Quantization of Gravity in the Black Hole Background

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Quantization of Gravity in the Black Hole background in the Regge-Wheeler harmonic basis

$$\mathcal{M}_4 = \mathcal{M}_2 \times \mathbb{S}^2$$

- We perform a covariant (Lagrangian) quantization of gravity in the Schwarzschild black hole background. We use the Regge-Wheeler gauge in the spherical harmonics basis for $l > 1$ modes.

- For low multipoles Regge-Wheeler gauge is not valid, we propose a background-covariant gauge condition for monopoles and dipoles well defined in perturbative quantum gravity.

- We find that in a covariant quantization Faddeev-Popov (FP) ghosts are non-propagating for $l > 1$ modes, but monopoles and dipoles, in general, have FP ghosts.

- In Schwarzschild coordinates all time derivatives acting on ghosts drop out, therefore monopoles and dipoles ghosts have instantaneous propagators.

- Up to subtleties related to quantizing gravity in a space with a horizon, **Faddeev's theorem** suggests a possibility of a canonical (Hamiltonian) quantization with ghost-free Hilbert space.

Lagrangian/Hamiltonian quantization of gauge theories

The canonical Hamiltonian quantization procedure is fundamental: it involves the issue of the physical states and unitarity

Covariant Lagrangian quantization is simpler, it is manifestly independent on the choice of the gauge-fixing condition and choice of coordinates

According to Faddeev 1969, Fradkin, Tyutin, 1970, Fradkin, Batalin, Vilkovisky, 1977

Hamiltonian quantization gives the same results for gravitational observables, as the Lagrangian quantization, under certain conditions

There are 2 types of gauge-fixing conditions

1. corresponds to the case of the **unitary ghosts-free Hamiltonian** in gauge theories and Hilbert space of states has a definite metric

2. corresponds to the case of the **pseudo-unitary Hamiltonian**: the S-matrix is pseudo-unitary in the Hilbert space of states with the **indefinite metric**.

We have performed covariant Lagrangian quantization of gravity in the black hole background. We use Regge-Wheeler gauge for l >1 modes and a new class of gauges for l < 2 where Regge-Wheeler is not valid.

We found evidence that the canonical Hamiltonian, when constructed, will belong to a class of unitary ghost free Hamiltonians in our gauge.

The Background Field Method for Gravity

Due to gauge symmetries the naive Feynman path integral over quantum fluctuation of the metric h in the gravitational background g is not well defined

$$\int dh\, e^{iS(g+h)}$$

1967

De Witt-Faddeev-Popov covariant Lagrangian quantization of gravity in the background field method involves the set of constraints on gravitational fields of the form

$$\chi_\alpha(g,h)=0$$

The well defined path integral, suitable for the Feynman diagram computations involves a **Jacobian**

The role of the Jacobian is to make the path integral independent on the choice of the gauge-fixing function

$$\int dh J_\chi(g,h)\delta\Big(\chi_\alpha(g,h)\Big)e^{iS(g+h)}$$

the Jacobian

is defined by the variation of the gauge-fixing function under the gauge symmetry

$$\delta\chi_\alpha = Q_\alpha{}^\beta(g,h)\xi_\beta$$

$$J_\chi(g,h)=\exp Tr\ln\, Q_\alpha{}^\beta(g,h)$$

This Jacobian can be also presented with the help of anti-commuting **FP ghosts**

$$J_\chi=\int d\bar C^\alpha dC_\beta \exp^{i\int d^4x\,\bar C^\alpha(x)Q_\alpha{}^\beta(g,h)C_\beta(x)}$$

BRST-BFV quantization in application to gravity

Becchi, Rouet, Stora,Tyutin, Fradkin,Batalin, Vilkovisky
1975-1977

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}$$

$$S_{BRST} = S_{cl}(g+h) + S_{g.f.}(g,h) + S_{FP}(\bar{C}, C, g, h)$$

$$S_{g.f.}(g,h) = B^\alpha \chi_\alpha(g,h)$$

Same path integral as in De Witt-Faddeev-Popov method

$$S_{FP} = \bar{C}^\alpha \frac{\delta\chi_\alpha}{\delta h_{\beta\gamma}} \nabla_\beta^{(g+h)} C_\gamma$$

$$\int dh\, dB\, d\bar{C}\, dC e^{iS_{BRST}(g,h,B,\bar{C}C)}$$

Our purpose is to construct the De Witt-Faddeev-Popov Feynman integral, and a corresponding BRST action for gravity in the black hole background in the Regge-Wheeler set up with expansion of gravitational perturbations in **spherical harmonics**.

