Towards a ghost-free theory of quantum gravity

Luca Buoninfante

In collaboration with
A.S. Koshelev, G. Lambiase, J. Marto,
A. Mazumdar, M. Yamaguchi,…

Quantum Gravity and Cosmology
(dedicated to A.D. Sakharov’s centennial)
4°-8° June 2021

東京工業大学
Tokyo Institute of Technology

JSPS

Unitarity in nonlocal quantum field theories

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4th order gravity

- The 4th order gravitational action quadratic in the curvature is power-counting renormalizable:

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\left(\mathcal{R} + \alpha\mathcal{R}^2 + \beta\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}\right)$$

- Unitarity is violated at the tree-level:

$$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{8\pi G}\,h_{\mu\nu}$$

$$\Pi(k) = \Pi_{GR} + \frac{1}{2}\frac{\mathcal{P}^0}{k^2 + m_0^2} - \frac{\mathcal{P}^2}{k^2 + m_2^2},$$

$$m_0 := (3\alpha + \beta)^{-1/2}$$

$$m_2 := \left(-\tfrac{1}{2}\beta\right)^{-1/2}$$

Spin-2 ghost degree of freedom

- **Conflict: Unitarity VS Renormalizability!**

[Stelle, 1977, PRD]

Unitarity VS Renormalizability

- Einstein's GR is **unitary but non-renormalizable**, while 4th order quadratic gravity is power-counting **renormalizable but non-unitary!**

Several (recent) attempts:

- Asymptotically safe gravity [Reuter, Wetterich, Eichhorn, Saueressig, Platania,......]
- 4th order gravity with Fakeons [Anselmi & Piva 2017+]
- 4th order gravity with unstable ghosts [Donoghue, Menezes, Salvio, Strumia…]
- Lee-Wick gravity theories [Modesto & Shapiro 2016+; Anselmi & Piva 2017+]
- **Nonlocal gravity theories** [Born, Pais, Yukawa, Efimov, Krasnikov, Kuz'min, Moffat, Woodard, Tomboulis, Dragovich, Aref'eva, Volovich, Koshelev, Siegel, Biswas, Mazumdar, Modesto, Frolov, Zelnikov, Rachwal, Starobinsky, Kumar, Tokareva, Boos,......]

Unitarity VS Renormalizability

- Einstein's GR is **unitary but non-renormalizable**, while 4th order quadratic gravity is power-counting **renormalizable but non-unitary!**

Several (recent) attempts:

- Asymptotically safe gravity [**Saueressig's yesterday talk**]

- **Nonlocal gravity theories** [**Koshelev's and Kumar's yesterday talks**]

Ghosts

- 4-derivative theory (-+++):

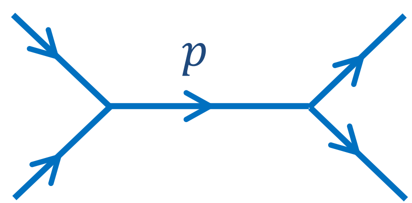
GHOST!

$$\mathcal{L} = \frac{1}{2}\phi\Box\left(1 - \frac{\Box}{m^2}\right)\phi - V(\phi) \quad \Longrightarrow \quad i\Pi(p) = \frac{1}{p^2 - i\epsilon} - \frac{1}{p^2 + m^2 - i\epsilon}$$

- Optical theorem:

$$S^+S = 1, \qquad S = 1 + iT \qquad \Longrightarrow \qquad 2Im\{T\} = T^+T \quad (\text{"}{\geq}0\text{"})$$

- Tree-level amplitude:



$$Im\{T\} = \pi\theta(p^0)[\delta(p^2) - \delta(p^2 + m^2)]$$

Non-positive definite: **violation of unitarity!**

Beyond 4-derivative theories

- 4-derivative theory (-+++):

GHOST!

$$\mathcal{L} = \frac{1}{2}\phi\Box\left(1 - \frac{\Box}{m^2}\right)\phi - V(\phi) \Longrightarrow \quad i\Pi(p) = \frac{1}{p^2 - i\epsilon} - \frac{1}{p^2 + m^2 - i\epsilon}$$

- Generalized higher-derivative theory:

$$\mathcal{L} = \frac{1}{2}\phi F(\Box)(\Box - m^2)\phi \quad \Longrightarrow \quad i\Pi(p) = \frac{1}{F(-p^2)}\frac{1}{p^2 + m^2}$$

- **Question:** Is there any higher-derivative operator $F(-p^2)$ such that the propagator is ghost-free? **YES!**

- **Nonlocality can help us!**

Local VS Nonlocal

- Local (polynomial) Lagrangians:

$$\mathcal{L}_L \equiv \mathcal{L}_L(\phi, \partial\phi, \partial^2\phi, \dots, \partial^n\phi)$$

- Nonlocal (non-polynomial) Lagrangians:

$$\mathcal{L}_{NL} \equiv \mathcal{L}_{NL}\left(\phi, \partial\phi, \partial^2\phi, \dots, \partial^n\phi, \dots, e^{\Box}\phi, \ln(\Box)\,\phi, \frac{1}{\Box}\phi, \dots\right)$$

Local VS Nonlocal

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e.g. string field theory and p-adic string

[Witten, Freund, Zwiebach, Aref'eva, Volovich, Dragovich, Koshelev, Sen, Siegel,....]

Generalized higher-derivative Lagrangian

- Scalar field Lagrangian:

$$\mathcal{L} = -\frac{1}{2}\phi F(\Box)\phi - V(\phi),$$

Entire function
(good IR limit $F(\Box) \to -\Box + m^2$)

- Weierstrass' theorem:

$$\boxed{F(\Box) = e^{-\gamma(\Box)} \prod_{i=1}^{N} \left(-\Box + m_i^2\right)^{r_i}, \qquad N \leq \infty,}$$

- $\gamma(\Box)$ is another entire function.

- N is the number of zeroes m_i^2; r_i is the multiplicity of each zero

Generalized higher-derivative Lagrangian

- Scalar field Lagrangian:

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Entire function

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- Weierstrass' theorem:

$$\boxed{F(\Box) = e^{-\gamma(\Box)} \prod_{i=1}^{N} \left(-\Box + m_i^2\right)^{r_i}, \qquad N \leq \infty,}$$

- Propagator:

$$i\Pi(-p^2) = \frac{e^{\gamma(-p^2)}}{p^2 + m^2} \prod_{i=2}^{N} \frac{1}{\left(p^2 + m_i^2\right)^{r_i}}$$

Generalized higher-derivative Lagrangians

$$\boxed{F(\Box) = e^{-\gamma(\Box)} \prod_{i=1}^{N} \left(-\Box + m_i^2\right)^{r_i}}$$

- $N = 1,\ \ r_i = 1,\ \gamma(\Box) = 0 \implies$ 2-derivative theory (Klein-Gordon)

$$F(\Box) = -\Box + m^2$$

- $N = 2,\ \ r_i = 1,\ \gamma(\Box) = 0 \implies$ 4-derivative theory with ghost

$$F(\Box) = (-\Box + m^2)\left(1 - \frac{\Box}{M^2}\right)$$

- $N \geq 2$ and/or $r_i \geq 2$ (with m_i real) $\implies$ ghosts!

Generalized higher-derivative Lagrangians

$$\boxed{F(\Box) = e^{-\gamma(\Box)} \prod_{i=1}^{N} \left(-\Box + m_i^2\right)^{r_i}}$$

- $N = 1,\ r_i = 1,\ \gamma(\Box) \neq 0$

$\Longrightarrow$ infinite-derivative theory with one real zero

$$F(\Box) = e^{-\gamma(\Box)}(-\Box + m^2)$$

- Propagator:

$$i\Pi(p) = \frac{e^{\gamma(-p^2)}}{p^2 + m^2 - i\epsilon}$$

Generalized higher-derivative Lagrangians

$$\boxed{F(\Box) = e^{-\gamma(\Box)} \prod_{i=1}^{N} \left(-\Box + m_i^2\right)^{r_i}}$$

- $N = 3,\ r_i = 1,\ \gamma(\Box) \neq 0$ [local case $\gamma(\Box) = 0$: Lee & Wick; Modesto & Shapiro 2016+; Anselmi & Piva 2017+]

$\Longrightarrow$ infinite-derivative theory with a pair of complex conjugate zeroes

$$F(\Box) = \frac{i}{M^4} e^{-\gamma(\Box)}(-\Box + m^2)(\Box + iM^2)(\Box - iM^2)$$
$$= e^{-\gamma(\Box)}(-\Box + m^2)\left(1 + \frac{\Box^2}{M^4}\right)$$

- Propagator:

$$i\Pi(p) = \frac{M^4 e^{\gamma(-p^2)}}{m^4 + M^4}\left[\frac{1}{p^2 + m^2 - i\epsilon} + \frac{p^2 + m^2}{p^4 + M^4}\right]$$

Generalized higher-derivative Lagrangians

$$\boxed{F(\Box) = e^{-\gamma(\Box)} \prod_{i=1}^{N} \left(-\Box + m_i^2\right)^{r_i}}$$

- $N = \infty, \;\; r_i = 1, \;\; \gamma(\Box) = \frac{\Box}{M_s^2}$

$\Longrightarrow$ infinite-derivative theory

with infinite pairs of complex conjugate zeroes

$$F(\Box) = M_s^2\left(e^{-\Box/M_s^2} - 1\right)$$

$$= -e^{-\gamma(\Box)}\Box \prod_{\ell=1}^{\infty}\left(\frac{\Box}{i2\pi M_s^2 \ell} + 1\right)\left(\frac{\Box}{i2\pi M_s^2 \ell} - 1\right)$$

- Propagator:

$$i\Pi(p) = \frac{1}{M_s^2\left(e^{p^2/M_s^2} - 1\right)} = \frac{1}{p^2 - i\epsilon} - \frac{1}{2M_s^2} + 2p^2 \sum_{\ell=1}^{\infty} (-1)^{\ell}\left(\frac{1}{p^4 + 4\pi^2 M_s^4 \ell^2}\right)$$

[LB, Lambiase, Yamaguchi PRD]

Perturbative unitarity

$$S^+S = 1, \qquad S = 1 + iT \qquad \Longrightarrow \qquad i(T^+ - T) = T^+T$$

$$i[\langle b|T^+|a\rangle - \langle b|T|a\rangle] = \sum_n \langle b|T^+|n\rangle\langle n|T|a\rangle$$

$$\langle b|T|a\rangle = (2\pi)^4\delta^{(4)}(P_b - P_a)\langle b|\mathcal{M}|a\rangle$$

$$\boxed{\begin{aligned} &i[\langle b|\mathcal{M}^+|a\rangle - \langle b|\mathcal{M}|a\rangle] \\ &= \sum_{\{n\}}\prod_{l=1}^{n}\int \frac{d^3k_l}{(2\pi)^3}\frac{1}{2\omega_l}\,(2\pi)^4\,\delta^{(4)}\left(P_a - \sum_{l=1}^{n} k_l\right)\langle b|\mathcal{M}^+|\{k_l\}\rangle\langle\{k_l\}|\mathcal{M}|a\rangle \end{aligned}}$$

Perturbative unitarity

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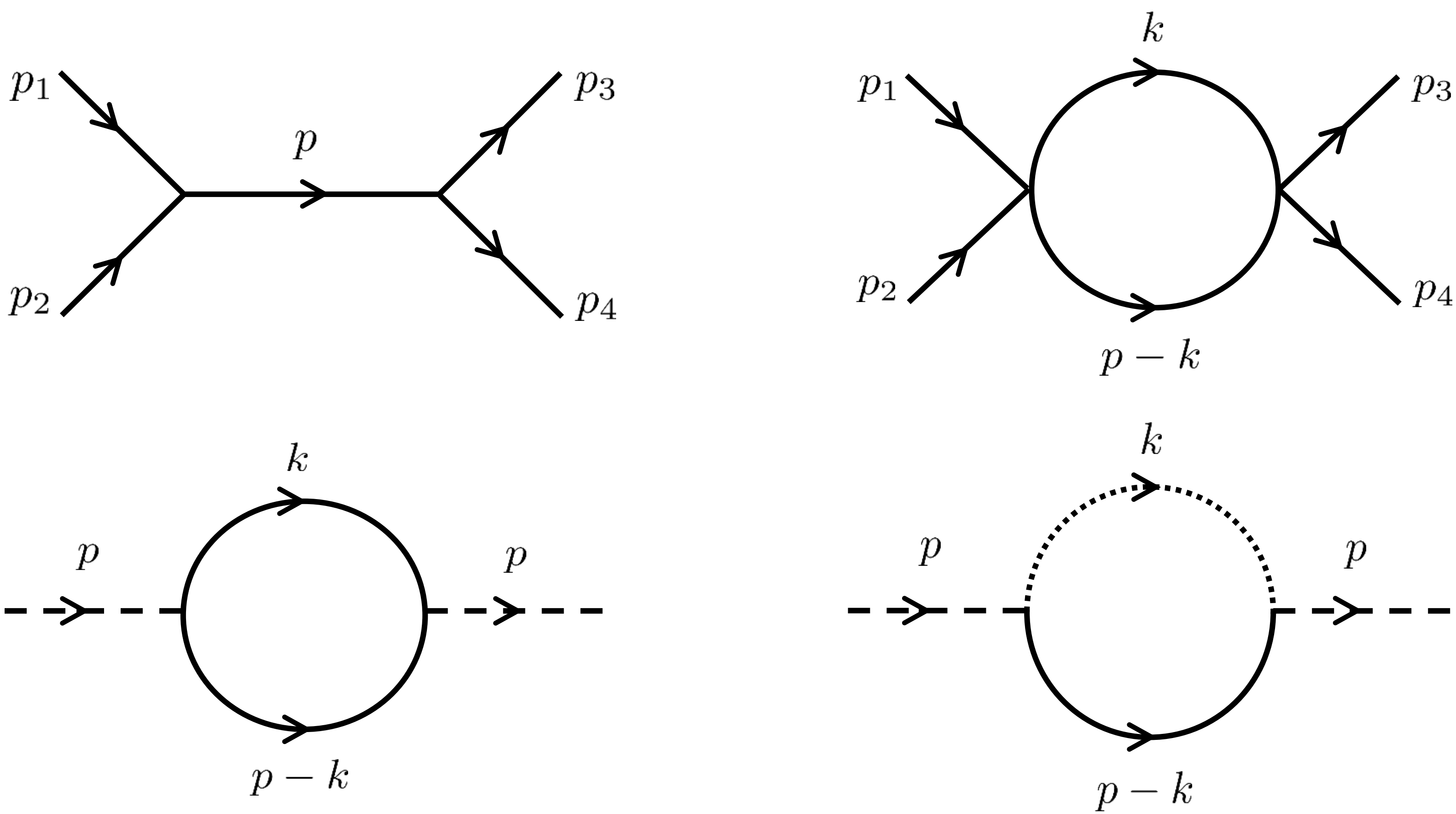
$$\langle b|T|a\rangle = (2\pi)^4\delta^{(4)}(P_b - P_a)\langle b|\mathcal{M}|a\rangle$$

$$\overbrace{i[\langle b|\mathcal{M}^+|a\rangle - \langle b|\mathcal{M}|a\rangle]}^{\text{LHS}}$$

$$\underbrace{= \sum_{\{n\}} \prod_{l=1}^{n} \int \frac{d^3k_l}{(2\pi)^3}\frac{1}{2\omega_l}\,(2\pi)^4\,\delta^{(4)}\left(P_a - \sum_{l=1}^{n} k_l\right)\langle b|\mathcal{M}^+|\{k_l\}\rangle\langle\{k_l\}|\mathcal{M}|a\rangle}_{\text{RHS}}$$

Perturbative unitarity

$$\mathcal{L} = \frac{1}{2}\phi F(\Box)\phi - \frac{\lambda}{3!}\phi^3 - \frac{g}{4!}\phi^4 + \kappa\,\phi\psi^2 + \cdots$$



