Initial conditions for inflation

Andrei Linde

Why Inflation?

Hard Art of the Universe Creation

According to the standard hot Big Bang universe, the total number of particles during its expansion did not change much, so the universe at the Planck time was supposed to contain about 10^{90} particles. At the Planck time $t = O(1)$, there was one particle per Planck length $ct = O(1)$.

Thus, at the Planck time $t = 1$, the universe consisted of 10^{90} causally disconnected parts of size $ct = O(1)$. These parts did not know about each other. If someone wanted to create the universe at the Planck time, he/she could only make **a Very Small Bang** in his/her own tiny part of the universe of a Planck size $ct = O(1)$. **Everything else was beyond causal control.**

Other aspects of this problem:

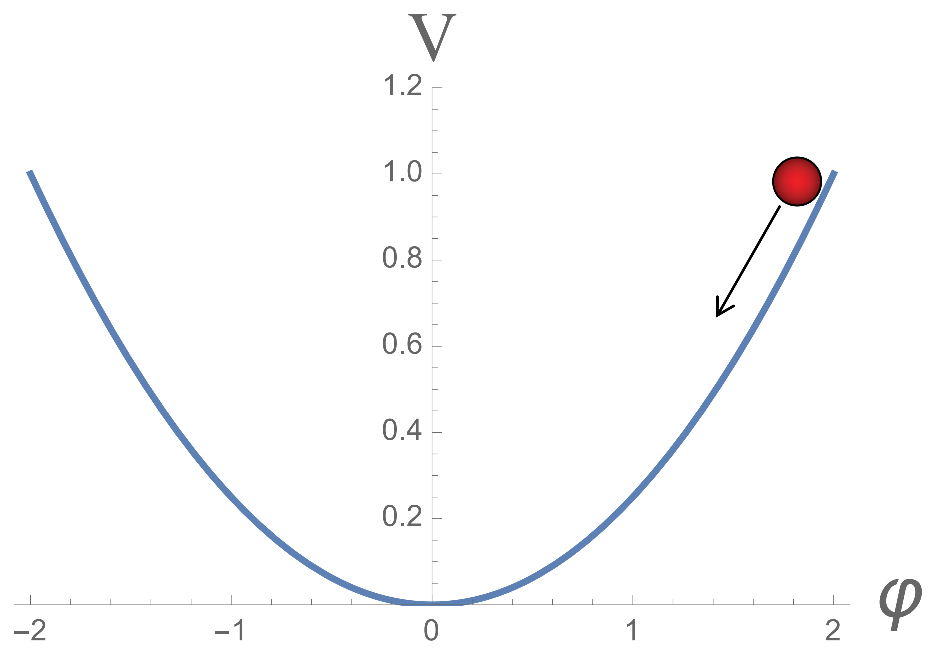
The original entropy S and mass M of the universe were greater than 10^{90}. Why? These are different aspects of the **flatness problem**.

If the size of the causally connected part of the universe at the Planck time was 1, then how the universe “knew” that there is anything beyond this Planckian size domain? (E.g. the universe may be a Planck size closed universe, or an open or flat topologically non-trivial Planck size universe.)

An assumption that the universe was born big leads to the **horizon problem**.

Simplest inflationary model: $V = \frac{m^2\phi^2}{2}$

Inflation can start at the Planck density if there is **a single Planck size domain** with a potential energy V of the same order as kinetic and gradient density. This is the minimal requirement, compared to standard Big Bang, where it is assumed that **10^{90}** Planck size domains simultaneously emerged from the singularity.



But this simple model is disfavored by Planck. What can we do?

One can fit all Planck data by a polynomial, with inflation starting at the Planck density

