

# Black hole induced false vacuum decay from first principles

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2105.09331



Quantum gravity and Cosmology

June 5, 2021

# Motivation

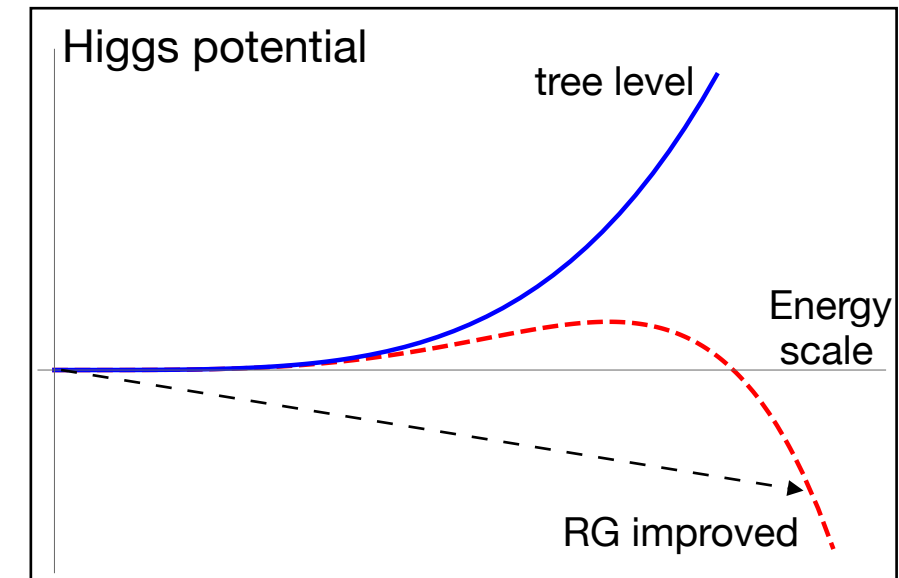
Vacuum decay in field theory is relevant for phenomenology.

- Standard Model Higgs vacuum may not be absolutely stable.

In the present-day Universe, the decay probability is low.

The situation can be different in different environments.

- Stability of the Higgs vacuum has been studied in various setups:



D. Buttazzo, G. Degrandi, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio, and A. Strumia, JHEP 12 (2013) 089  
M. Herranen, T. Markkanen, S. Nurmi, and A. Rajantie, Phys. Rev. Lett. 115 (2015) 241301  
A. Salvio, A. Strumia, N. Tetradis, and A. Urbano, JHEP 09 (2016) 054  
A. Rajantie, S. Stoprya, Phys.Rev.D95 (2017) 2, 025008  
A. Andreassen, W. Frost, M. Schwartz, Phys.Rev.D97 (2018) 5, 056006 ....

In the Standard Model and its modifications

In flat spacetime

in thermal bath

seeded by local spatial  
inhomogeneities...

... such as black holes

with gravity

during inflation

R. Gregory, I. Moss, B. Withers, JHEP 03 (2014) 081 [1401.0017]  
P. Burda, R. Gregory, I. Moss, Phys. Rev. Lett. 115 (2015) 071303 [1501.04937]  
P. Burda, R. Gregory, I. Moss, JHEP 08 (2015) 114 [1503.07331]  
P. Burda, R. Gregory, I. Moss, JHEP 06 (2016) 025 [1601.02152]  
N. Tetradis, JCAP 1609 (2016) 036 [1606.04018]  
D. Gorbunov, D. Levkov, A. Panin, JCAP 1710 (2017) 016 [1704.05399]  
K. Mukaida, M. Yamada, Phys. Rev. D96 (2017) 103514 [1706.04523]  
K. Kohri, H. Matsui, Phys. Rev. D98 (2018) 123509 [1708.02138]  
T. Hayashi, K. Kamada, N. Oshita, J. Yokoyama, JHEP 08 (2020) 088 [2005.12808]

# Motivation

Inhomogeneities accelerate phase transition... What's so special about black holes?

- 🌍 It's a simple gravitational impurity — curved geometry
- 🌍 It's a simple source of thermal radiation — quantum vacuum

In general, both effects are equally important.

The problem is not new...

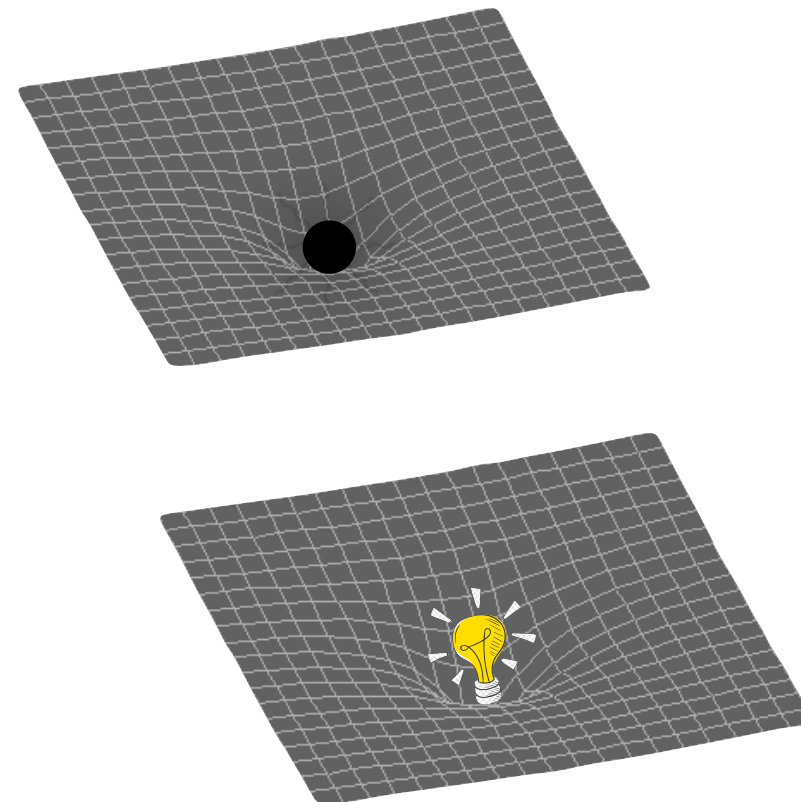
W. Hiscock, Phys. Rev. D35 (1987) 1161  
V. Berezin, V. Kuzmin and I. Tkachev, Phys. Lett. B207 (1988) 397  
P. Arnold, Nucl. Phys. B346 (1990) 160  
V. Berezin, V. Kuzmin and I. Tkachev, Phys. Rev. D43 (1991) 3112