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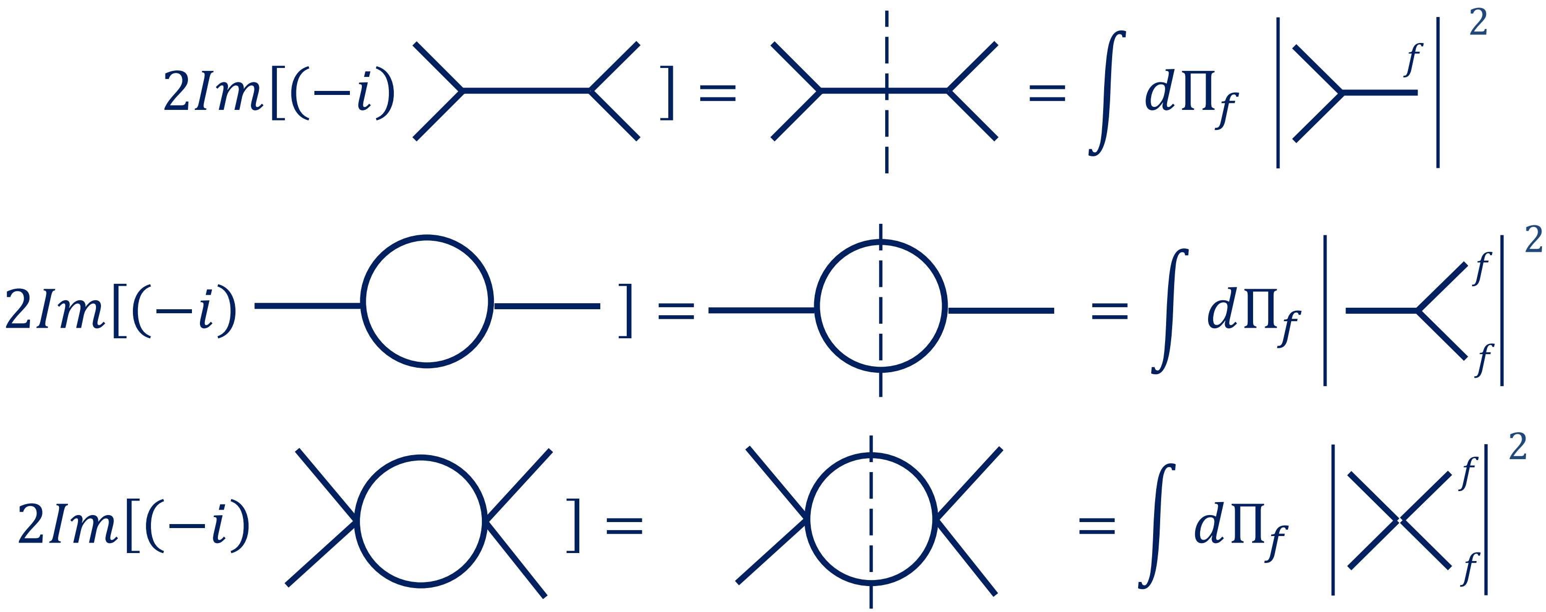
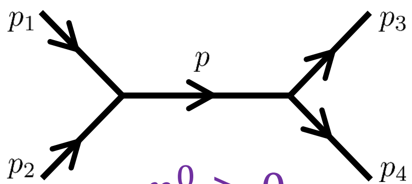


Tree-level: nonlocality + real poles

$$\mathcal{L} = \frac{1}{2}\phi F(\Box)\phi - \frac{\lambda}{3!}\phi^3 - \frac{g}{4!}\phi^4 + \kappa\,\phi\psi^2 + \cdots$$



$p^0 > 0$

$$F(\Box) = e^{-\gamma(\Box)}(-\Box + m^2), \qquad i\Pi(p) = \frac{e^{\gamma(-p^2)}}{p^2 + m^2 - i\epsilon}, \qquad \gamma(m^2) = 0$$

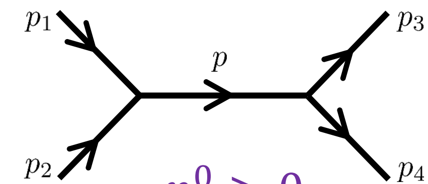
$$LHS = i[\langle p_3, p_4|\mathcal{M}^+|p_1, p_2\rangle - \langle p_3, p_4|\mathcal{M}|p_1, p_2\rangle]$$

$$= i\lambda^2 e^{\gamma(-p^2)}\left[\frac{1}{p^2 + m^2 + i\epsilon} - \frac{1}{p^2 + m^2 - i\epsilon}\right]$$

$$= 2\pi\lambda^2\theta(p^0)\delta(p^2 + m^2)$$

Tree-level: nonlocality + real poles

$$\mathcal{L} = \frac{1}{2}\phi F(\Box)\phi - \frac{\lambda}{3!}\phi^3 - \frac{g}{4!}\phi^4 + \kappa\,\phi\psi^2 + \cdots$$



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$$F(\Box) = e^{-\gamma(\Box)}(-\Box + m^2), \qquad i\Pi(p) = \frac{e^{\gamma(-p^2)}}{p^2 + m^2 - i\epsilon}, \qquad \gamma(m^2) = 0$$

$$RHS = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega}(2\pi)^4\,\delta^{(4)}(p-k)\langle p_3, p_4|\mathcal{M}^+|k\,\rangle\langle k\,|\mathcal{M}|p_1, p_2\rangle$$

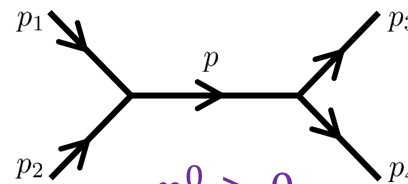
$$= 2\pi\lambda^2\theta(p^0)\delta(p^2 + m^2)$$

$$= LHS$$

Tree-level unitarity!

Tree-level: nonlocality + complex conjugate poles

$$\mathcal{L} = \frac{1}{2}\phi F(\Box)\phi - \frac{\lambda}{3!}\phi^3 - \frac{g}{4!}\phi^4 + \kappa\,\phi\psi^2 + \cdots$$



$p^0 > 0$

$$F(\Box) = e^{-\gamma(\Box)}(-\Box + m^2)\left(1 + \frac{\Box^2}{M^4}\right), \qquad i\Pi(p) = \frac{M^4 e^{\gamma(-p^2)}}{m^4 + M^4}\left[\frac{1}{p^2 + m^2 - i\epsilon} + \frac{p^2 + m^2}{p^4 + M^4}\right]$$

$[\gamma(m^2) = 0]$

$$LHS = i[\langle p_3, p_4|\mathcal{M}^+|p_1, p_2\rangle - \langle p_3, p_4|\mathcal{M}|p_1, p_2\rangle]$$

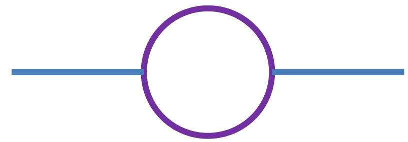
$$= i\lambda^2 \frac{M^4 e^{\gamma(-p^2)}}{m^4 + M^4}\left[\frac{1}{p^2 + m^2 + i\epsilon} - \frac{1}{p^2 + m^2 - i\epsilon}\right]$$

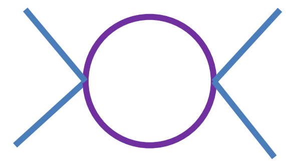
$$= 2\pi\lambda^2 \frac{M^4}{m^4 + M^4}\theta(p^0)\delta(p^2 + m^2)$$

Perturbative unitarity

- So far, we have shown tree-level unitarity

- What about loops?





- Unitarity of nonlocal theories with standard poles

  [Sen & Pius 2015; Carone 2017; Briscese & Modesto 2018; Chin & Tomboulis 2018; Koshelev & Tokareva 2021]

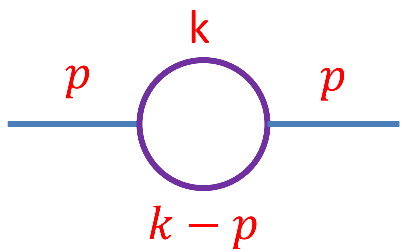
- Unitarity of nonlocal theories with complex conjugate poles

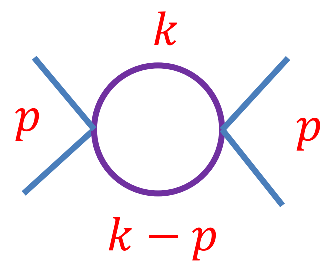
  [LB, Yamaguchi – arXiv:21XX.XXXXX]

- We only consider one-loop bubble diagrams

$$\mathcal{L} = \frac{1}{2}\phi F(\Box)\phi - \frac{\lambda}{3!}\phi^3 - \frac{g}{4!}\phi^4 + \kappa\, \phi\psi^2 + \cdots$$

Bubble diagram: local 2-derivative case





$$\mathcal{M}(p) \equiv \langle p|\mathcal{M}|p\rangle = (-i)\lambda^2 \int_{\mathcal{C}} \frac{dk^0}{2\pi} \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2+m^2-i\epsilon} \frac{1}{(k-p)^2+m^2-i\epsilon}$$

$$Q_1 = -\omega_k + i\epsilon$$

$$Q_2 = p^0 - \omega_{k-p} + i\epsilon$$

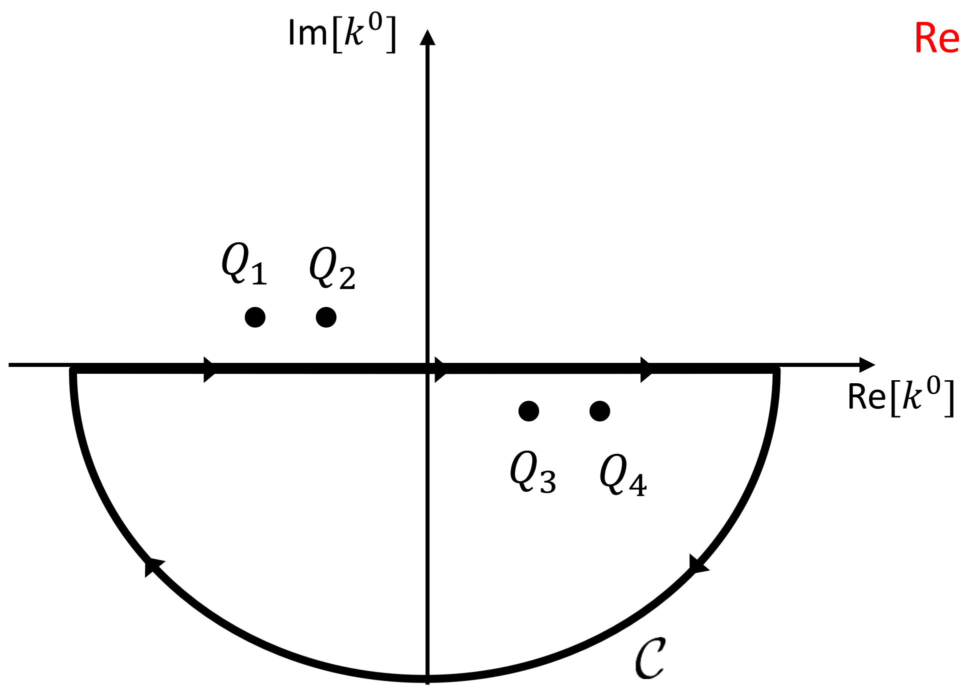
$$Q_3 = \omega_k - i\epsilon$$

$$Q_4 = p^0 + \omega_{k-p} - i\epsilon$$

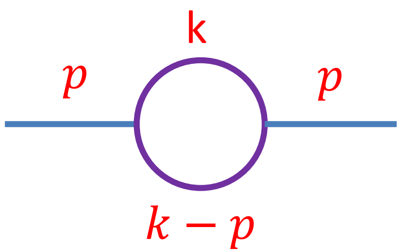
$$\omega_k = \sqrt{\vec{k}^2 + m^2}$$

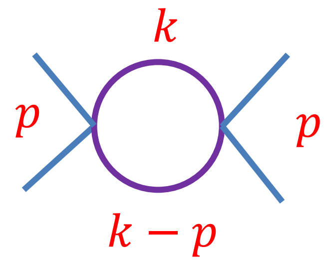
$$\omega_{k-p} = \sqrt{(\vec{k}-\vec{p})^2 + m^2}$$

$\mathrm{Re}(p^0) > 0$



Bubble diagram: local 2-derivative case





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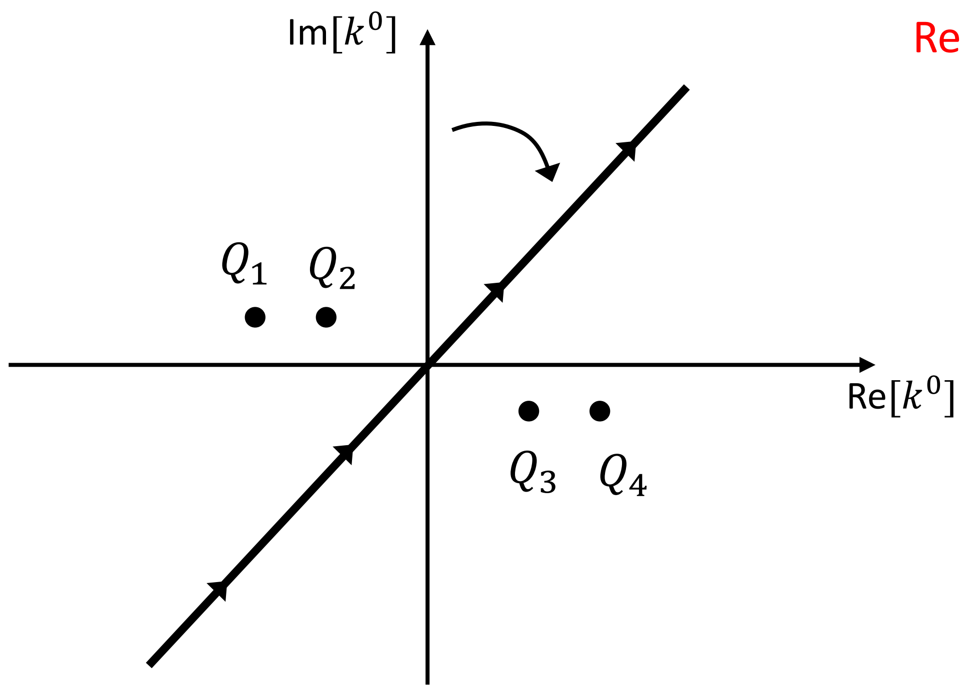
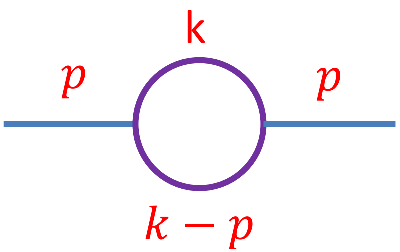
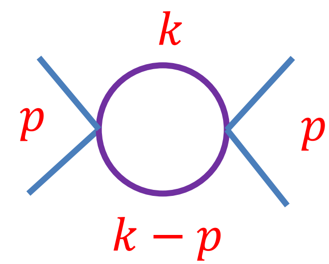


$\mathrm{Re}(p^0) > 0$

Bubble diagram: local 2-derivative case





$\mathrm{Re}(p^0) > 0$

$$\mathcal{M}(p) = \lambda^2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k 2\omega_{k-p}} \left[\frac{1}{p^0 - \omega_k - \omega_{k-p} + i\epsilon} - \frac{1}{p^0 + \omega_k + \omega_{k-p}} \right]$$

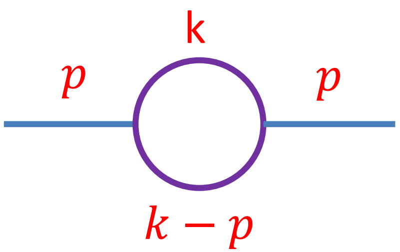
$$\mathrm{L}HS = 2Im\{\mathcal{M}(p)\}$$

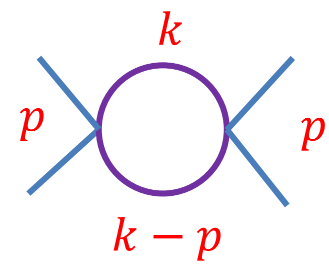
$$= 2\pi\lambda^2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k 2\omega_{k-p}} \theta(p^0 - \omega_{k-p})\delta(p^0 - \omega_k - \omega_{k-p}) = RHS$$

Optical theorem (unitarity) is satisfied!

$$\boxed{\frac{1}{x \pm \epsilon} = P.V.\left(\frac{1}{x}\right) \mp i\pi\delta(x)}$$

Bubble diagram: nonlocal with real poles

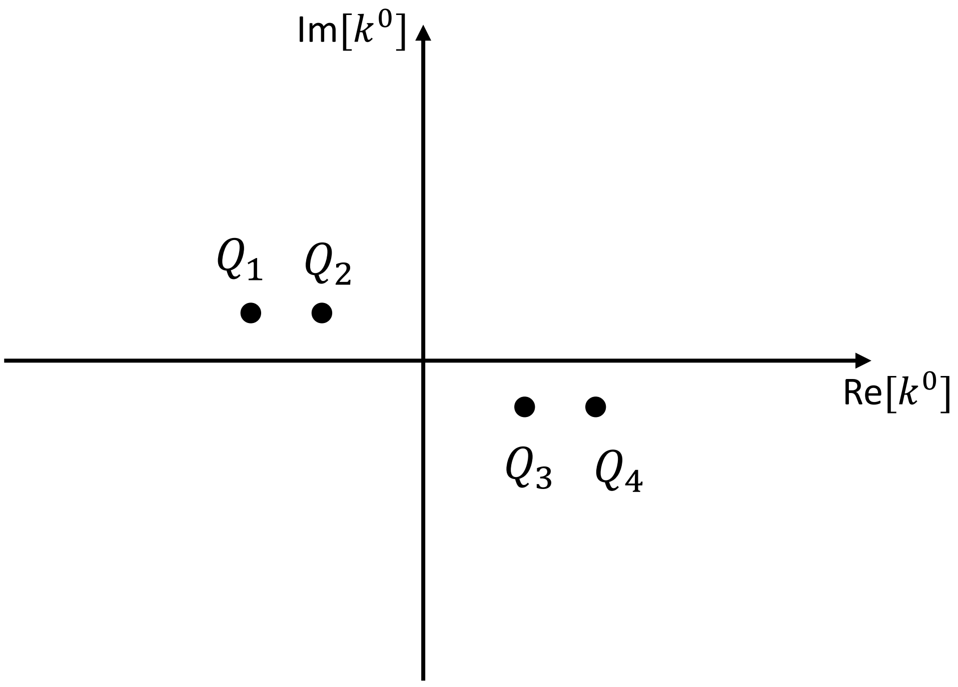




$\mathrm{Re}(p^0) > 0$

$$\mathcal{M}(p) \equiv \langle p|\mathcal{M}|p\rangle = (-i)\lambda^2 \int_{\mathcal{C}} \frac{dk^0}{2\pi} \int \frac{d^3k}{(2\pi)^3} \frac{e^{\gamma(-k^2)}}{k^2+m^2-i\epsilon} \frac{e^{\gamma(-(k-p)^2)}}{(k-p)^2+m^2-i\epsilon}$$

- Liouville's theorem implies the presence of singularities at infinity!