$$V = \frac{m^2\phi^2}{2}\left(1 - a\phi + b\phi^2\right)$$

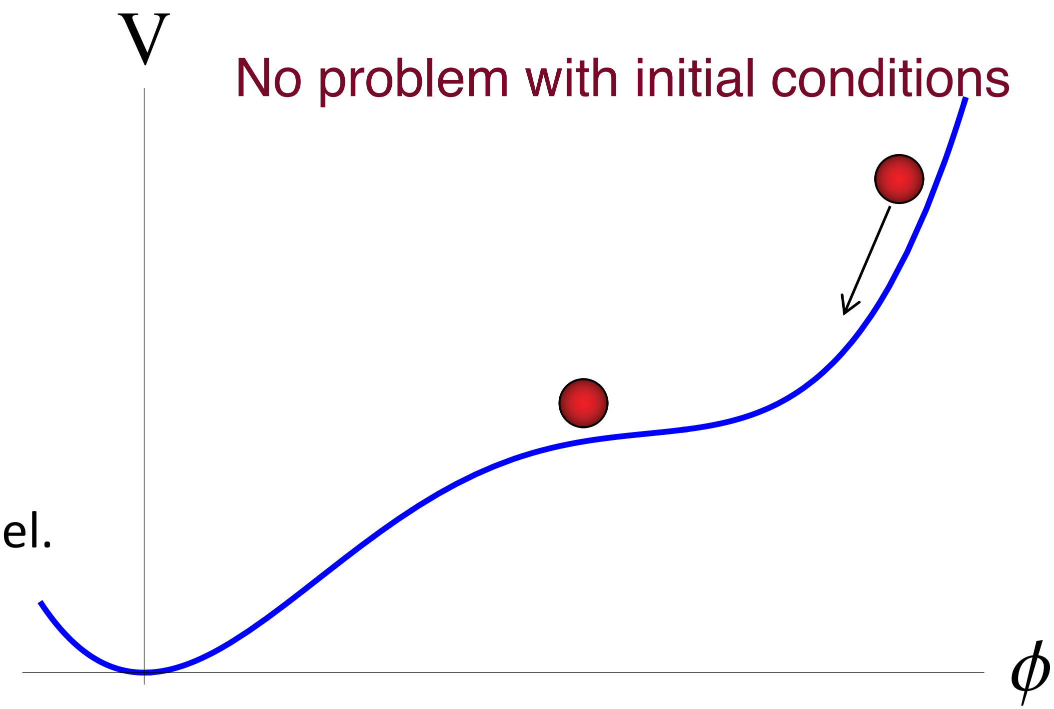
Destri, de Vega, Sanchez, 2007
Nakayama, Takahashi and Yanagida, 2013
Kallosh, AL, Westphal 2014
Kallosh, AL, Roest, Yamada **1705.09247**

3 observables: A_s, n_s, r

3 parameters: m, a, b

Example: $m = 10^{-5}$, $a = 0.12$, $b = 0.29$

If you think that it is complicated, compare it with the Standard model.



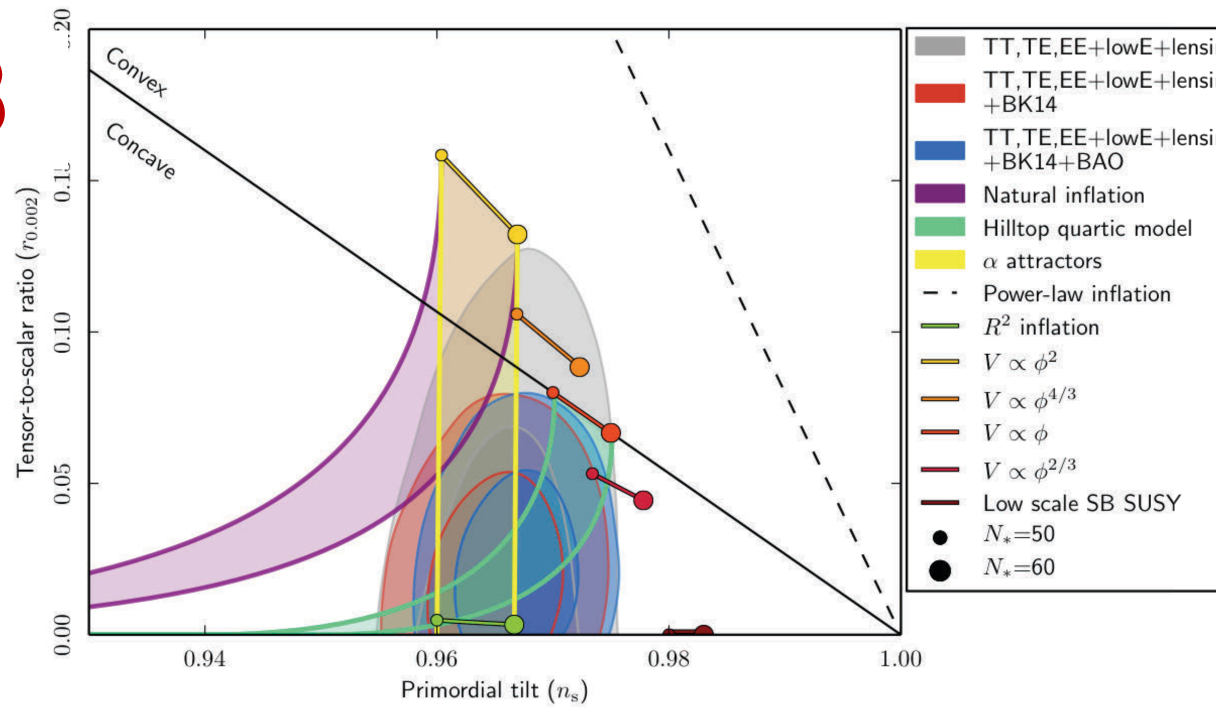
But it is better to have models which require no more than 1 or 2 free parameters