...but the interest has been revived recently.



- 🌍 Small (ergo hot) black holes (BHs) make vacuum decay unsuppressed.
- 🌍 There could be small primordial BHs in the early Universe.

If true, this would put constraints on primordial BH models or imply that the Standard Model is completed in the way to prevent the electroweak vacuum instability.



# Burning questions

- Is it possible to formulate the decay problem referring only to the region outside the horizon?
- Which (complexified) spacetime coordinates one should use?
- What are the vacuum states characterising a BH?
- What are the boundary conditions imposed for the tunnelling solution by a given vacuum state?

# Geometry, setup

Let's be as simple as possible:

- descend to 1+1 dimensions,
- no back reaction on background geometry,
- scalar field with unstable potential.

The metric:  $ds^2 = \Omega(x) (-dt^2 + dx^2)$

! We neglect BH evaporation and do not consider BH formation.

$\Omega(x) \approx 1$   $x \rightarrow +\infty$  — spatial infinity

$\Omega(x) \approx e^{2\lambda x}$   $x \rightarrow -\infty$  — horizon

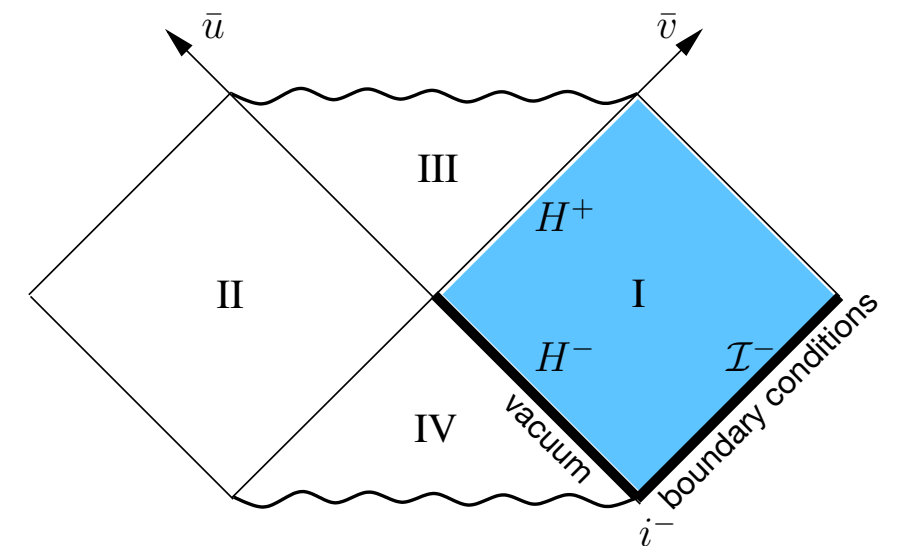
i The near-horizon geometry is approximated by the Rindler spacetime.

In what follows, we'll make  $t$  complex.

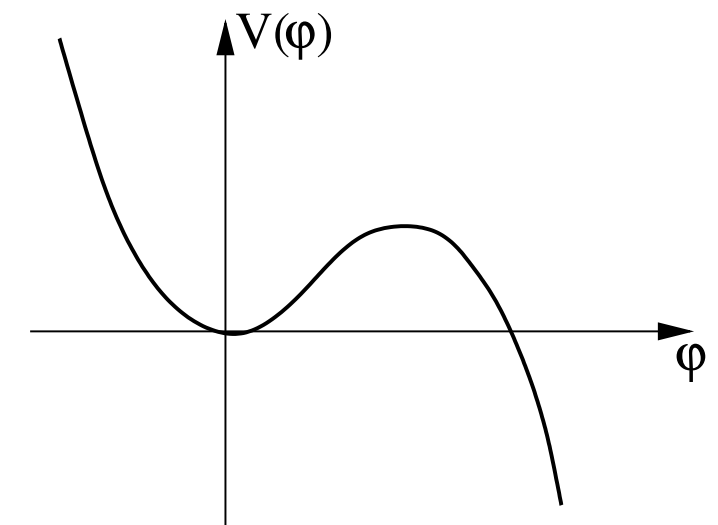
$$\lambda = 2\pi T'_{\text{BH}}$$

The theory:  $\mathcal{S} = \frac{1}{\tilde{g}} \int d^2x \sqrt{-g} \left( -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right)$

$\tilde{g} \ll 1$  — coupling constant



Penrose diagram of the maximally-extended BH spacetime.  
The tortoise coordinates  $(t, x)$  cover the exterior region I.



Scalar potential with false vacuum at  $\varphi = 0$ .