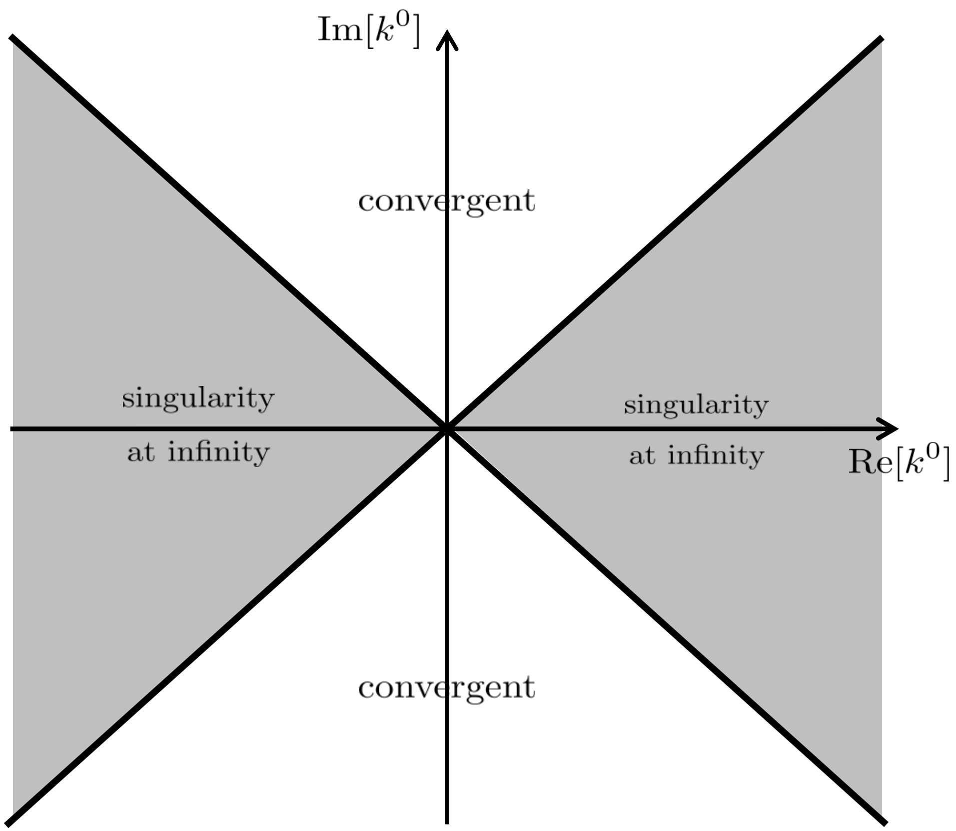


Singularities at infinity

- Example of singularity at infinity: $\gamma(-k^2) = -k^2 = (k^0)^2 - \vec{k}^2$

$$k^0 = \kappa e^{i\vartheta}, \qquad \kappa \in \mathbb{R}_+$$

$$e^{-k^2} = e^{(k^0)^2 - \vec{k}^2} = e^{-\vec{k}^2} e^{i\,\kappa^2 \sin 2\vartheta} e^{\kappa^2 \cos 2\vartheta}$$



Contour prescription

- In local two-derivative theories starting from Minkowski is equivalent to starting from Euclidean
- In nonlocal theories starting from Minkowski is NOT well-defined because of divergencies at infinity

---

- Define the contour $\mathcal{C}$ to be the imaginary k^0 −axis
- Complexify internal and external energies: $k^0 \in \mathbb{C}, \quad p^0 \in \mathbb{C}$
- To avoid poles and pinchings deform the contour in finite-distance region of the complex plane by keeping the ends fixed at $\pm i\infty$
- Analitically continue external energies to real values

- Nonlocal theories + real poles [Sen & Pius 2015]
- Nonlocal theories + complex conjugate poles [LB, Yamaguchi - arXiv:21XX.XXXXX]

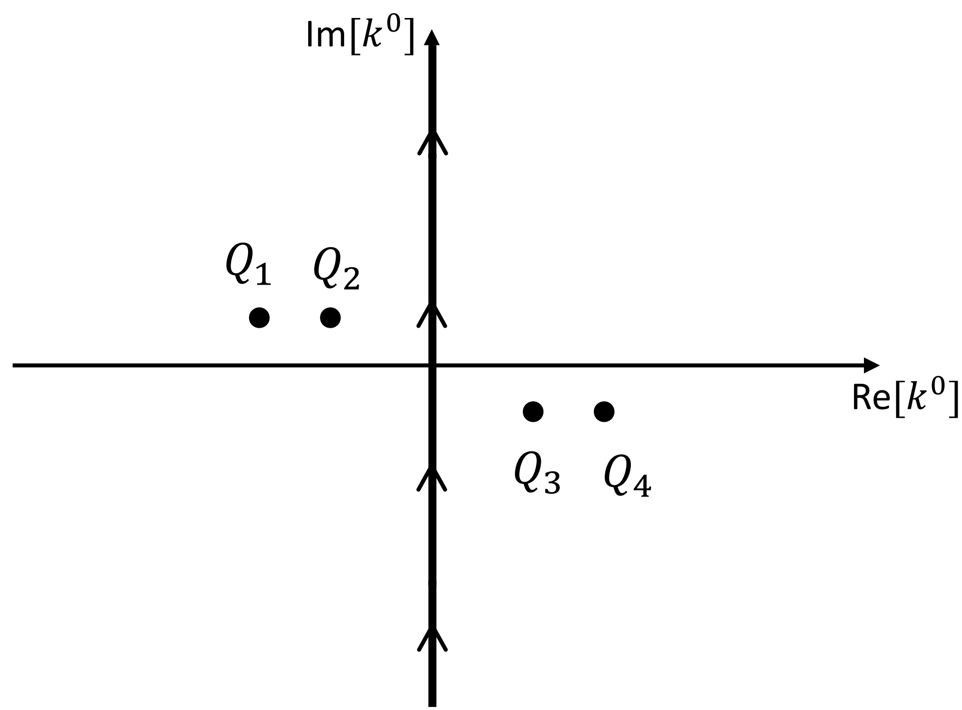
Bubble diagram: nonlocal with real poles

$\mathrm{Re}(p^0) > 0$

$$\mathcal{M}(p) \equiv \langle p|\mathcal{M}|p\rangle = (-i)\lambda^2 \int_{\mathcal{C}} \frac{dk^0}{2\pi} \int \frac{d^3k}{(2\pi)^3} \frac{e^{\gamma(-k^2)}}{k^2+m^2-i\epsilon} \frac{e^{\gamma(-(k-p)^2)}}{(k-p)^2+m^2-i\epsilon}$$

- Pinching:

$$Q_2 = Q_3 \quad \Leftrightarrow \quad p^0 = \omega_k + \omega_{k-p}$$

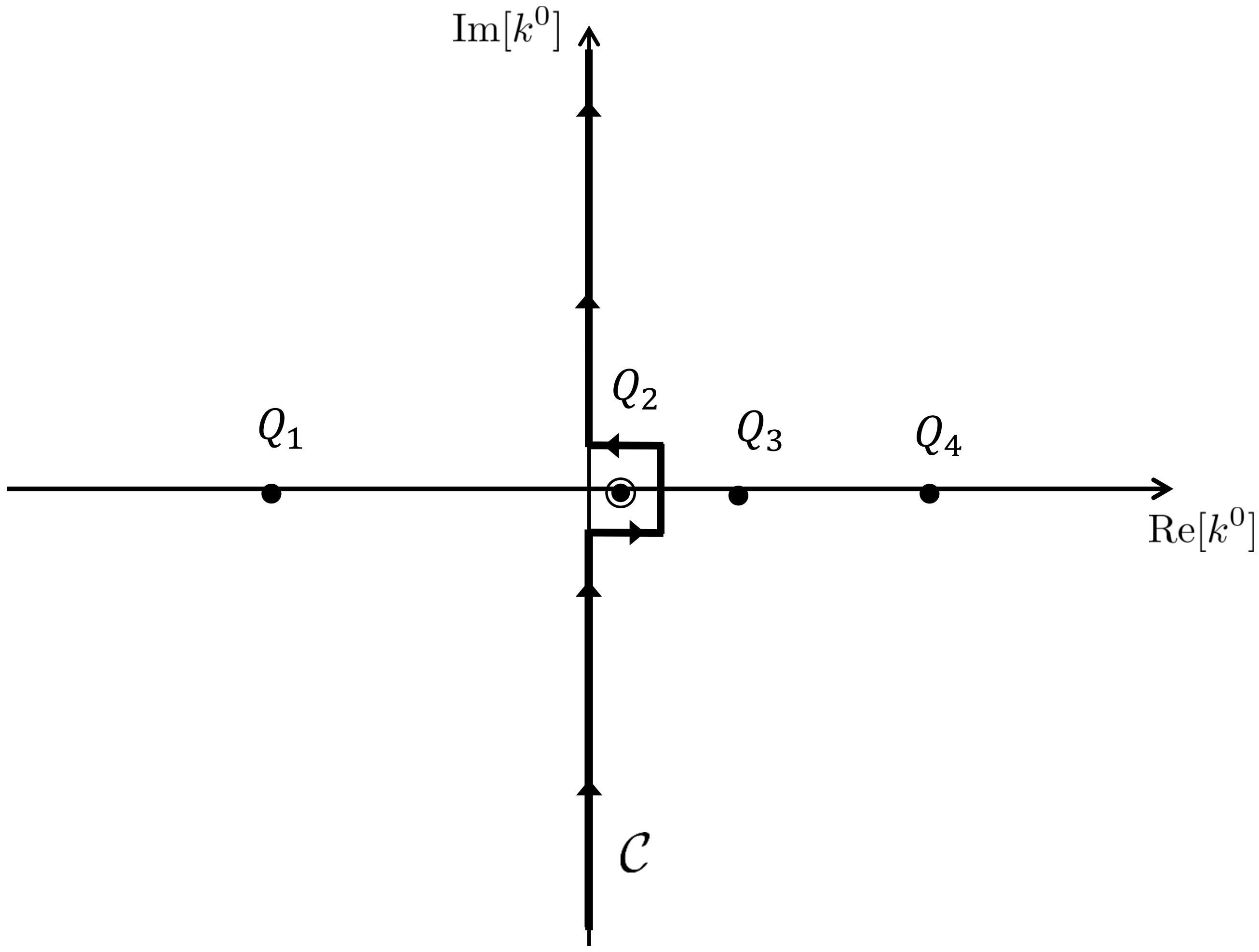


Bubble diagram: nonlocal with real poles

- Region I:

$\mathrm{Re}(p^0) > 0$

$$Re[Q_2] < Re[Q_3] \quad \Leftrightarrow \quad p^0 < \omega_k + \omega_{k-p}$$

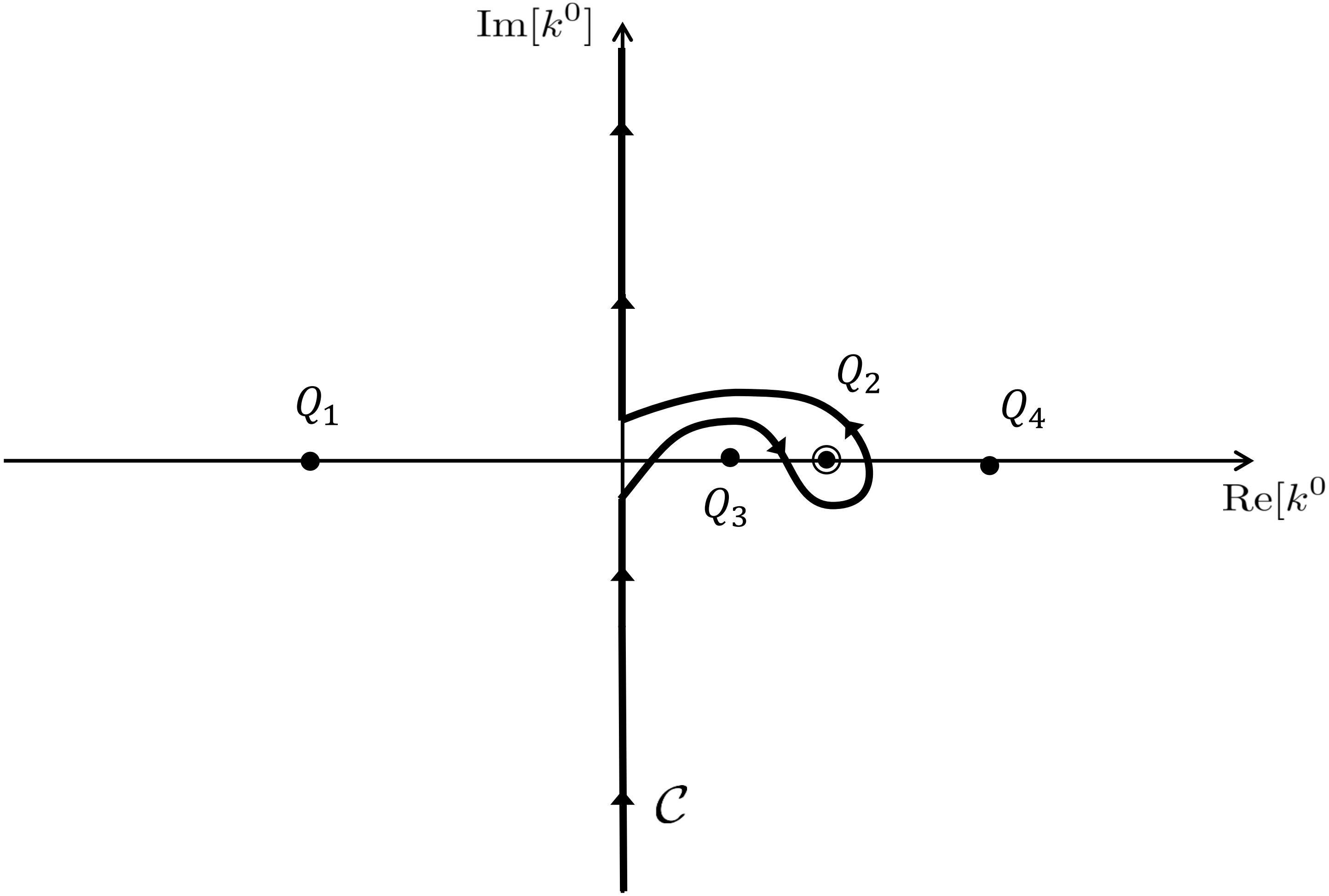


Bubble diagram: nonlocal with real poles

- Region II:

$\mathrm{Re}(p^0) > 0$

$$Re[Q_2] > Re[Q_3] \quad \Leftrightarrow \quad p^0 > \omega_k + \omega_{k-p}$$

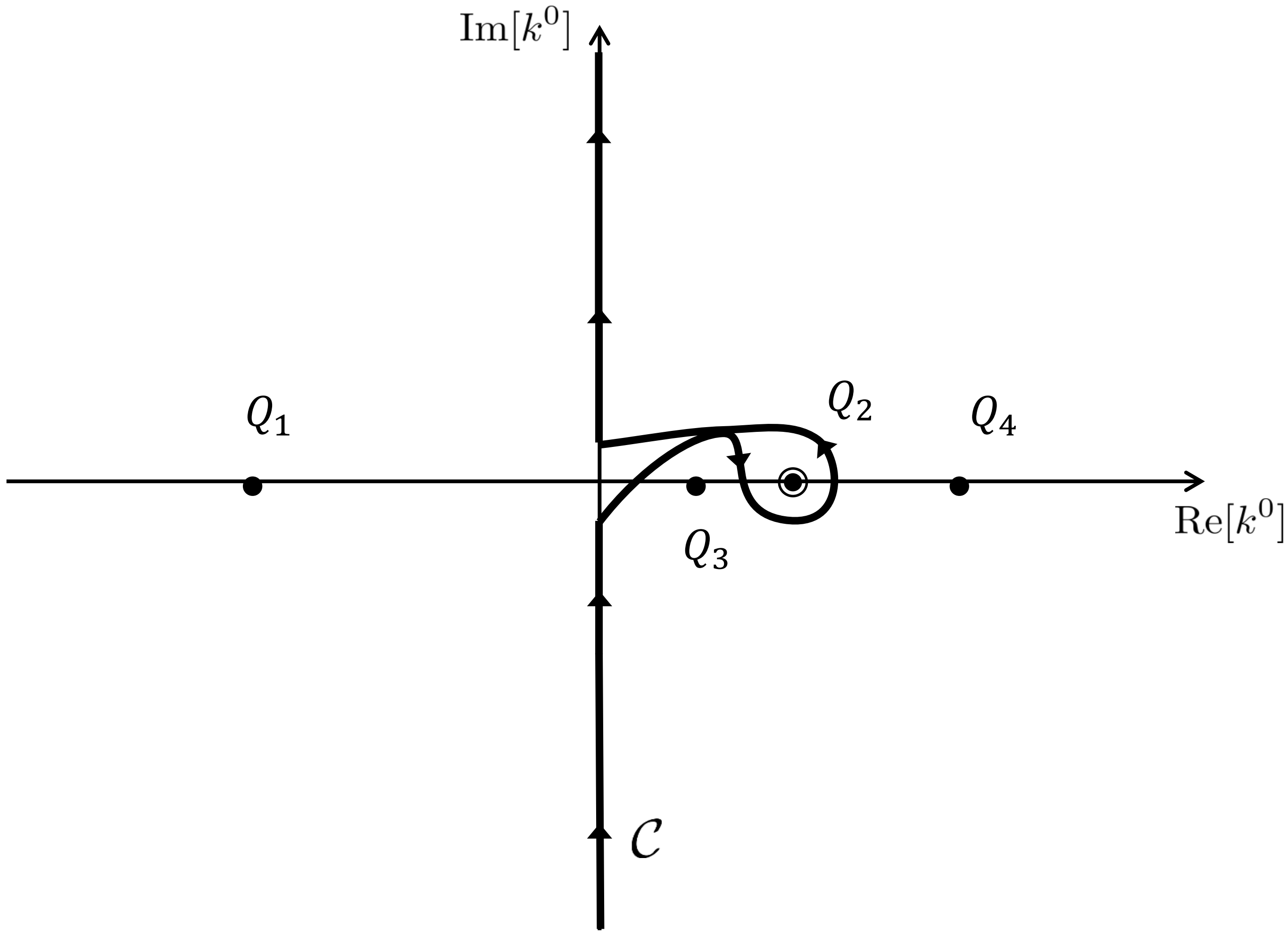


Bubble diagram: nonlocal with real poles

- Region II:

$\mathrm{Re}(p^0) > 0$

$Re[Q_2] > Re[Q_3] \quad \Leftrightarrow \quad p^0 > \omega_k + \omega_{k-p}$

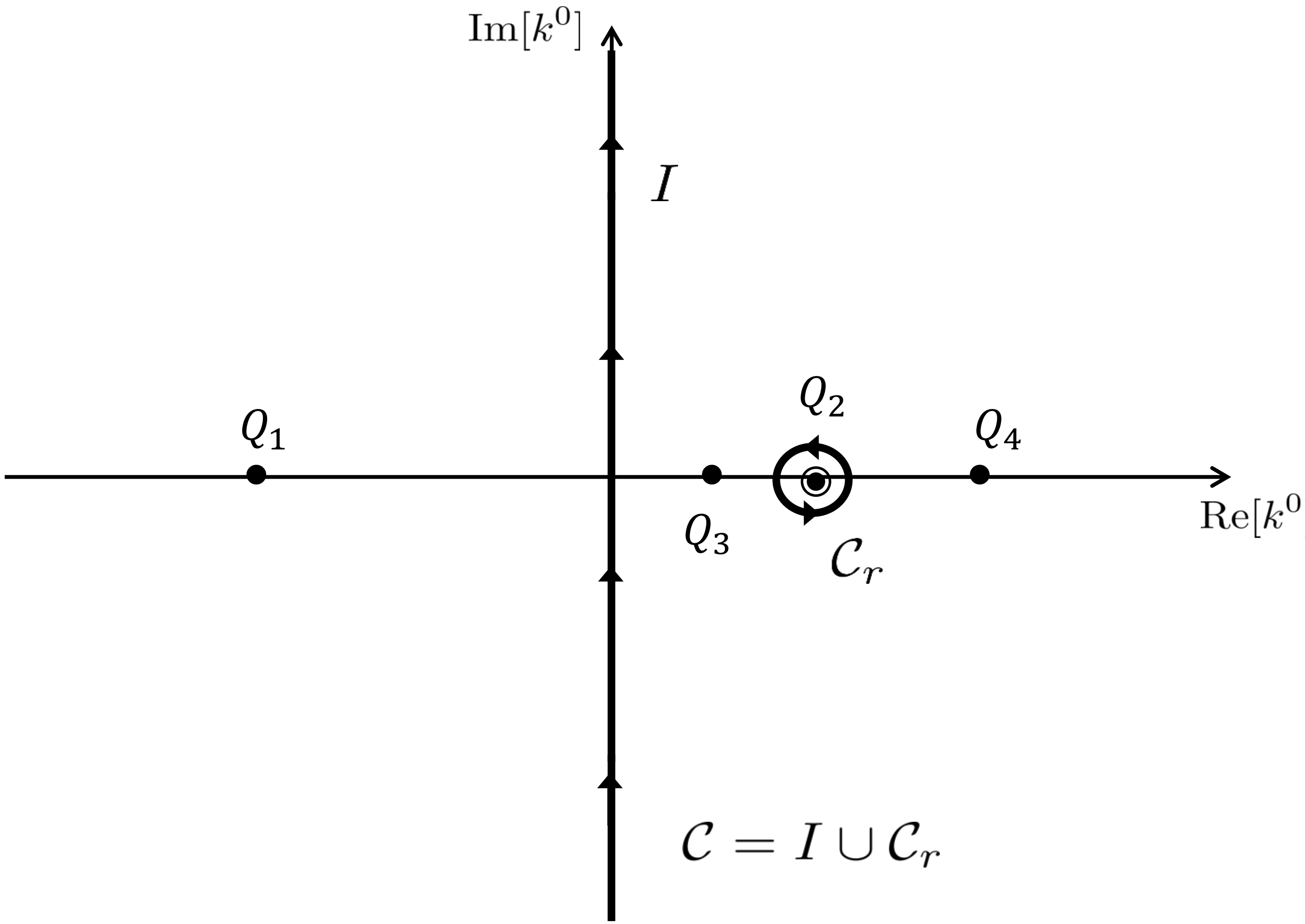


Bubble diagram: nonlocal with real poles

- Region II:

$\mathrm{Re}(p^0) > 0$

$$Re[Q_2] > Re[Q_3] \quad \Leftrightarrow \quad p^0 > \omega_k + \omega_{k-p}$$

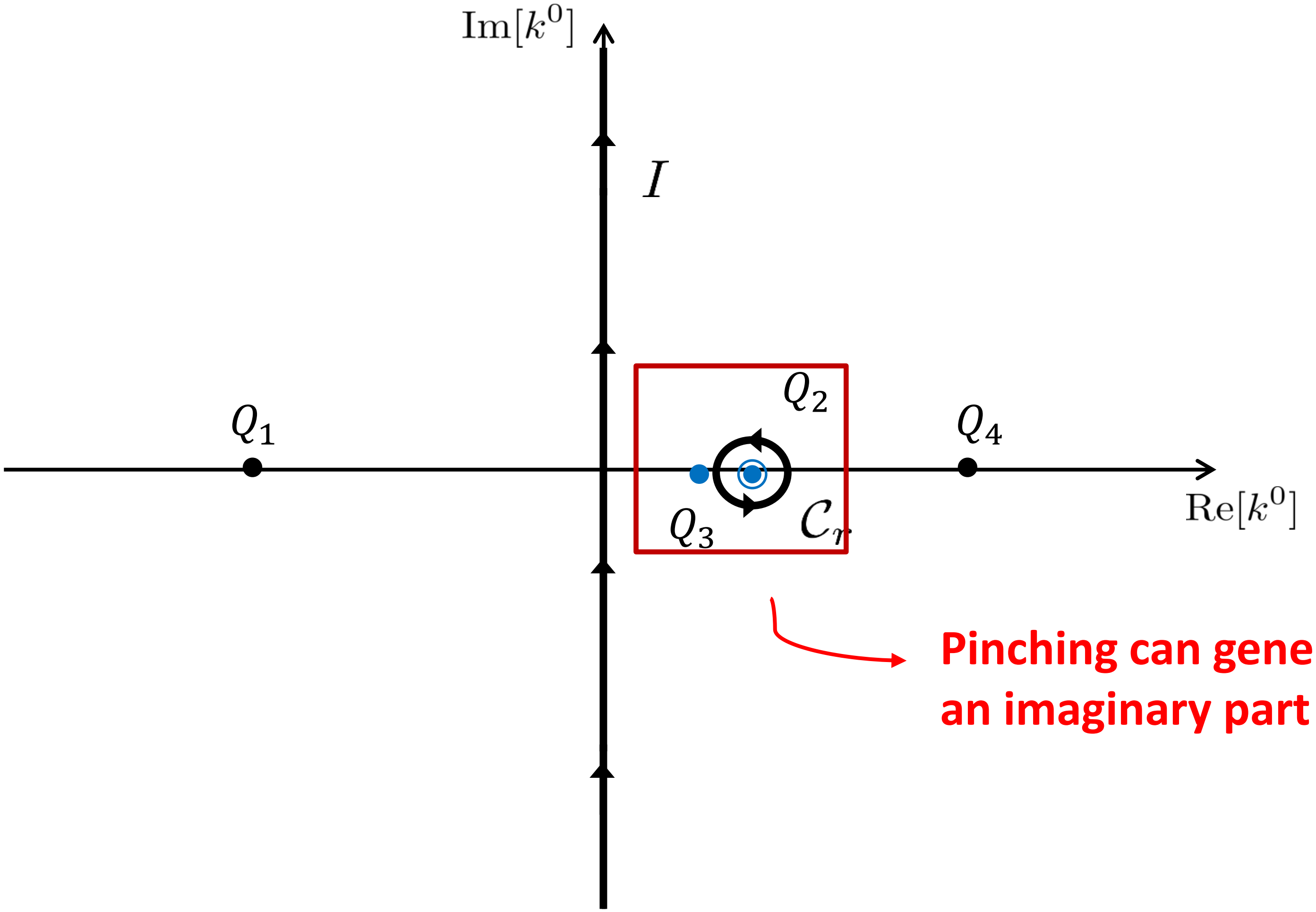


Bubble diagram: nonlocal with real poles

- Region III: close to the pinching

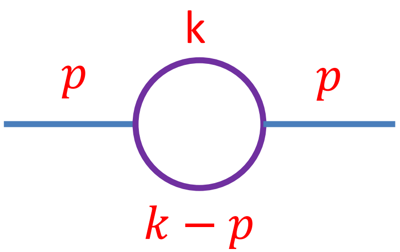
$\mathrm{Re}(p^0) > 0$

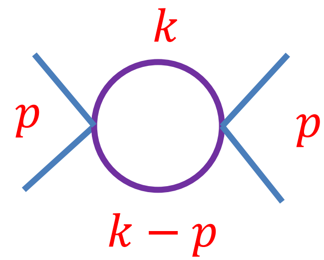
$$Re[Q_2] \approx Re[Q_3] \quad \Leftrightarrow \quad p^0 \approx \omega_k + \omega_{k-p}$$



Pinching can generate an imaginary part!

Bubble diagram: nonlocal with real poles





$\mathrm{Re}(p^0) > 0$

$$\mathcal{M}(p) = (-i)\lambda^2 \int_{I\cup\mathcal{C}_r} \frac{dk^0}{2\pi} \int \frac{d^3k}{(2\pi)^3} \frac{e^{\gamma(-k^2)}}{k^2+m^2-i\epsilon} \frac{e^{\gamma(-(k-p)^2)}}{(k-p)^2+m^2-i\epsilon}$$

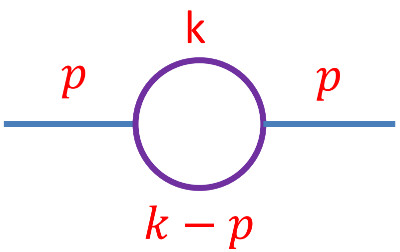
$$= \mathcal{M}_I(p) + \mathcal{M}_{\mathcal{C}_r}(p)$$

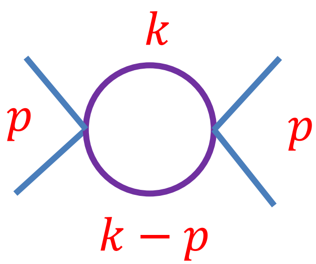
Gives an imaginary part!

$$LHS = 2Im\{\mathcal{M}(p)\} = 2\pi\lambda^2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k 2\omega_{k-p}} \theta(p^0 - \omega_{k-p})\delta(p^0 - \omega_k - \omega_{k-p})$$

Same Cutkosky rules of
local two-derivative case!

Bubble diagram: nonlocal with real poles





$$RHS = \int \frac{d^3k_2}{(2\pi)^3} \int \frac{d^3k_2}{(2\pi)^3} \frac{(2\pi)^4}{2\omega_1 2\omega_2} \delta^{(4)}(p-k_1-k_2)\langle p\,|\mathcal{M}^+|k_1,k_2\rangle\langle k_1,k_2|\mathcal{M}|p\,\rangle$$

$$= 2\pi\lambda^2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k 2\omega_{k-p}} \theta(p^0-\omega_{k-p})\delta(p^0-\omega_k-\omega_{k-p})$$

$$= LHS$$

Optical theorem
(unitarity) is satisfied!

