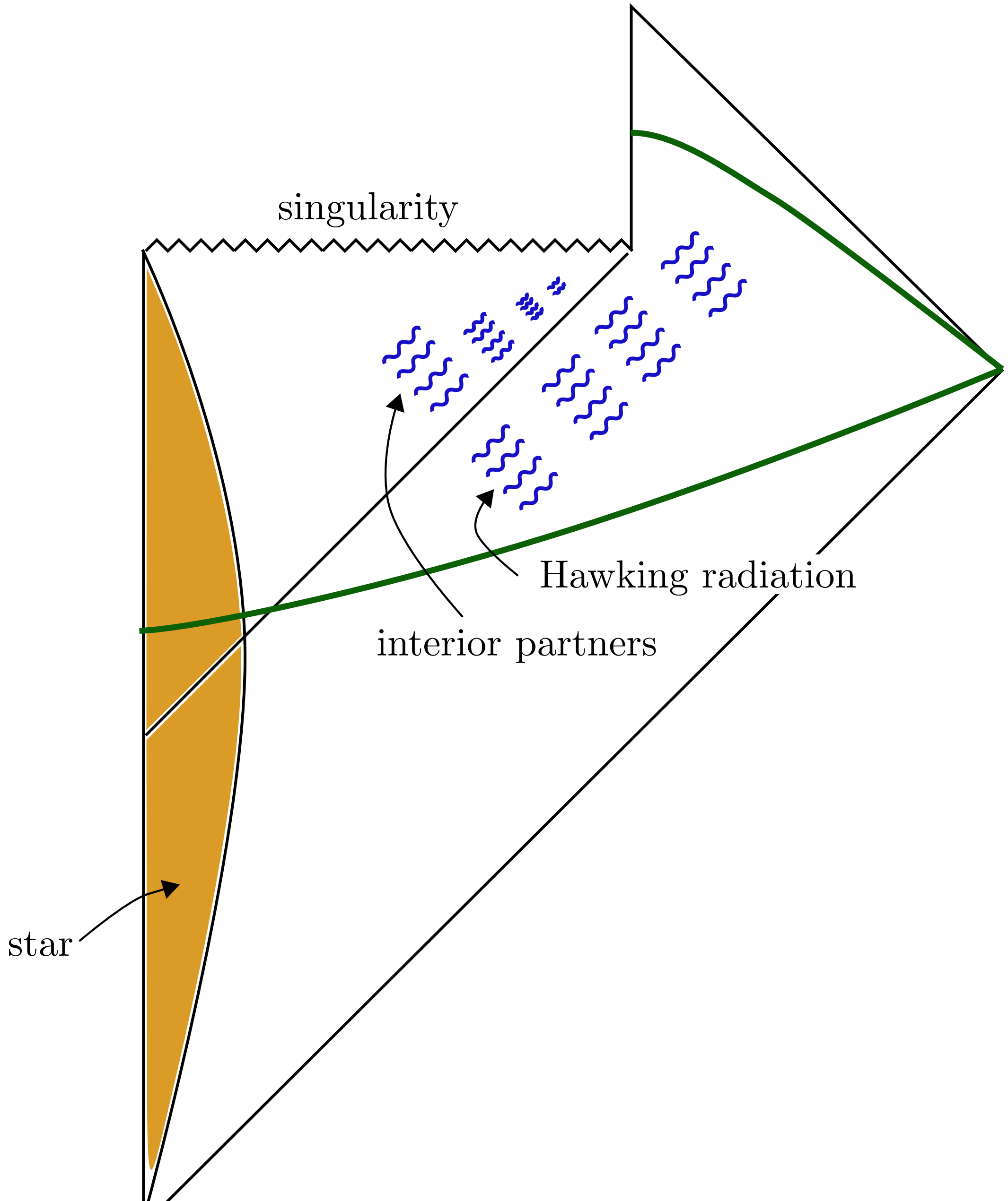
Replica wormholes and the information paradox

Tom Hartman
Cornell University

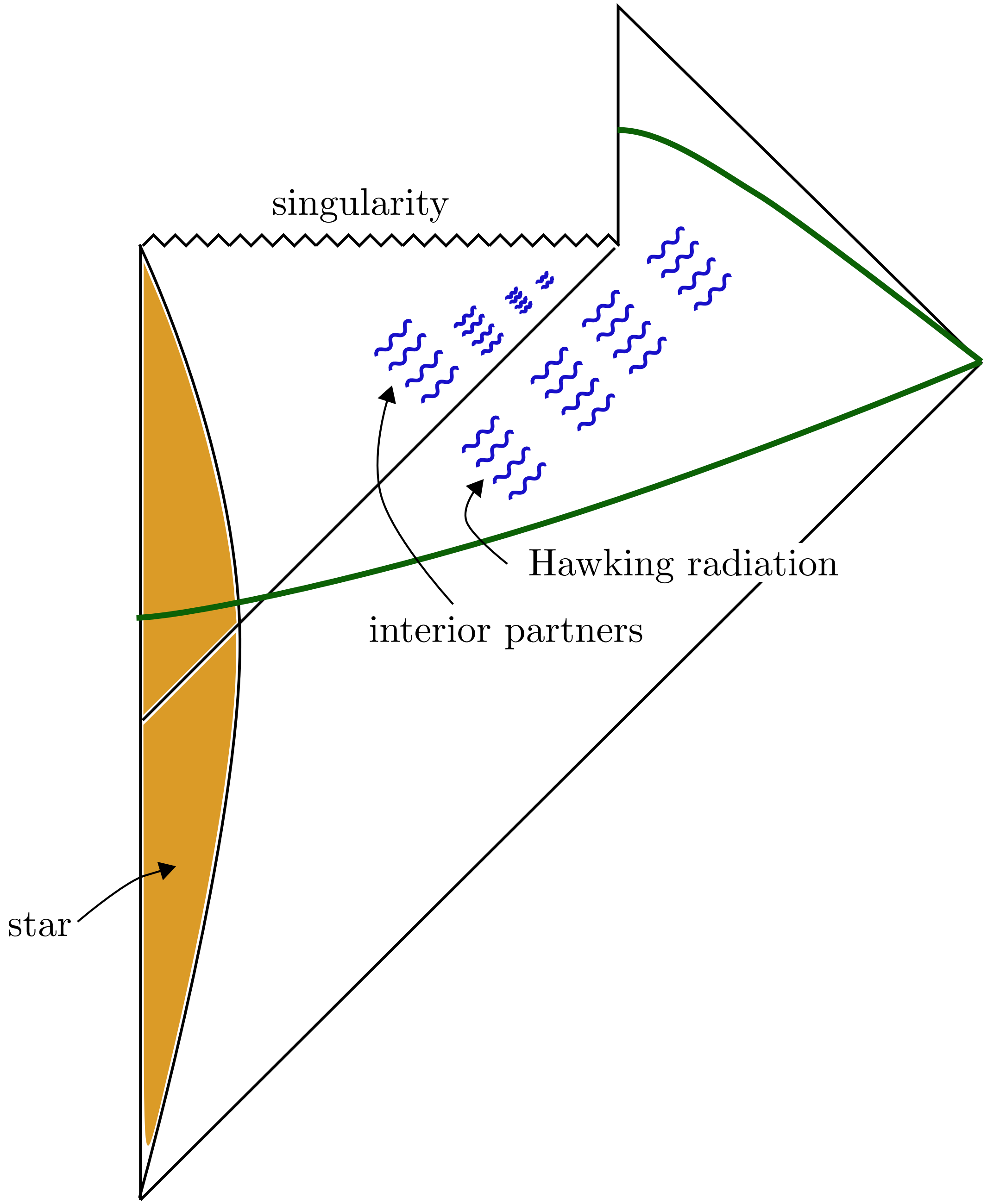
Quarks Conference ✦ June 5, 2021

Background:
The Page Curve

The information paradox

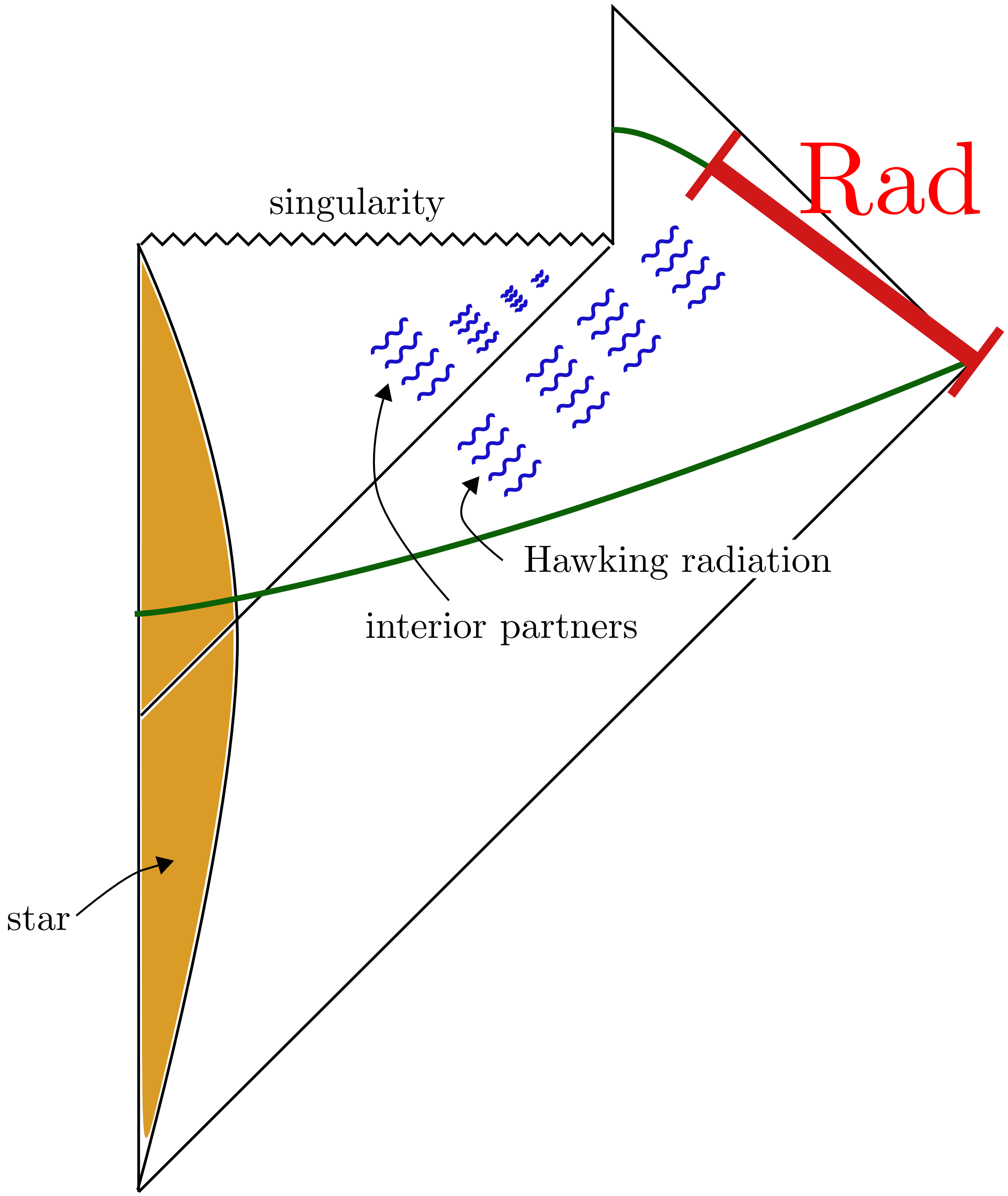


The information paradox



Hawking radiation is a process of *entanglement production* between the black hole interior and the radiation.