# Vacuum states

The quantum field: 
$$\hat{\psi}(t,x) = g \int_0^\infty \frac{d\omega}{\sqrt{4\pi\omega}} \sum_{I=L,R} \left( \hat{a}_{I,\omega} \psi_{I,\omega}^+(t,x) + \hat{a}_{I,\omega}^\dagger \psi_{I,\omega}^-(t,x) \right)$$

**L** — left-moving modes


**R** — right-moving modes

Consider the following vacuum states:

## Boulware:

$$\hat{a}_{R,\omega} |0\rangle_B = \hat{a}_{L,\omega} |0\rangle_B = 0$$

D. G. Boulware, Phys. Rev. D11 (1975) 1404


 (eternal BH; BH mimickers)

 It's known how to compute the tunnelling solution.

## Hartle-Hawking:

$$\langle \hat{a}_{R,\omega}^\dagger \hat{a}_{R,\omega'} \rangle_{HH} = \langle \hat{a}_{L,\omega}^\dagger \hat{a}_{L,\omega'} \rangle_{HH} = \frac{\delta(\omega - \omega')}{e^{\frac{2\pi\omega}{\lambda}} - 1}$$

 (BH in thermal equilibrium)

 ... also known.

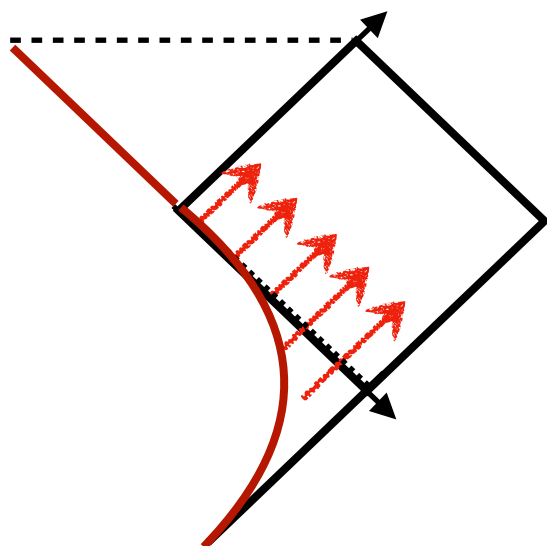
J. B. Hartle and S. W. Hawking, Phys. Rev. D13 (1976) 2188

## Unruh:

$$\langle \hat{a}_{R,\omega}^\dagger \hat{a}_{R,\omega'} \rangle_U = \frac{\delta(\omega - \omega')}{e^{\frac{2\pi\omega}{\lambda}} - 1} \quad \langle \hat{a}_{L,\omega}^\dagger \hat{a}_{L,\omega'} \rangle_U = 0$$

W. G. Unruh, Phys. Rev. D14 (1976) 870

 (BH once formed in a collapse of matter)



Even without an explicit computation, one can expect that the decay rates are

$$\Gamma_B < \Gamma_U < \Gamma_{HH}$$

See, e.g., in P. Arnold, Nucl.Phys.B346 (1990):

... I do not know how to handle the question of false vacuum decay in a nonequilibrium situation such as this one. I merely note that, since radiation helps the system cross the barrier, the result should lie somewhere between the two extremes of zero radiation and thermal equilibrium.

# Decay probability

Very generally, we have  $\langle f|i\rangle$  – transition amplitude

$|i\rangle$  – initial state associated with vacuum of the free theory

$|f\rangle$  – final state somewhere around true vacuum (TV)

Decay probability:

$$\mathcal{P}_{\text{decay}} = \sum_{|f\rangle \in \text{TV}} \langle i|f\rangle \langle f|i\rangle$$

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Going path-integral:

$$\langle f|i\rangle = \int D\varphi_i(x) D\varphi_f(x) D\varphi(t,x) e^{iS[\varphi]} \langle f|\varphi_f t_f\rangle \langle \varphi_i t_i|i\rangle$$

$$\begin{aligned}\varphi(t_i, x) &= \varphi_i(x) \\ \varphi(t_f, x) &= \varphi_f(x)\end{aligned}$$

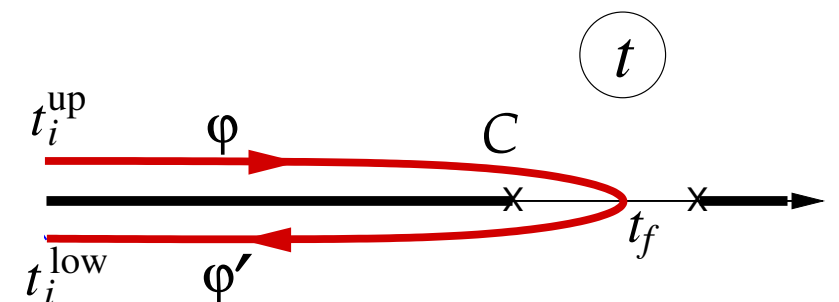
where  $\hat{\varphi}(t, x) |\varphi, t\rangle = \varphi(x) |\varphi, t\rangle$  – eigenstates of the field operator

$$\mathcal{P}_{\text{decay}} = \int D\varphi_i(x) D\varphi'_i(x) D\varphi_c(t,x) e^{iS[\varphi_c]} \langle \varphi_i t_i^{\text{up}}|i\rangle \langle i|\varphi'_i t_i^{\text{low}}\rangle$$

$$\begin{aligned}\varphi(t_i^{\text{up}}, x) &= \varphi_i(x) \\ \varphi(t_i^{\text{low}}, x) &= \varphi'_i(x)\end{aligned}$$

“In-in” formalism for tunneling, cf.

Miller'74; Rubakov, Son, Tinyakov'92; Bonini, Cohen, Rebbi, Rubakov'99; Bezrukov, Levkov'04  
Bramberger, Lavrelashvili, Lehnert, 16; Turok'13; Cherman, Unsal'14; Andreassen, Fahri, Frost, Schwartz'16



Contour in the complex time plane for the calculation of the false vacuum decay probability in the in-in formalism.