Inflation after Planck 2018

with Renata Kallosh and Yusuke Yamada, 1811.01023, 1906.02156, 1906.04729, 1909.04687

The main goal was to use Planck results and identify possible CMB targets for future observational missions

Planck 2018

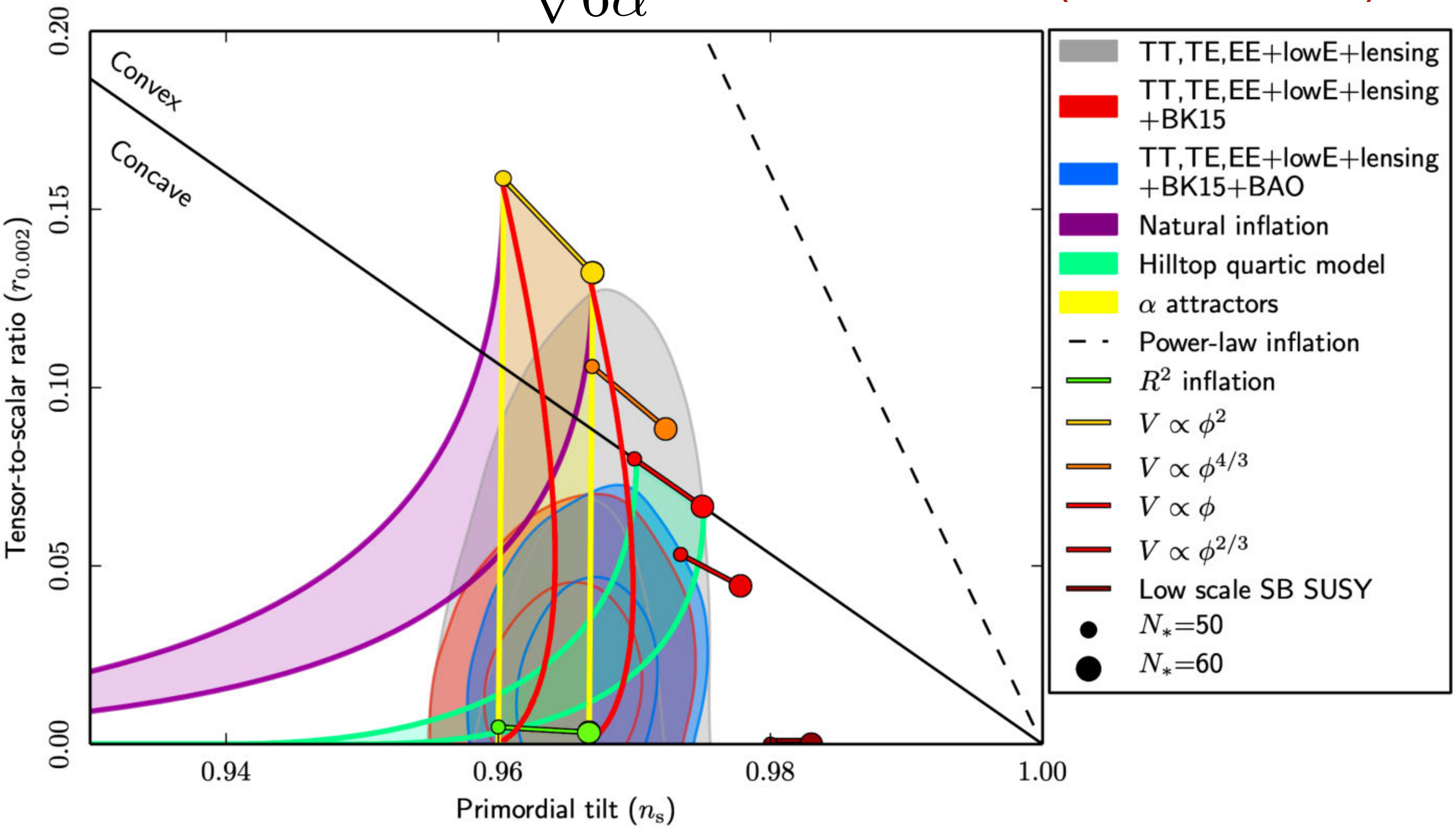


$R + R^2/(6M^2)$				
Power-law potential				
Power-law potential				
Power-law potential				
Power-law potential				
Power-law potential				
Power-law potential				
Non-minimal coupling				
Natural inflation				
Hilltop quadratic model				
Hilltop quartic model	$\Lambda^4\left(1-\phi^4/\mu_4^4+\ldots\right)$	$-2 < \log_{10}(\mu_4/M_{\rm Pl}) < 2$	-0.3	-1.4
D-brane inflation ($p = 2$)	$\Lambda^4\left(1-\mu_{{\rm D}\,2}^2/\phi^p+\ldots\right)$	$-6 < \log_{10}(\mu_{{\rm D}\,2}/M_{\rm Pl}) < 0.3$	-2.3	1.6
D-brane inflation ($p = 4$)	$\Lambda^4\left(1-\mu_{{\rm D}\,4}^4/\phi^p+\ldots\right)$	$-6 < \log_{10}(\mu_{{\rm D}\,4}/M_{\rm Pl}) < 0.3$	-2.2	0.8
Potential with exponential tails	$\Lambda^4\left[1-\exp\left(-q\phi/M_{\rm Pl}\right)+\ldots\right]$	$-3 < \log_{10} q < 3$	-0.5	-1.0
