

Vacuum Energy of the Universe, Large scale magnetic field and nontrivial topology in QFT

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This talk is mostly based on two recent papers where we attempt to reveal the nature of vacuum energy in QFT

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Inflation and gauge field holonomy

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Cosmological magnetic field and dark energy as two sides of the same coin

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1. PRELIMINARY: ENERGY IN GAUGE THEORIES

■ WE WANT TO ARGUE THAT THERE IS A NOVEL TYPE OF ENERGY IN GAUGE THEORIES. THIS ENERGY HAS “NON-DISPERSIVE” NATURE, AND CAN NOT BE EXPRESSED IN TERMS OF CONVENTIONAL SCATTERING AMPLITUDES.

■ IT EXPLICITLY CONTRADICTS TO THE “FOLK THEOREM” THAT THE S-MATRIX CONTAINS ALL THE INFORMATION ABOUT ALL PHYSICAL OBSERVABLES.

■ ALL THESE NOVEL EFFECTS ARE DUE TO THE NONTRIVIAL TOPOLOGICAL SECTORS IN THE GAUGE SYSTEMS AND TUNNELLING TRANSITIONS BETWEEN THEM.

■ THE EFFECT IS NON-LOCAL IN NATURE, AND CAN NOT BE EXPRESSED IN TERMS OF LOCAL CURVATURE IN GRADIENT EXPANSION. IT IS EXPRESSED IN TERMS OF A NON-LOCAL CHARACTERISTICS OF THE SYSTEM -THE HOLONOMY.

2. PLAN OF THE TALK

- WE APPLY THESE IDEAS TO COSMOLOGY (DESITTER BEHAVIOUR DURING INFLATIONARY STAGE AND DARK ENERGY AT PRESENT EPOCH).
- WE WANT TO TEST THESE IDEAS WITH EXPLICIT COMPUTATIONS IN HYPERBOLIC SPACE AND THE SO-CALLED “DEFORMED QCD” MODEL.
- WE WANT TO APPLY THESE IDEAS TO THE INFLATIONARY EPOCH. WE ALSO WANT TO APPLY THESE IDEAS TO THE PRESENT TIME (THE DARK ENERGY EPOCH)
- WE ALSO WANT TO ARGUE THAT THE OBSERVED MAGNETIC FIELD (WITH HUGE CORRELATION LENGTH ON THE LEVEL OF 1 GPS) IS THE DIRECT MANIFESTATION OF THIS DE.

3. TOPOLOGICAL SUSCEPTIBILITY

A CONVENIENT WAY TO EXPLAIN THE NATURE OF NEW TYPE OF VACUUM ENERGY IS TO STUDY THE TOPOLOGICAL SUSCEPTIBILITY (*it is the key element in the resolution of the so-called $U(1)$ problem in QCD, Witten, Veneziano, 1979*).

$$\chi_{YM} = \int d^4x \langle q(x), q(0) \rangle \neq 0 \qquad \frac{\partial^2 E_{\text{vac}}(\theta)}{\partial \theta^2} = \chi_{YM}$$

To avoid confusion: This is the Wick's T -product, not Dyson's

χ_{YM} DOES NOT VANISH, THOUGH $q(x) \sim \partial_\mu K^\mu(x)$. IT HAS "WRONG SIGN", SEE BELOW. IT CAN NOT BE RELATED TO ANY PHYSICAL PROPAGATING DEGREES OF FREEDOM. FURTHERMORE, IT HAS A POLE IN MOMENTUM SPACE

$$\lim_{k \rightarrow 0} \int d^4x e^{ikx} \langle K_\mu(x), K_\nu(0) \rangle \sim \frac{k_\mu k_\nu}{k^4}$$

THERE IS A MASSLESS POLE (VENEZIANO GHOST), BUT THERE ARE NO ANY PHYSICAL MASSLESS STATES IN THE SYSTEM.

$$\chi_{dispersive} \sim \lim_{k \rightarrow 0} \sum_n \frac{\langle 0|q|n\rangle \langle n|q|0\rangle}{-k^2 - m_n^2} < 0,$$

■ CONVENTIONAL PHYSICAL DEGREES OF FREEDOM ALWAYS CONTRIBUTE WITH SIGN (-) WHILE ONE NEEDS SIGN (+) TO SATISFY WI AND RESOLVE THE U(1) PROBLEM

$$\chi_{non-dispersive} = \int d^4x \langle q(x), q(0) \rangle = \frac{1}{N^2} |E_{vac}| > 0$$

■ CONVENTIONAL TERMS (RELATED TO PROPAGATING DEGREES OF FREEDOM) ALWAYS PRODUCE $\exp(-\Lambda_{QCD}L)$ BEHAVIOUR AT LARGE DISTANCES.

■ WITTEN SIMPLY POSTULATED THIS TERM, WHILE VENEZIANO ASSUMED THE UNPHYSICAL FIELD, THE SO-CALLED THE “VENEZIANO GHOST” TO SATURATE “WRONG” SIGN IN χ .

■ IN SOME MODELS THIS CONTACT NON-DISPERSIVE TERM WITH “WRONG” SIGN (+) CAN BE EXPLICITLY COMPUTED. IT IS ORIGINATED FROM THE TUNNELLING EFFECTS BETWEEN THE DEGENERATE TOPOLOGICAL SECTORS OF THE THEORY.