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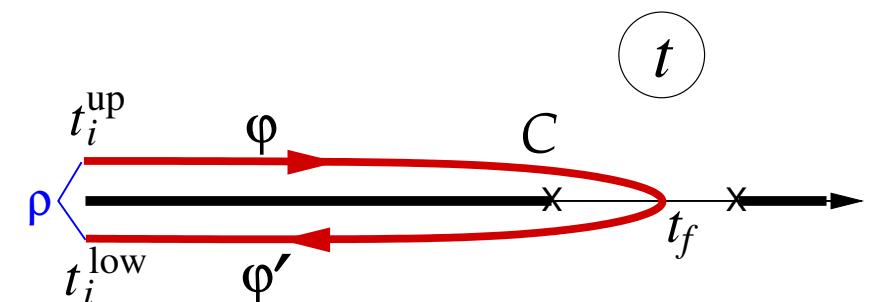
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$$\mathcal{P}_{\text{decay}} = \int D\varphi_i(x) D\varphi_i'(x) D\varphi_c(t,x) e^{iS[\varphi_c]} \langle \varphi_i t_i^{\text{up}} | \varphi_i' t_i^{\text{low}} \rangle$$

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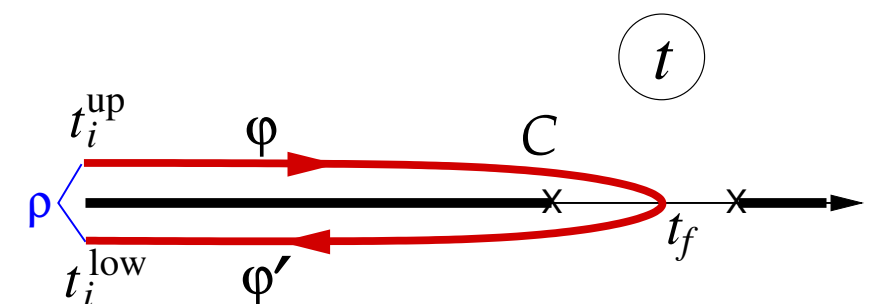
# Bounce solution

$g \ll 1 \rightarrow$  saddle-point approximation is applicable

Denote the saddle-point configuration by  $\varphi_b(t, x)$ .

Assume it is unique.

- It is a solution of the classical equation of motion  $\square \varphi_b - m^2 \varphi_b - \Omega V'_{int}(\varphi_b) = 0$ .
- It lives on the contour  $C$ , its values on the upper and lower parts of the contour are complex conjugate.
- It is real at  $t = t_f$ . At  $t > t_f$  it describes the evolution of the field after tunnelling.
- It must linearise in the limit  $t \rightarrow t_i^{up/down}$ .
- In this limit, it must satisfy the vacuum boundary conditions.



Contour in the complex time plane for the calculation of the false vacuum decay probability in the in-in formalism.

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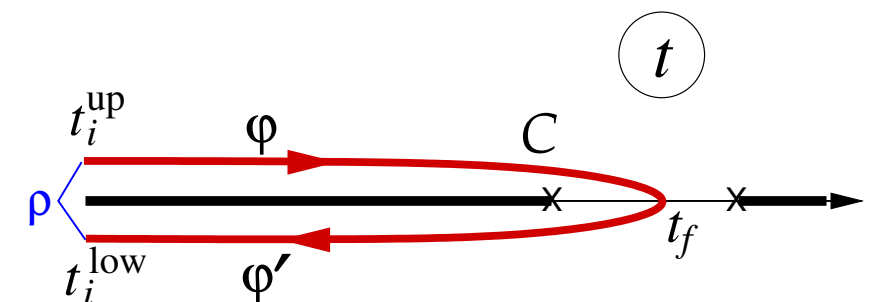
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What are they?

Long story short, they are the same as for the **time-ordered Green's function** in the corresponding vacuum  $G_x$ .

$$G_x(t, x, t', x') = \frac{1}{g^2} \langle T(\hat{\varphi}(t, x) \hat{\varphi}(t', x')) \rangle$$

$$(\square - m^2 \Omega(x)) G_x(t, x, t', x') = i \delta(t - t') \delta(x - x')$$



Contour in the complex time plane for the calculation of the false vacuum decay probability in the in-in formalism.

# Vacuum boundary conditions

Equation of motion for the bounce:  $\square \varphi_b - m^2 \Omega \varphi_b - \Omega V'_{int}(\varphi_b) = 0$

One can write the solution via the Green's function:

$$\varphi_b(t, x) = -i \int_C dt' dx' G(t, x, t', x') \Omega(x') V'_{int}(\varphi_b(t', x'))$$

↑  
It provides boundary conditions for  $\varphi_b(t, x)$ .

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$$X = B, HH, U$$

Finally, the decay rate is evaluated as  $\Gamma \sim e^{-B}$ ,

where  $B = -i \oint [\varphi_B] (+ \text{boundary terms})$ .