Spontaneously broken SUSY	$\Lambda^4\left[1+\alpha_h\log\left(\phi/M_{\rm Pl}\right)+\ldots\right]$	$-2.5 < \log_{10}\alpha_h < 1$	9.0	-5.0
E-model ($n = 1$)	$\Lambda^4\left\{1-\exp\left[-\sqrt{2}\phi\left(\sqrt{3\alpha_1^{\rm E}}M_{\rm Pl}\right)^{-1}\right]\right\}^{2n}$	$-2 < \log_{10}\alpha_1^{\rm E} < 4$	0.2	-1.0
E-model ($n = 2$)	$\Lambda^4\left\{1-\exp\left[-\sqrt{2}\phi\left(\sqrt{3\alpha_2^{\rm E}}M_{\rm Pl}\right)^{-1}\right]\right\}^{2n}$	$-2 < \log_{10}\alpha_2^{\rm E} < 4$	-0.1	0.7
T-model ($m = 1$)	$\Lambda^4\tanh^{2m}\left[\phi\left(\sqrt{6\alpha_1^{\rm T}}M_{\rm Pl}\right)^{-1}\right]$	$-2 < \log_{10}\alpha_1^{\rm T} < 4$	-0.1	0.1
T-model ($m = 2$)	$\Lambda^4\tanh^{2m}\left[\phi\left(\sqrt{6\alpha_2^{\rm T}}M_{\rm Pl}\right)^{-1}\right]$	$-2 < \log_{10}\alpha_2^{\rm T} < 4$	-0.4	0.1

α-attractors and Planck 2018

T-models (yellow) and E-models (red)

$$V_T = V_0 \tanh^2 \frac{\varphi}{\sqrt{6\alpha}} \qquad V_E = V_0 \left(1 - e^{\sqrt{\frac{2}{3\alpha}}\varphi}\right)^2$$



T-models (yellow) and E-models (red)