Bubble diagram: nonlocal case + complex poles

$\mathrm{Re}(p^0) > 0$

$$\mathcal{M}(p) = (-i)\lambda^2 M^8 \int_{\mathcal{C}} \frac{dk^0}{2\pi} \int \frac{d^3k}{(2\pi)^3} \frac{e^{\gamma(-k^2)}}{(k^2+m^2-i\epsilon)(k^4+M^4)} \frac{e^{\gamma(-(k-p)^2)}}{[(k-p)^2+m^2-i\epsilon][(k-p)^4+M^4]}$$

$Q_5 = -\Omega_k \qquad Q_6 = p^0 - \Omega_{k-p}$

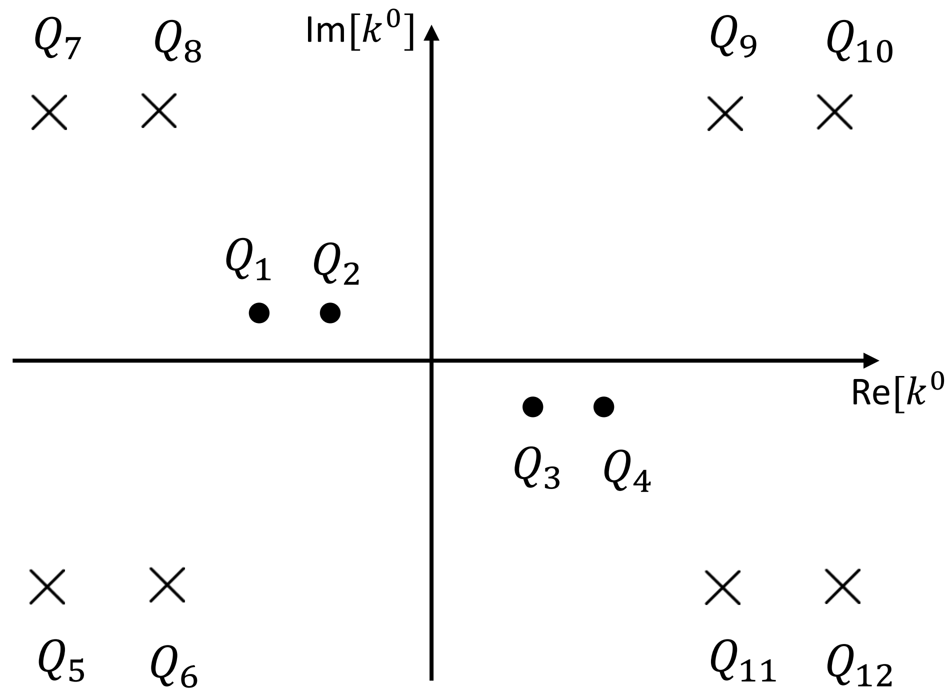
$Q_7 = -{\Omega_k}^* \qquad Q_8 = p^0 - {\Omega_{k-p}}^*$

$Q_9 = \Omega_k \qquad Q_{10} = p^0 + \Omega_{k-p}$

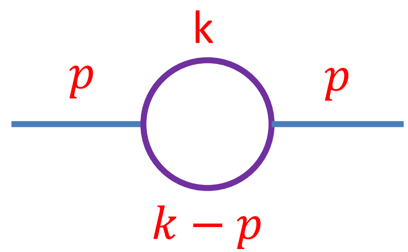
$Q_{11} = {\Omega_k}^* \qquad Q_{12} = p^0 + {\Omega_{k-p}}^*$

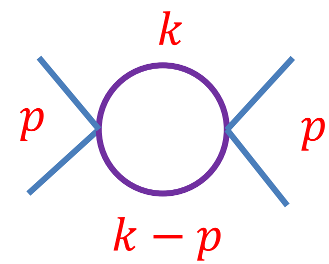
$$\Omega_k = \sqrt{\vec{k}^2 + iM^2}$$

$$\Omega_{k-p} = \sqrt{(\vec{k}-\vec{p})^2 + iM^2}$$



Bubble diagram: nonlocal case + complex poles





$\mathrm{Re}(p^0) > 0$

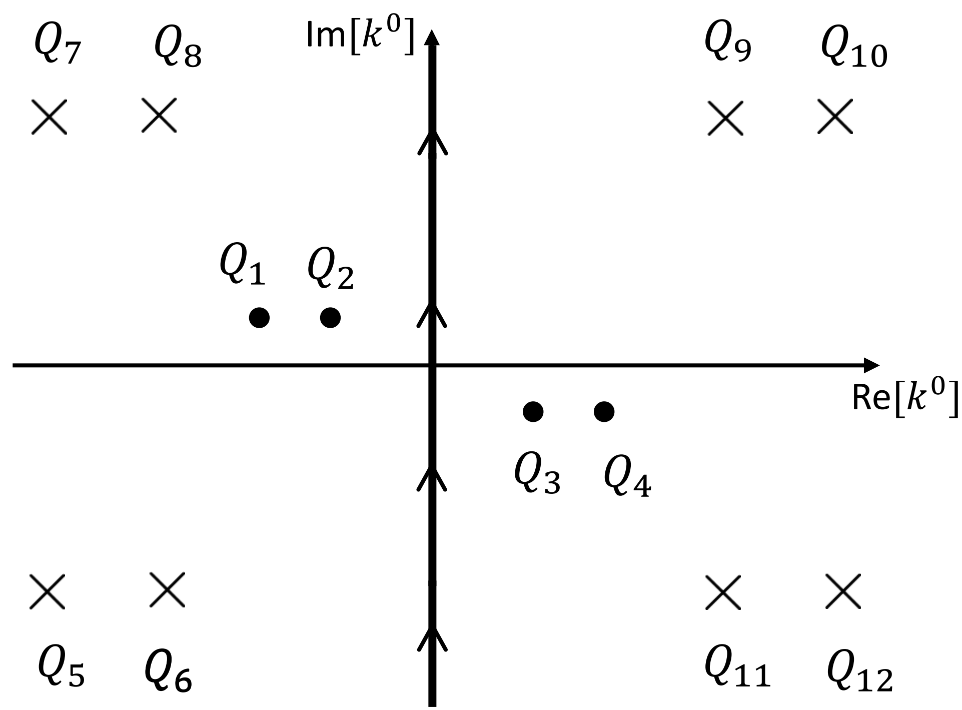
$$\mathcal{M}(p) = (-i)\lambda^2 M^8 \int_{\mathcal{C}} \frac{dk^0}{2\pi} \int \frac{d^3k}{(2\pi)^3} \frac{e^{\gamma(-k^2)}}{(k^2+m^2-i\epsilon)(k^4+M^4)} \frac{e^{\gamma(-(k-p)^2)}}{[(k-p)^2+m^2-i\epsilon][(k-p)^4+M^4]}$$

- Pinchings with real and positive external momenta:

$$Q_2 = Q_3 \quad \Leftrightarrow \quad p^0 = \omega_k + \omega_{k-p}$$

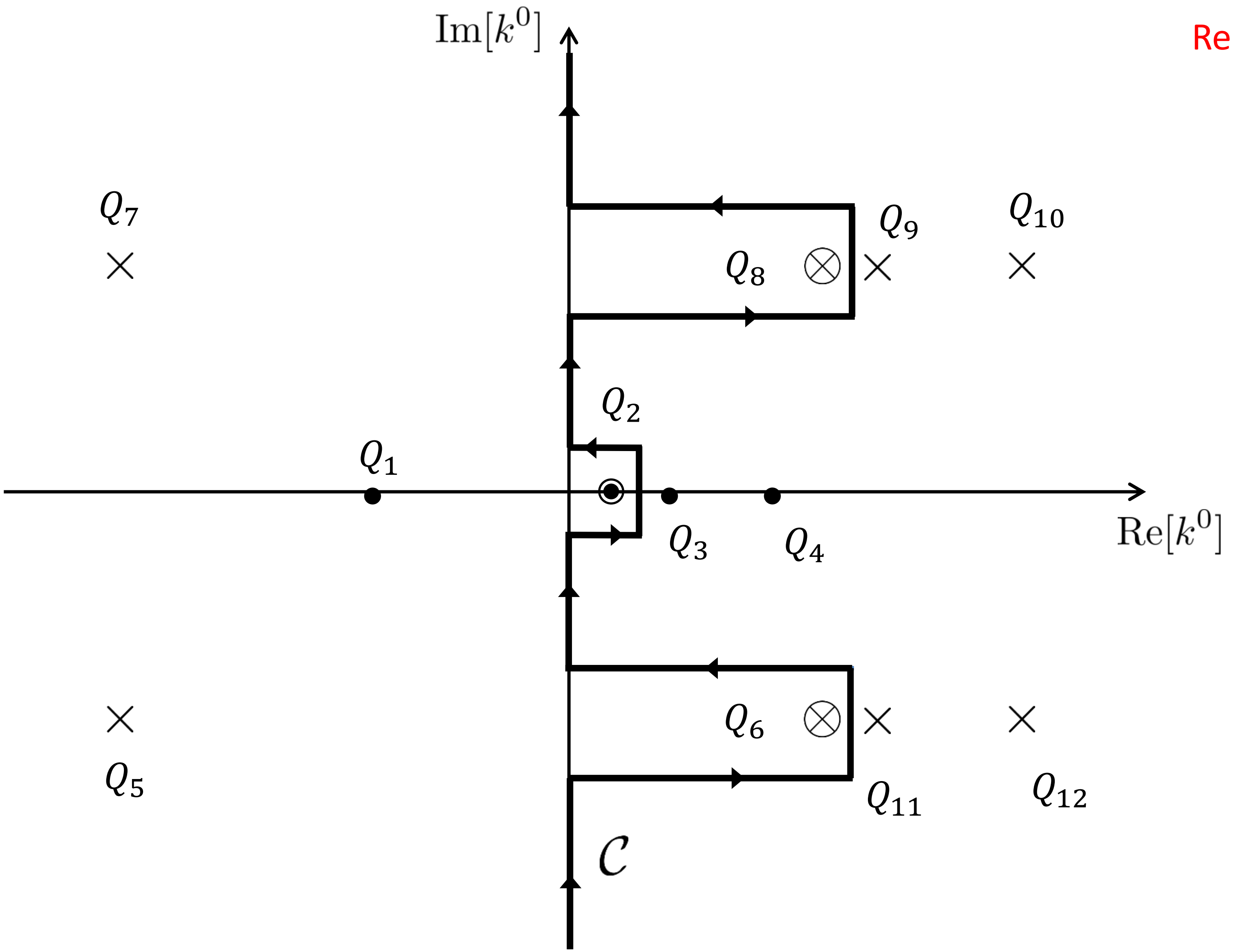
$$Q_8 = Q_9 \quad \Leftrightarrow \quad p^0 = \Omega_k{}^* + \Omega_{k-p}$$