# OK, but can we proceed further?

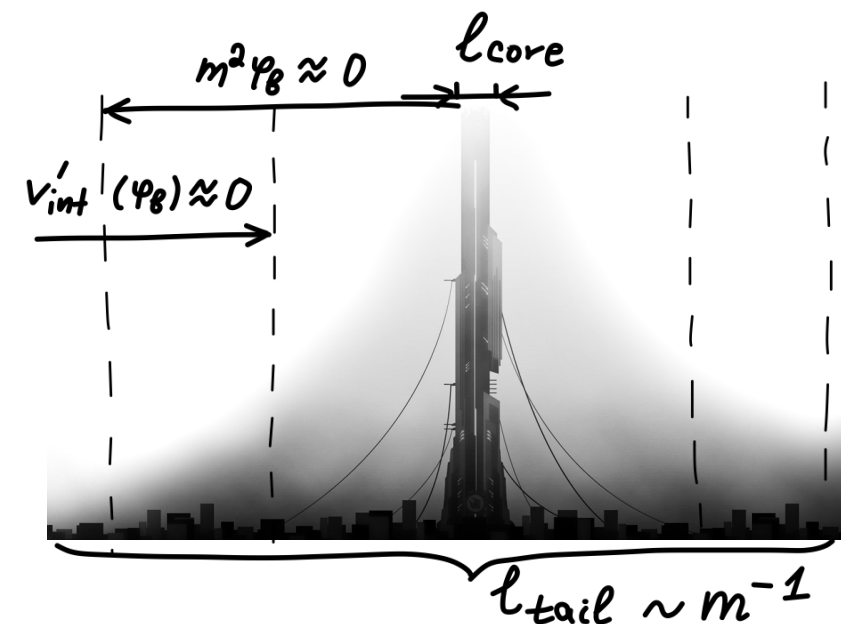
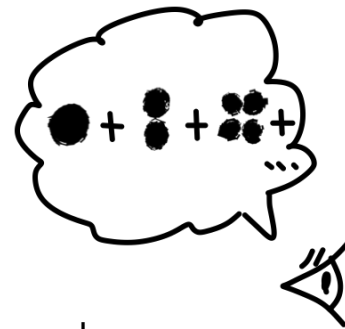
In general, the integral equation  $\psi_B(t, x) = -i \int_C dt' dx' G_X(t, x, t', x') \Omega(x') V'_{int}(\psi_B(t', x'))$  is hard to solve.

But let's devise a model in which the bounce has a narrow nonlinear core and a broad linear tail.

! It's quite opposite to the widely used thin-wall approximation.  
In fact, our solution is closer to the realistic bounce in the Higgs potential.

Then  $V'_{int}(\psi_B(t', x')) \propto \delta^{(2)}(t', x')$ .  
far enough from the core.

One can solve the equation separately in the regions where the mass term and the interaction term can be neglected, then match in the overlap.



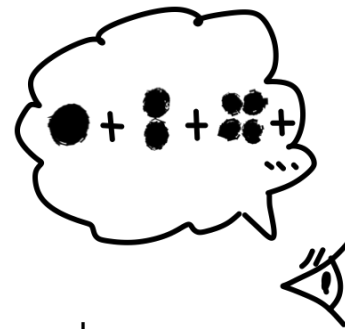
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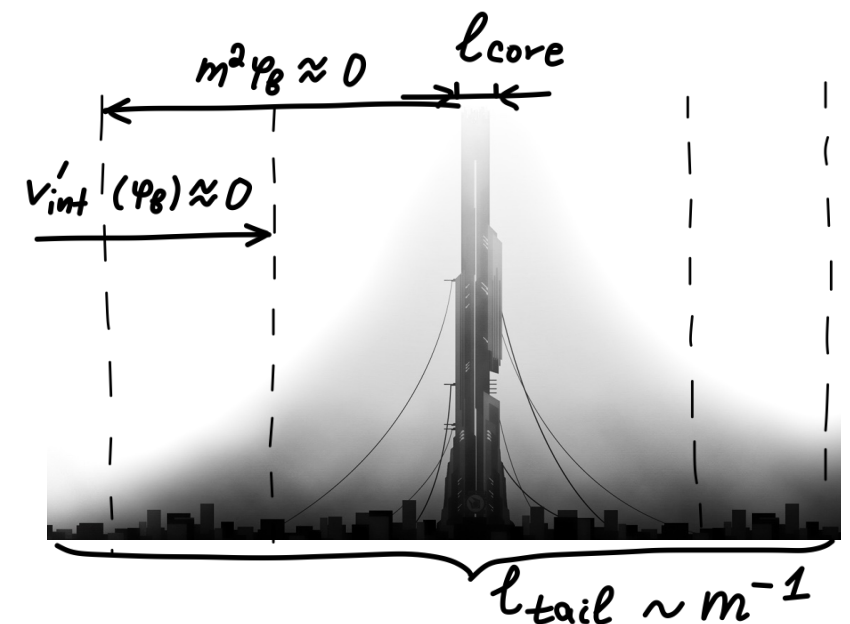
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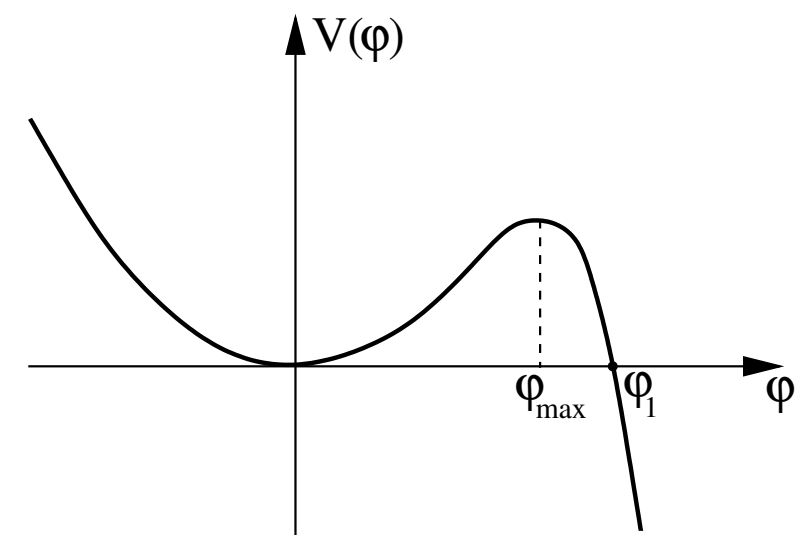