The information paradox



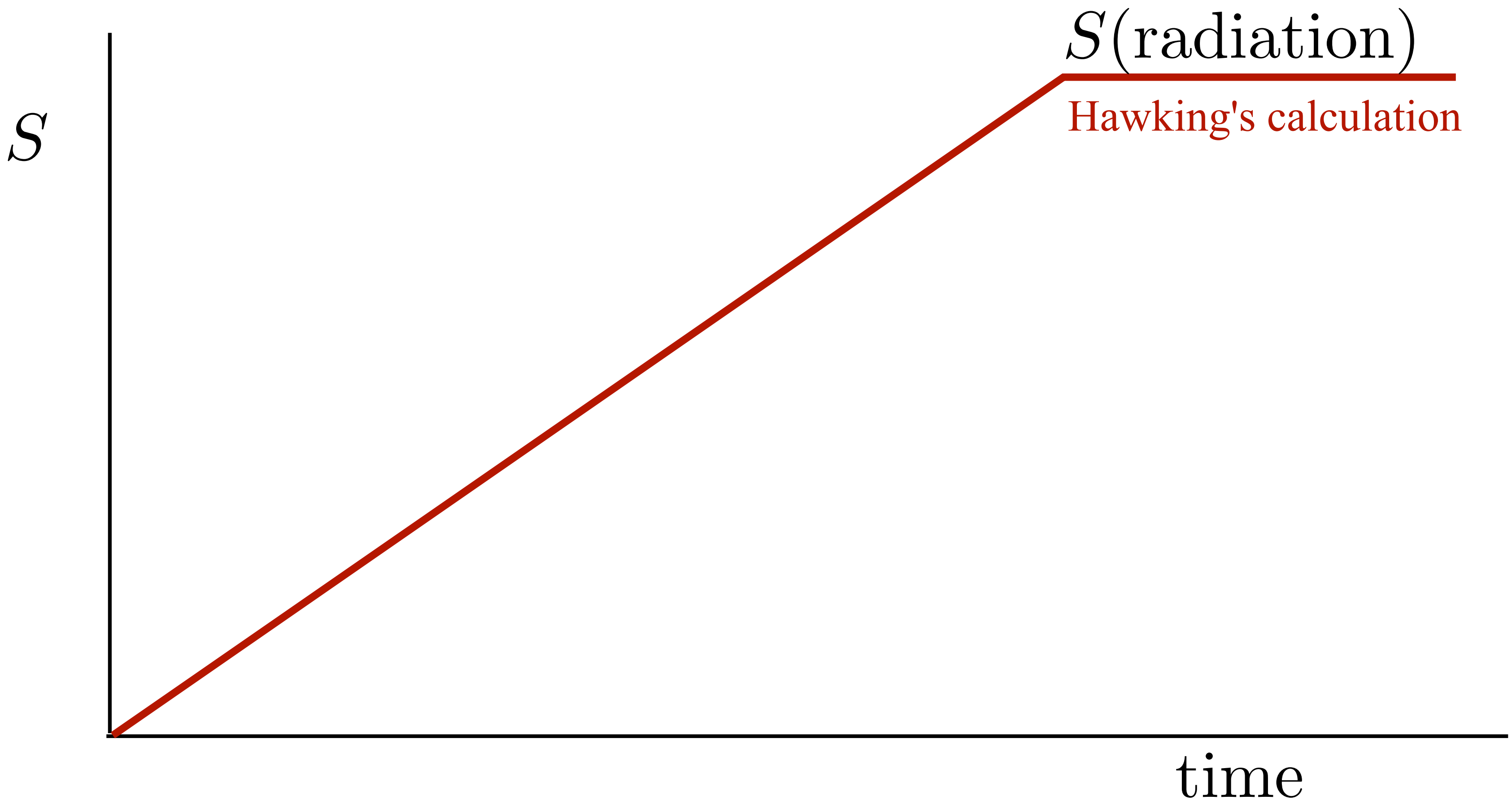
Consider the fine-grained (von Neumann) entropy of the radiation

$$S(\mathrm{Rad}) = -\mathrm{tr}\rho_R \log \rho_R$$

Fine-grained vs. coarse-grained

The Page Curve

The Page Curve



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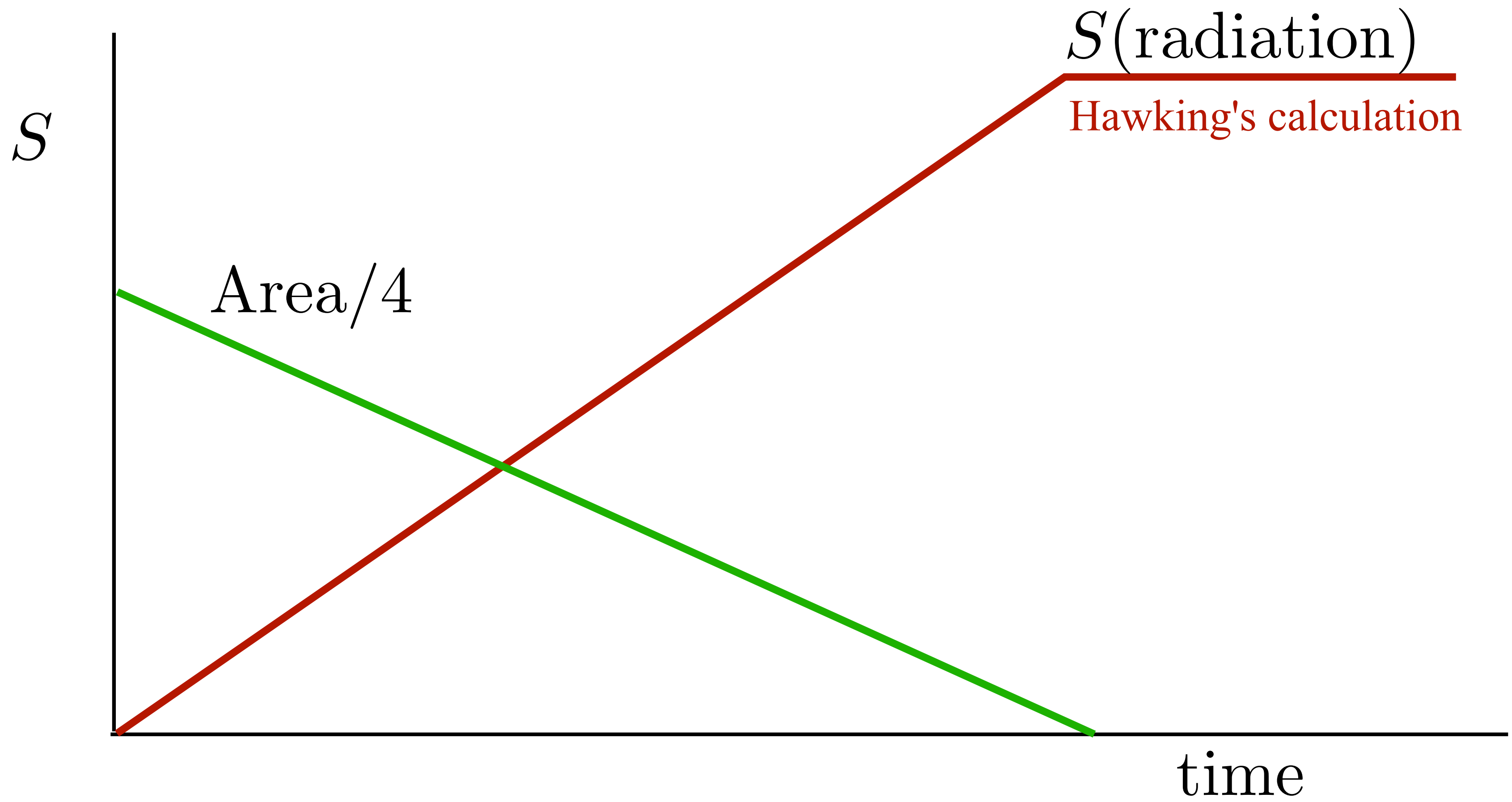

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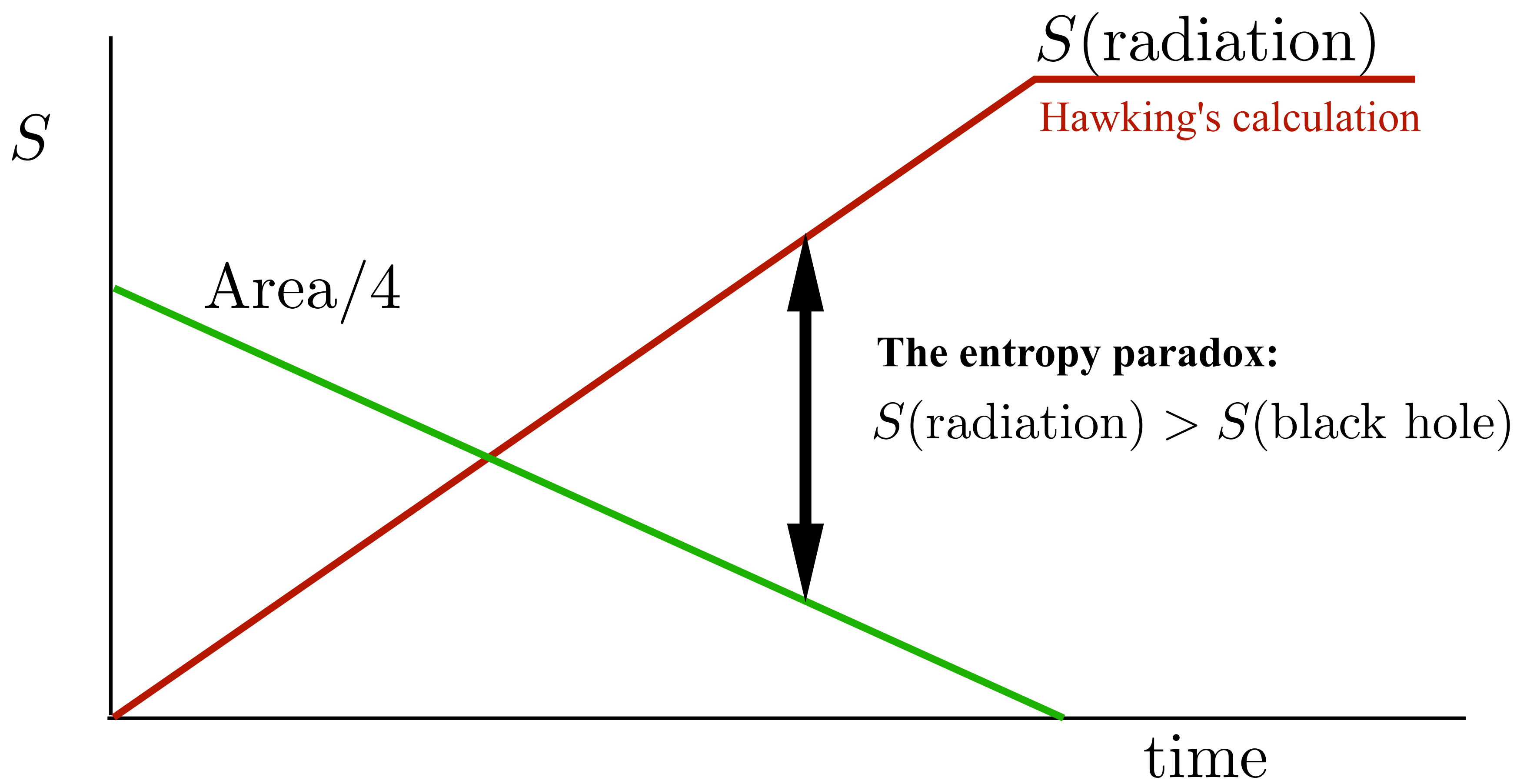