This could have been done any time after 1967

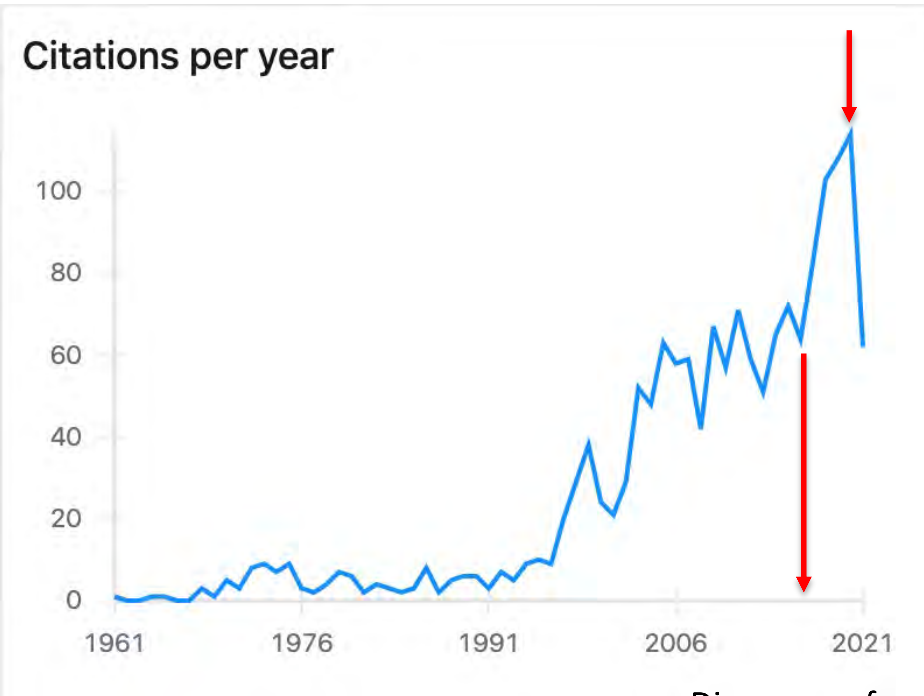
De Witt-Faddeev-Popov

Why now?

Stability of a Schwarzschild singularity

Regge, Wheeler, 1957

COVID-19



Discovery of gravitational waves from black hole mergers

The interest to Regge-Wheeler set up with expansion of gravitational perturbations in spherical harmonics was gradually increasing from about 1995.

It was rapidly increasing after detection of gravity waves

Gaddam,Groenenboom,'t Hooft 2012.02357

In the list of shortcomings:
we do not know FP ghosts actions in Regge-Wheeler gauge

'‘t Hooft: discreteness on BH micro-states in terms of partial waves

$$[u^{\pm}_{\ell m},\, p^{\mp}_{\ell' m'}] = i\,\delta_{\ell\ell'}\,\delta_{mm'}$$

Example of QFT loop computations in gravity in the background covariant generalized de Donder gauge

$$S_{gf} = \sqrt{-g}\frac{1}{2\xi}(\nabla_\nu h^\nu_\mu - \frac{1}{2}\nabla_\mu h^\nu_\nu)(\nabla_\lambda h^{\lambda\mu} - \frac{1}{2}\nabla^\mu h^\lambda_\lambda)$$

where FP ghosts are propagating

$$\boxed{S_{FP} = \sqrt{-g}\,\bar{C}^\mu(\nabla_\lambda\nabla^\lambda C^\mu - R_{\nu\mu}C^\nu) + \dots}$$

’t Hooft, Veltman;
RK, Tarasov, Tyutin; Barvinsky, Vilkovisky;
Goroff, Sagnotti; Van de Ven
1974-1992

$\bar{C}\Box C$ and give important contribution

Dirac gauge, 1959

$$\psi_3{}^0 \equiv (\sqrt{-g})e^{ik}\Gamma_{ik}{}^0 = 0\,,$$

$$\psi_3{}^k \equiv \partial_i[(-g^{(3)})^{1/3}e^{ik}] = 0\,.$$

Fradkin, Tyutin, Faddeev, Popov:
canonical quantization, there is a **unitarizing ghost-free Hamiltonian**

In the Lagrangian quantization one finds FP actions with **simultaneous ghost propagators**, like in Coulomb gauge in Yang-Mills: kinetic term has no time derivatives, only space derivatives.