**Toy model:** inverted Liouville potential with a mass term in the dilaton BH background

$$V(\varphi) = \frac{m^2 \varphi^2}{2} - 2\kappa (e^\varphi - 1), \quad m^2, \kappa > 0 \quad \ln \frac{m}{\sqrt{\kappa}} \gg 1$$

$$\Omega(x) = \frac{1}{1 + e^{-2\lambda x}}$$

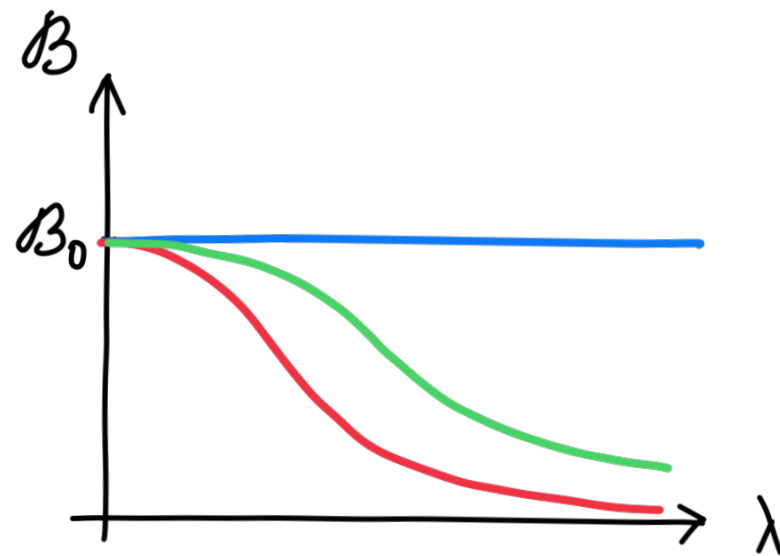
- The model admits **analytic solution** both for the core and tail of the bounce.
- The bounce can be found analytically in the near-horizon and asymptotically-flat regions.
- The bounce can be found in all three vacuum states (in a certain range of temperatures).



The toy model potential.

# Toy model: results

- For the **Boulware** and **Hartle-Hawking** vacua our method is equivalent to the known prescriptions of looking for vacuum and finite-temperature bounces. The method allowed us to compute the **Unruh** bounce in a certain range of temperatures.
- The suppressions are in agreement with expectations:  $\beta_{\text{HH}} < \beta_{\text{U}} < \beta_{\text{B}}$  and  $\beta_{\text{near}} < \beta_{\text{far}}$ .
- The catalysing effect is both due to geometry and due to excitations of the field modes by the BH. Both effects are of the same order and closely intertwined.
- Note that  $\beta_{\text{U}} \rightarrow 0$  at  $\lambda \rightarrow \infty$ . We think it's an artefact of working in two dimensions.



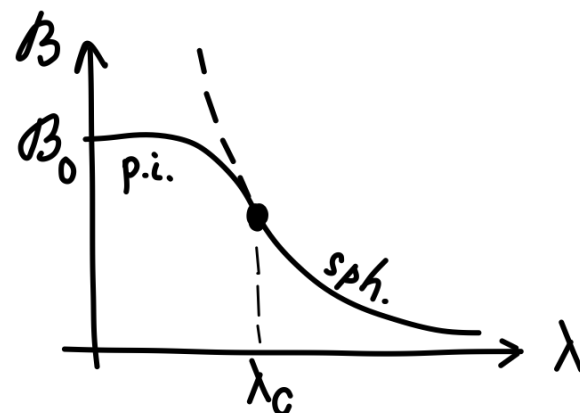
Suppression of the **Boulware**, **Hartle-Hawking** and **Unruh** vacuum decay as a function of BH temperature.



# Outlook: troubles with analytic solution

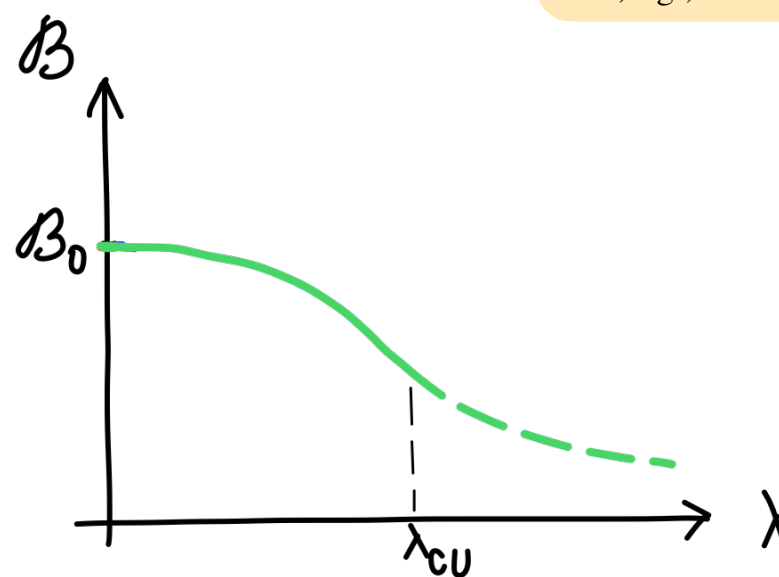
- It is known that in flat spacetime, finite-temperature periodic instanton degenerates into the time-independent solution — sphaleron — at some critical temperature. The transition regime changes from tunnelling to classical jumps over the potential barrier.

See, e.g., A. D. Linde, Nucl. Phys. B 216 (1983) 421



- Something similar happens with the Unruh bounce: at some temperature it disappears, giving way to the “Unruh sphaleron”. It’s not completely clear what the latter is: our cut-and-match procedure breaks down.
- In **our particular two-dimensional model**, we were able to find the high-temperature suppression by a simple analytical stochastic estimate:  $\Gamma \sim \exp\left(-\frac{\varphi_{\text{max}}^2}{2\delta\varphi^2}\right)$ . It doesn’t work in general. Classical simulation is needed.