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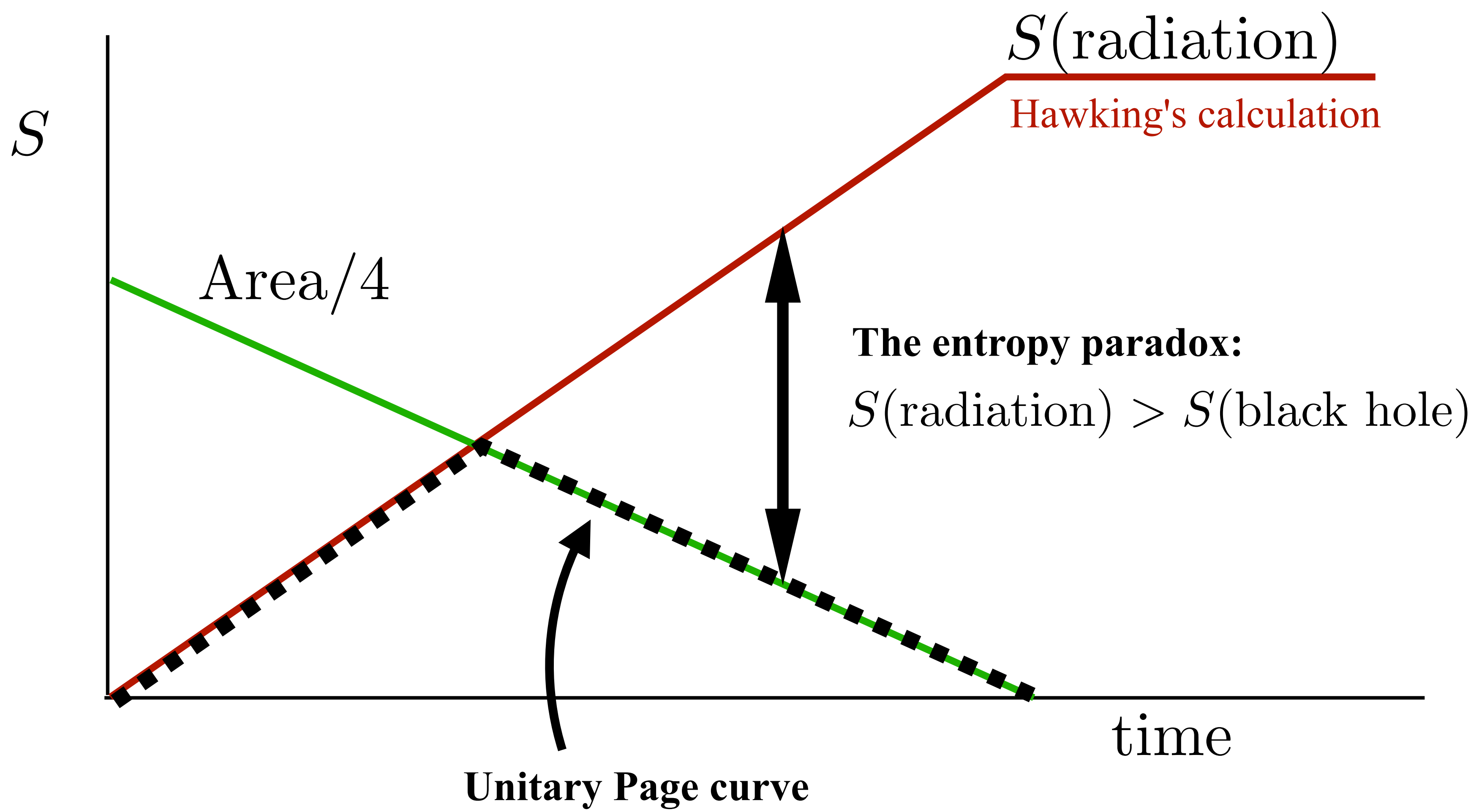

Hawking:

$$\rho_R = \rho_{\text{thermal}} + \text{perturbative} + \mathcal{O}(e^{-\#S})$$

e^{-S} corrections to each matrix element $(\rho_R)_{mn}$ are big enough to fix the entropy

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Why is it a "paradox"?

- Local / perturbative corrections don't help
- No known *mechanism* for information escape

Summary of new developments

e^{-S} corrections to the gravitational path integral produce large corrections to the entropy.

This calculation gives a small entropy, consistent with unitary evaporation.

Nothing in this calculation requires string theory or AdS/CFT.

Path integral methods sidestep some of the most difficult aspects of the paradox.

So this addresses just one piece of the information puzzle.

Holographic entanglement entropy

[Ryu and Takayanagi ’06], [Hubeny, Rangamani, Takayanagi ’07], [Lewkowycz, Maldacena ’13], [Barella, Dong, Hartnoll, Martin ’13], [Faulkner, Lewkowycz, Maldacena ’13], [Engelhardt, Wall ’14], [Dong, Lewkowycz ’17]

The “Island formula” for the radiation entropy

[Penington ’19]
[Almheiri, Engelhardt, Marolf, Maxfield ’19]
[Almheiri, Mahajan, Maldacena, Zhao ’19]

Replica wormholes

[Almheiri, TH, Maldacena, Shaghoulian, Tajdini ’19]
[Penington, Shenker, Stanford, Yang ’19]

Conceptual review article: arXiv 2006.06872 [Almheiri, TH, Maldacena, Shaghoulian, Tajdini]

Islands

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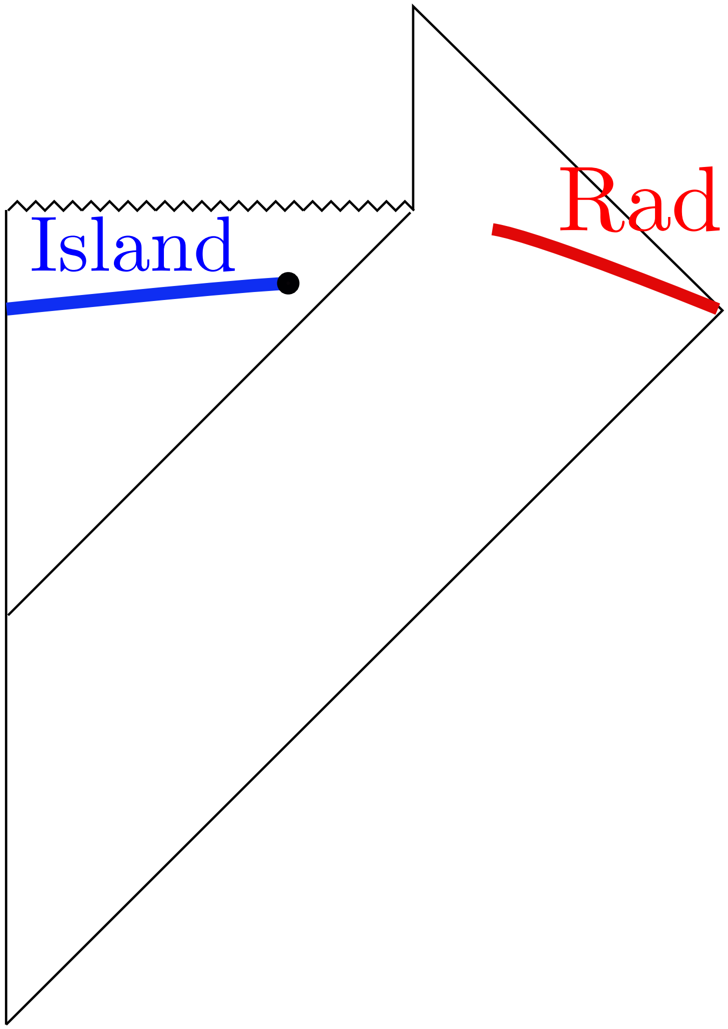

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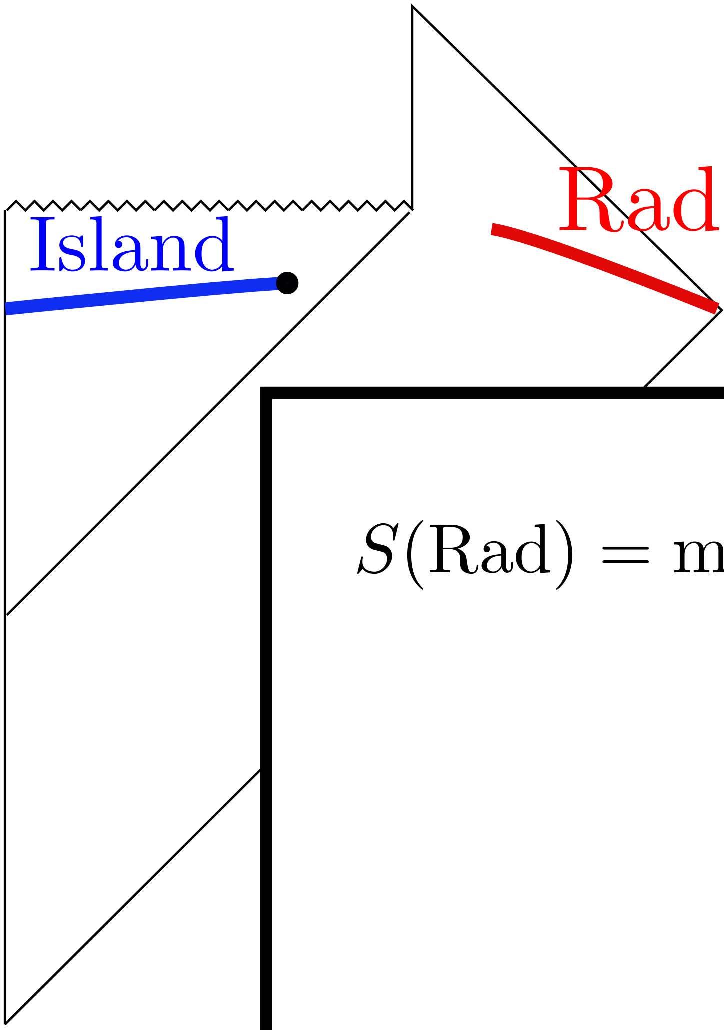

$$S(\mathrm{Rad}) = \mathrm{min}_I \mathrm{ext}_I \left[\frac{\mathrm{Area}(\partial I)}{4} + S_{\mathrm{QFT}}(I \cup \mathrm{Rad}) \right]$$

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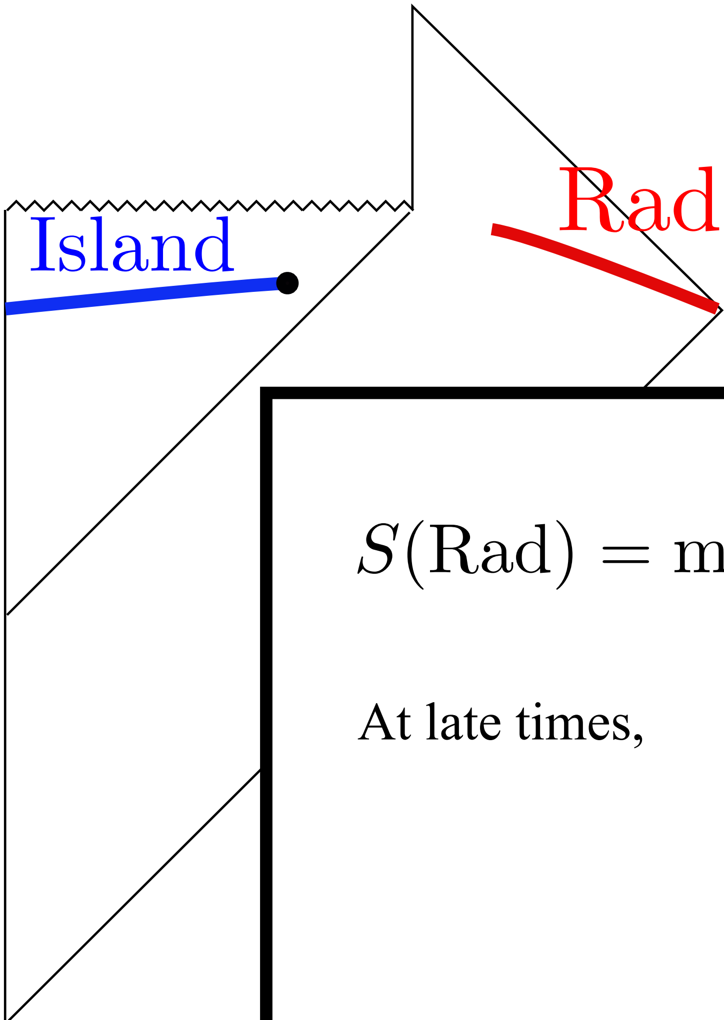