$$V_T = V_0 \tanh^2 \frac{\varphi}{\sqrt{6\alpha}}$$

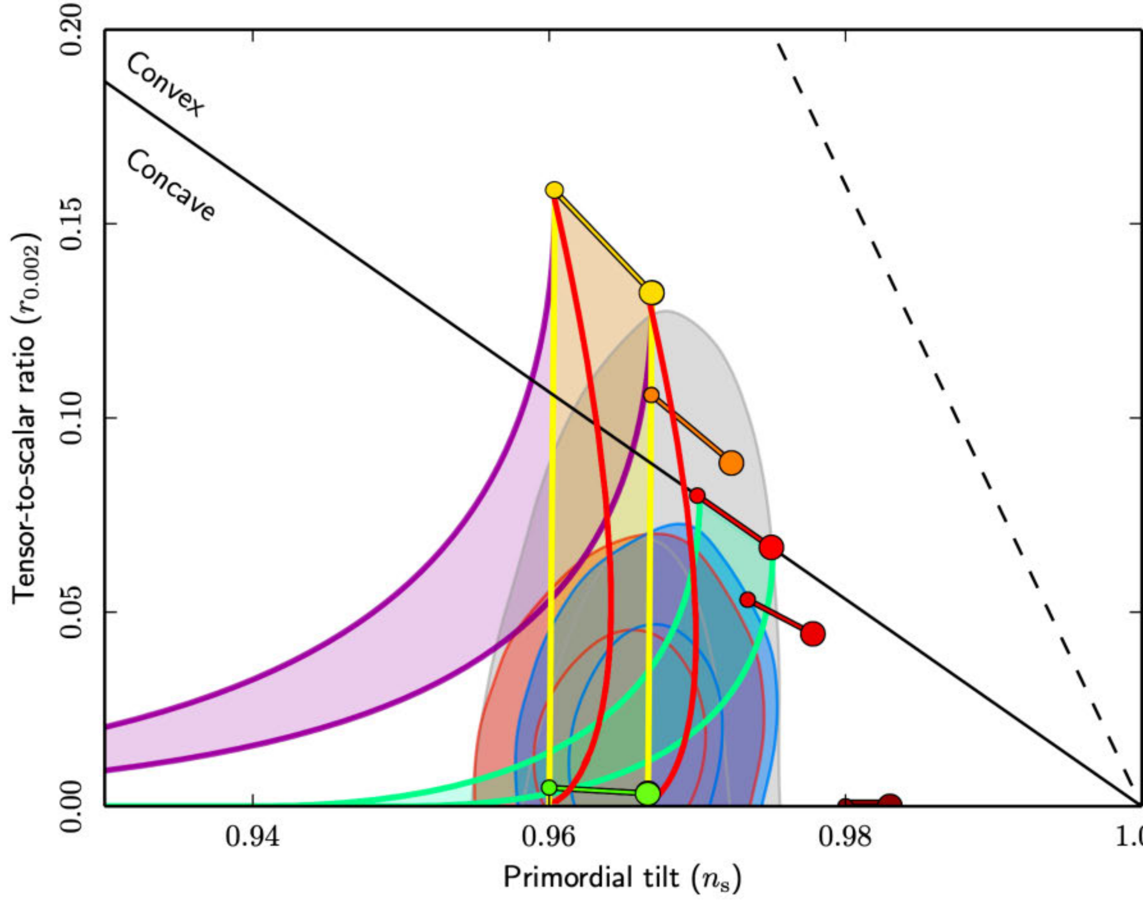
$$V_E = V_0 \left(1 - e^{\sqrt{\frac{2}{3\alpha}}\varphi}\right)^2$$

Looking at the 1σ area (dark pink or dark blue), we see that most of it is covered by two simplest models of α-attractors. The two circles correspond to the Starobinsky model and Higgs inflation

The green area in the Planck figure was supposed to describe hilltop inflation

$$V = V_0 \left(1 - \frac{\varphi^4}{m^4}\right)$$

but this model leads to collapse of the universe after inflation. Its improved versions are complicated and lead to different, model dependent results. Thus, the green area in this figure is misleading.



Meaning of α-attractors

Kallosh, AL, Roest 2014

Start with the simplest chaotic inflation model

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\partial\phi^2 - \frac{1}{2}m^2\phi^2$$

Modify its kinetic term

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\frac{\partial\phi^2}{(1-\underline{\frac{\phi^2}{6\alpha}})^2} - \frac{1}{2}m^2\phi^2$$

Switch to canonical variables $\phi = \sqrt{6\alpha}\,\tanh\frac{\varphi}{\sqrt{6\alpha}}$

The kinetic term becomes canonical, and one finds a plateau potential

$$V = 3\alpha\, m^2 \tanh^2\frac{\varphi}{\sqrt{6\alpha}}$$

Another class of models favored by Planck 2018 is D-brane inflation

In classification of *Encyclopaedia Inflationaris*, there are two main types of such models. The simplest ones are **BI** (**b**rane **i**nflation) models. They have potentials which look like inverted hilltop

Hilltop: $$1-\frac{\varphi^n}{m^n}$$ BI: $$1-\frac{m^n}{\varphi^n}$$

Just like the simplest hilltop inflation, the BI models are ruled out since they lead to immediate collapse of the universe after inflation.

The second class is called **KKLTI** models (from **KKLT I**nflation).