See, e.g., D. Grigoriev, V. Rubakov, M. Shaposhnikov, Phys. Lett. B 326 (1989) 737

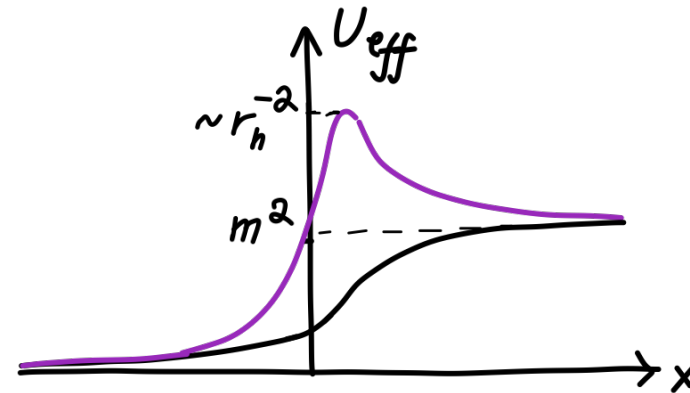


Suppression of the **Unruh** vacuum decay as a function of BH temperature.

# Outlook: graybody factors and more

- Our toy model captures some important features of a realistic BH in four dimensions. But not all of them:

- graybody factors:



- area growth  $\longrightarrow$  spreading of the particle flux.

- They are both expected to further **reduce** the **Unruh** vacuum decay rate. In principle, our method allows to take into account both of them.

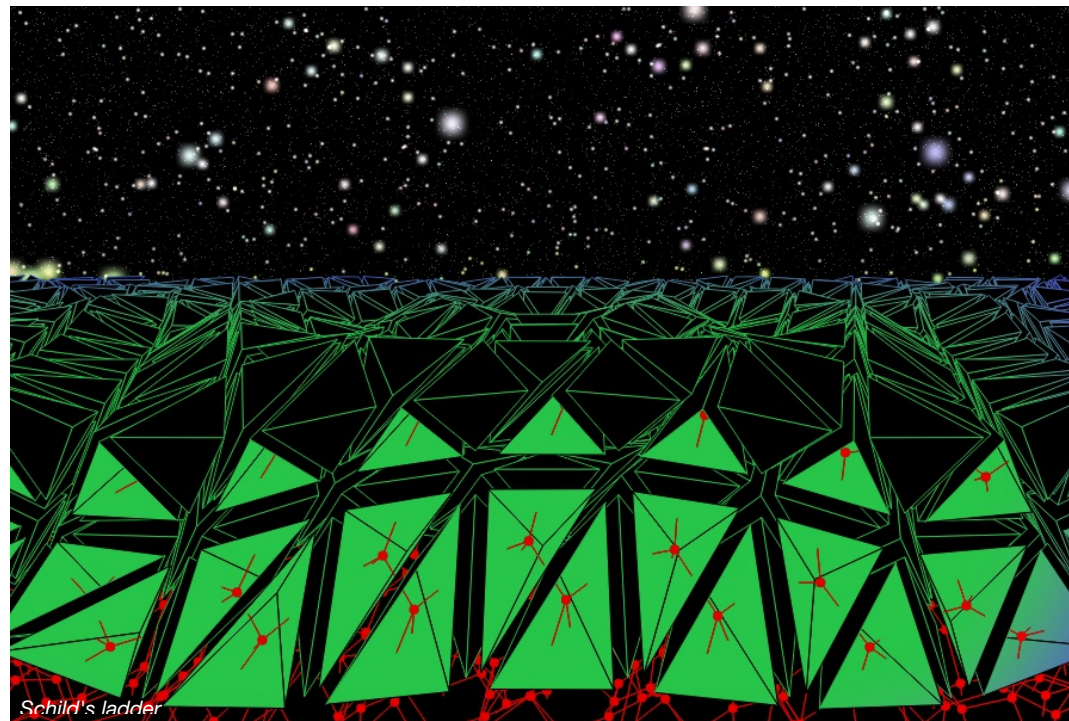
- Further research may include:

- pre-exponential factor,
- thermal corrections,
- dynamical gravity  $\longrightarrow$  window into semiclassical gravity, BH entropy,...

See, e.g., R. Gregory, I. Moss and B. Withers, JHEP 03 (2014) 081 [1401.0017]

- The method is quite general and can be applied to many other systems.

# Thank you!



# Appendix: linear modes

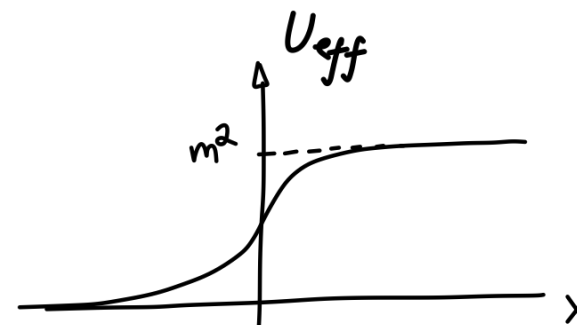
Positive/negative frequency modes:  $\varphi_{\omega}^{+}(t, x) = f_{\omega}(x) e^{-i\omega t}$

$$\varphi_{\omega}^{-}(t, x) = f_{\omega}^{*}(x) e^{i\omega t}$$

$$\omega > 0$$

EOM for modes:

$$-f_{\omega}'' + m^2 \Omega f_{\omega} = \omega^2 f_{\omega}$$



Mode asymptotics:

The effective potential for modes in the dilaton BH model.