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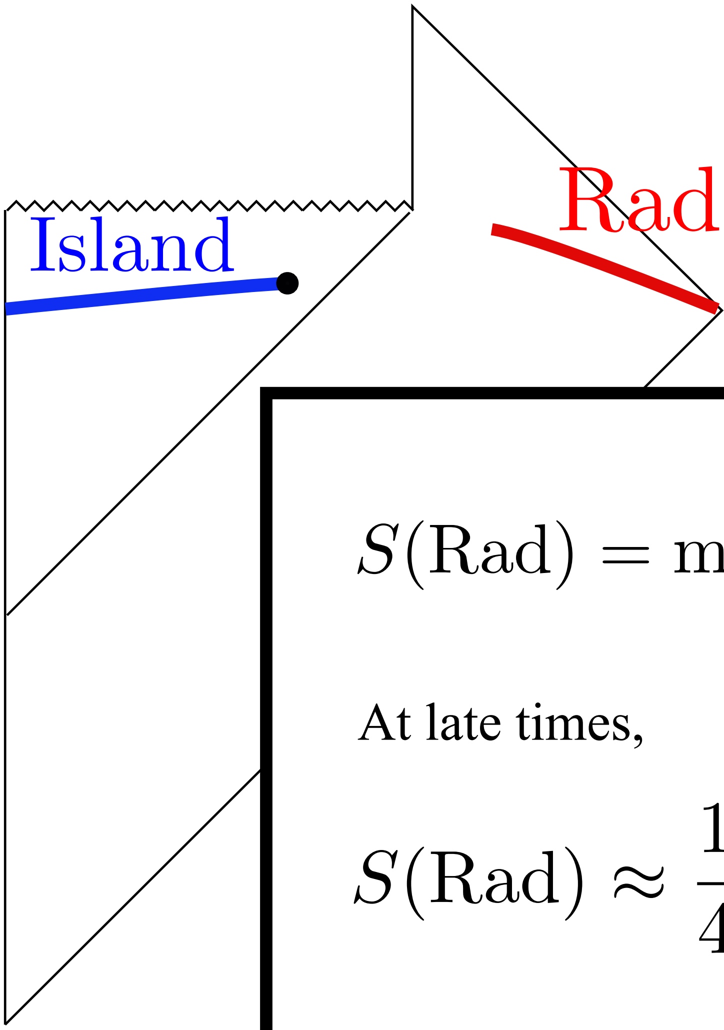


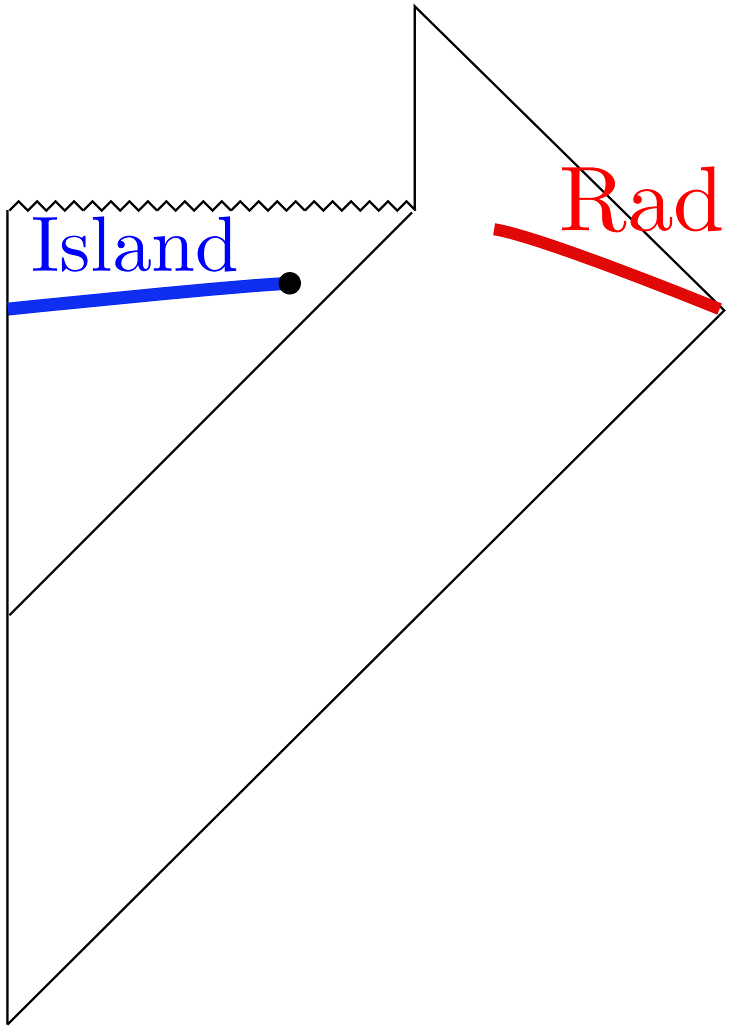
$$S(\mathrm{Rad}) = \min_I \mathrm{ext}_I \left[\frac{\mathrm{Area}(\partial I)}{4} + S_{\mathrm{QFT}}(I \cup \mathrm{Rad}) \right]$$

At late times,

$$S(\mathrm{Rad}) \approx \frac{1}{4}\mathrm{Area}(\mathrm{horizon})$$

$\to 0$ as the black hole evaporates

Interpretation



The island is actually "encoded" in the radiation, in the sense of holographic duality.

This encoding is very complex and needs to be understood better.

Entanglement wedge reconstruction:

[Wall '12], [Czech, Karczmarek, Nogueira, Van Raamsdonk '12], [Headrick, Hubeny, Lawrence, Rangamani '14], [Almheiri, Dong, Harlow '14], [Dong, Harlow, Wall '16]

Replica wormholes

Goal: Derive the island formula directly from the Euclidean gravitational path integral.

[Almheiri, TH, Maldacena, Shaghoulian, Tajdini ’19]
[Penington, Shenker, Stanford, Yang ’19]

Borrowing methods developed earlier in:
[Lewkowycz, Maldacena ’13]
[Barella, Dong, Hartnoll, Martin ’13]
[Faulkner, Lewkowycz, Maldacena ’13]
[Dong, Lewkowycz, Rangamani '16]
[Dong, Lewkowycz ’17]

