

$\overline{G}_N$ is merely an integration constant!

$$\frac{G_{\mu\nu}}{G} = \frac{T_{\mu\nu}}{T}$$

Jiroušek, Shimada, Vikman, Yamaguchi (2020)

invariant under *rescaling* of the energy-momentum tensor

$$T_{\mu\nu} \to \beta\, T_{\mu\nu}$$

invariant under *rescaling* of the Einstein tensor

$$G_{\mu\nu} \to \alpha\, G_{\mu\nu}$$

two distinct ways to write an action

Changing Gravity: Henneaux–Teitelboim analogy for G_N

$$S_{G_N}\left[g, C, \alpha\right] = \frac{1}{2}\int d^4x\sqrt{-g}\,\left(\nabla_\mu C^\mu - R\right)\alpha$$

cf. Kaloper, Padilla, Stefanyszyn, Zahariade (2016)

global shift-symmetric degree of freedom

canonically conjugated to $\alpha = M_{pl}^2$

$$\mathscr{R}\left(t\right) = \frac{1}{2}\int_\Sigma d^3\mathbf{x}\sqrt{-g}\,C^t\left(t, \mathbf{x}\right)$$

$$\dot{\mathscr{R}} = \frac{1}{2}\int_\Sigma d^3\mathbf{x}\partial_t\left(\sqrt{-g}\,C^t\left(t, \mathbf{x}\right)\right) = \frac{1}{2}\int_\Sigma d^3\mathbf{x}\left[\sqrt{-g}R - \cancel{\partial_i\left(\sqrt{-g}\,C^i\left(t, \mathbf{x}\right)\right)}\right]$$

$$\mathscr{R}\left(t_2\right) - \mathscr{R}\left(t_1\right) = \frac{1}{2}\int_{\mathscr{V}} d^4x\sqrt{-g}\,R$$ integrated Ricci scalar

Heisenberg uncertainty relation

$$\frac{\delta \ell_{Pl}}{\ell_{Pl}} \times \frac{\delta \int_{\mathscr{V}} d^4x \sqrt{-g}\, R}{\ell_{Pl}^2} \geq 4\pi$$

Frozen Axion for G_N

Canonically normalised ν instead of α

$$S\left[g, \mathscr{A}, \nu\right] = \int d^4x\sqrt{-g}\left[-\frac{1}{2}\nu^2 R + \frac{1}{2}(\partial\nu)^2 + \frac{\nu}{f_\alpha}\mathscr{F}_{\gamma\sigma}\mathscr{F}^{\star\gamma\sigma} - V_\alpha(\nu) \qquad - \frac{1}{4\mathbf{g}^2}\mathscr{F}_{\gamma\sigma}\mathscr{F}^{\gamma\sigma}\right]$$

Again formal limit / “confinement” $\mathbf{g} \to \infty$

Changing Matter via Henneaux–Teitelboim

$$S\left[g,\beta,L,\Phi_m\right] = \int d^4x\sqrt{-g}\,\beta\left(\mathscr{L}_m - \nabla_\lambda L^\lambda\right)$$

Momentum rescaling $\pi = \beta\pi^{(m)} = \beta\sqrt{-g}\dfrac{\partial\mathscr{L}_m}{\partial\dot{\phi}}$ $\Rightarrow$ $\left[\phi\left(\mathbf{x}\right),\pi^{(m)}\left(\mathbf{y}\right)\right] = \dfrac{i\hbar}{\beta}\delta\left(\mathbf{x}-\mathbf{y}\right)$

Effective Planck Quanta $\bar{\hbar} = \dfrac{\hbar}{\beta}$ Effective Newton Constant $\overline{G}_N = \beta\, G_N$

Planck length $\ell_{Pl} = \sqrt{\hbar G_N}$ remains invariant

Heisenberg uncertainty relation

$$\delta\bar{\hbar} \times \delta \int_{\mathscr{V}} d^4x \sqrt{-g}\, \mathscr{L}_m \geq \frac{1}{2}\bar{\hbar}^2$$

Frozen Axion for $\hbar$ and G_N

Canonically normalised η instead of β

$$S\left[g,\eta,A,\Phi_m\right] = \int d^4x\sqrt{-g}\left[-\frac{R}{2\varkappa}+\frac{1}{2}\left(\partial\eta\right)^2 - V_\beta\left(\eta\right)+\frac{\eta^2}{M_m^2}\mathscr{L}_m - \frac{\eta}{f_\beta}F_{\mu\nu}F^{\star\mu\nu} - \frac{1}{4\mathbf{g}^2}F_{\alpha\beta}F^{\alpha\beta}\right]$$

formal limit / "confinement"

$$\varkappa = 8\pi G_N$$

Unimodular, Unicurvature and Unimatter

for the globally dynamical Λ

Unimodular $$S[g, \Lambda] = \int d^4x \, \Lambda \left(1 - \sqrt{-g}\right)$$

Henneaux-Teitelboim construction with fixed $W^\mu = \delta^\mu_t \frac{t}{\sqrt{-g}}$

similarly one can write for the globally dynamical $\hbar$ and G_N

Unimatter $$S[g, \beta, \Phi_m] = \int d^4x \, \beta \left(\sqrt{-g}\mathcal{L}_m - 1\right)$$

Can be changed to $\pm$ as in cosmology $R = 3p - \varepsilon$

Unicurvature $$S[g, \alpha] = \frac{1}{2}\int d^4x \left(1 - \sqrt{-g}R\right)\alpha$$

each of these three constraints could be a gauge condition… but they yield dynamics!

Thanks a lot for attention!