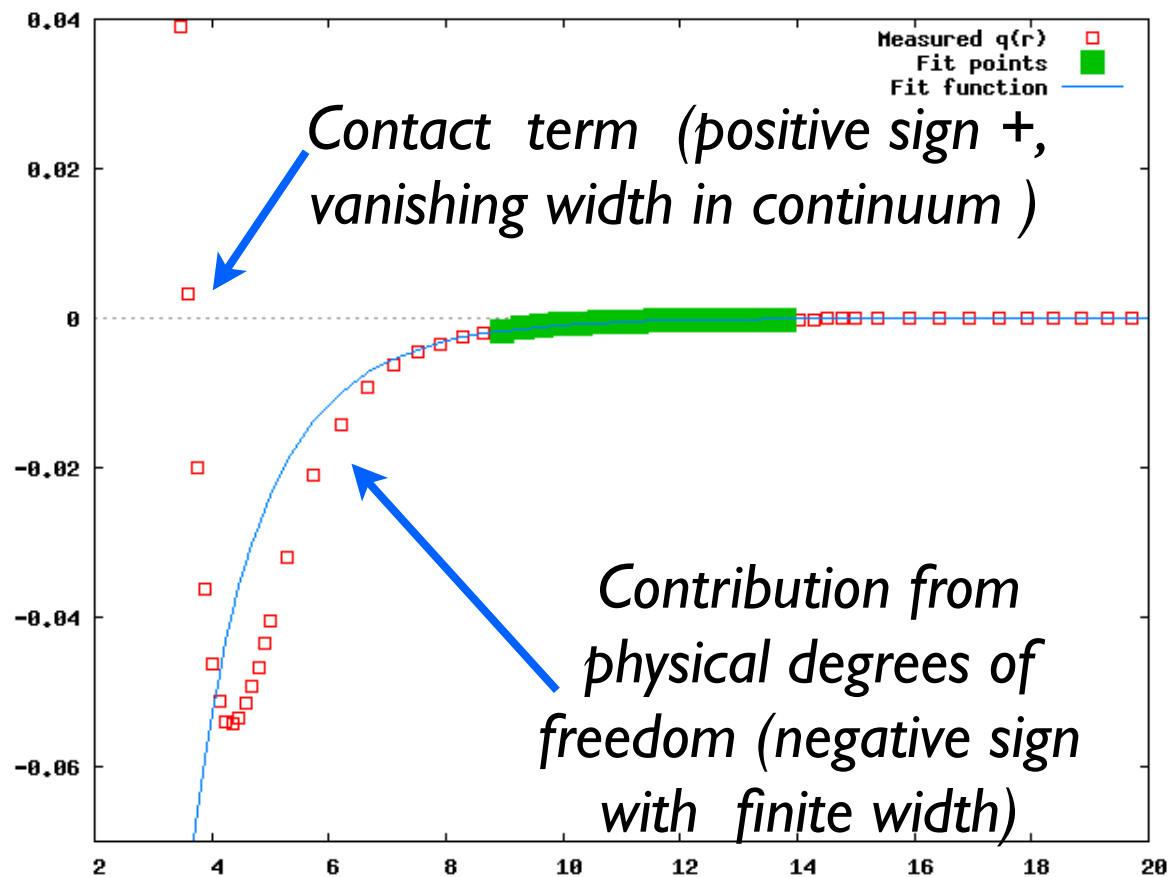
■ THESE CONTRIBUTIONS CAN NOT BE DESCRIBED IN TERMS OF CONVENTIONAL DEGREES OF FREEDOM (WRONG SIGN);

■ THEY ARE INHERENTLY NON-LOCAL IN NATURE AS THEY ARE RELATED TO THE TUNNELLING PROCESSES WHICH ARE FORMULATED IN TERMS OF THE NON-LOCAL LARGE GAUGE TRANSFORMATION OPERATOR AND HOLONOMY;

■ THESE TERMS MAY EXHIBIT THE LONG RANGE FEATURES EVEN THROUGH QCD HAS A GAP (SIMILAR TO THE CM TOPOLOGICAL INSULATORS);

■ THE EFFECTS HAVE BEEN EXPLAINED IN TERMS χ_{YM} . HOWEVER, THE θ -DEPENDENT PORTION OF ENERGY $E_{\text{vac}}(\theta)$ (RELEVANT FOR THE COSMOLOGICAL APPLICATIONS) HAS ALL THESE UNUSUAL FEATURES DUE TO THE RELATION

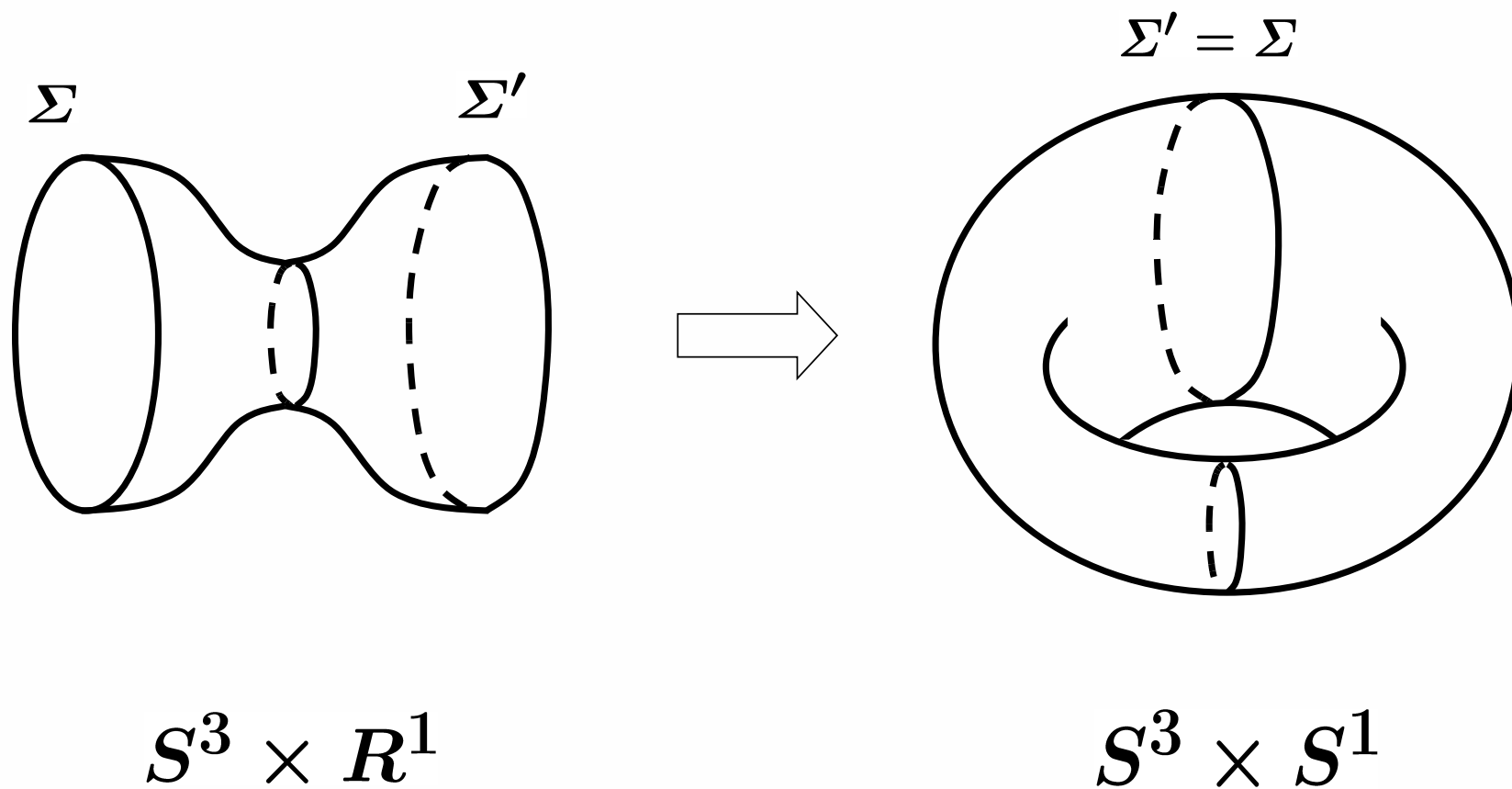
$$\chi_{YM} = \frac{\partial^2 E_{\text{vac}}(\theta)}{\partial^2 \theta} \Big|_{\theta=0}$$