$$Q_6 = Q_{11} \quad \Leftrightarrow \quad p^0 = \Omega_k + \Omega_{k-p}{}^*$$

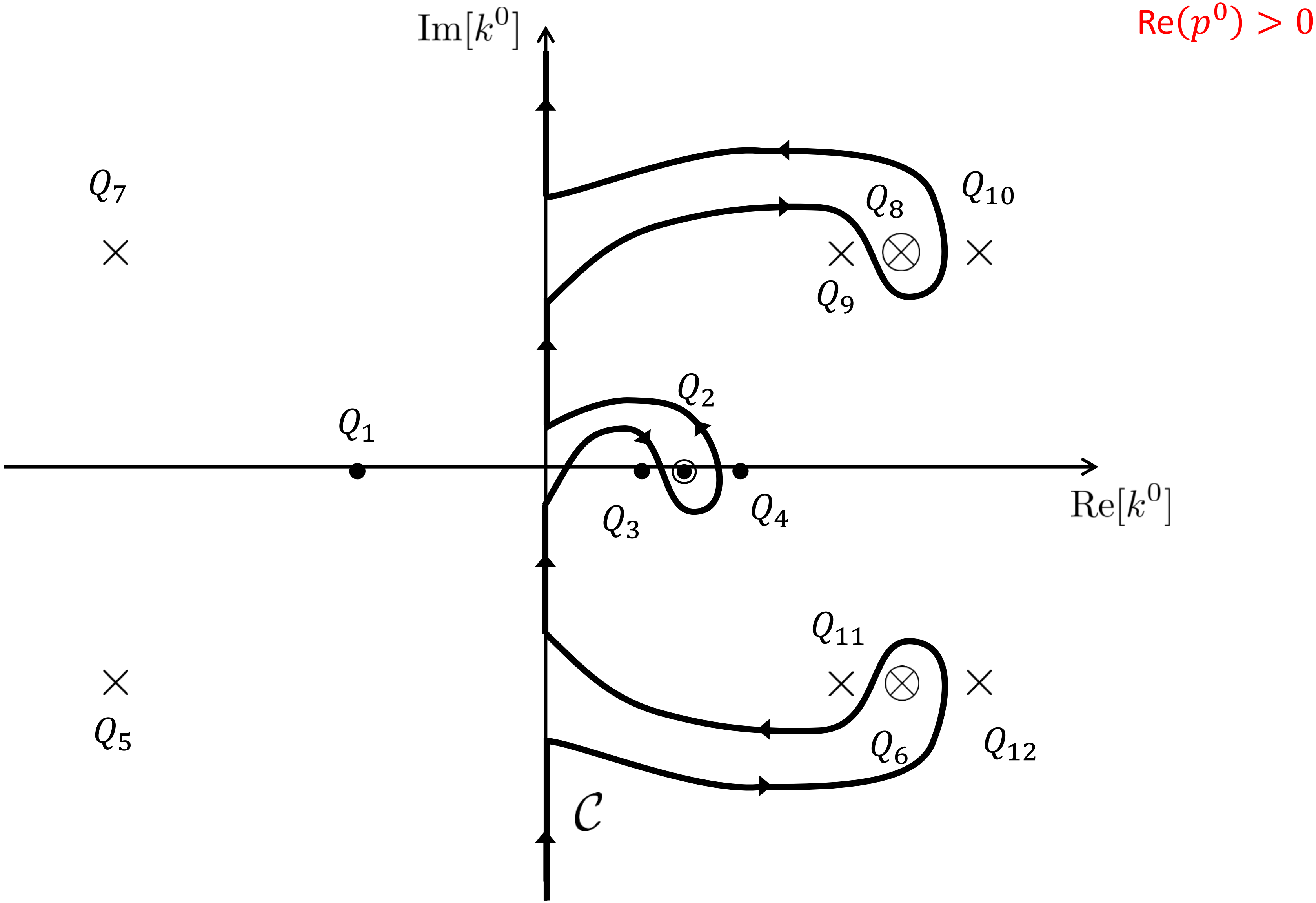


Bubble diagram: nonlocal case + complex poles

$\mathrm{Re}(p^0) > 0$

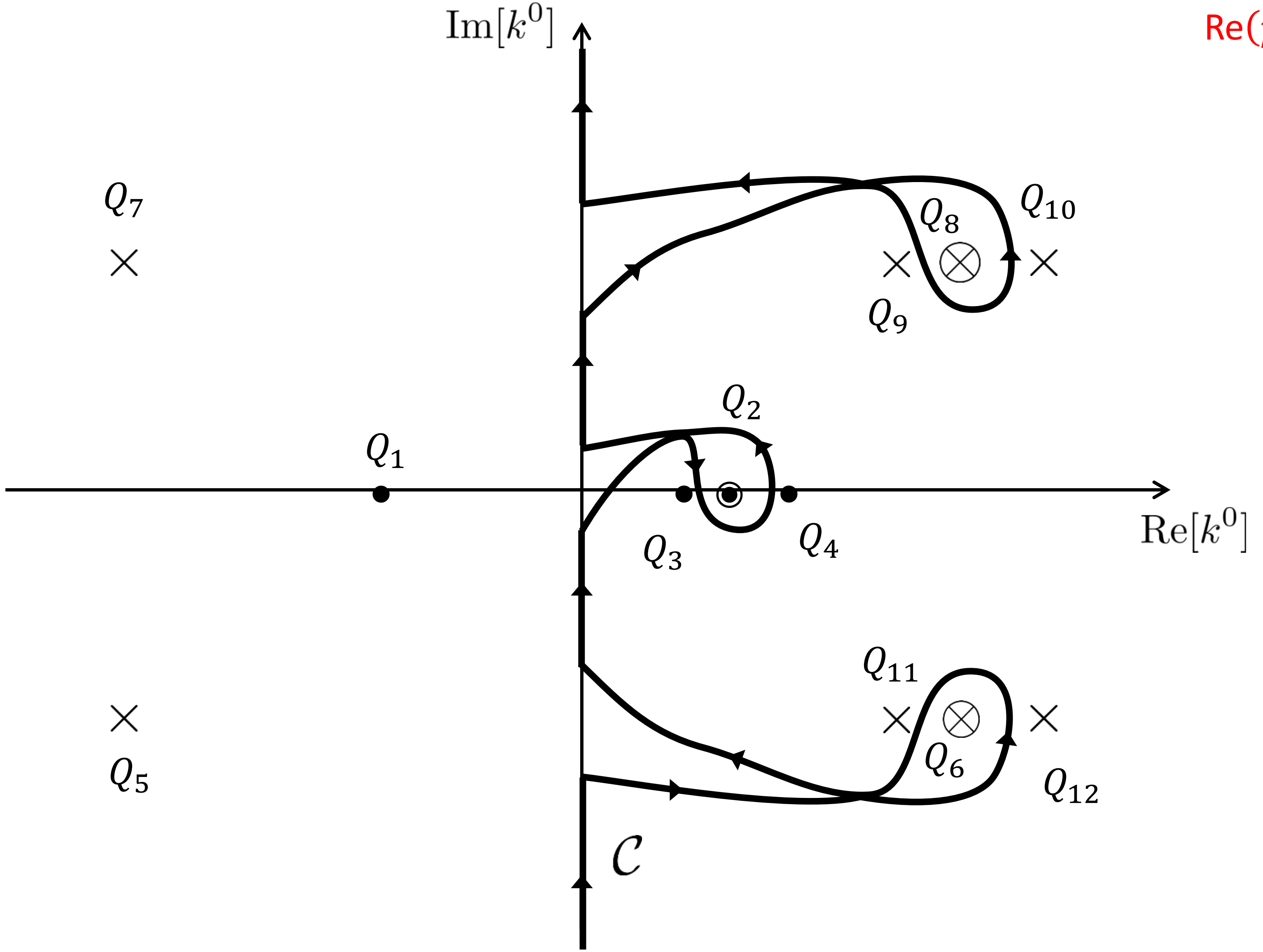


Bubble diagram: nonlocal case + complex poles

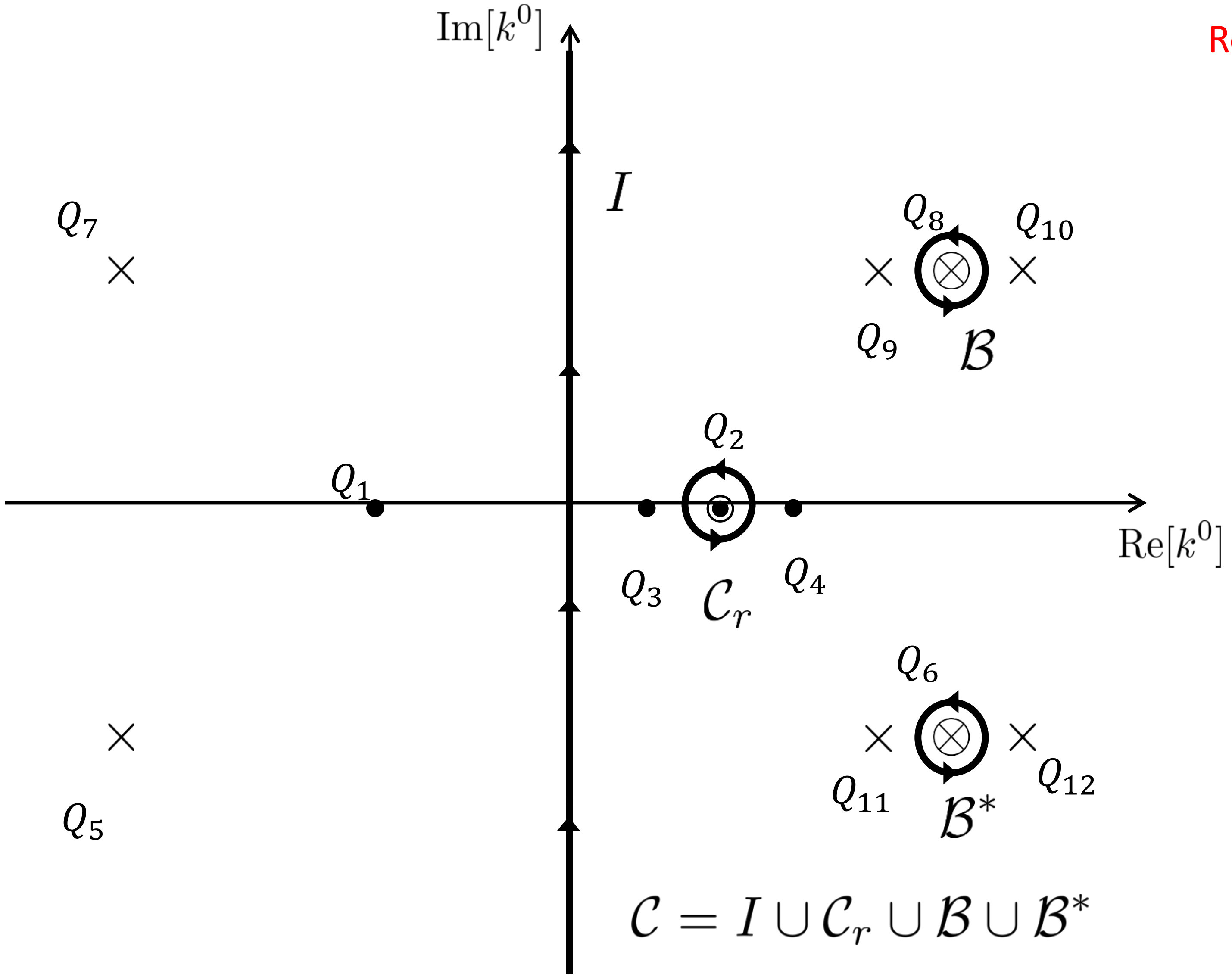


Bubble diagram: nonlocal case + complex poles

$\mathrm{Re}(p^0) > 0$

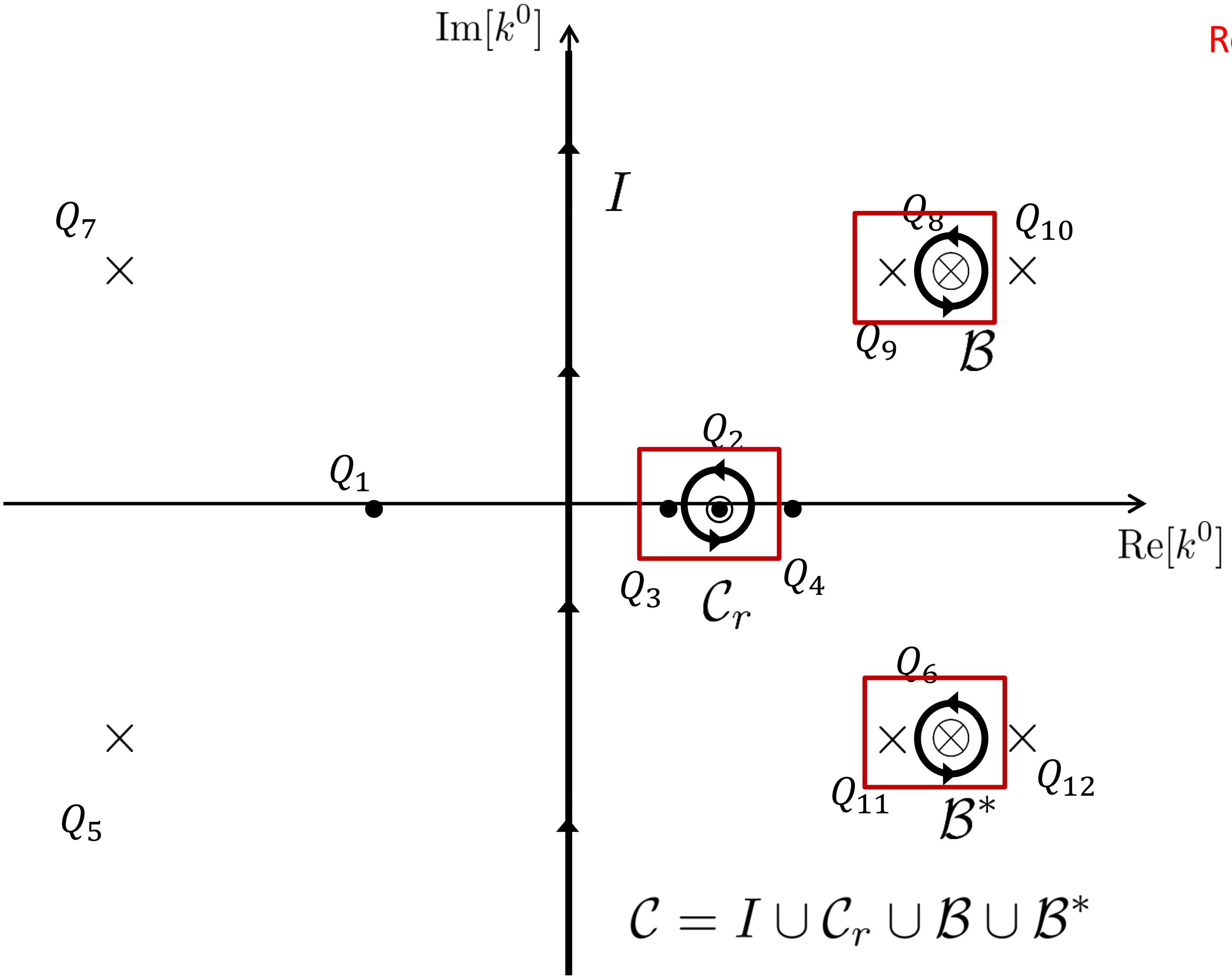


Bubble diagram: nonlocal case + complex poles



Bubble diagram: nonlocal case + complex poles

$\mathrm{Re}(p^0) > 0$



$$\mathcal{C} = I \cup \mathcal{C}_r \cup \mathcal{B} \cup \mathcal{B}^*$$

Bubble diagram: nonlocal case + complex poles

$\mathrm{Re}(p^0) > 0$

$$\mathcal{M}(p)$$

$$= (-i)\lambda^2 M^8 \int_{I\cup\mathcal{C}_r\cup\mathcal{B}\cup\mathcal{B}^*} \frac{dk^0}{2\pi} \int \frac{d^3k}{(2\pi)^3} \frac{e^{\gamma(-k^2)}}{(k^2+m^2-i\epsilon)(k^4+M^4)} \frac{e^{\gamma(-(k-p)^2)}}{[(k-p)^2+m^2-i\epsilon][(k-p)^4+M^4]}$$

$$= \mathcal{M}_I(p) + \mathcal{M}_{\mathcal{C}_r}(p) + \mathcal{M}_{\mathcal{B}}(p) + \mathcal{M}_{\mathcal{B}^*}(p)$$

- $\boxed{\mathcal{M}_{\mathcal{B}}^*(p) = \mathcal{M}_{\mathcal{B}^*}(p)}$ $\Rightarrow$ Imaginary contribution ONLY from the residue at Q_2

$$LHS = 2Im\{\mathcal{M}(p)\}$$

$$= 2\pi\lambda^2 \frac{M^8}{(m^4+M^4)^2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k 2\omega_{k-p}} \theta(p^0-\omega_{k-p})\delta(p^0-\omega_k-\omega_{k-p})$$

Same Cutkosky rules of
local two-derivative case!

Consistent Projection

- We have shown that the complex masses do not contribute to the discontinuity across the real axis

- In other words, the Cutkosky rules only applies to the standard propagator with one real mass:

$$i\Pi(p) = \frac{M^4 e^{\gamma(-p^2)}}{m^4+M^4}\left[\frac{1}{p^2+m^2-i\epsilon}+\frac{p^2+m^2}{p^4+M^4}\right] \quad\longrightarrow\quad 2\pi\frac{M^4}{m^4+M^4}\theta(p^0)\delta(p^2+m^2)$$

- However, in the RHS we can still have intermediate states with complex masses:

$$i[\langle b|T^+|a\rangle - \langle b|T|a\rangle] = \sum_{|n\rangle\in\mathcal{F}} \langle b|T^+|n\rangle\langle n|T|a\rangle$$

- We need to make a consistent projection onto the physical Fock space: $\mathcal{F}\rightarrow\mathcal{F}_{ph}$

Consistent Projection

- We need to make a consistent projection onto the physical Fock space: $\mathcal{F} \to \mathcal{F}_{ph}$

$$i[\langle b|T^+|a\rangle - \langle b|T|a\rangle] = \sum_{|n\rangle \in \mathcal{F}_{ph}} \langle b|T^+|n\rangle\langle n|T|a\rangle$$

- Thus, the RHS will be given by

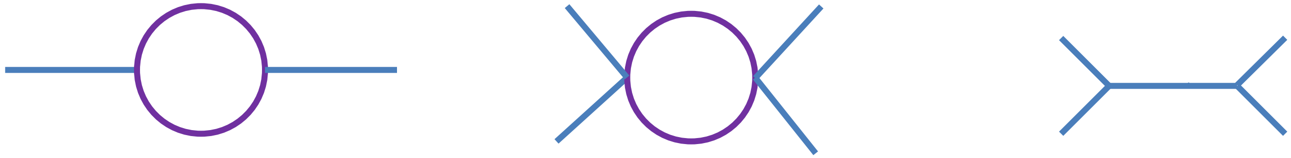
$$RHS = \frac{M^8}{(m^4+M^4)^2}\int\frac{d^3k_2}{(2\pi)^3}\int\frac{d^3k_2}{(2\pi)^3}\frac{(2\pi)^4}{2\omega_1 2\omega_2}\delta^{(4)}(p-k_1-k_2)\langle p\,|\mathcal{M}^+|k_1,k_2\rangle\langle k_1,k_2|\mathcal{M}|p\,\rangle$$

$$= 2\pi\lambda^2\frac{M^8}{(m^4+M^4)^2}\int\frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k 2\omega_{k-p}}\theta\left(p^0-\omega_{k-p}\right)\delta\left(p^0-\omega_k-\omega_{k-p}\right)$$

$$= LHS$$

Optical theorem (unitarity) is satisfied!

Some remarks



- Higher loops investigated in the case of real masses [Sen & Pius 2015]
- Higher loops complex conjugate masses not yet [Work in progress…]
- More complicated vertexes will not affect the result as long as they do not change the pole structure
- In the gravitational case, proving the Cutkosky rules is not sufficient. We also need to project away unphysical states due to gauge invariance
- What about infinite pairs of complex conjugate poles?
  The same prescription should apply.

Outlook

- Why do we care? Even about nonlocal theories with complex conjugate poles???

- First of all, it is very interesting that higher (infinite)-derivative theories can be unitary

- Nonlocal gravitational actions seem to be good candidate for quantum gravity [Koshelev's and Kumar's yesterday talks]

- Loop corrections (e.g. due to matter) in nonlocal theories introduce complex conjugate poles! Is then unitarity spoiled? [Shapiro PLB 2015]

  Our analysis 'suggests' that unitarity may still be satisfied.

- Unitarity of p-adic string Lagrangian around tachyon vacuum:

$$\mathcal{L}_{p-adic} = -\frac{1}{2}\phi(e^{-\Box} - 1)\phi + \cdots$$

Some open problems

- How to define a 'good' classical limit?
  Standard correspondence principle does not work especially in presence of complex masses

- How do define an Hamiltonian ? (is it really needed?)

- Quantification of the causality violation?
  Only at short distances…? [Tomboulis – private communications]

- Huge arbitrarity in the choice of the entire function !?!

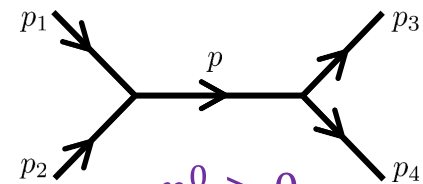
- Nonlocal Lagrangians from first principles…?

Спасибо за
Ваше внимание!

Thank you
for
your attention!

Tree-level: nonlocality + complex conjugate poles

$$\mathcal{L} = \frac{1}{2}\phi F(\Box)\phi - \frac{\lambda}{3!}\phi^3 - \frac{g}{4!}\phi^4 + \kappa\,\phi\psi^2 + \cdots$$



$p^0 > 0$

$$F(\Box) = e^{-\gamma(\Box)}(-\Box + m^2)\left(1 + \frac{\Box^2}{M^4}\right), \qquad i\Pi(p) = \frac{M^4 e^{\gamma(-p^2)}}{m^4 + M^4}\left[\frac{1}{p^2 + m^2 - i\epsilon} + \frac{p^2 + m^2}{p^4 + M^4}\right]$$

- In the RHS we need to project unphysical complex-mass states away

$$RHS = \frac{M^4}{m^4 + M^4}\int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega}(2\pi)^4\,\delta^{(4)}(p-k)\langle p_3, p_4|\mathcal{M}^+|k\rangle\langle k|\mathcal{M}|p_1, p_2\rangle$$

$$= 2\pi\lambda^2\frac{M^4}{m^4 + M^4}\theta(p^0)\delta(p^2 + m^2)$$

$$= LHS$$

Tree-level unitarity!

Generalized quadratic action

- Locality, causality, unitarity , renormalizability, positive norms, positive energies… too many requirements?