Plateau potential $$V_{KKLTI}=V_0\,\frac{\varphi^n}{\varphi^n+m^n}$$

They describe Dp-brane inflation, which (in the small r limit) predict

$$(1-n_s)|_{r\to 0}=\frac{2}{N}\,\frac{8-p}{9-p}$$

α-attractors, D-brane inflation and pole inflation

α-attractors and KKLTI models form a physically motivated (in SUGRA and string theory) subclass of models of pole inflation with

$$\mathcal{L} = \mathcal{L}_{\rm kin} - V = -\frac{1}{2}\frac{a_q}{\rho^q}(\partial\rho)^2 - V(\rho)$$

α-attractors correspond to pole inflation with $q = 2$ (supported by SUGRA)

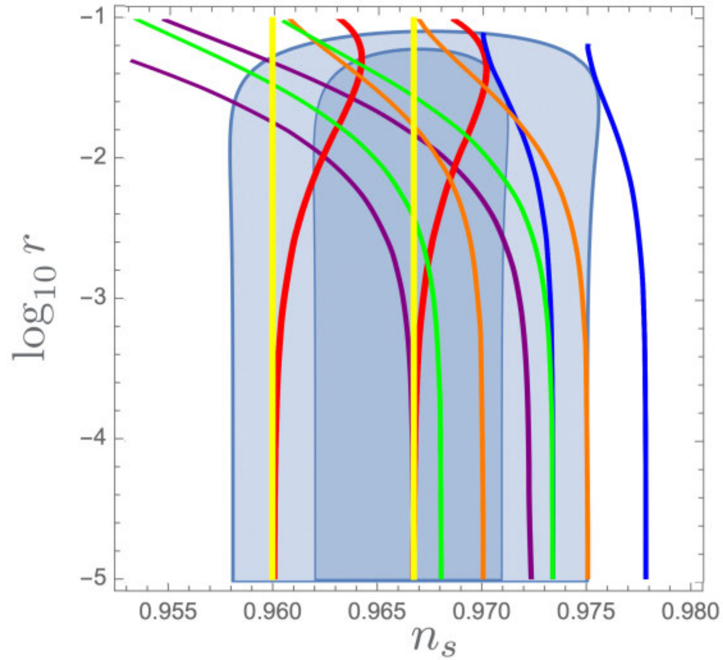
D-brane inflation forms a subclass of physically motivated models of pole inflation with

$$q \;=\; \frac{5}{3}, \quad \frac{8}{5}, \quad \frac{4}{3}, \quad \frac{3}{2}$$

These models at small r describe a set of β – stripes with

$$1 - n_s = \frac{\beta}{N} \qquad\qquad \beta = \frac{q}{q-1}$$

T-models, E-models and KKLTI models on Log r scale:



α-attractors and KKLTI models of Dp-brane inflation with p = 3, 4, 5, 6 form a set of stripes, which become vertical at small r:

$$1 - n_s = \frac{\beta}{N}\,, \qquad \beta = 2,\ \frac{5}{3},\ \frac{8}{5},\ \frac{4}{3},\ \frac{3}{2}$$

A combination of α-attractors and KKLTI models covers most of the area favored by Planck 2018, all the way down to r = 0.

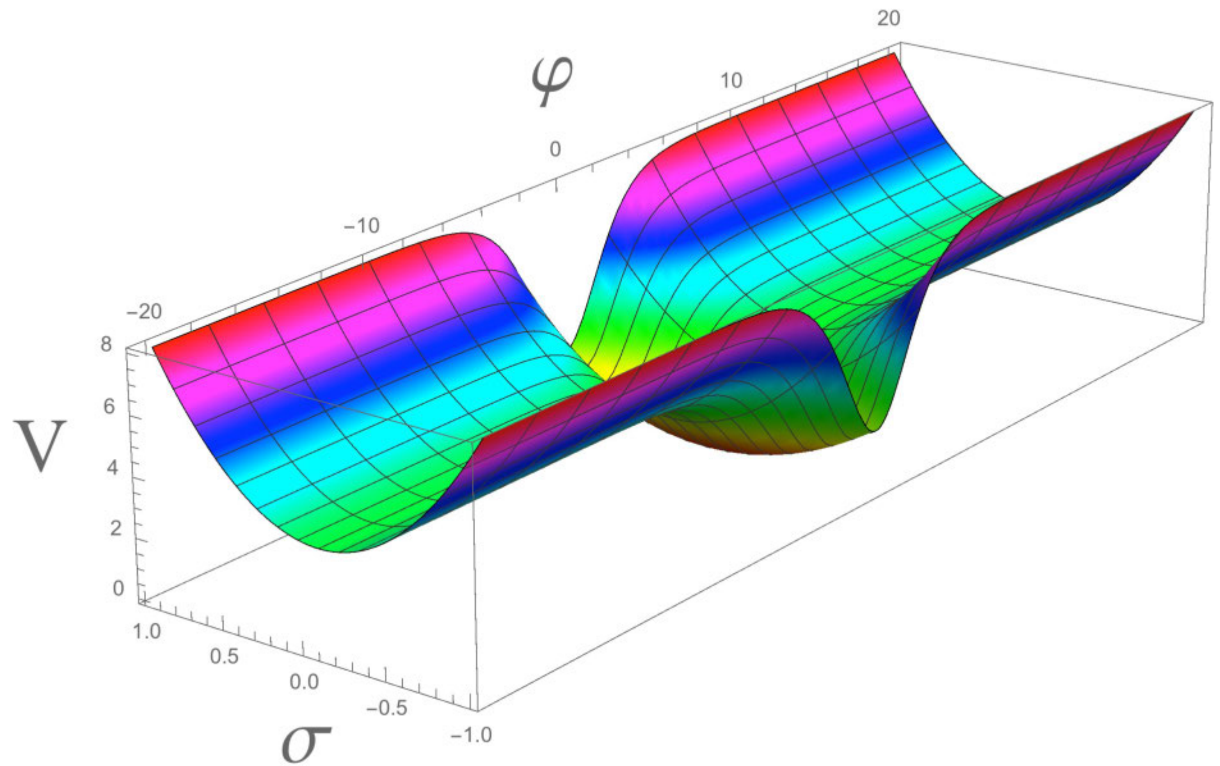
In all of these models, as well as in the Starobinsky model and in Higgs inflation, the inflaton potential is a plateau 10 orders of magnitude below Planck density.