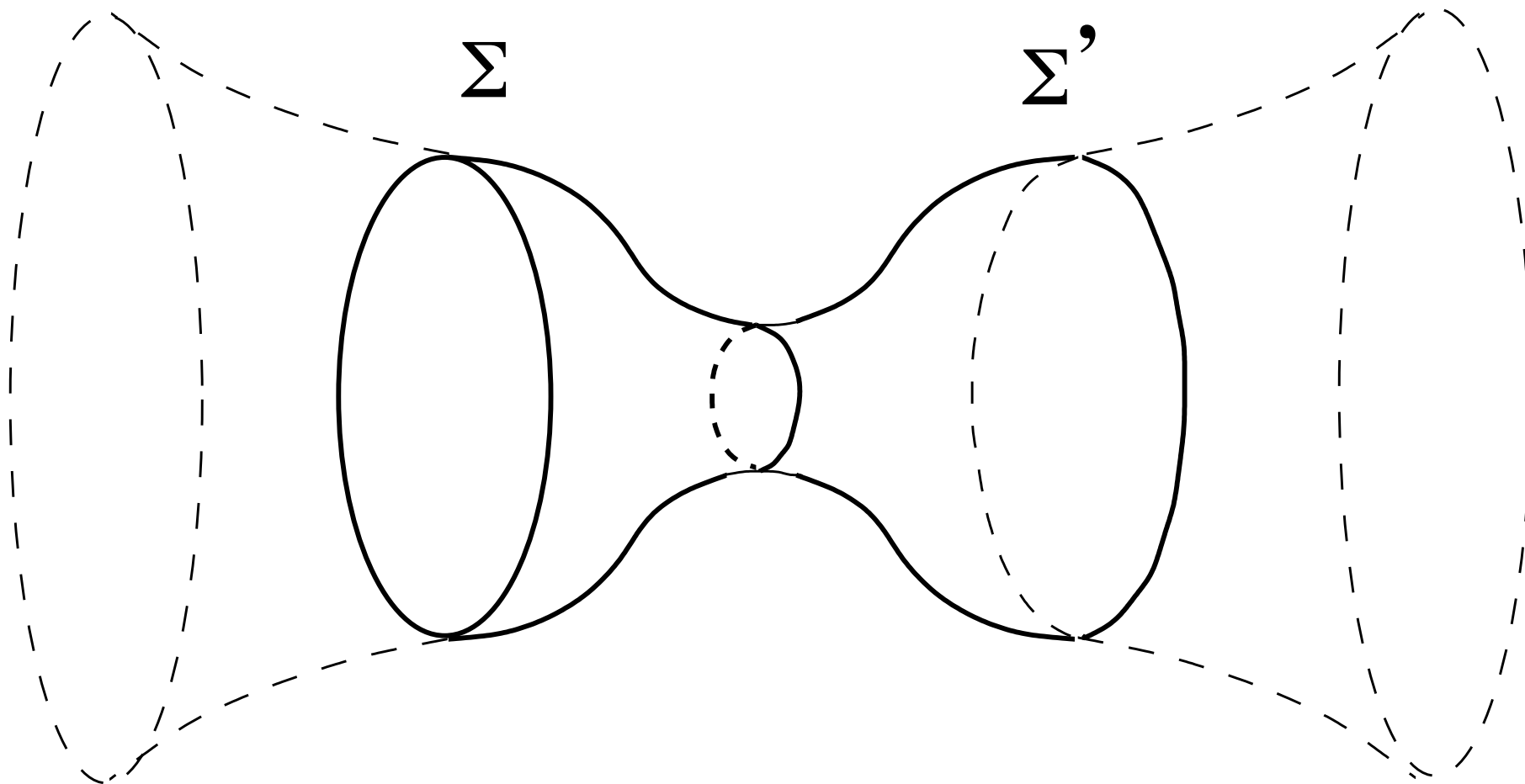
The topological susceptibility $\chi(r)$ as a function of r . Wrong sign for χ is well established phenomenon; it has been tested on the lattice (plot above is from C. Bernard et al, LATTICE 2007). This $\chi(r=0)$ contribution is not related to any physical degrees of freedom, and can be interpreted as a contact term.

4. COSMOLOGICAL APPLICATION: INFLATION, DE

- THIS PROPOSAL REPRESENTS A SYNTHESIS OF TWO PREVIOUS, AND NAIVELY UNRELATED, IDEAS:
- 1. SELF-CONSISTENT GENERATION OF THE DE SITTER BEHAVIOUR BASED ON THE EUCLIDEAN $S^3 \times S^1$ GRAVITATIONAL INSTANTONS, GARLANDS. THEY CAN BE THOUGHT AS THE THERMAL (SO IS THE S^1) VERSION OF HARTLE-HAWKING INSTANTONS (*Barvinsky and Co*), SEE NEXT SLIDE WITH A FIGURE.
- INFLATION STAGE STARTS AFTER “NUCLEATION” OF THE SYSTEM FROM $S^3 \times S^1$ INSTANTON. ANALYTICAL CONTINUATION OF THE SCALE FACTOR LEADS TO THE DE SITTER BEHAVIOUR (*Barvinsky and Co*), SEE NEXT SLIDE WITH A FIGURE.



Previous papers (Barvinsky & Co): S^1 was treated as the system with finite temperature. Novel element: the same S^1 is a key element to generate a nontrivial holonomy.