It is known in the example of a Coulomb gauge in Yang-Mills theory that the ghosts loops with instantaneous propagators are cancelled by the loops of the instantaneous part of the gluon part of the propagator, in all orders of perturbation theory. In this case the equivalence of the Hamiltonian perturbative Feynman rules in QCD to the Lagrangian De Witt-Faddeev-Popov rules is clearly established.

What are ghosts actions in Regge-Wheler gauge?

Regge-Wheeler, 1957 Zerilli, 1970, Martel and Poisson, 2004

Gravity in a spherically symmetric Schwarzschild Black Hole Background

$$g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu = g_{ab}\,\mathrm{d}x^a\mathrm{d}x^b + r^2(x)\,\mathrm{d}\Omega_2^2$$

$$\mathcal{M}_4 = \mathcal{M}_2 \times \mathbb{S}^2$$

split

$$4\pi r(x)^2 \equiv \mathrm{Area}\Big(\mathrm{SO}(3)\text{ orbit of } x \text{ in } \mathcal{M}\Big)$$

2D geometry

metric	covariant derivative	volume form
g_{ab}	$\mathcal{D}_a$	ϵ_{ab}

$$r_a(x) = \mathcal{D}_a r(x) \qquad t^a(x) = -\epsilon^{ab} r_b(x)$$

$$g^{ab} = \frac{1}{f(r)}\Big(-t^a t^b + r^a r^b\Big) \qquad \epsilon^{ab} = \frac{1}{f(r)}\Big(-t^a r^b + r^a t^b\Big)$$

$$f(r) \equiv g_{ab} r^a r^b = -g_{ab} t^a t^b = 1 - \frac{2GM}{r}$$

Due to the **warp factor $r^2(x)$**, there are "cross-terms" Christoffel symbols for the coordinate system which prevent the simple factorization

$$\mathrm{d}x^\mu\,\nabla_\mu^{(g)} \neq \mathrm{d}x^a \mathcal{D}_a + \mathrm{d}\theta^A D_A$$

D_A denote the covariant derivative on $(\mathbb{S}^2, \Omega_{AB})$

$$\Gamma^a{}_{AB} = -r\,g^{ab} r_b\,\Omega_{AB}, \qquad \Gamma^\mu{}_{aB} = \frac{1}{r}\,r_a\,\delta^\mu_B$$

It is **almost 2D gravity**

Regge-Wheeler ansatz

$$Y^{\ell m}(\theta^A)$$

$$Y^{\ell m}_A \equiv D_A Y^{\ell m}$$

$$X^{\ell m}_A \equiv -\epsilon_A{}^B D_B Y^{\ell m}$$

etc

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = g_{ab}\,dx^a dx^b + r^2\Omega_{AB}\,d\theta^A d\theta^B$$

Decomposition into Spherical Harmonics on **S**^{2}

$$h_{\mu\nu} = \underbrace{\begin{pmatrix} p_{ab} & p^{(+)}_{aB} \\ p^{(+)}_{bA} & p^{(+)}_{AB} \end{pmatrix}}_{\text{parity even}} + \underbrace{\begin{pmatrix} 0 & p^{(-)}_{aB} \\ p^{(-)}_{Ab} & p^{(-)}_{AB} \end{pmatrix}}_{\text{parity odd}}$$

$$p_{ab} = \sum_{\ell=0}^{\infty}\sum_{|m|\leq\ell} h^{\ell m}_{ab} Y^{\ell m}$$

$$p^{(+)}_{aA} = \sum_{\ell=1}^{\infty}\sum_{|m|\leq\ell} j^{\ell m}_a\, Y^{\ell m}_A \qquad p^{(-)}_{aA} = \sum_{\ell=1}^{\infty}\sum_{|m|\leq\ell} h^{\ell m}_a\, X^{\ell m}_A$$