- Beyond fourth-order derivatives? Diffeomorphism invariance allows more…

- Generalized quadratic gravitational action, parity-invariant and torsion-free:

$$S = S_{EH} + \frac{1}{32\pi G}\int d^4x\,\sqrt{-g}\left(RF_1(\Box)R + R_{\mu\nu}F_2(\Box)R^{\mu\nu} + R_{\mu\nu\rho\sigma}F_3(\Box)R^{\mu\nu\rho\sigma}\right)$$

$$F_i(\Box/M_S^2) = \sum_{n=0}^{N\leq\infty} f_{i,n}\left(\frac{\Box}{M_S^2}\right)^n, \qquad [\boldsymbol{N=\infty \quad Nonlocal}]$$

Generalized quadratic action

- Generalized quadratic gravitational action, parity-invariant and torsion-free:

$$S = S_{EH} + \frac{1}{32\pi G}\int d^4x\sqrt{-g}\big(RF_1(\Box)R + R_{\mu\nu}F_2(\Box)R^{\mu\nu} + R_{\mu\nu\rho\sigma}F_3(\Box)R^{\mu\nu\rho\sigma}\big)$$

- By using

$$R_{\mu\nu\rho\sigma}\Box^n R^{\mu\nu\rho\sigma} = 4R_{\mu\nu}\Box^n R^{\mu\nu} - R\Box^n R + \mathcal{O}(R^3) + tot.div.$$

- We can write

$$S = S_{EH} + \frac{1}{32\pi G}\int d^4x\sqrt{-g}\left(R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + \mathcal{O}(R^3)\right)$$

Generalized quadratic action

- Up to quadratic order around Minkowski the relevant part of the action is:

$$S = S_{EH} + \frac{1}{32\pi G}\int d^4x\,\sqrt{-g}\big(R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu}\big)$$

- Gauge independent part of the graviton propagator:

$$\boxed{\Pi_{\mu\nu\rho\sigma}(k) = \frac{\mathcal{P}^2_{\mu\nu\rho\sigma}}{f(k)k^2} + \frac{\mathcal{P}^0_{\mu\nu\rho\sigma}}{(f(k)-3g(k))k^2}}$$

$$f(\Box) = 1 + \frac{1}{2}\mathcal{F}_2(\Box)\Box,$$

$$g(\Box) = 1 - 2\mathcal{F}_1(\Box)\Box - \frac{1}{2}\mathcal{F}_2(\Box)\Box$$

Ghost-free higher derivative gravity

- Up to quadratic order around Minkowski the relevant part of the action is:

$$S = S_{EH} + \frac{1}{32\pi G}\int d^4x\,\sqrt{-g}\big(R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu}\big)$$

- Gauge independent part of the graviton propagator:

$$\Pi_{\mu\nu\rho\sigma}(k) = \frac{\mathcal{P}^2_{\mu\nu\rho\sigma}}{f(k)k^2} + \frac{\mathcal{P}^0_{\mu\nu\rho\sigma}}{(f(k)-3g(k))k^2}$$

- **Ghost-freeness condition:**

$$f(k) = e^{\gamma_1(k)}, \qquad f(k) - 3g(k) = e^{\gamma_2(k)},$$

Entire functions

[Biswas et al., PRL 2012]

Ghost-free higher derivative gravity

- Nonlocal higher-derivative gravity theories can be ghost-free; also known as Infinite Derivative Gravity (IDG)

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\left(\mathcal{R} + G_{\mu\nu}F(\Box)\mathcal{R}^{\mu\nu}\right), \qquad \mathcal{F}_2(\Box) = -2\mathcal{F}_1(\Box) \equiv F(\Box) = \frac{f(\Box)-1}{\Box}$$

- Ghost-free propagator:

$$\Pi_{\mu\nu\rho\sigma}(k) = \frac{1}{f(k)}\Pi^{GR}_{\mu\nu\rho\sigma}(k) = \frac{1}{f(k)}\left(\frac{\mathcal{P}^2_{\mu\nu\rho\sigma}}{k^2} - \frac{\mathcal{P}^0_{\mu\nu\rho\sigma}}{2k^2}\right)$$

$$\boxed{f(\Box) = e^{-\gamma\left(\Box/M_s^2\right)}}$$

[Krasnikov 1987; Kuz'min, 1989; Tomboulis 1997; Biswas et al., 2006,2011; Modesto et al. 2011+]

Ghost-free higher derivative gravity

- Non-local higher derivative theories can be ghost-free:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(\mathcal{R} + G_{\mu\nu}F(\Box)\mathcal{R}^{\mu\nu}\right), \qquad F(\Box) = \frac{e^{-\gamma(\Box/M_s^2)} - 1}{\Box}$$

- Ghost-free propagator:

$$\Pi_{\mu\nu\rho\sigma}(k) = e^{\gamma(-k^2/M_s^2)}\Pi^{GR}_{\mu\nu\rho\sigma}(k) = e^{\gamma(-k^2/M_s^2)}\left(\frac{\mathcal{P}^2_{\mu\nu\rho\sigma}}{k^2} - \frac{\mathcal{P}^0_{\mu\nu\rho\sigma}}{2k^2}\right)$$

- Entire function, e.g.

$$\boxed{e^{-\gamma(\Box/M_s^2)} = e^{-\Box/M_s^2}}$$

[Krasnikov 1987; Kuz'min, 1989; Tomboulis 1997; Biswas et al., 2006,2011; Modesto et al. 2011+]

Nonlocal Lee-Wick: gravity sector

- Nonlocal gravitational action:

$$\boxed{S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\left(\mathcal{R} - G_{\mu\nu}\frac{1}{\Box}\mathcal{R}^{\mu\nu} + G_{\mu\nu}e^{-\gamma(\Box)}\frac{\Box^2 - M^4}{M^4\Box}\mathcal{R}^{\mu\nu}\right)}$$

- Nonlocal graviton propagator:

$$\Pi_{\mu\nu\rho\sigma}(p) = \frac{e^{\gamma(-p^2)}M^4}{p^4 + M^4}\Pi^{GR}_{\mu\nu\rho\sigma}(p) = \frac{e^{\gamma(-p^2)}M^4}{p^4 + M^4}\left(\frac{\mathcal{P}^2_{\mu\nu\rho\sigma}}{p^2} - \frac{\mathcal{P}^0_{\mu\nu\rho\sigma}}{2p^2}\right)$$

- Ghost-free graviton propagator! Poles: massless spin-2 graviton pole + 1 pair of Lee-Wick poles

Nonlocal case: infinite complex poles

- Nonlocal scalar theory with infinite complex conjugate poles:

$$\mathcal{L} = -\frac{M_s^2}{2}\phi\left(e^{-\Box/M_s^2} - 1\right)\phi, \qquad F(\Box) = M_s^2\left(e^{-\Box/M_s^2} - 1\right)$$

- Ghost-free propagator:

$$\Pi(p) = \frac{1}{M_s^2\left(e^{p^2/M_s^2} - 1\right)} = \frac{e^{-\frac{p^2}{2M_s^2}}}{p^2} + e^{-\frac{p^2}{2M_s^2}}\sum_{\ell=1}^{\infty}(-1)^\ell\left(\frac{1}{p^2 + i2\pi M_s^2\ell} + \frac{1}{p^2 - i2\pi M_s^2\ell}\right)$$

$$\boxed{\frac{1}{\sinh(iz/2)} = \frac{2}{i}\sum_{\ell=-\infty}^{\infty}(-1)^\ell\frac{1}{z + i2\pi\ell}}$$

Infinite pairs of complex conjugate poles: $p^2 = i2\pi M_s^2\ell$

[LB, Lambiase, Yamaguchi, PRD]

Nonlocal (infinite-derivative) case: gravity sector

- Nonlocal gravitational action:

$$\boxed{S=\frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(\mathcal{R}-G_{\mu\nu}\frac{1}{\Box}\mathcal{R}^{\mu\nu}-M_s^2G_{\mu\nu}\frac{e^{-\Box/M_s^2}-1}{\Box^2}\mathcal{R}^{\mu\nu}\right)}$$

- Nonlocal graviton propagator:

$$\Pi_{\mu\nu\rho\sigma}(p)=\frac{p^2}{M_s^2\left(e^{p^2/M_s^2}-1\right)}\Pi^{GR}_{\mu\nu\rho\sigma}(p)=\frac{1}{M_s^2\left(e^{p^2/M_s^2}-1\right)}\left(\mathcal{P}^2_{\mu\nu\rho\sigma}-\frac{1}{2}\mathcal{P}^0_{s,\mu\nu\rho\sigma}\right)$$

- Ghost-free graviton propagator!

Poles: massless spin-2 graviton propagator + infinite pairs of complex conjugate poles

[LB, Lambiase, Yamaguchi, PRD]

Ghost-free higher derivative gravity

- Linearized metric for a static point-like source:

$$ds^2 = -\big(1+2\phi(r)\big)dt^2 + \big(1-2\phi(r)\big)(dr^2 + r^2 d\Omega^2)$$

$$e^{-\frac{\nabla^2}{M_s^2}}\nabla^2\phi(\vec{r}) = 4\pi G m \delta^{(3)}(\vec{r}) \;\Longrightarrow\; \phi(r) = -\frac{Gm}{r}Erf\left(\frac{M_s r}{2}\right)$$

IR

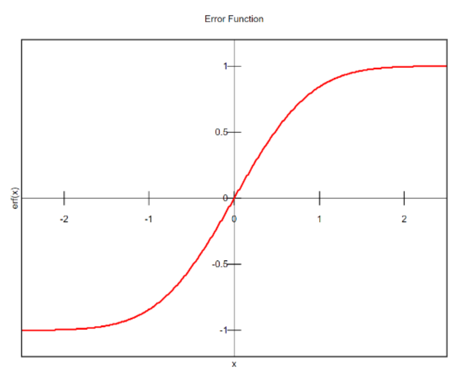
$$\phi(r) \sim -\frac{Gm}{r}$$

UV

$$\phi(r) \sim -\frac{GmM_s}{\sqrt{\pi}} < \infty$$

Singularity-free!

$$Erf(x) := \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$$



[Tseytlin, 1995; Biswas et al., 2006; Biswas et al., 2012]

Ghost-free higher derivative gravity

- Scalar curvature:

$$R = \frac{GmM_s^3 e^{-M_s^2 r^2/4}}{\sqrt{\pi}}$$

- Non-singular curvature invariants!
- Conformally-flat at r=0!
- Smearing of the (delta-) source due to non-locality!
- The UV/short distances behavior is ameliorated!

[LB, Koshelev, Lambiase, Marto, Mazumdar, JCAP]

Ghost-free higher derivative gravity

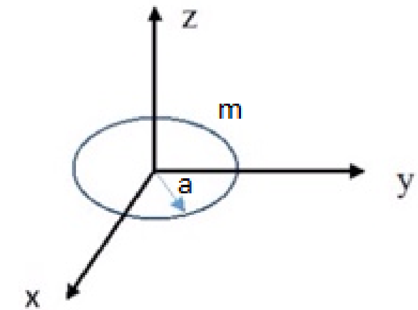
- Linearized metric for a rotating ring source in IDG:

$$ds^2 = -\big(1+2\phi(r)\big)dt^2 + 2\vec{h}\cdot d\vec{r}dt + \big(1-2\phi(r)\big)(dr^2 + r^2 d\Omega^2)$$

- Stress-energy tensor:

$$T_{00} = m\delta(z)\frac{\delta(x^2+y^2-a^2)}{\pi}, \qquad T_{0i} = T_{00}v_i,$$

$$v_x = -y\omega, \;\; v_y = x\omega, \;\; v_z = 0$$



- Differential equations:

$$e^{-\frac{\nabla^2}{M_s^2}}\nabla^2\phi(\vec{r}) = 4Gm\delta(z)\delta(x^2+y^2-a^2),$$

$$e^{-\frac{\nabla^2}{M_s^2}}\nabla^2 h_{0x}(\vec{r}) = -16Gm\omega y\delta(z)\delta(x^2+y^2-a^2),$$

$$e^{-\frac{\nabla^2}{M_s^2}}\nabla^2 h_{0y}(\vec{r}) = 16Gm\omega x\delta(z)\delta(x^2+y^2-a^2)$$

[LB, et al. PRD]

Ghost-free higher derivative gravity

- Linearized metric for a rotating ring source in IDG:

$$ds^2 = -\big(1 + 2\phi(r)\big)dt^2 + 2\vec{h} \cdot d\vec{r}dt + \big(1 - 2\phi(r)\big)(dr^2 + r^2 d\Omega^2)$$

- Solutions:

$$\boxed{\begin{aligned} \phi(\rho) &= -Gm \int_0^\infty d\zeta\, I_0(ia\zeta) I_0(i\rho\zeta) Erfc\left(\frac{\zeta}{M_s}\right), \\ h_{0x}(x,y) &= 4Gm\omega a \frac{y}{\rho} \int_0^\infty d\zeta\, I_1(ia\zeta) I_1(i\rho\zeta) Erfc\left(\frac{\zeta}{M_s}\right), \\ h_{0y}(x,y) &= -4Gm\omega a \frac{x}{\rho} \int_0^\infty d\zeta\, I_1(ia\zeta) I_1(i\rho\zeta) Erfc\left(\frac{\zeta}{M_s}\right) \end{aligned}}$$

$$\rho = \sqrt{x^2 + y^2}$$

[LB, et al. PRD]

Ghost-free higher derivative gravity

- Linearized metric for a rotating ring source in IDG:

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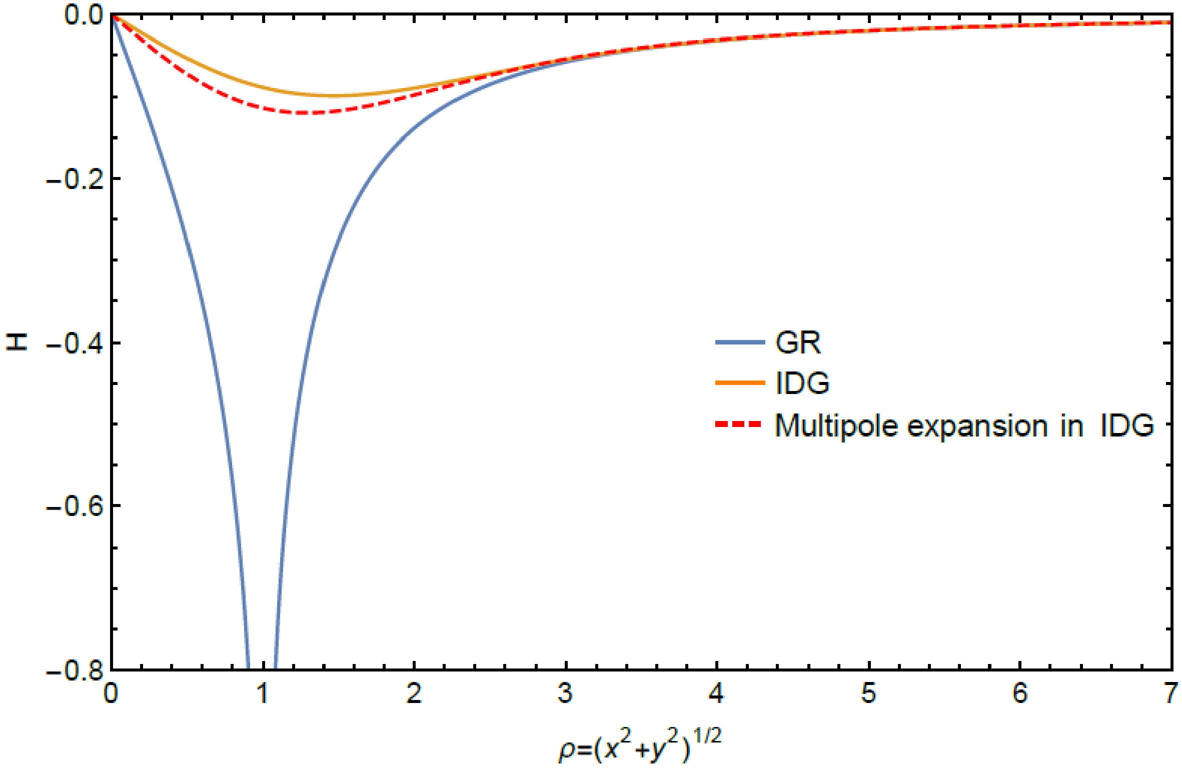
- Non-singular solutions:



[LB, et al. PRD]

Ghost-free higher derivative gravity

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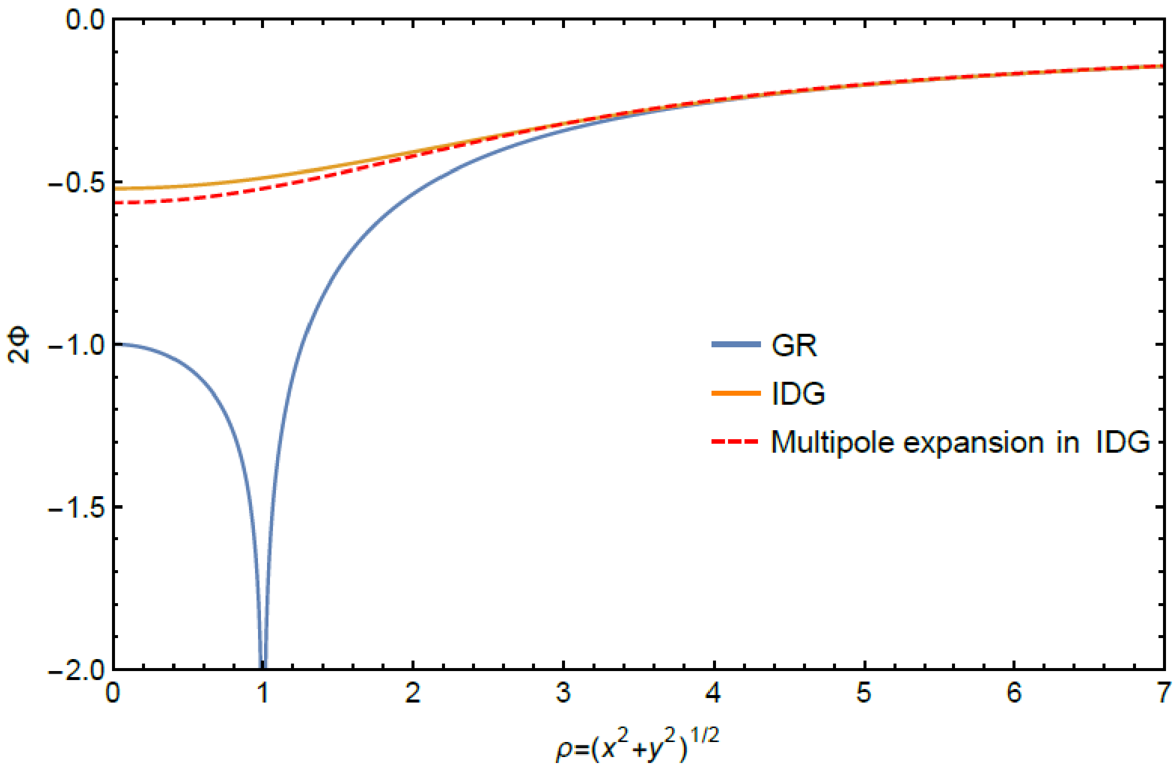


[LB, et al. PRD]

Extra Slides: nonlocality

- **Nonlocal action:**

$$S = \int d^4x d^4y\, \phi(x) K(x-y) \phi(y)$$

$$= \int d^4x d^4y\, \phi(x) \int \frac{d^4k}{(2\pi)^4} F(-k^2) e^{ik\cdot(x-y)} \phi(y)$$

$$= \int d^4x d^4y\, \phi(x) F(\Box) \int \frac{d^4k}{(2\pi)^4} e^{ik\cdot(x-y)} \phi(y)$$

$$= \int d^4x d^4y\, \phi(x) F(\Box) \phi(x)$$

Stringy inspired nonlocal theories

- Nonlocal scalar field:

$$\boxed{\mathcal{L} = \frac{1}{2}\phi e^{-\gamma(\Box/M_s^2)}(\Box - m^2)\phi - V(\phi),}$$

$$\gamma(\Box/M_s^2) = \sum_{n=0}^{N\leq\infty} \gamma_n \left(\frac{\Box}{M_s^2}\right)^n$$

Entire function:
No extra poles!