The “nucleation” of the system occurs in both cases in the same way after analytical continuation from Euclidean to Lorentzian spacetime:

$$\tau = 2m\pi/\Omega + it, \quad a^2 \sim \cos(\Omega\tau) \rightarrow \cosh(\Omega t) \sim \exp(\Omega t)$$

- 2. NAIVELY UNRELATED IDEA IS THAT THE SAME S^1 PLAYS A KEY ROLE IN GENERATION OF THE VACUUM ENERGY IN A CONJECTURED $\overline{\text{QCD}}$ WITH $\Lambda_{\overline{\text{QCD}}} \ll M_P$ WHEN HOLONOMY ASSUMES A NONTRIVIAL VALUE

$$L = \mathcal{P} \exp \left(i \int_0^{\mathcal{T}} dx_4 A_4(x_4, |\mathbf{x}| \rightarrow \infty) \right), \quad \mathcal{T} = \oint_{S^1} d\tau, \quad \mathcal{T} = \frac{2\pi m}{\Omega}$$

- IN OUR FRAMEWORK THE VACUUM ENERGY ENTERING THE FRIEDMAN EQUATION IS DEFINED AS THE DIFFERENCE BETWEEN THE ENERGY IN NONTRIVIAL BACKGROUND AND FLAT SPACE TIME GEOMETRY, I.E.

$$\rho \equiv \rho_{\text{vac}}[S^3 \times S^1] - \rho_{\text{vac}}[\mathbb{R}^4] = \frac{\bar{c} \mathcal{T} \bar{\Lambda}_{\text{QCD}}^3}{\mathcal{T}}$$

- THIS SUBTRACTION PRESCRIPTION IS CONSISTENT WITH CONVENTIONAL SUBTRACTION OF THE UV DIVERGENCES. NON-LOCAL FUNCTIONS OF GEOMETRY (SUCH AS \mathcal{T}^{-1}) CANNOT BE REMOVED BY ANY UV COUNTER-TERMS.

- HISTORICAL COMMENTS: MANY PEOPLE FROM DIFFERENT FIELDS HAD ADVOCATED (AFTER *Zeldovich, 1967*) A SIMILAR IDEA ON THE RHS FOR THE FRIEDMAN'S EQUATION

$$\Delta E(L) = [E(L) - E_{\text{Mink}}]$$

- $$E(L) \equiv -(\beta V)^{-1} \ln \mathcal{Z}$$

- *James Bjorken (particle physics), 2001,*
Ralf Schuetzhold (GR), PRL, 2002;
Grisha Volovik (CM physics), 2008 +many more

- I PERSONALLY ADOPTED THIS IDEA IN 2009, MOSTLY DUE TO THE INTENSE (AND NEVER ENDING) DISCUSSIONS WITH GRISHA VOLOVIK IN THE RELATION WITH HIS COSLAB (COSMOLOGY IN A LABORATORY) ACTIVITIES.

5. HOLONOMY AND THE LINEAR CORRECTION

$$\kappa \sim 1/\mathcal{T} \quad \text{IN HYPERBOLIC SPACE} \quad \mathbb{H}_{\kappa}^3 \times \mathbb{S}_{\kappa^{-1}}^1$$

■ NORMALLY IT IS EXPECTED THAT ALL CORRECTIONS DUE TO THE TIME-DEPENDENT (CURVED) BACKGROUND ARE PROPORTIONAL TO THE LOCAL CURVATURE $R[\mathbb{H}_{\kappa}^3] \sim \kappa^2$

■ WE WANT TO TEST THESE IDEAS IN GAUGE THEORIES WITH NONTRIVIAL HOLONOMY. IN THIS CASE CORRECTIONS ARE NOT REDUCED TO THE LOCAL OBSERVABLES. THE IR REGULARIZATION PLAYS KEY ROLE IN ALL COMPUTATIONS.

■ SPECIFICALLY, WE COMPUTE THE RATIO WHICH EXPLICITLY SHOWS THE LINEAR CORRECTION $\sim \kappa$

$$\frac{E_{\text{vac}}[\mathbb{H}_{\kappa}^3 \times \mathbb{S}_{\kappa^{-1}}^1]}{E_{\text{vac}}[\mathbb{R}^3 \times \mathbb{S}^1]} \simeq \left(1 - \frac{\nu \bar{\nu}}{2} \cdot \frac{\kappa}{\Lambda_{QCD}} \right) \cdot E_{\text{vac}} \equiv -\frac{1}{\beta V} \ln \mathcal{Z}$$

■ THE COMPUTATIONS ARE BASED ON KVBLT CALORONS WITH NONTRIVIAL HOLONOMY (KRAAN-VAN BAAL-LEE-LU)

$$\frac{1}{2}TrL = \frac{1}{2}Tr\mathcal{P} \exp \left(i \int_0^\beta dx_4 A_4(x_4, |\mathbf{x}| \rightarrow \infty) \right) = \cos(\pi\nu)$$

■ NORMALLY, NONTRIVIAL HOLONOMY ($\nu \neq 0, 1$) GENERATES ZERO CONTRIBUTION TO THE PARTITION FUNCTION IN THERMODYNAMICAL LIMIT. HOWEVER, THE KVBLT CONFIGURATIONS ARE KNOWN TO GENERATE IR - FINITE CONTRIBUTION TO THE FREE ENERGY (IN HUGE CONTRAST WITH CONVENTIONAL INSTANTONS).

■ THE KVBLT CONFIGURATIONS CAN BE THOUGHT AS A SUPERPOSITION OF “N” DIFFERENT MONOPOLES WHICH CARRY THE FRACTIONAL TOPOLOGICAL CHARGE $Q = \pm 1/N$

■ CONFINEMENT CAN BE UNDERSTOOD AS PERCOLATION OF THESE FRACTIONALLY CHARGED MONOPOLES WHICH ENTER THE PARTITION FUNCTION IN SETS OF “N” .

THE CRUCIAL ROLE IN GENERATING THIS RESULT IS ZERO-MODE DETERMINANT. THESE MODES ARE DRASTICALLY DIFFERENT IN HYPERBOLIC AND IN EUCLIDEAN SPACES.