It could seem that we have a problem with initial conditions.

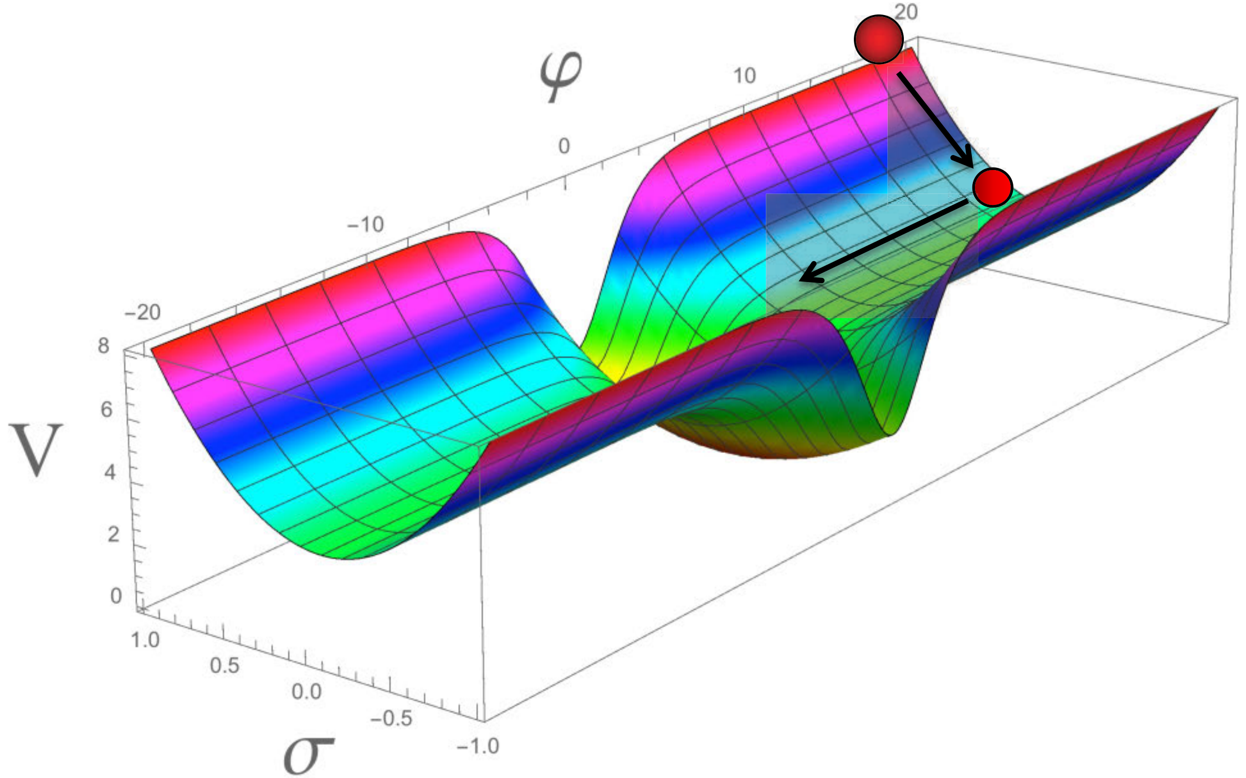
α-attractors and the simplest quadratic model

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\frac{(\partial\phi)^2}{(1-\frac{\phi^2}{6\alpha})^2} - \frac{1}{2}m^2\phi^2 - \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}M^2\sigma^2$$

Potential in canonical variables has a plateau at large values of the inflaton field, and it is quadratic with respect to σ.