$$p^{(+)}_{AB} = r^2\left(\sum_{\ell=0}^{\infty}\sum_{|m|\leq\ell} K^{\ell m}\Omega_{AB}Y^{\ell m} + \sum_{\ell=2}^{\infty}\sum_{|m|\leq\ell} G^{\ell m}Y^{\ell m}_{AB}\right) \qquad p^{(-)}_{AB} = \sum_{\ell=2}^{\infty}\sum_{|m|\leq\ell} h^{\ell m}_2 X^{\ell m}_{AB}$$

the coefficient functions with fixed l, m depend only on 2d coordinates x^a

Ansatz with 10 functions of x^1, x^2

$$h^{\ell m}_{ab},\, j^{\ell m}_a,\, K^{\ell m},\, G^{\ell m},\, h^{\ell m}_a,\, h^{\ell m}_2$$

one 2D tensor, two 2D vectors, three 2D scalars

7 in even sector, 3 in odd

Gauge Symmetry Parameters in Spherical Harmonics

$$\xi_\mu = \underbrace{(\Xi_a, \Xi_A^{(+)})}_{\text{parity even}} + \underbrace{(0, \Xi_A^{(-)})}_{\text{parity odd}}$$

$$\Xi_a = \sum_{\ell=0}^{\infty} \sum_{|m|\le\ell} \xi_a^{\ell m} Y^{\ell m}$$

$$\Xi_A^{(+)} = \sum_{\ell=1}^{\infty} \sum_{|m|\le\ell} \xi^{(+)\ell m} Y_A^{\ell m}, \qquad \Xi_A^{(-)} = \sum_{\ell=1}^{\infty} \sum_{|m|\le\ell} \xi^{(-)\ell m} X_A^{\ell m}$$

All 4 functions with fixed l, m depend only on 2D coordinates x^a

Gauge transformations

$$\delta p_{ab} = \mathcal{D}_a \Xi_b + \mathcal{D}_b \Xi_a - 2\widehat{\Gamma}^{\mu}_{ab} \Xi_\mu$$

$$\delta p_{aB} = \mathcal{D}_a \Xi_B + D_B \Xi_a - \frac{2}{r} r_a \Xi_b - 2\widehat{\Gamma}^{\mu}_{aB} \Xi_\mu$$

$$\delta p_{AB} = D_A \Xi_B + D_B \Xi_A + 2 r g^{ab} r_a \Xi_b \Omega_{AB} - 2\widehat{\Gamma}^{\mu}_{AB} \Xi_\mu$$

∇_μ is the connection on the four-dimensional spacetime (M_4, $g_{\mu\nu}$),

$\mathcal{D}_a$ is the connection on the two-dimensional spacetime (M_2,g_{ab}), and

D_A is the connection on the round unit two-sphere (S^2,Ω_{AB}).

For modes with $\ell \geq 2$,

Even sector

$$\delta h_{ab} = \mathscr{D}_a \xi_b + \mathscr{D}_b \xi_a + f_{ab}[h, \xi]$$

$$\delta j_a = \mathscr{D}_a \xi^{(+)} + \xi_a - \frac{2}{r} r_a \xi^{(+)} + f_a^{(+)}[h, \xi]$$

$$\delta K = -\frac{\ell(\ell+1)}{r^2} \xi^{(+)} + \frac{2}{r} r^a \xi_a^{(+)} + f_{(K)}[h, \xi]$$

$$\delta G = \frac{2}{r^2} \xi^{(+)} + f_{(G)}[h, \xi]$$

Odd sector

$$\delta h_a = \mathscr{D}_a \xi^{(-)} - \frac{2}{r} r_a \xi^{(-)} + f_a^{(-)}[h, \xi]$$

$$\delta h_2 = 2\xi^{(-)} + f_2[h, \xi]$$

Regge-Wheeler gauge

$$G = j_a = h_2 = 0$$

$$S_{\text{ghost}}[h, \tilde{C}, \bar{C}; g] = \int_{\mathcal{M}} \mathrm{d}^4 x \sqrt{g} \left[\bar{C}^{(+)} \tilde{C}^{(+)} + \bar{C}^a \tilde{C}_a + \bar{C}^{(-)} \tilde{C}^{(-)} \right]$$