THIS DIFFERENCE IN THESE TWO CASES IS DETERMINED BY ASYMPTOTIC BEHAVIOUR (DIFFERENT CUTOFF: $\nu \longleftrightarrow \rho$)

$$A_4^M(r) = \left(v \coth(vr) - \frac{1}{r} \right) \frac{\tau^3}{2} \quad \text{on } \mathbb{R}^3$$

$$A_\chi^M(\rho) = \left((\nu + 1) \coth [(\nu + 1)\kappa\rho] - \coth \kappa\rho \right) \frac{\kappa\tau^3}{2} \quad \text{on } \mathbb{H}_\kappa^3$$

EVENTUALLY, THIS DIFFERENCE TRANSLATES INTO THE DIFFERENCE IN FUGACITIES (AND VACUUM ENERGIES) AS CLAIMED ABOVE

$$f^2 = \left[\frac{4\pi\beta\Lambda_{QCD}^4}{g^4} \right]^2 \cdot \left\langle \frac{[1 + 2\pi\nu\bar{\nu}\frac{r_{12}}{\beta}]}{(\Lambda_{QCD} r_{12})^{2/3}} [1 + 2\pi\nu\frac{r_{12}}{\beta}]^{\frac{8}{3}\nu-1} [1 + 2\pi\bar{\nu}\frac{r_{12}}{\beta}]^{\frac{8}{3}\bar{\nu}-1} \right\rangle$$

THE CORRECTION \mathcal{T}^{-1} CAN BE EXPLICITLY COMPUTED IN SOME SIMPLIFIED SETTINGS (HYPERBOLIC SPACE, WEAKLY COUPLED DEFORMED QCD MODEL, ETC):

$$P = -\frac{\partial F}{\partial V} = +\frac{32\pi^2}{g^4}\bar{\Lambda}_{QCD}^4 \left(1 - \frac{c\mathcal{T}}{\mathcal{T}\bar{\Lambda}_{QCD}}\right) \quad \rho = \frac{F}{V} = -\frac{32\pi^2}{g^4}\bar{\Lambda}_{QCD}^4 \left(1 - \frac{c\mathcal{T}}{\mathcal{T}\bar{\Lambda}_{QCD}}\right)$$

THESE CORRECTIONS DO NOT MODIFY THE EQUATION OF STATES WHICH IS NORMALLY ASSOCIATED WITH THE COSMOLOGICAL CONSTANT CONTRIBUTION:

$$\Delta\rho \equiv \rho_{\text{vac}}[\mathbb{S}^3 \times \mathbb{S}^1] - \rho_{\text{vac}}[\mathbb{R}^4] = \frac{\bar{c}\mathcal{T}\bar{\Lambda}_{QCD}^3}{\mathcal{T}}, \quad w = \frac{\Delta P}{\Delta\rho} = -1$$

$$H \sim \frac{1}{\mathcal{T}} = \frac{\bar{c}\mathcal{T}\bar{\Lambda}_{QCD}^3}{3\pi M_P^2}, \quad \Delta\rho = \frac{\bar{c}^2\mathcal{T}\bar{\Lambda}_{QCD}^6}{3\pi^2 M_P^2}, \quad \frac{H}{M_P} \ll 1$$

THE DRIVING FORCE FOR THE DESITTER BEHAVIOUR IS NOT A LOCAL DYNAMICAL INFLATON FIELD $\Phi(x)$. RATHER, THE DRIVING FORCE SHOULD BE THOUGHT AS A CASIMIR TYPE VACUUM ENERGY GENERATED BY TUNNELLING TRANSITIONS

■ Q: *How a system with a gap could be ever sensitive to arbitrary large distances?*

■ A1: THE LONG RANGE ORDER IN GAPPED QCD IS SIMILAR TO AHARONOV -CASHER EFFECT. IF ONE INSERTS AN EXTERNAL CHARGE INTO SUPERCONDUCTOR WHEN ELECTRIC FIELD IS SCREENED $\exp(-r/\lambda)$ A NEUTRAL MAGNETIC FLUXON WILL BE STILL SENSITIVE TO EXTERNAL CHARGE AT ARBITRARY LARGE DISTANCES.

■ A2: LONG RANGE ORDER IN THE SYSTEM EMERGES BECAUSE THE LARGE GAUGE TRANSFORMATION OPERATOR AND HOLONOMY ARE NON-LOCAL OPERATORS SENSITIVE TO FAR IR-PHYSICS, SIMILAR TO “MODULAR OPERATOR” IN AHARONOV -CASHER EFFECT.

6. HOW THE INFLATION ENDS. THE REHEATING

■ IN A SIMPLIFIED CASE (NO SM PARTICLES) THE HUBBLE CONSTANT H AND THE ENERGY DENSITY REMAIN CONSTANT AFTER THE “NUCLEATION” FROM THE GRAVITATIONAL INSTANTON. THE SYSTEM ASSUMES CONVENTIONAL LORENTZIAN SIGNATURE.

■ THE SOLUTION AFTER NUCLEATION CORRESPONDS TO INFLATIONARY DE-SITTER BEHAVIOUR

$$w = \frac{\Delta P}{\Delta \rho} = -1, \quad a(t) \sim \exp(Ht), \quad H \sim \mathcal{T}^{-1},$$

$$\Delta \rho \equiv \rho_{\text{vac}}[\mathbb{S}^3 \times \mathbb{S}^1] - \rho_{\text{vac}}[\mathbb{R}^4], \quad \Delta p \equiv p_{\text{vac}}[\mathbb{S}^3 \times \mathbb{S}^1] - p_{\text{vac}}[\mathbb{R}^4],$$

■ THIS WOULD BE THE FINAL DESTINATION OF THE UNIVERSE IF THE INTERACTION WITH SM PARTICLES IS SWITCHED OFF

■ WHEN THE INTERACTION WITH SM PARTICLES IS SWITCHED BACK ON THE INFLATION ENDS AS A RESULT OF THIS INTERACTION WITH SM PARTICLES.

■ THE COMPUTATIONAL PROCEDURE IS WELL DEFINED IN PRINCIPLE (PROFOUNDLY COMPLICATED IN PRACTISE):

■ 1. ONE SHOULD DESCRIBE THE RELEVANT EUCLIDEAN CONFIGURATIONS SATISFYING PROPER BOUNDARY CONDITIONS (SIMILAR TO CALORONS WITH NONTRIVIAL HOLONOMY DEFINED ON $\mathbb{R}^3 \times S^1$)

■ 2. ONE SHOULD COMPUTE THE CORRESPONDING PATH INTEGRAL ACCOUNTING FOR THE TUNNELLING TRANSITIONS;

■ 3. ONE SHOULD COMPUTE (ρ, p) IN THE PRESENCE OF ALL MASSLESS SM GAUGE FIELDS (γ, W, Z, g)

■ 4. ONE SHOULD SUBTRACT THE CORRESPONDING EXPRESSIONS $\rho(\mathbb{R}^4), p(\mathbb{R}^4)$ TO DERIVE $(\Delta\rho, \Delta p)$

■ WHILE THESE STEPS ARE WELL DEFINED IN PRINCIPLE, IT IS NOT FEASIBLE NOW TO PERFORM THE COMPUTATIONS

■ THERE IS ANALOGY WITH THE DYNAMICAL CASIMIR EFFECT (DCE) WHEN PHOTONS ARE RADIATED FROM TIME-DEPENDENT BACKGROUND.