$$f_{R,\omega} = \begin{cases} \alpha_{\omega} e^{i\omega x} + \beta_{\omega} e^{-i\omega x}, & x \rightarrow -\infty \\ \gamma_{\omega} e^{ikx}, & x \rightarrow +\infty \end{cases}$$

$$f_{L,\omega} = \begin{cases} \tilde{\beta}_{\omega} e^{-i\omega x}, & x \rightarrow -\infty \\ \tilde{\gamma}_{\omega} e^{ikx} + \tilde{\delta}_{\omega} e^{-ikx}, & x \rightarrow +\infty \end{cases} \quad k = \sqrt{\omega^2 - m^2}$$

Orthonormality relations for modes:

$$\int_{-\infty}^{+\infty} dx f_{R,\omega}(x) f_{L,\omega'}^{*}(x) = 0$$

$$\int_{-\infty}^{+\infty} dx f_{R,\omega}(x) f_{R,\omega'}^{*}(x) = 2\pi \delta(\omega - \omega')$$

$$\int_{-\infty}^{+\infty} dx f_{L,\omega}(x) f_{L,\omega'}^{*}(x) = 2\pi \delta(\omega - \omega'), \quad \omega > m$$

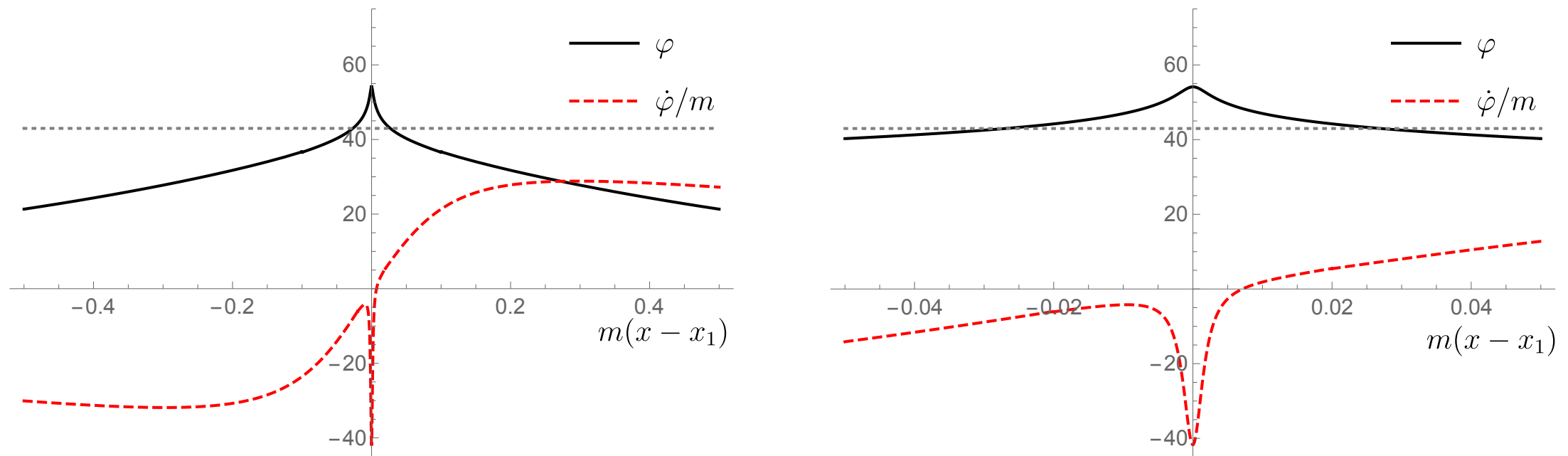
$|\beta_{\omega}|^2$  — reflection coefficient

$|\gamma_{\omega}|^2$  — transmission coefficient

# Appendix: Unruh Green's function

$$\begin{aligned}
 G_{\text{U}}(t, x, t', x') = & \int_0^\infty \frac{d\omega}{4\pi\omega} \left\{ f_{R,\omega}(x) f_{R,\omega}^*(x') \left[ \frac{e^{-i\omega|t-t'|}}{1 - e^{-\frac{2\pi\omega}{\lambda}}} + \frac{e^{i\omega|t-t'|}}{e^{\frac{2\pi\omega}{\lambda}} - 1} \right] \right. \\
 & + f_{L,\omega}(x) f_{L,\omega}^*(x') e^{-i\omega|t-t'|} \\
 & + (|\beta_\omega|^2 - 1) [f_{R,\omega}(x) f_{R,\omega}^*(x') - f_{L,\omega}(x) f_{L,\omega}^*(x')] \frac{e^{i\omega(t-t')}}{e^{\frac{2\pi\omega}{\lambda}} - 1} \\
 & \left. + \sqrt{\frac{R}{\omega}} [\gamma_\omega \beta_\omega^* f_{R,\omega}(x) f_{L,\omega}^*(x') + \gamma_\omega^* \beta_\omega f_{L,\omega}(x) f_{R,\omega}^*(x')] \frac{e^{i\omega(t-t')}}{e^{\frac{2\pi\omega}{\lambda}} - 1} \right\}
 \end{aligned}$$

# Appendix: Unruh instanton far from horizon



Bounce solution describing tunneling from the Unruh vacuum far away from the BH.

**Left:** Profiles of the bounce (black solid) and its time derivative (red dashed) at  $t=0$  for  $\lambda = 0.87 \Lambda_{U1}$ .

**Right:** Zoom-in on the central region of the left plot. We take  $\ln \frac{m}{\sqrt{\kappa}} = 20$ .

The grey dotted line marks the field value  $\varphi_{\max}$  at the maximum of the potential barrier.

$$\Lambda_{U1} = \frac{3\pi m}{4} \left( \ln \frac{m}{\sqrt{\kappa}} + \gamma_E - \frac{1}{4} \right)$$