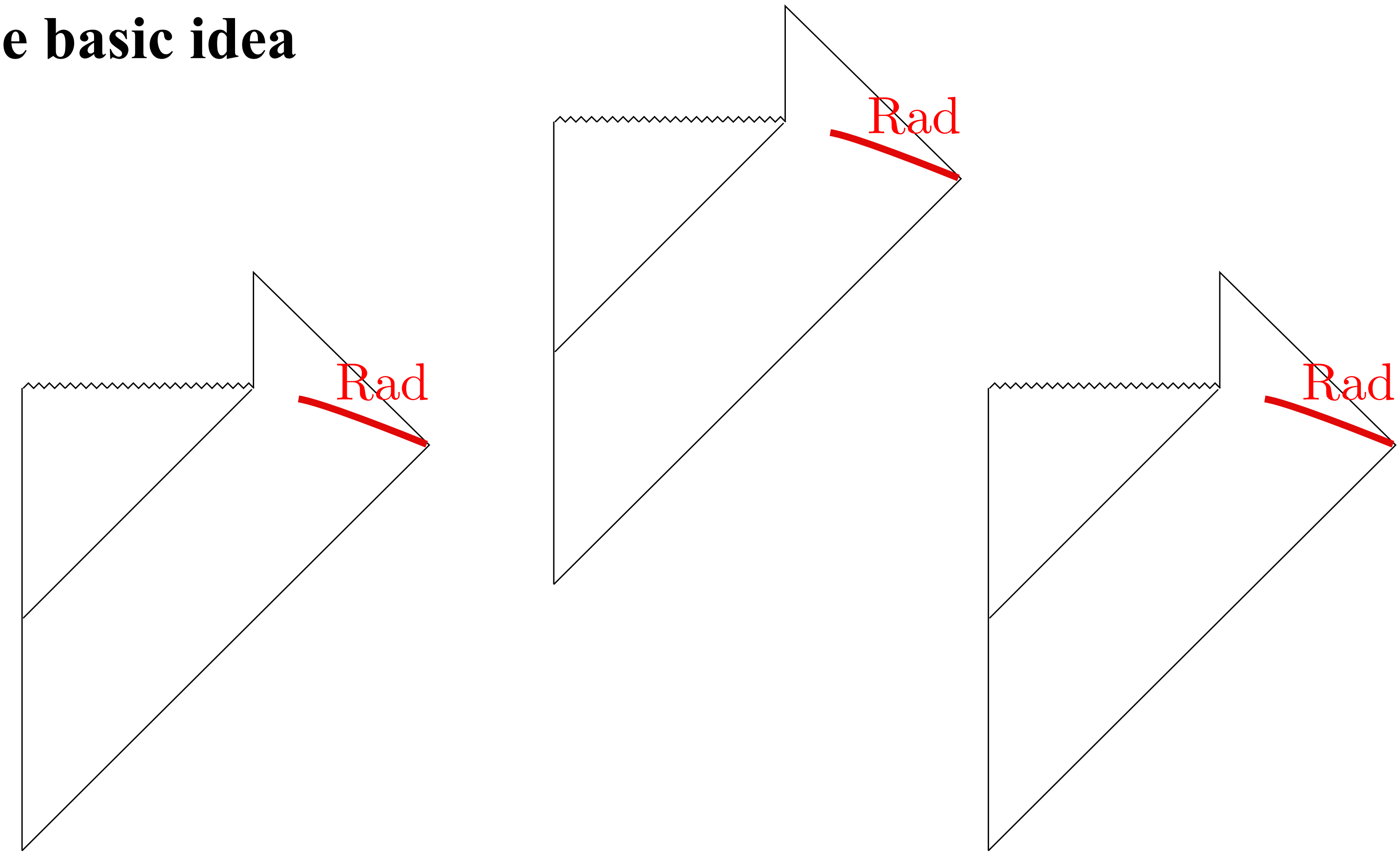
Replica method

$$S(\rho_R) = -\mathrm{tr}\,\rho_R \log \rho_R$$

$$Z(n) = \mathrm{tr}(\rho_R)^n, \qquad n = 1, 2, 3, \ldots$$

$$S(\rho_R) = -Z'(1)$$

The basic idea



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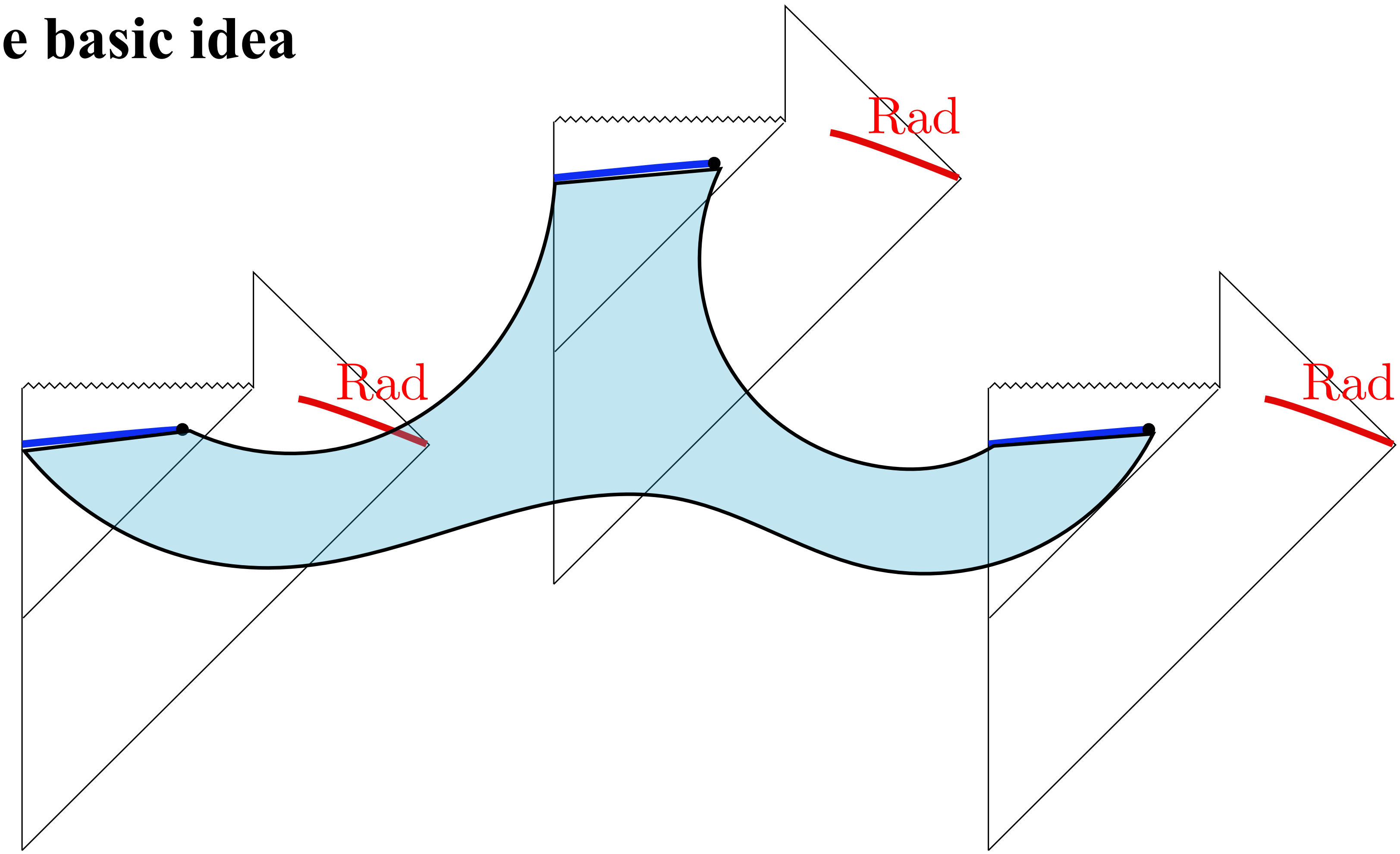

In the replica method, dynamical wormholes appear connecting the black hole interiors.

These are complex saddles (instantons) that we can construct by explicit solution of the EOM for gravity+matter in certain simple cases.

In the replica limit, these saddles leave an imprint on certain observables, including the von Neumann entropy.

Replica Calculation

$$Z(n) = \mathrm{tr}(\rho_R)^n$$

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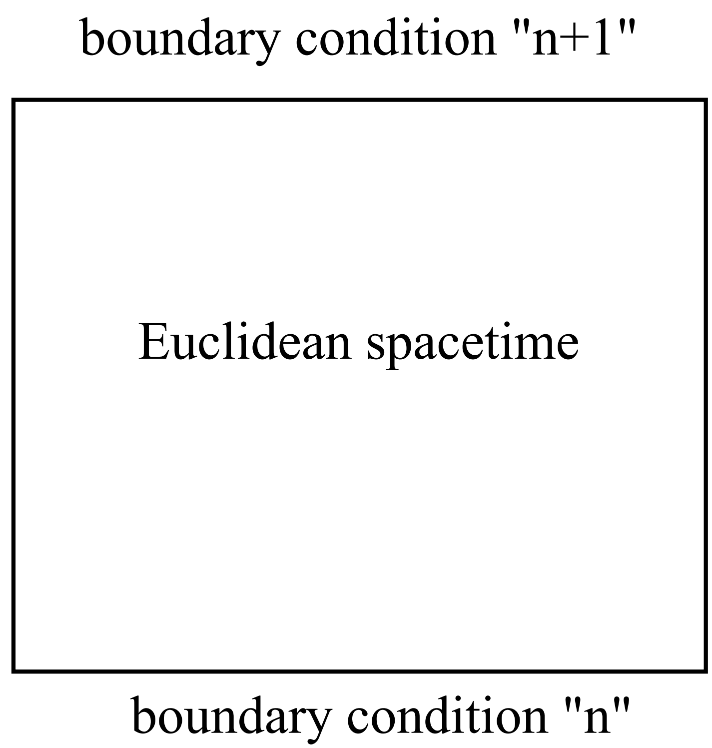
Recall the path integral calculation of a transition amplitude:

$$\langle n+1|n\rangle =$$



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n=1 replica in Euclidean signature

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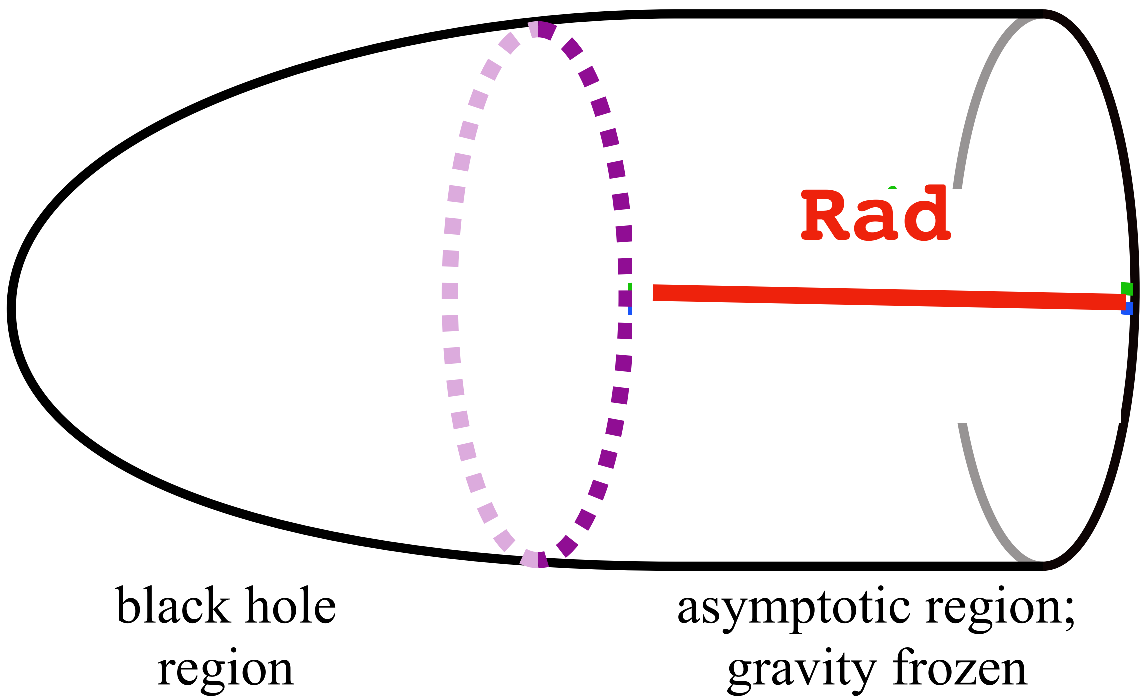

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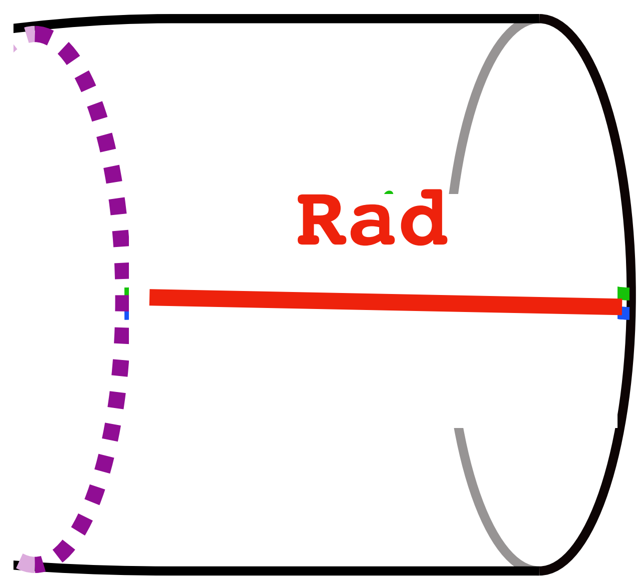

black hole region

asymptotic region; gravity frozen

Replica Calculation

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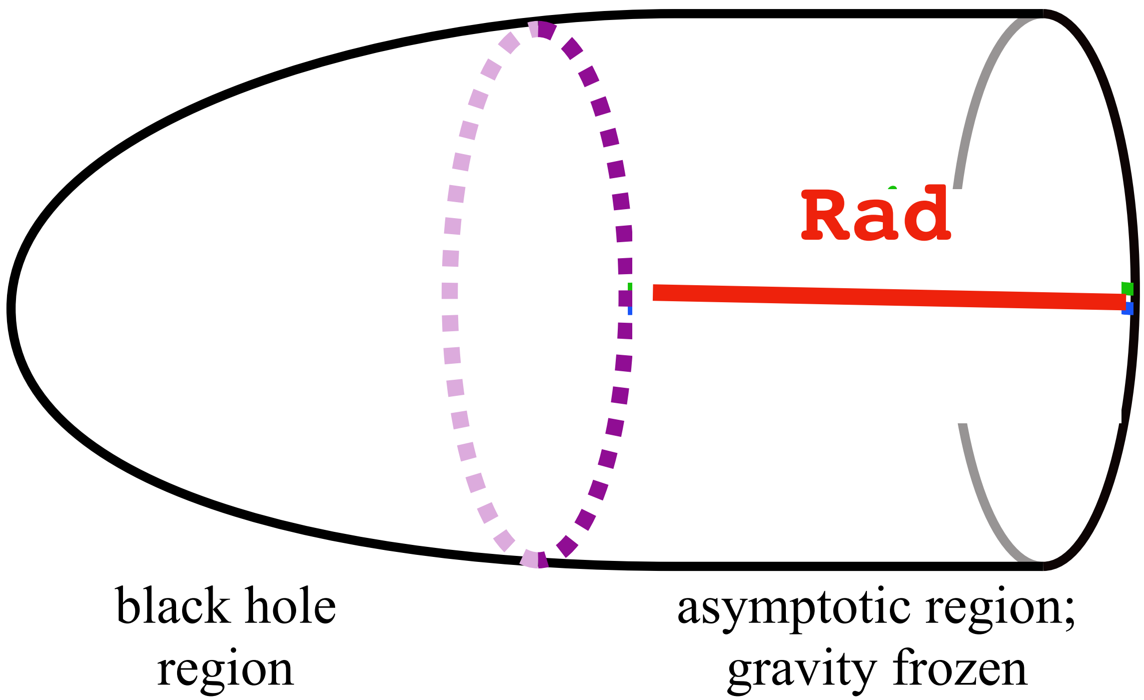

n=2 replicas

$$\mathrm{tr}\,(\rho_R)^2 =$$

_n=2_ replicas

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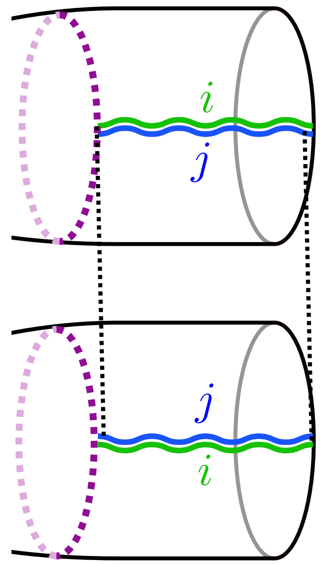