+ non-linear h-dependent terms

Ghosts do not propagate

Compare with Standard Model in a unitary gauge where Goldstone bosons are absent

$$\chi_\alpha = -i(t_\alpha)_{nm} \phi'_m v_n \qquad \bar{C}_\alpha (t_\alpha v)_n (t_\beta \phi)_n C_\beta = \bar{C}_\alpha \mu^2_{\alpha\beta} C_\beta \qquad \tilde{C}_\alpha = \mu^2_{\alpha\beta} C_\beta$$

$$S_{ghost} = \bar{C}_\alpha \tilde{C}_\alpha$$

Monopoles and dipoles: no G, no j_a, no h_2

RW gauge $$G^{lm} = j_a^{lm} = h_2^{lm} = 0$$

However $$G^{lm} = j_a^{lm} = h_2^{lm} = 0$$ at l=0

$$G^{lm} = h_2^{lm} = 0$$ at l=1

These fields come in the RW ansatz contracted with higher harmonics $Y_{AB}^{\ell m}$ $Y_A^{\ell m}$ $X_A^{\ell m}$ on the sphere, absent in low multipoles

Therefore RW gauge cannot be used for l=0,1

Also the number of gauge symmetries/required gauge-fixing conditions is different

$\xi^{l\geq 2} \quad \Rightarrow \quad \{\xi_a^{\ell m(+)}, \xi^{\ell m(+)}, \xi^{\ell m(-)}\}$ RW $\quad j_a = G = h_2 = 0$

$\xi^{l=1(+)} \quad \Rightarrow \quad \{\xi_a^{1m(+)}, \xi^{1m(+)}\} \quad K = j_a = 0$

$\xi^{l=1(-)} \quad \Rightarrow \quad \{\xi^{1m(-)}\} \quad r^a h_a = 0$

$\xi^{l=0} \quad \Rightarrow \quad \{\xi_a^{00(+)}\} \quad K = t^a r^b h_{ab} = 0$

Our choice of the 2D background covariant gauge-fixing for monopoles and dipoles

Now we have all information to build FP ghost actions => covariant quantization of gravity in the black hole background

$$\bar{C}^\alpha X(g,h)_\alpha{}^\beta C_\beta$$

- Kinetic terms defining FP ghost propagators $\bar{C}^\alpha X(g,h=0)_\alpha{}^\beta C_\beta$

- Interaction terms $\bar{C}^\alpha \Big(X(g,h) - X(g,h=0) \Big)_\alpha{}^\beta C_\beta$

l > 1 ghost kinetic terms

$$\bar{C}^a \Big(C_a + (\mathcal{D}_a - \frac{2}{r} r_a) C^{(+)} \Big) + \bar{C}^+ \frac{2}{r^2} C^{(+)} + \bar{C}^- 2 C^{(-)}$$

We can integrate out $\bar{C}^+ \; , \; \bar{C}^- \longrightarrow C^+ = C^- = 0$

$$\bar{C}^\alpha X(g,h=0)_\alpha{}^\beta C_\beta \longrightarrow \bar{C}^a C_a$$

All ghosts in Regge-Wheeler gauge in l>1 sector are non-propagating!

Low multipoles

Kinetic terms for dipoles **l=1 even**

$$\bar{C}^a\left(C_a+(\mathcal{D}_a-\frac{2}{r}\,r_a)C^{(+)}\right)-\bar{C}^+\frac{2}{r^2}(C^{(+)}-r\,r^aC_a)$$

We can integrate out $\bar{C}^+ \longrightarrow C^{(+)}=r\,r^aC_a$

$$\bar{C}^\alpha X(g,h=0)_\alpha{}^\beta C_\beta \longrightarrow \bar{C}^a\left(C_a+(\mathcal{D}_a-2\,r_a)r^bC_b\right)$$

Kinetic terms for dipoles **l=1 odd**

$$\bar{C}^\alpha X(g,h=0)_\alpha{}^\beta C_\beta \longrightarrow \bar{C}^{(-)}r^a\partial_aC^{(-)}$$

Kinetic terms for monopoles **l=0**

$$\bar{C}^K\frac{2}{r}\,r^aC_a+\bar{C}^h\left(t^ar^b(\mathcal{D}_aC_b+\mathcal{D}_bC_a)\right)$$