- Ghost-free propagator:

$$\Pi(p) = \frac{e^{\gamma(-p^2/M_s^2)}}{p^2 + m^2}$$

Perturbative unitarity (optical theorem and Cutkosky rules)
[Pius & Sen 2015; Briscese & Modesto 2018; Chin & Tomboulis 2018]

Causality violation at microscopic scales (acausal Green functions and local commutativity violation)
[Tomboulis 2015; LB, Lambiase, Mazumdar 2018]

Microcausality violation

- **NO time-ordered propagator:**

$$e^{-\gamma(\Box)}(\Box - m^2)\Pi(x) = i\delta^{(4)}(x),$$

$$\boxed{\Pi(x) = \Pi_c(x) + \Pi_{nc}(x),}$$

$$\Pi_c(x) = \langle T\{\phi(x)\phi(0)\}\rangle$$

$$\Pi_{nc}(x) = i\sum_{q=1}^{\infty}\frac{i^{q-1}}{q!}\partial_{x^0}^{(q-1)}[W^{(q)}(x) - W^{(q)}(-x)]$$

$$W^{(q)}(x) = \int\frac{d^4k}{(2\pi)^4}e^{ik\cdot x}\theta(k^0)\delta(k^2+m^2)\frac{\partial^{(q)}e^{-\gamma(-k^2)}}{\partial k^{0(q)}}$$

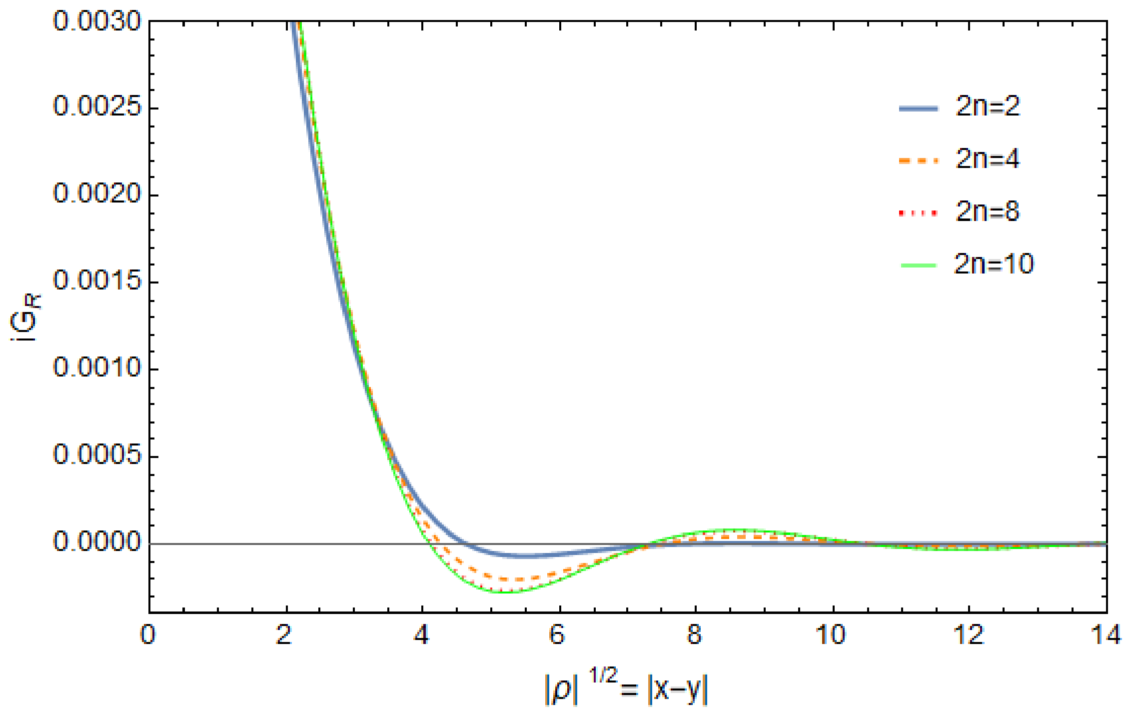
[Tomboulis, PRD; LB, Lambiase, Mazumdar, NPB]

Microcausality violation

- **Acausal Green function (principal value + residues):**

$$e^{\left(-\frac{\Box}{M_s^2}\right)^{2n}}(\Box - m^2)G_R(x-y) = i\delta^{(4)}(x-y),$$

$$G_R(x-y) \neq 0, \qquad for \quad (x-y)^2 > 0$$



[LB, Lambiase, Mazumdar, NPB]

Extra slides: spin projection operators

$$\mathcal{P}^2_{\mu\nu\rho\sigma} = \frac{1}{2}\left(\theta_{\mu\rho}\theta_{\nu\sigma} + \theta_{\mu\sigma}\theta_{\nu\rho}\right) - \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma},$$

$$\mathcal{P}^1_{\mu\nu\rho\sigma} = \frac{1}{2}\left(\theta_{\mu\rho}\omega_{\nu\sigma} + \theta_{\mu\sigma}\omega_{\nu\rho} + \theta_{\nu\rho}\omega_{\mu\sigma} + \theta_{\nu\sigma}\omega_{\mu\rho}\right),$$

$$\mathcal{P}^0_{s,\mu\nu\rho\sigma} = \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma}, \qquad \mathcal{P}^0_{w,\mu\nu\rho\sigma} = \omega_{\mu\nu}\omega_{\rho\sigma}$$

$$\mathcal{P}^0_{sw,\mu\nu\rho\sigma} = \frac{1}{\sqrt{3}}\theta_{\mu\nu}\omega_{\rho\sigma}, \qquad \mathcal{P}^0_{ws,\mu\nu\rho\sigma} = \frac{1}{\sqrt{3}}\omega_{\mu\nu}\theta_{\rho\sigma},$$

$$\theta_{\mu\nu} = \eta_{\mu\nu} - \omega_{\mu\nu}, \qquad \omega_{\mu\nu} = \frac{k_\mu k_\nu}{k^2}.$$

Extra Slides: Renormalizable nonlocal gravity

- **Kuzmin/Tomboulis' entire function:**

$$\gamma(z) = \Gamma\big(0, P(z)\big) + \gamma_E + log\big(P(z)\big)$$

- **UV behavior:**

$$\mathrm{z} \rightarrow \infty \qquad \Longrightarrow \qquad e^{\gamma(z)} \longrightarrow e^{\gamma_E} P(z)$$

[Kuzmin 1989, Tomboulis 1977]

Enlarging the class of ghost-free operators

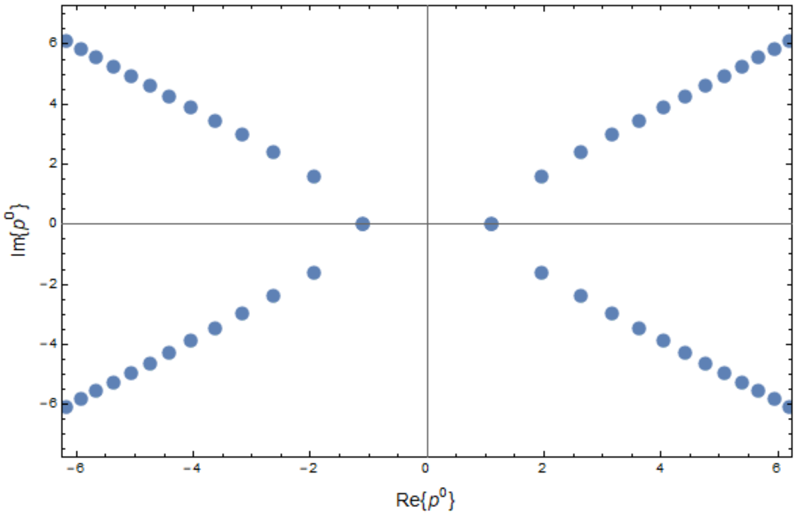
- Nonlocal propagator:

$$\Pi(p) = \frac{1}{M_s^2\left(e^{\frac{p^2}{M_s^2}} - 1\right)}$$

- Poles:

$$p^2 = i2\pi M_s^2 \ell \Longrightarrow p^0 = \pm\sqrt{\vec{p}^2 - i2\pi M_s^2 \ell}$$

$$p^0 = \pm\left(\sqrt{\frac{\vec{p}^2 + \sqrt{\vec{p}^4 + i4\pi^2 M_s^4 \ell^2}}{2}} - i\varepsilon(\ell)\sqrt{\frac{-\vec{p}^2 + \sqrt{\vec{p}^4 + i4\pi^2 M_s^4 \ell^2}}{2}}\right)$$



[LB, Lambiase, Yamaguchi, PRD]

Enlarging the class of ghost-free operators

- Non-local gravitational action:

$$\boxed{S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(\mathcal{R} - G_{\mu\nu}\frac{1}{\Box}\mathcal{R}^{\mu\nu} - M_s^2 G_{\mu\nu}\frac{e^{-\Box/M_s^2}-1}{\Box^2}\mathcal{R}^{\mu\nu}\right)}$$

- Nonlocal graviton propagator:

$$\Pi_{\mu\nu\rho\sigma}(k) = \frac{k^2}{M_s^2\left(e^{\frac{k^2}{M_s^2}}-1\right)}\Pi^{GR}_{\mu\nu\rho\sigma}(k) = \frac{1}{M_s^2\left(e^{\frac{k^2}{M_s^2}}-1\right)}\left(\mathcal{P}^2_{\mu\nu\rho\sigma} - \frac{1}{2}\mathcal{P}^0_{s,\mu\nu\rho\sigma}\right)$$

- Ghost-free graviton propagator! Poles: massless spin-2 graviton propagator + infinite pairs of complex conjugate poles

[LB, Lambiase, Yamaguchi, PRD]

Different type of nonlocal Lagrangians

- P-adic string: [Freund, Witten, Frampton, Dragovich....]

$$\mathcal{L}_{p-adic} = \frac{M_s^2}{2}\phi e^{-\Box/M_s^2}\phi, \qquad \Pi_{p-adic}(k) = e^{-\frac{k^2}{M_s^2}}$$

- SFT inspired nonlocal Lagrangian: [Krasnikov, Arefeva, Koshelev, Biswas, Mazumdar, Modesto...]

$$\mathcal{L}_{SFT} = \frac{1}{2}\phi e^{-\Box/M_s^2}\Box\phi, \qquad \Pi_{SFT}(k) = \frac{e^{-k^2/M_s^2}}{k^2}$$

- Nonlocal model with complex conjugate poles: [LB, Lambiase, Yamaguch PRD]

$$\mathcal{L} = -\frac{M_s^2}{2}\phi(e^{-\Box/M_s^2} - 1)\phi, \qquad \Pi(k) = \frac{1}{M_s^2(e^{k^2/M_s^2} - 1)}$$

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Reciprocity Theory of Elementary Particles

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I. INTRODUCTION

THE theory of elementary particles which I propose in the following pages is based on the current conceptions of quantum mechanics and differs widely from the ideas which Einstein himself has developed in regard to this problem. I hope that it may nevertheless be acceptable as a contribution to this volume in honor of his 70th birthday, as it is based on his famous relation between energy E and mass m of a physical system, $E=mc^2$, and as it can be interpreted as a rational generalization of his ("special") theory of relativity.

Relativity postulates that all laws of nature are invariant with respect to such linear transformations of space time $x^k=(\mathbf{x}, t)$ for which the quadratic form $R=x^kx_k=t^2-\mathbf{x}^2$ is invariant (the velocity of light is taken to be unity). The underlying physical assumption is that the 4-dimensional distance $r=R^{\frac{1}{2}}$ has an absolute significance and can be measured. This is a natural and plausible assumption as long as one has to do with macroscopic dimensions where measuring rods and clocks can be applied. But is it still plausible in the domain of atomic phenomena?

Doubts have been expressed a long time ago, e.g., by Lindemann (Lord Cherwell) (14) in his instructive little book. I think that the assumption of the observability of the 4-dimensional distance of two events inside atomic dimensions is an extrapolation which can only be justified by its consequences; and I am inclined to interpret the difficulties which quantum mechanics encounters in describing elementary particles and their interactions as indicating the failure of that assumption.

The well-known limits of observability set by Heisenberg's uncertainty rules have little to do with this question; they refer to the measurements of coordinates sponding to the particles with which one has possibly to do. This is the problem which is now in the center of interest: by estimating $\mathbf{p}$ and E for a particle observed in the Wilson chamber or in a photographic emulsion, one obtains a rough value of the rest mass which may permit one to recognize the kind of particle with which one has to do. If the value of P thus obtained is however incompatible with the known particles a new one is discovered. During the last year this has happened several times, and one gets the impression that there may be no end of it. New types of mesons are found almost every week, and it seems to be not an extravagant extrapolation to suppose that there is an infinite number.

It looks, therefore, as if the distance P in momentum space is capable of an infinite number of discrete values which can be roughly determined while the distance R in coordinate space is not an observable quantity at all.

This lack of symmetry seems to me very strange and rather improbable. There is strong formal evidence for the hypothesis, which I have called *the principle of reciprocity*, that the laws of nature are symmetrical with regard to space-time and momentum-energy, or more precisely, that they are invariant under the transformation

$$x_k \rightarrow p_k, \quad p_k \rightarrow -x_k. \tag{I.1}$$

The most obvious indications are these: The canonical equations of classical mechanics

$$\dot{x}^k = \partial H/\partial p_k, \quad \dot{p}_k = -\partial H/\partial x^k \tag{I.2}$$

are indeed invariant under the transformation (1), if only the first 3 components of the 4-vectors x^k and p_k are considered. These equations hold also in the matrix or

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analogy with classical theory a current density 4-vector j_k is introduced one has instead of (VII.4)

$$e^{-p^2}p^2A_k=4\pi j_k. \tag{VII.5}$$

For a point charge at rest one has

$$j_1=j_2=j_3=0, \quad j_4=e\delta(r), \tag{VII.6}$$

where $\mathbf{r}=\mathbf{x}-\mathbf{x}_0$ and δ is a Dirac symbolic function.

Using ordinary units the solution of (VII.5) for this case, which can be easily obtained by Fourier transformation, reads

$$A_1=A_2=A_3=0, \quad A_4=\frac{e}{a}Y\left(\frac{r}{a}\right), \tag{VII.7}$$

where

$$Y(x)=\frac{1}{x}\frac{1}{(\pi)^{\frac{1}{2}}}\int_0^x e^{-x^2/4}dx. \tag{VII.8}$$

This modification of Coulomb's law has already been suggested by myself and published in collaboration with Rumer (7) as long ago as 1931, and it appears later sporadically in the literature, as a more or less arbitrary assumption, while in the reciprocity theory it is a necessity. The main features of this potential are these: for $x\gg a$ it goes over in the Coulomb potential, for $x\to 0$ it tends to the finite value $A_4\to e/a(\pi)^{\frac{1}{2}}$ and it leads to a

$$2Im[(-i) \qquad] = \qquad = \int d\Pi_f$$