Hawking saddle

***n*=2 replicas**

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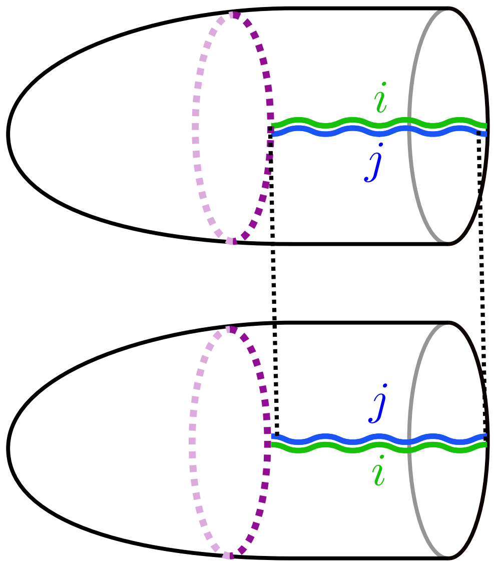

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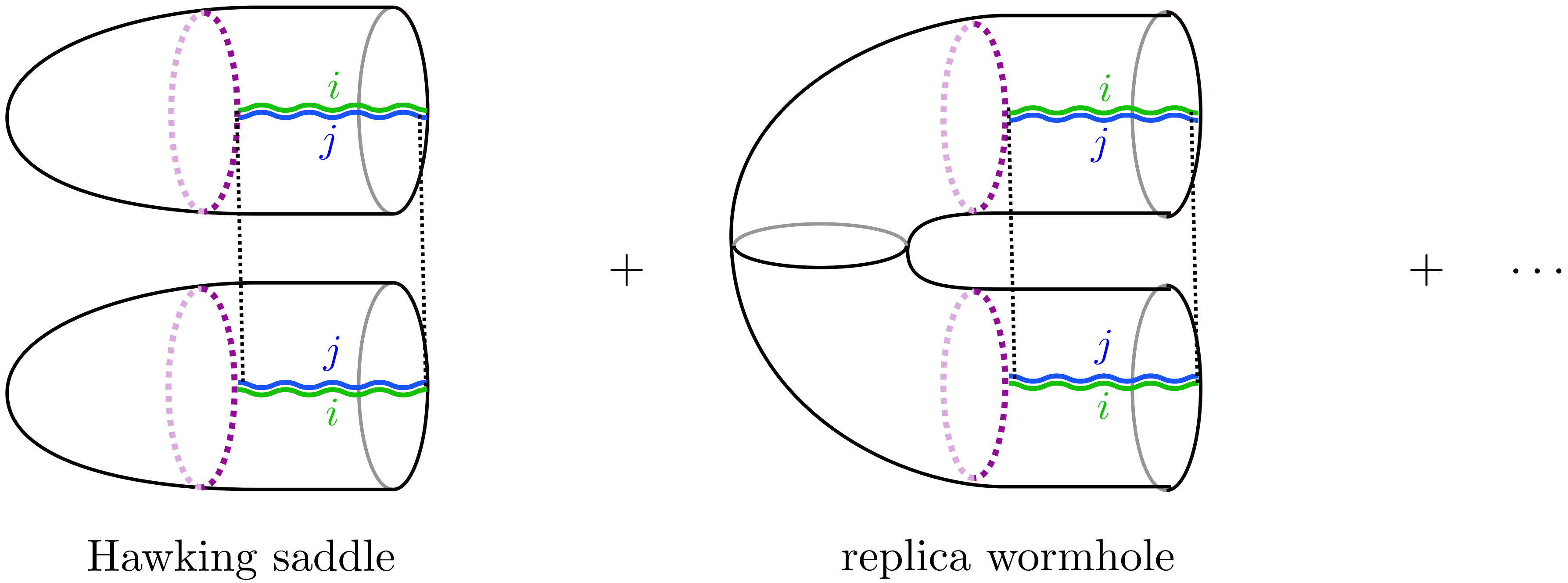

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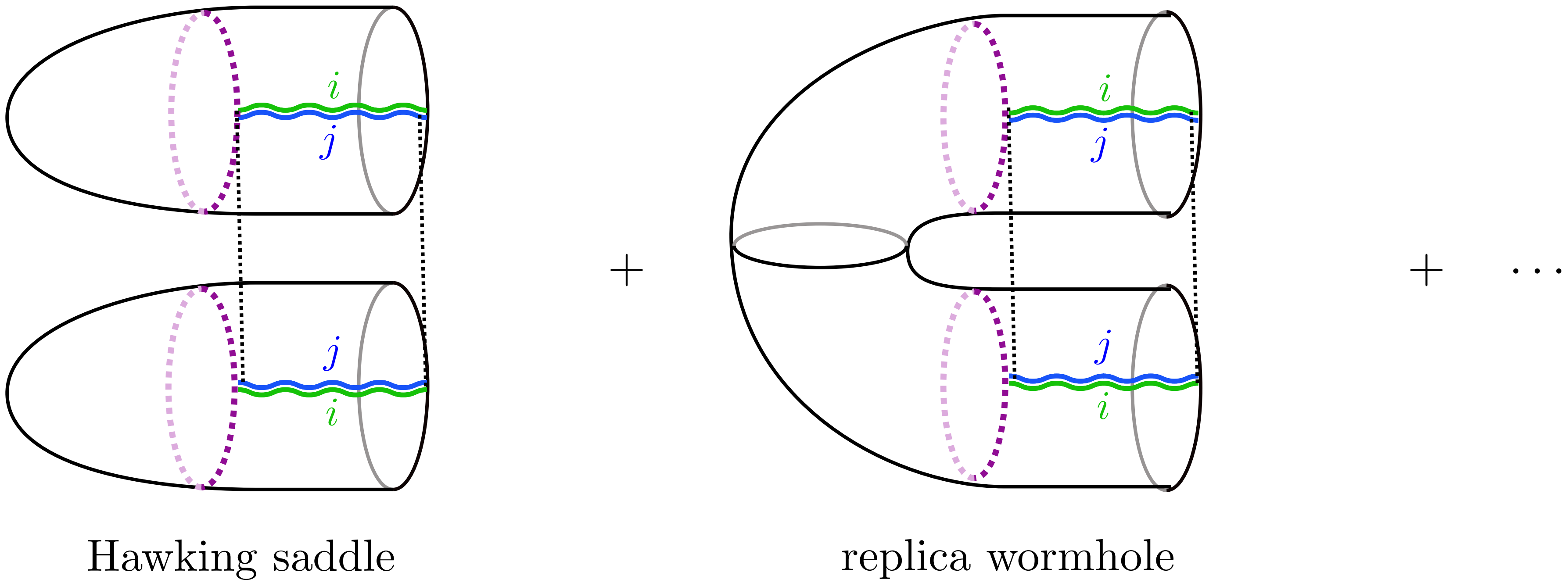

$$= \quad e^{-S_2^{\text{Hawking}}} \quad + \quad e^{-S_2^{\text{Wormhole}}} \quad + \quad \cdots$$

n=2 replicas

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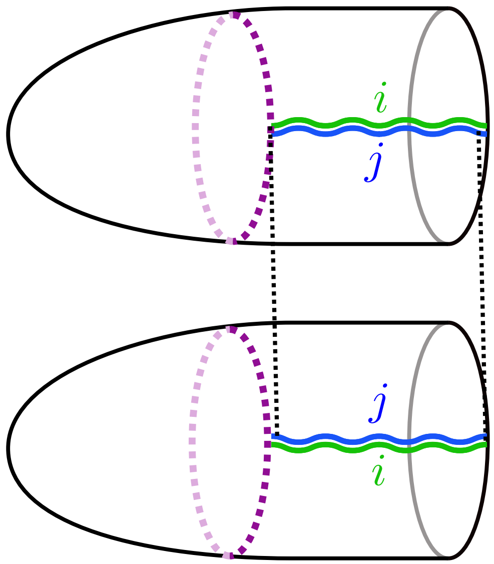
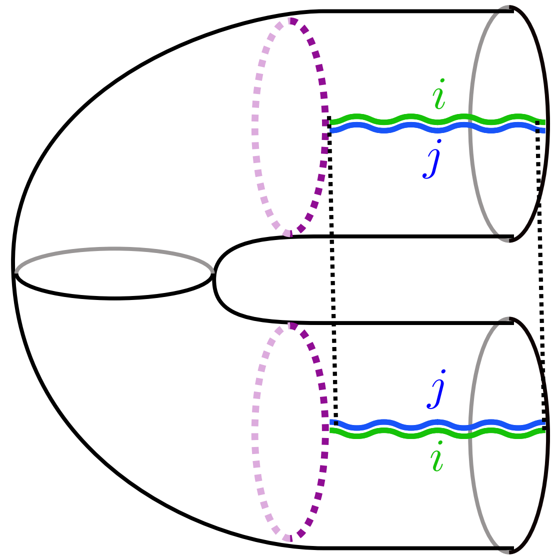


Hawking saddle

$$+$$



replica wormhole

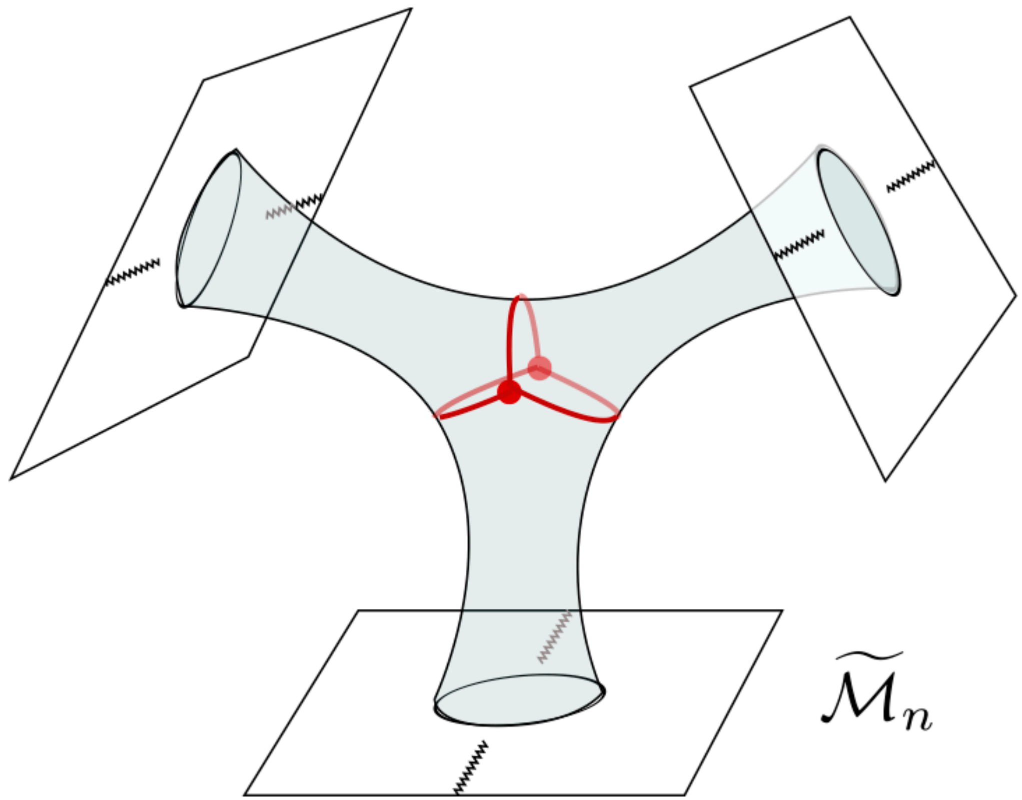
$$+ \quad \cdots$$

$$= \quad e^{-S_2^{\mathrm{Hawking}}} \quad + \quad e^{-S_2^{\mathrm{Wormhole}}} \quad + \quad \cdots$$