We can integrate out $\bar{C}^K \longrightarrow r^aC_a=0$

$$\bar{C}^\alpha X(g,h=0)_\alpha{}^\beta C_\beta \longrightarrow \bar{C}^h\left(t^ar^b(\mathcal{D}_aC_b+\mathcal{D}_bC_a)\right)$$

l=0, 1 monopole and dipole ghosts are propagating, in general
Non-linear terms do not change this fact

In Schwarzschild coordinates

all time derivatives on ghosts drop

$$r^a C_a \Rightarrow f(r) C_r \quad r^a \mathcal{D}_a \Rightarrow f(r) \mathcal{D}_r$$

$$t^a C_a \Rightarrow C_t \qquad t^a \mathcal{D}_a \Rightarrow \mathcal{D}_t$$

l=1 even

$$\bar{C}^a \Big(C_a + (\mathcal{D}_a - 2\, r_a) r^b C_b \Big)$$

$$\bar{C}^r C_r + \bar{C}^t C_t + \bar{C}^t \mathcal{D}_t\, f\, C_r + \bar{C}^r (\mathcal{D}_r - 2)\, f\, C_r$$

Intregrate out C_t $\Longrightarrow$ $\bar{C}^t = 0$ $\Longrightarrow$ $\bar{C}^r C_r + \bar{C}^r (\mathcal{D}_r - 2)\, f\, C_r$

l=1 odd

$$\bar{C}^{(-)} r^a \partial_a C^{(-)} \quad \Longrightarrow \quad \bar{C}^{(-)} f \partial_r C^{(-)}$$

l=0

$$\bar{C}^h \Big(t^a r^b (\mathcal{D}_a C_b + \mathcal{D}_b C_a) \Big) \quad \Longrightarrow \quad \bar{C}^h f(r) \Big(\mathcal{D}_r - \frac{f'}{f} \Big) C_t$$

$$r^a C_a = 0$$

The analysis of non-linear terms is more complicated, but confirms that **all ghost actions in covariant quantization have no time derivatives acting on ghosts**!

Evidence for unitarity of H in our gauge for gravity in the black hole background

In general, the classical action with gauge symmetries $\mathcal{L}(q,\dot{q})$

Can be given in the form with Lagrange multipliers λ and 1st class constraints $\phi^\alpha(q,p)$

$$S(q,p,\lambda) = \int dt(p_i\dot{q}^i - H(q,p) - \lambda_\alpha\phi^\alpha(q,p))$$

1=1, ..., n
α=1, ..., m

An additional set of conditions has to be added to perform the canonical quantization,

1. corresponds to the case of the **unitary ghosts-free Hamiltonian** in gauge theories and Hilbert space of states has a **definite metric**

$$\chi_\alpha(p,q) \qquad H(p^*,q^*)$$

$\boxed{* = 1, \ldots, (n-m)}$

2. corresponds to the case of the **pseudo-unitary Hamiltonian**: the S-matrix is pseudo-unitary in the Hilbert space of states with the **indefinite metric**.

$$\chi_\alpha(p,q,\dot{\lambda},\lambda) \qquad H(p_A,q^A,\underbrace{\mathcal{P}_a,\eta^a}_{\text{2m FP anti-commuting ghosts}})$$

A=1,...,(n+m)
a=1,..., 2m

$\boxed{n+m - 2m = n-m}$

> In our covariant quantization we have found that in Schwarzschild coordinates there are no time derivative acting on ghosts, therefore under canonical quantization there will be no canonical degrees of freedom associated with FP ghosts
>
> A unitary ghost-free Hamiltonian is expected

The presence of the **horizon** is not taken into account yet,
Eddington-Finkelstein and Kruskal-Szekeres coordinates?

Faddeev's theorem and Hamiltonian origin of the FP ghosts in covariant quantization

In canonical quantization, once the 1st class constraints $\phi^\alpha(q,p)$ are established, an additional set of conditions $\chi_\alpha(p,q)$ has to be added to perform the canonical quantization

It is required that the Poisson brackets of constraints with additional conditions have a non-vanishing determinant

$$\det ||\{\chi_\alpha, \phi^\beta\}|| \neq 0$$

The Poisson bracket defines a differential operator $M_\alpha{}^\beta$

$$\{\chi_\alpha(t,\vec{x}), \phi^\beta(t,\vec{y})\} = M_\alpha{}^\beta \delta^3(\vec{x}-\vec{y})$$

In case of unitary Hamiltonian this operator does not involve time derivatives
However, it might involve space derivatives.