■ THE DIFFERENCE IS: IN OUR CASE PHOTONS ARE EMITTED NOT FROM CONVENTIONAL QUANTUM VACUUM FLUCTUATIONS, BUT FROM CONFIGURATIONS DESCRIBING THE VACUUM TUNNELLING PROCESSES.

■ THIS IS A HARD TECHNICAL PROBLEM: TUNNELLING IS DESCRIBED IN EUCLIDEAN PATH INTEGRAL WHILE THE EMISSION REPRESENTS INHERENT MINKOWSKI PROCESS

■ FORTUNATELY, THE KEY FEATURES CAN BE UNDERSTOOD USING ALTERNATIVE TECHNIQUE FORMULATED IN TERMS OF THE AUXILIARY TOPOLOGICAL NON-PROPAGATING FIELD, SIMILAR TO THE VENEZIANO GHOST FIELD IN QCD.

■ THE END OF INFLATION IS DESCRIBED BY SUCH EFFECTIVE AUXILIARY FIELD. IT IS FIXED BY TRIANGLE ANOMALY:

$$\mathcal{L}_{b\gamma\gamma} = \frac{\alpha(H_0)}{8\pi} N Q^2 [\theta - b(x)] \cdot F_{\mu\nu} \tilde{F}^{\mu\nu},$$

■ WHERE $b(x)$ IS TOPOLOGICAL NON-PROPAGATING FIELD (LAGRANGE MULTIPLIER). THIS TECHNIQUE HAS BEEN EXPLICITLY TESTED IN SIMPLIFIED SOLVABLE SYSTEMS

■ IN CONDENSED MATTER THIS TECHNIQUE IS WELL KNOWN WHEN SUMMATION OVER TOPOLOGICAL SECTORS IS REPLACED BY AUXILIARY (NON-PROPAGATING) FIELD, E.G. IN TOPOLOGICALLY ORDERED SYSTEMS

■ IN QCD CONTEXT $\dot{b} = \mu_5$ IMPLIES THAT THE HELICAL INSTABILITY WILL BE DEVELOPED (STUDIES WERE DONE IN RELATION WITH THE CHIRAL MAGNETIC EFFECT IN QCD WHERE μ_5 IS THE CHIRAL CHEMICAL POTENTIAL).

■ IN QCD CONTEXT ($\mu_5 \sim H_0$) THE TYPICAL TIME SCALE FOR DEVELOPMENT OF THE HELICAL INSTABILITY IS

$$\tau_{\text{instability}} \sim \frac{1}{\alpha_s^2 \mu_5} \rightarrow \tau_{\text{inflation}} \sim \frac{1}{\alpha_s^2(H_0) H_0} \rightarrow N_{\text{e-folds}} \sim \alpha_s^{-2}(H_0) \sim 10^2$$

■ WE IDENTIFY $\tau_{\text{instability}}$ WITH τ_{inflaton}

■ THE NUMBER OF E-FOLDS $N_{\text{e-folds}} \sim \alpha_s^{-2}(H_0) \sim 10^2$

■ THE DEVIATION FROM PURE DE-SITTER IS RELATED TO THE COUPLING WITH SM (WE KNOW AND LOVE) AND NOT TO SOME AD-HOC LOCAL INFLATON POTENTIAL $V(\Phi)$

7. APPLICATIONS TO THE DARK ENERGY

■ LESSON 1: THERE IS A FUNDAMENTALLY NEW TYPE OF THE VACUUM ENERGY WHICH CAN NOT BE EXPRESSED IN TERMS OF THE SCATTERING AMPLITUDES (THE S-MATRIX ELEMENTS).

■ LESSON 2: IT EMERGES AS A RESULT OF TUNNELLING PROCESSES BETWEEN DEGENERATE TOPOLOGICAL SECTORS, AND FORMULATED IN TERMS OF THE “NON-DISPERSIVE” CONTACT TERMS AND NONLOCAL HOLONOMY.

■ LESSON 3: WE IDENTIFY THIS NEW TYPE OF ENERGY WITH COSMOLOGICAL VACUUM ENERGY.

■ INSTEAD OF UNKNOWN GAUGE THEORY (WE CALLED $\overline{\text{QCD}}$) RELEVANT FOR INFLATIONARY SCALE WE APPLY THE SAME IDEAS TO QCD (WE KNOW AND LOVE)

■ IN THIS CASE WE SIMPLY MODIFY ALL OUR PREVIOUS FORMULAE AND COMPUTATIONS BY REPLACING

$$\text{Inflation} \rightarrow \text{DE} \qquad \bar{\Lambda}_{\text{QCD}} \rightarrow \Lambda_{\text{QCD}}$$

■ THE OBTAINED RELEVANT PARAMETERS ARE AMAZINGLY CLOSE TO THE OBSERVED DE VALUES:

$$\mathcal{T}^{-1} \sim H \sim \frac{\Lambda_{\text{QCD}}^3}{M_{\text{PL}}^2} \sim 10^{-33} \text{eV}, \quad \rho_{\text{DE}} \sim H \Lambda_{\text{QCD}}^3 \sim (10^{-3} \text{eV})^4, \quad \mathcal{T} \sim H^{-1} \sim \frac{M_{\text{PL}}^2}{\Lambda_{\text{QCD}}^3} \sim 10 \text{ Gyr},$$

■ IN CASE OF $\bar{\Lambda}_{\text{QCD}}$ THE COUPLING WITH SM PARTICLES LEADS TO THE REHEATING (ENERGY TRANSFER FROM THE VACUUM TO GAUGE FIELDS)

■ IN CASE OF Λ_{QCD} THE COUPLING WITH MAXWELL EM FIELD LEADS TO SIMILAR ENERGY (VERY SLOW) TRANSFER.

■ RESULT: THE LARGE SCALE MAGNETIC FIELD (WITH CORRELATION LENGTH OF ENTIRE VISIBLE UNIVERSE) WILL BE GENERATED.