Suppressed by topology

Suppressed by large entanglement of radiation with interior

Higher replicas



von Neumann entropy

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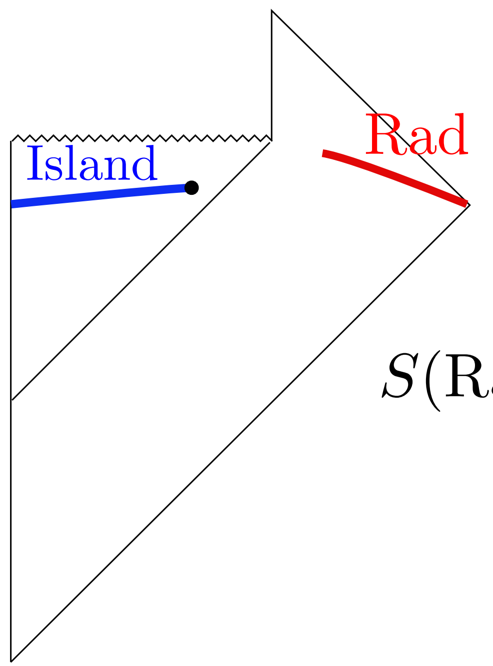
==> Island formula



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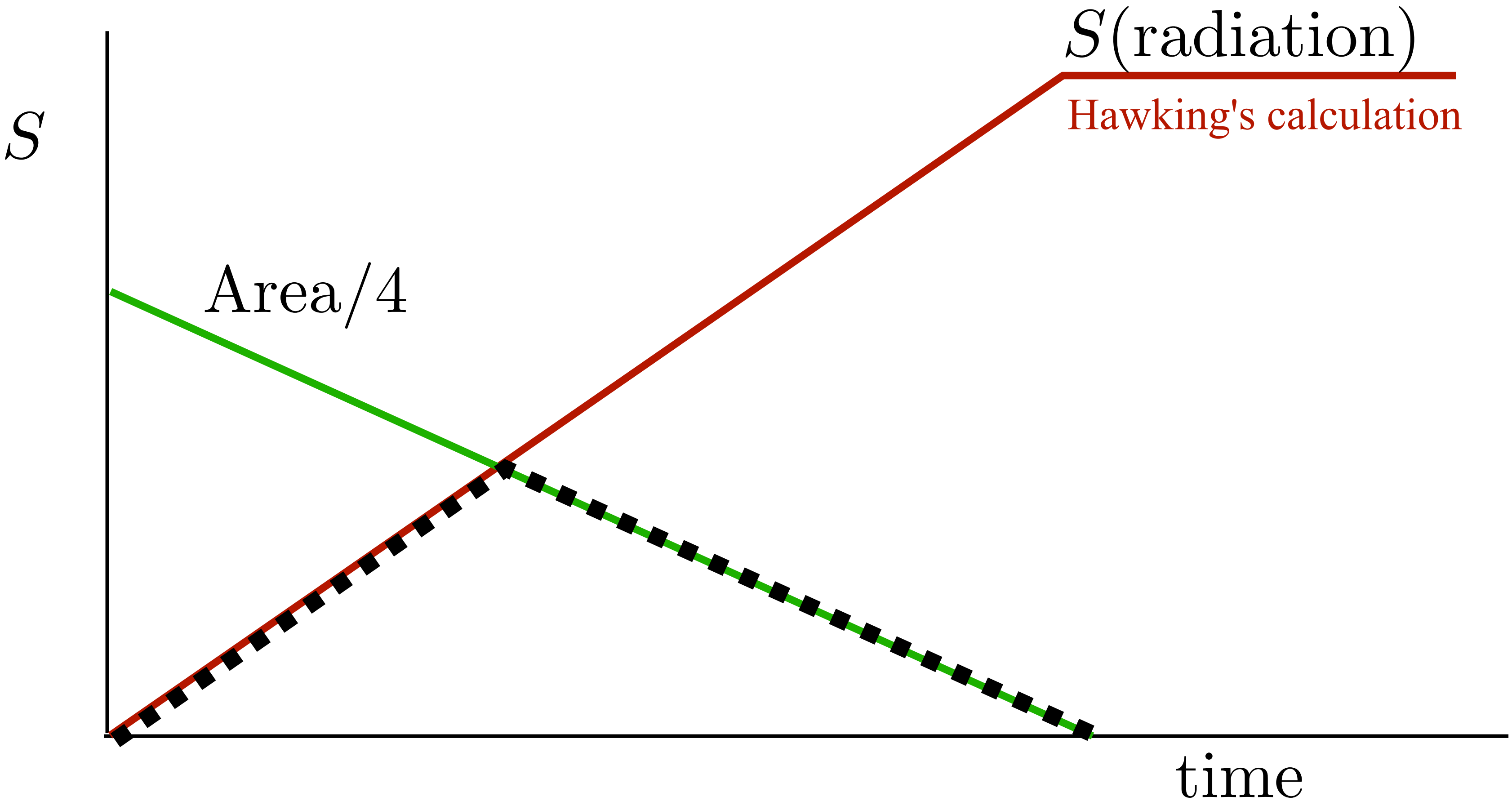
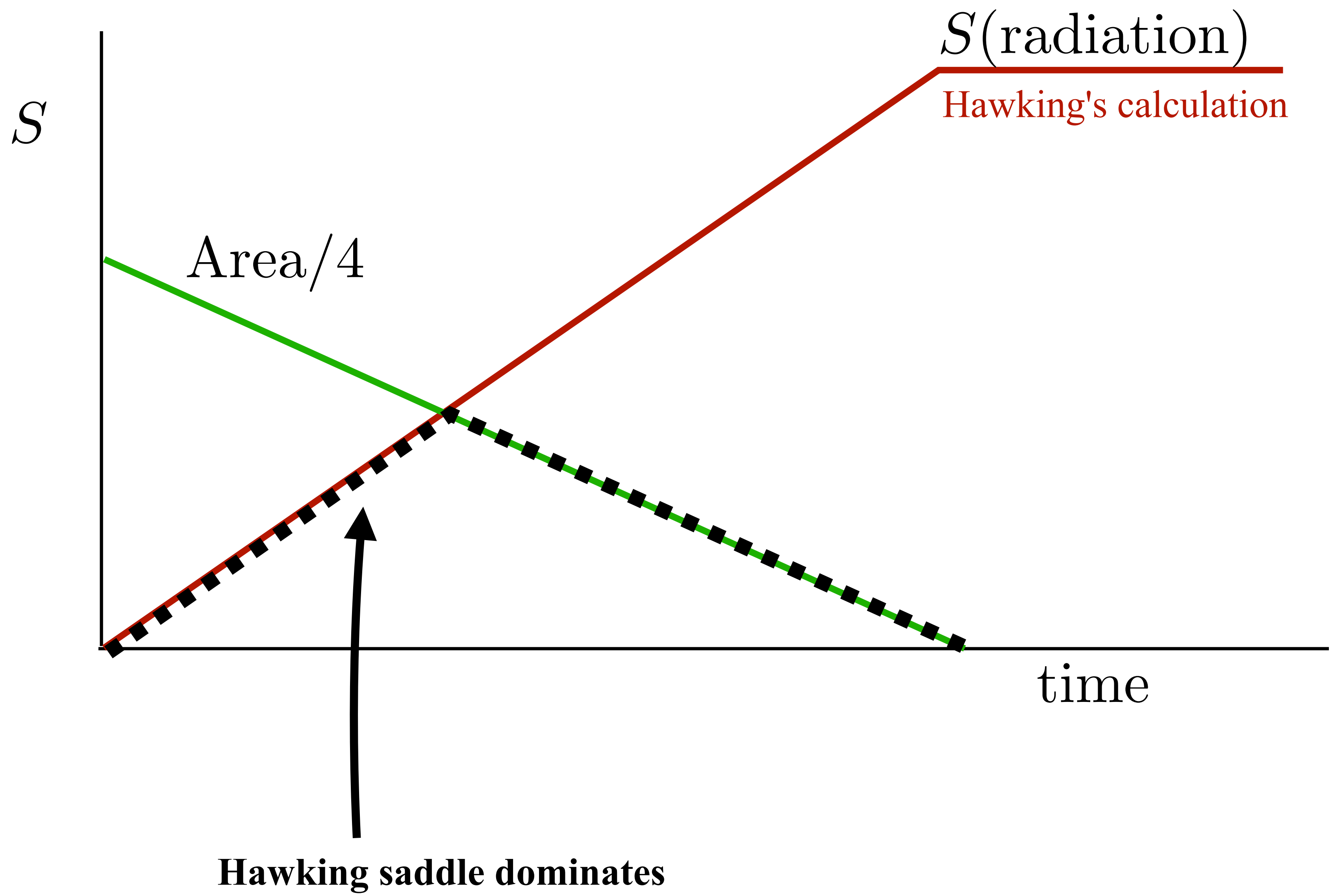

$S(\text{radiation})$
Hawking's calculation
S
Area/4
time



S(radiation)
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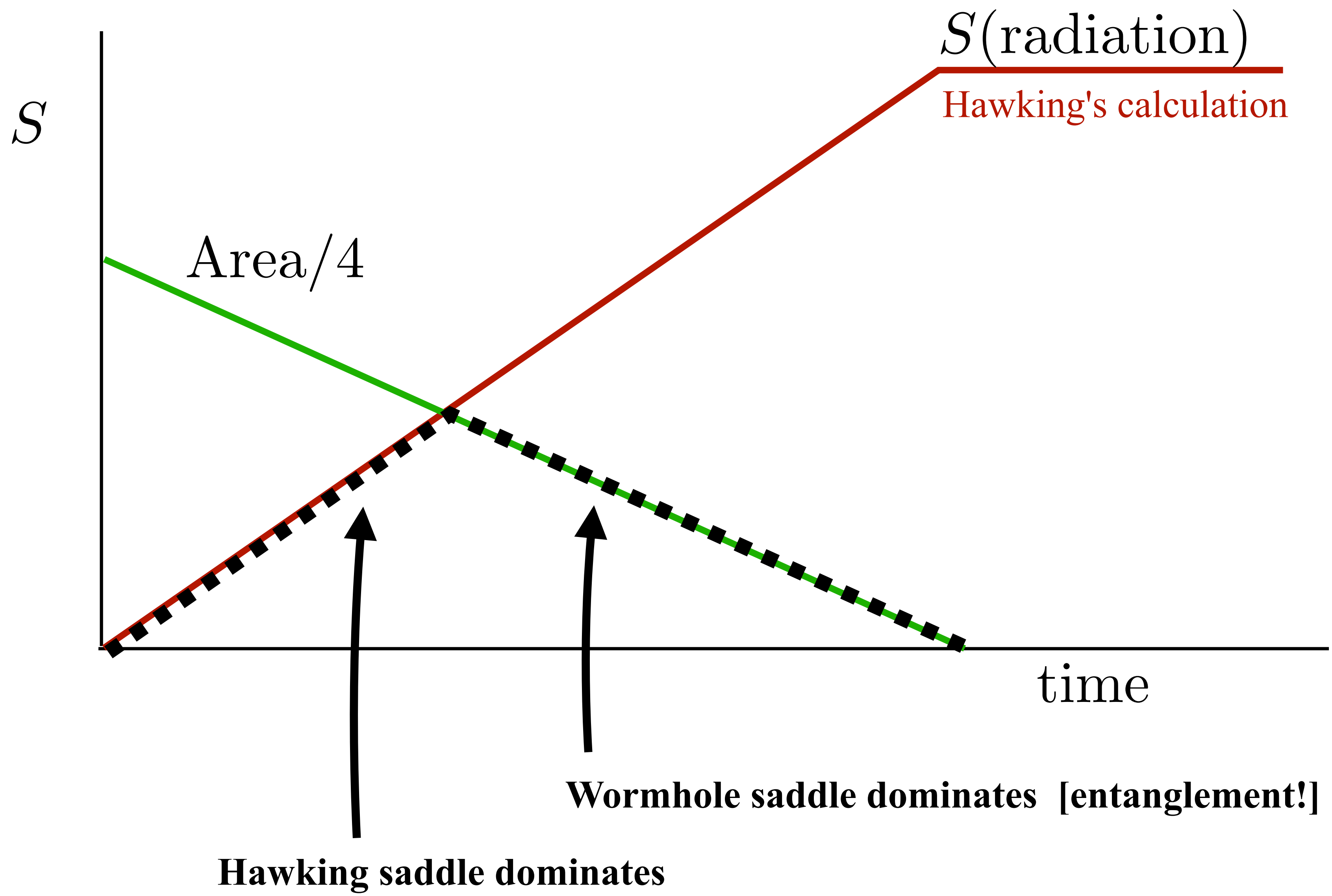
Hawking saddle dominates