In such case it offers a Hamiltonian origin of the FP ghost actions with simultaneous ghost propagators in Lagrangian quantization.

We have computed det $M_\alpha{}^\beta$ for all partial waves l. We have confirmed that, in general, there are time derivatives. However, all of them drop in Schwarzschild coordinates. For example in l=1 even

In Schwarzschild coordinates $r_a = (0,1)$ and $r^a = (0,f)$

$$\det \begin{pmatrix} 1 & 0 & \mathscr{D}_1 - \frac{2}{r} r_1 \\ 0 & 1 & \mathscr{D}_2 - \frac{2}{r} r_2 \\ \frac{2}{r} r^1 & \frac{2}{r} r^2 & -\frac{2}{r^2} \end{pmatrix} = -\frac{2}{r^2} - \frac{2}{r} r^a \left(\mathscr{D}_a - \frac{2}{r} r_a\right) \qquad \det|_{Sch} \begin{pmatrix} 1 & 0 & \mathscr{D}_1 - \frac{2}{r} r_1 \\ & 1 & \mathscr{D}_2 - \frac{2}{r} r_2 \\ 0 & \frac{2}{r} r^2 & -\frac{2}{r^2} \end{pmatrix} = -\frac{2}{r^2} - \frac{2f}{r}\left(\mathscr{D}_2 - \frac{2}{r}\right)$$

In l=0 we studied the theory to all order in **h** in Schwarzschild coordinates. The corresponding Poisson bracket and its determinant are

$$\det\begin{pmatrix} \frac{2}{r}\frac{f(r)}{1+f(r)h_{rr}} & 0 \\ \partial_t - \frac{f(r)\partial_t h_{rr}}{1+f(r)h_{rr}} & \partial_r - \frac{f'(r)-\partial_r h_{tt}}{f(r)-h_{tt}} \end{pmatrix} = \det\begin{pmatrix} \frac{2}{r}\frac{f(r)}{1+f(r)h_{rr}} & 0 \\ 0 & \partial_r - \frac{f'(r)-\partial_r h_{tt}}{f(r)-h_{tt}} \end{pmatrix}$$

Time derivatives drop in Schwarzschild coordinates, but space derivatives are present at l=0,1 $\Rightarrow$ l=0,1 FP ghosts have simultaneous propagators

The canonical quantization of gravity and the constructions of the Feynman path integral was so far performed only in a flat Minkowski space.

Meanwhile in the black hole background in Schwarzschild coordinates there is a singular horizon. The concept of the Hamiltonian and of the physical states might be more complicated, moreover, the black hole puzzles may not be resolved in the context of a perturbative Feynman path integral.

Nevertheless, according to Faddeev's theorem valid in the flat background, the ghost free unitary Hamiltonian exists in the class of gauges studied here in the black hole background in Schwarzschild coordinates.

Conclusion

We have found that in the covariant quantization **in Schwarzschild coordinates** there are no time derivatives acting on the ghost fields. This suggests that such a Hamiltonian, if explicitly constructed, might belong to the class of

unitary ghost-free Hamiltonian $H(p_*, q_*)$, $\quad * = 1, \ldots, (n-m)$

The reason this is a likely outcome of the canonical quantization is that the case of pseudo-unitary Hamiltonian in a Hilbert space of states with the indefinite metric,

pseudo-unitary Hamiltonian $H(q^A, p_A, \eta^a, P_a)$ $\quad n+m-2m = n-m$

would be inconsistent with the absence of time derivatives on the ghosts, which we found in this work. Here $A = 1, \ldots, n+m$ involves commuting fields, and $a = 1, \ldots, 2m$ involves anti-commuting fields. But we have just shown that all of our anti-commuting fields (FP ghosts and anti-ghosts) have no time derivatives, so they are not expected to contribute to a Hilbert space of states with a negative metric in a process of canonical quantization.

Note that in **Eddington-Finkelstein and Kruskal-Szekeres** coordinates the situation is different and has to be studied. Although we have performed a covariant quantization in any of these coordinates, the canonical quantization, choices of gauges, and the issue of unitarity are still to be explored.