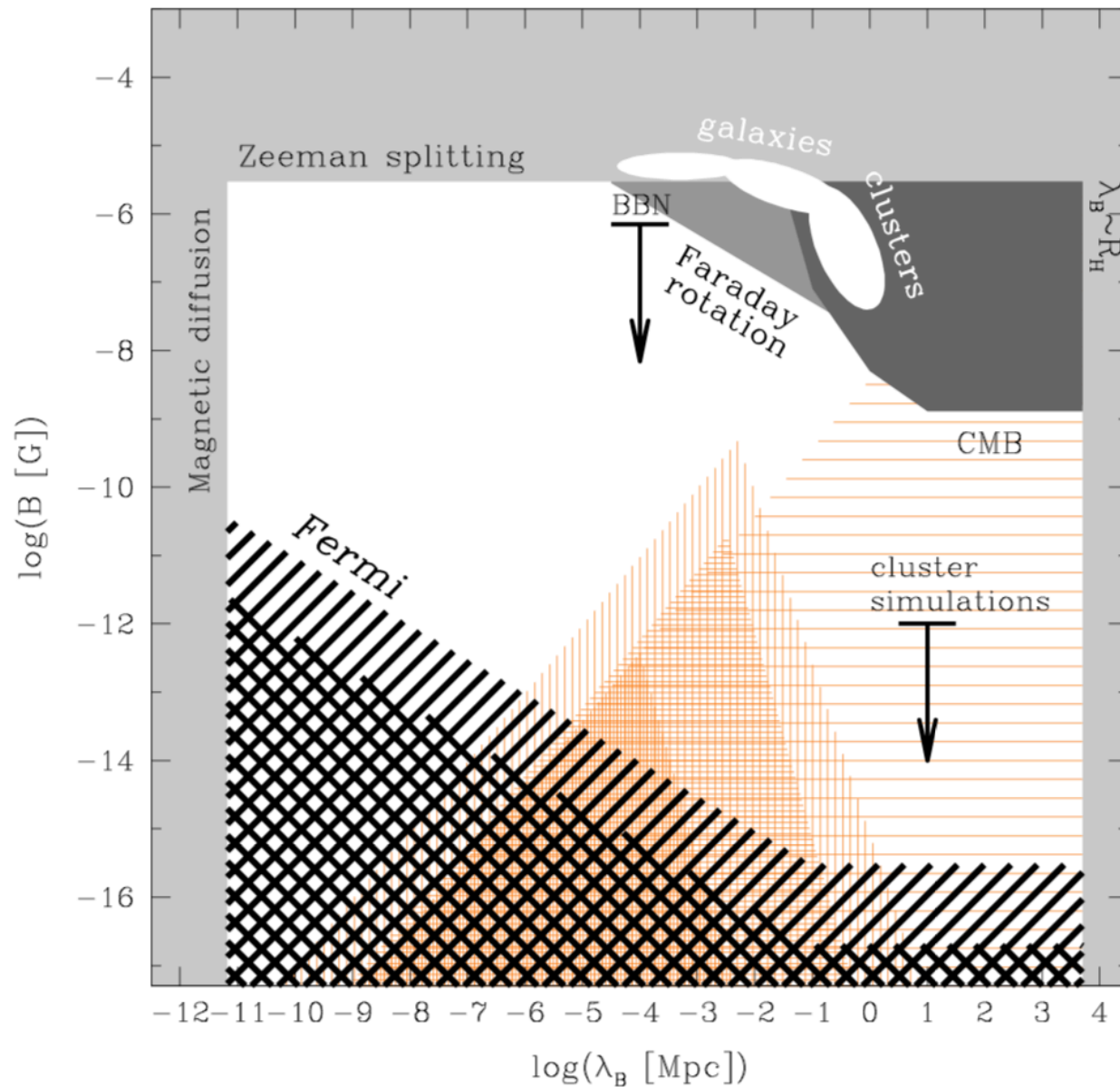
8. LARGE SCALE MAGNETIC FIELD

- THERE ARE MANY IDEAS HOW THE OBSERVED COSMOLOGICAL MAGNETIC FIELD IS GENERATED INCLUDING PRIMORDIAL MECHANISMS

- WE ADVOCATE UNORTHODOX MECHANISM WHICH IS DRAMATICALLY DIFFERENT FROM ALL PREVIOUS APPROACHES: THE B- FIELD IS GENERATED WITH ENORMOUS COHERENCE SCALE FROM DE, SIMILAR TO REHEATING IN THE INFLATION STUDIES

- NO NEED FOR ANY AMPLIFICATIONS AS IT IS CHARACTERIZED BY THE LARGEST POSSIBLE SCALE AT THE MOMENT OF FORMATION

- THE B FIELD CORRELATED ON ENORMOUS (GPC) SCALE MUST EXIST. WHAT IS THE ORIGIN OF SUCH CORRELATION?



Constraints on the B field [From Neronov & Vovk]. The B-field correlated on Gpc scales must exist: $10^{-15} G \lesssim B \lesssim 10^{-9} G$

■ THE STARTING POINT IS THE SAME EFFECTIVE LAGRANGIAN IN TERMS OF THE AUXILIARY FIELD

$$\mathcal{L}_{b\gamma\gamma}(x) = \frac{\alpha}{4\pi} N \frac{\sum_i Q_i^2}{N_f} [\theta + b(x, H)] \cdot F_{\mu\nu} \tilde{F}^{\mu\nu}(x)$$

■ IT GENERATES WELL KNOWN EXTRA TERM WITH $\mu_5 \sim H_0$

$$\vec{\nabla} \times \vec{B} = \sigma \vec{E} + \frac{\alpha}{2\pi} N \frac{\sum_i Q_i^2}{N_f} \cdot (\mu_5 \vec{B}), \quad \mu_5 \equiv \langle \dot{b}(x, H) \rangle$$

■ SIMILAR EQUATIONS HAVE BEEN STUDIED BEFORE (E.G. DYNAMICAL AXION FIELD). THE DIFFERENCE HERE IS THAT THE μ_5 DOES NOT SATISFY ANY CLASSICAL EQUATION OF MOTION AS THERE IS NO CANONICAL KINETIC TERM FOR AUXILIARY FIELD $b(x, H)$. THE μ_5 IS BACKGROUND FIELD

■ THE $b(x, H)$ FIELD WAS INTRODUCED AS THE LAGRANGE MULTIPLIER TO ACCOUNT FOR TUNNELLING EVENTS

■ IT IS KNOWN THAT THE PRESENCE OF THE μ_5 TERM LEADS TO THE HELICAL INSTABILITY. IN THE PRESENT CONTEXT IT IMPLIES THE GENERATION OF THE MAGNETIC FIELD ON THE HUGE SCALES WHERE μ_5 IS CORRELATED.

■ THE INSTABILITY DEVELOPS FOR LARGE WAVELENGTHS:

$$B(t) = B_0 \exp(\gamma t), \quad k < \frac{\alpha}{\pi} \mu_5, \quad \mu_5 \sim H$$

■ IN CASE OF THE INFLATION THIS COUPLING LEADS TO THE REHEATING. IN CASE OF DE AT PRESENT TIME THE SAME PHYSICS LEADS TO THE GENERATION OF THE MAGNETIC FIELD CORRELATED ON THE ENORMOUS SCALES.