S
S(radiation)
Hawking's calculation
Area/4
time
Wormhole saddle dominates [entanglement!]
Hawking saddle dominates

Remarks

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No, it does not. But it is evidence in this direction.

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We used the Euclidean path integral to calculate this without calculating ρ_R

or even showing that it exists.

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$$S(\mathrm{Rad}) = -\mathrm{tr}\rho_R \log \rho_R$$

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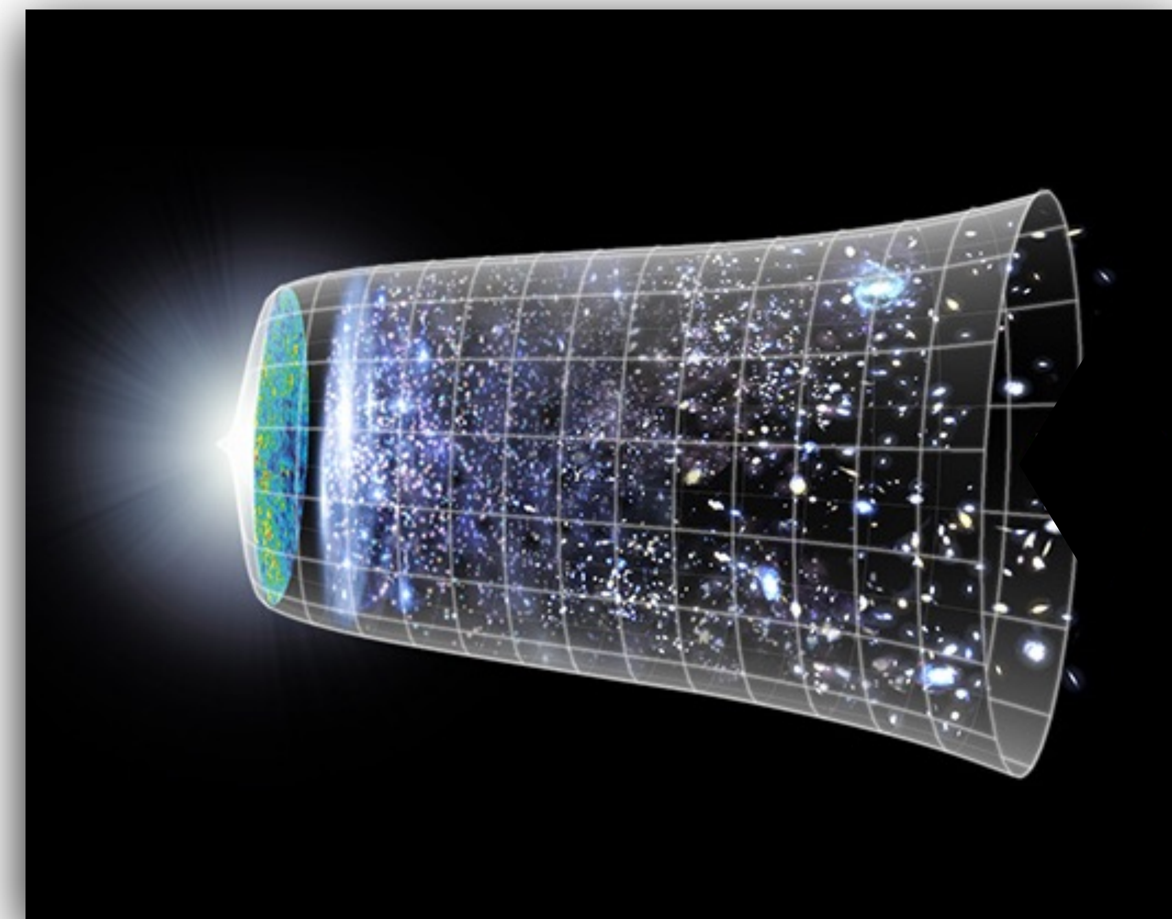
or even showing that it exists.

This is on the same footing as the Euclidean calculation of the black hole entropy by Gibbons and Hawking,

$$S(\text{black hole}) = \frac{\text{Area}}{4}$$

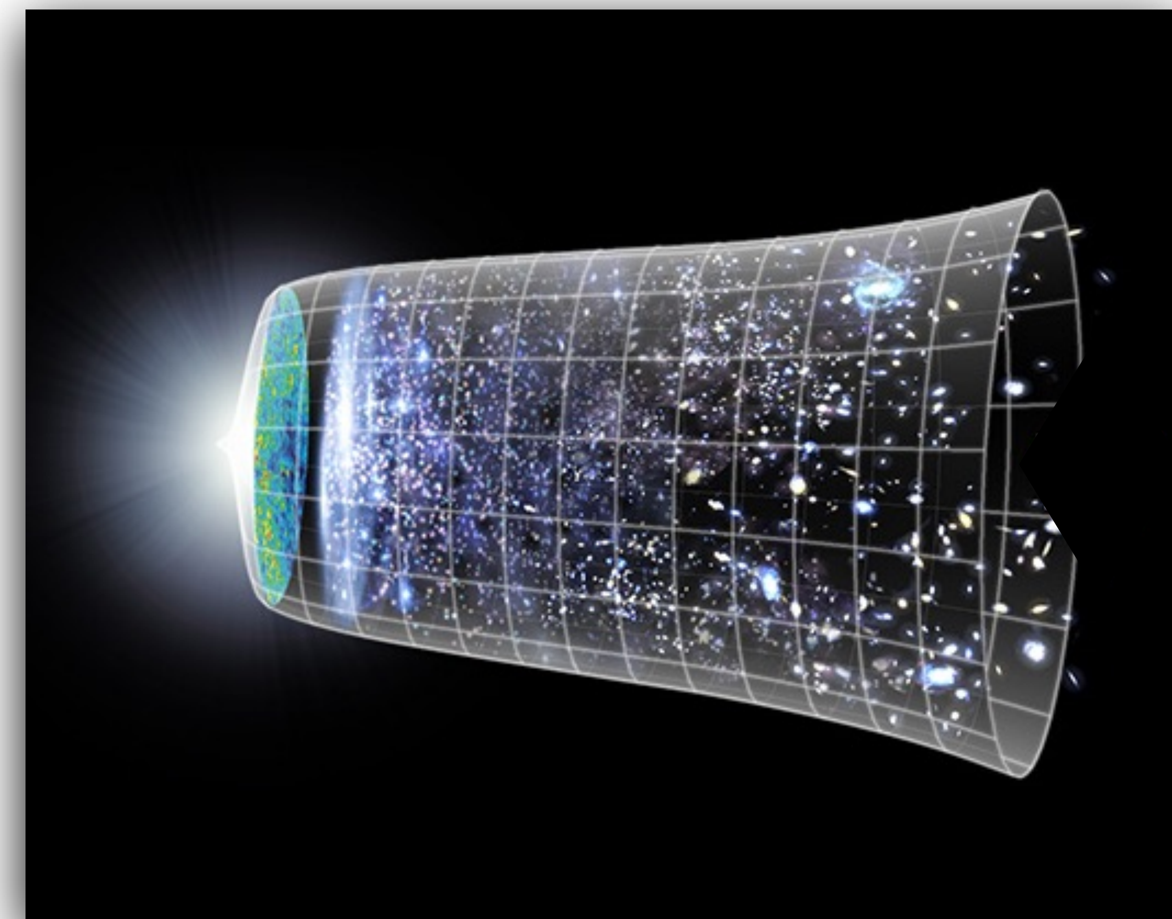
which we believe gives the right answer but does not exhibit the microstates.

Quantum cosmology revisited?



Cosmology also has horizons, large entropy, etc. Does it have islands?

Quantum cosmology revisited?



Cosmology also has horizons, large entropy, etc. Does it have islands?

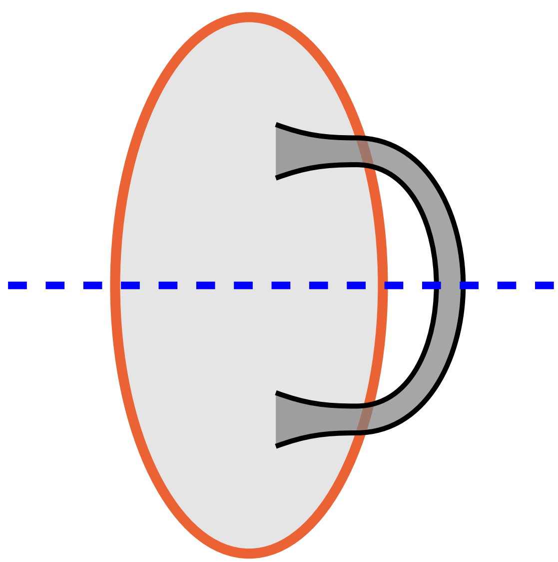
Maybe. The interpretation is unclear.

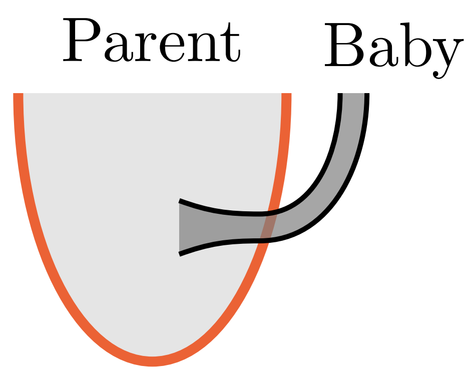
[Anous, Kruthoff, Mahajan ’20], [Chen, Gorbenko, Maldacena ’20], [TH, Jiang, Shaghoulian ’20],
[Balasubramanian, Kar, Ugajin ’20], [Van Raamsdonk ’20]
etc.

Do wormholes violate quantum mechanics?

In other situations, wormholes seem to violate some basic properties of quantum mechanics.

"Factorization paradoxes"





[Old work by Coleman, Giddings, Strominger, etc.]
[Saad Shenker Stanford ’19]
[Marolf, Maxfield ’20]
etc.

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Thank you