■ THE ORDER OF MAGNITUDE ESTIMATES SUGGEST (PRESENT TIME)

$$B \sim 10^{-10} G$$

Concluding comments on Inflation

■ WE SPECULATE THAT A LINER CORRECTION $\mathcal{T}^{-1} \sim H$ TO THE ENERGY COULD BE GENERATED AS A RESULT OF DYNAMICS OF TOPOLOGICAL CONFIGURATIONS WITH NONTRIVIAL HOLONOMY. THE IDEA IS TESTED IN “DEFORMED QCD” AND IN THE SYSTEM DEFINED ON HYPERBOLIC SPACE $\mathbb{H}_{\kappa}^3 \times \mathbb{S}_{\kappa-1}^1$

■ APPLICATION TO THE INFLATION WITH $\bar{\Lambda}_{\text{QCD}}$. IT GENERATES THE DE SITTER BEHAVIOUR WHICH ENDS (REHEATING) AS A RESULT OF COUPLING WITH SM PARTICLES:

$$H_0 \sim \frac{1}{\mathcal{T}} = \frac{\bar{c}_{\mathcal{T}} \bar{\Lambda}_{\text{QCD}}^3}{3\pi M_P^2}, \quad \tau_{\text{inflation}} \sim \frac{1}{\alpha_s^2(H_0) H_0} \rightarrow N_{\text{e-folds}} \sim \alpha_s^{-2}(H_0) \sim 10^2$$

■ TECHNICALLY: EFFECT IS NON-LOCAL IN NATURE, AND CAN NOT BE EXPRESSED IN TERMS OF PROPAGATING DOF. THERE ARE MANY ANALOGIES WITH TOPOLOGICALLY -ORDERED SYSTEMS, INCLUDING AUXILIARY FIELDS

Concluding comments on Dark Energy & B field

■ IF THE SAME IDEA IS APPLIED TO THE PRESENT DE WHEN THE GAUGE FIELD IS THE QCD (WE KNOW AND LOVE), IT PRODUCES CORRECT ORDER OF MAGNITUDE ESTIMATE WHICH IS CONSISTENT WITH THE OBSERVATIONS:

$$H \sim \frac{\Lambda_{\text{QCD}}^3}{M_{\text{PL}}^2} \sim 10^{-33} \text{eV}, \quad \rho_{\text{DE}} \sim H \Lambda_{\text{QCD}}^3 \sim (10^{-3} \text{eV})^4$$

■ SIMILAR TO THE REHEATING (IN CASE OF INFLATION) THE VACUUM ENERGY WILL BE EVENTUALLY TRANSFERRED TO THE MAGNETIC ENERGY IN $\alpha^{-2} H^{-1}$ years

■ THE MAGNETIC FIELD AT PRESENT TIME COULD BE LARGE: $B \sim 10^{-10} G$. IT MUST BE CORRELATED ON THE SCALE OF THE VISIBLE UNIVERSE (ORDER OF MAGNITUDE ESTIMATE FOR B-FIELD, AT THE VERY BEST).

Extra Slides

PHYSICAL MEANING OF S^1 (THE SIZE OF \mathcal{T})

(appendix A3 from paper with Barvinsky)

ORIGINALLY \mathcal{T} WAS INTRODUCED IN EUCLIDEAN SPACE FOR COMPUTATIONS IN THE WEAK COUPLING REGIME WITH GIVEN HOLONOMY.

$$L = \mathcal{P} \exp \left(i \int_0^{\mathcal{T}} dx_4 A_4(x_4, |\mathbf{x}| \rightarrow \infty) \right).$$

IT SHOULD NOT BE CONFUSED WITH REAL SIZE IN 4D IN MINKOWSKI SPACE.

IN WEAKLY COUPLED GAUGE THEORIES (SUCH AS DEFORMED QCD) ALL COMPUTATIONS CAN BE CARRIED OUT EXPLICITLY WITH FIXED \mathcal{T} . ONE CAN SEE EXPLICITLY CONFINEMENT, FRACTIONALLY $1/N$ CHARGED MONOPOLES (INSTANTON QUARKS), GENERATION OF THE VACUUM ENERGY EXPRESSED IN TERMS OF THE AUXILIARY FIELD $b(x, H)$, ETC

- IN STRONGLY COUPLED REAL QCD (WHEN YOU START FROM THE VERY BEGINNING FROM $T \rightarrow \infty$) SUCH COMPUTATIONS CANNOT BE DONE AS CALORONS WITH NONTRIVIAL HOLONOMY CANNOT BE CONSTRUCTED
- HOW DO WE KNOW ABOUT ANYTHING ABOUT HOLONOMY DEFINED ON S^1 IF IT WAS NOT A PART OF CONSTRUCTION TO BEGIN WITH?
- IT TURNS OUT THAT THE HOLONOMY CAN BE DYNAMICALLY GENERATED (EMERGING) IN STRONGLY COUPLED REGIME.
- THE WELL KNOWN EXAMPLE IS $2d$ CP^{N-1} MODEL DEFINED ON \mathbb{R}^2 WHEN THE ONLY INTEGER VALUED INSTANTONS WITH TRIVIAL HOLONOMY WERE INTRODUCED INTO THE SYSTEM

- HOWEVER, WHEN ALL THE INSTANTS ARE TAKEN INTO ACCOUNT (GRAND CANONICAL ENSEMBLE) THE FRACTIONAL TOPOLOGICAL CHARGE $1/N$ DYNAMICALLY EMERGES. IT LOOKS EXACTLY AS WE WERE STARTED WITH NONTRIVIAL HOLONOMY DEFINED ON S^1 . HOWEVER, THE SEMICLASSICAL DESCRIPTION IS NOT JUSTIFIED.
- THEREFORE, IN THIS “EMERGENT” CASE DEFINED ON \mathbb{R}^2 THE EFFECTIVE SIZE S^1 IS GENERATED DYNAMICALLY.
- THE MAIN LESSON IN THE PRESENT CONTEXT: THE EFFECTIVE SIZE S^1 CAN BE UNLINKED FROM THE SO-CALLED BOOTSTRAP EQUATIONS AND SHOULD BE TREATED AS FREE PARAMETER TO BE FIXED FROM OBSERVATIONS: $\mathcal{T} \sim H^{-1}$
- SUFFICIENTLY LARGE SIZE OF S^1 IS CONSISTENT WITH PRESENTLY AVAILABLE CMB OBSERVATIONS FOR $\mathcal{T} \gtrsim H^{-1}$