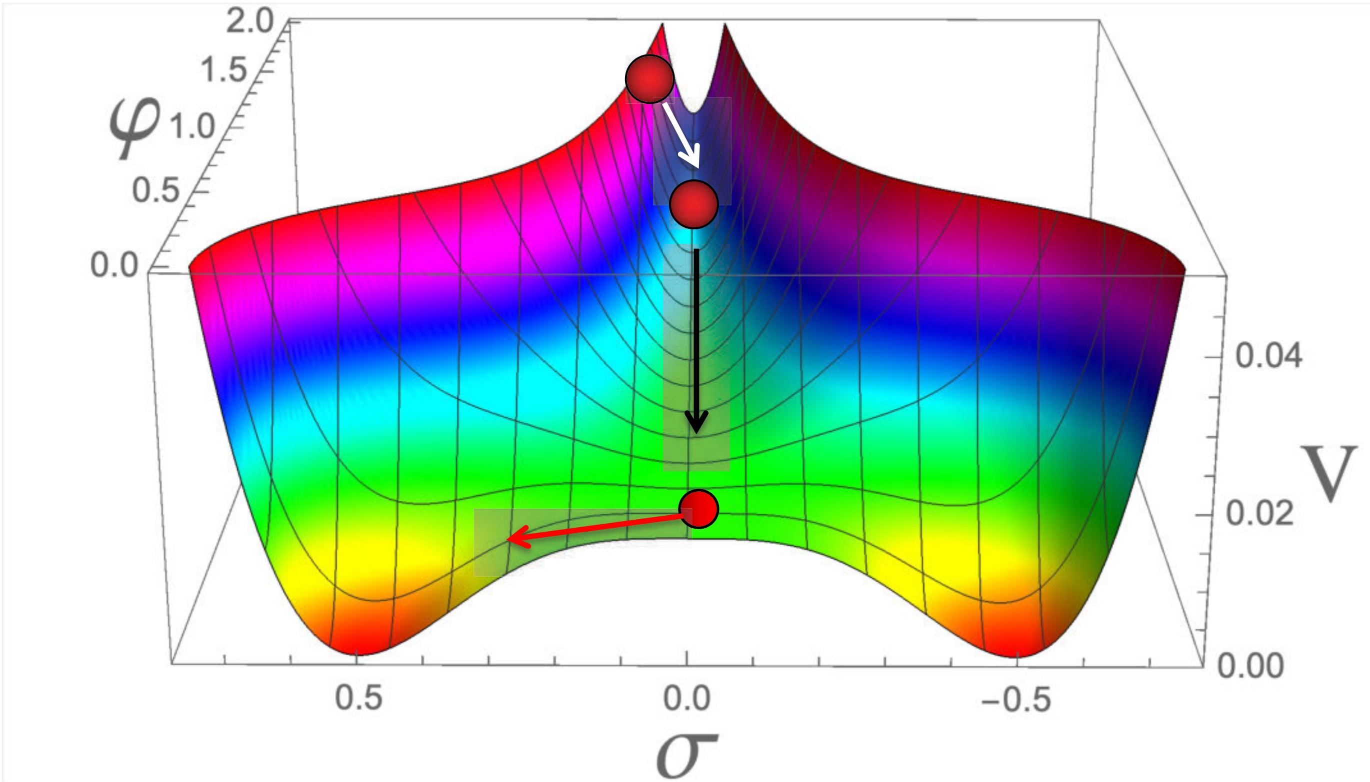
Initial conditions for plateau inflation



Chaotic inflation with a parabolic potential goes first, starting at nearly Planckian density. When the field σ rolls down, the plateau inflation begins.

No problem with initial conditions

Initial conditions for small field inflation



Chaotic inflation with a parabolic potential goes first, starting at nearly Planckian density. Then the field ϕ rolls down, and the small field "new" inflation begins.

No problem with initial conditions

There is another, simple and general way to solve the problem of initial conditions for low energy scale inflation, without using additional fields

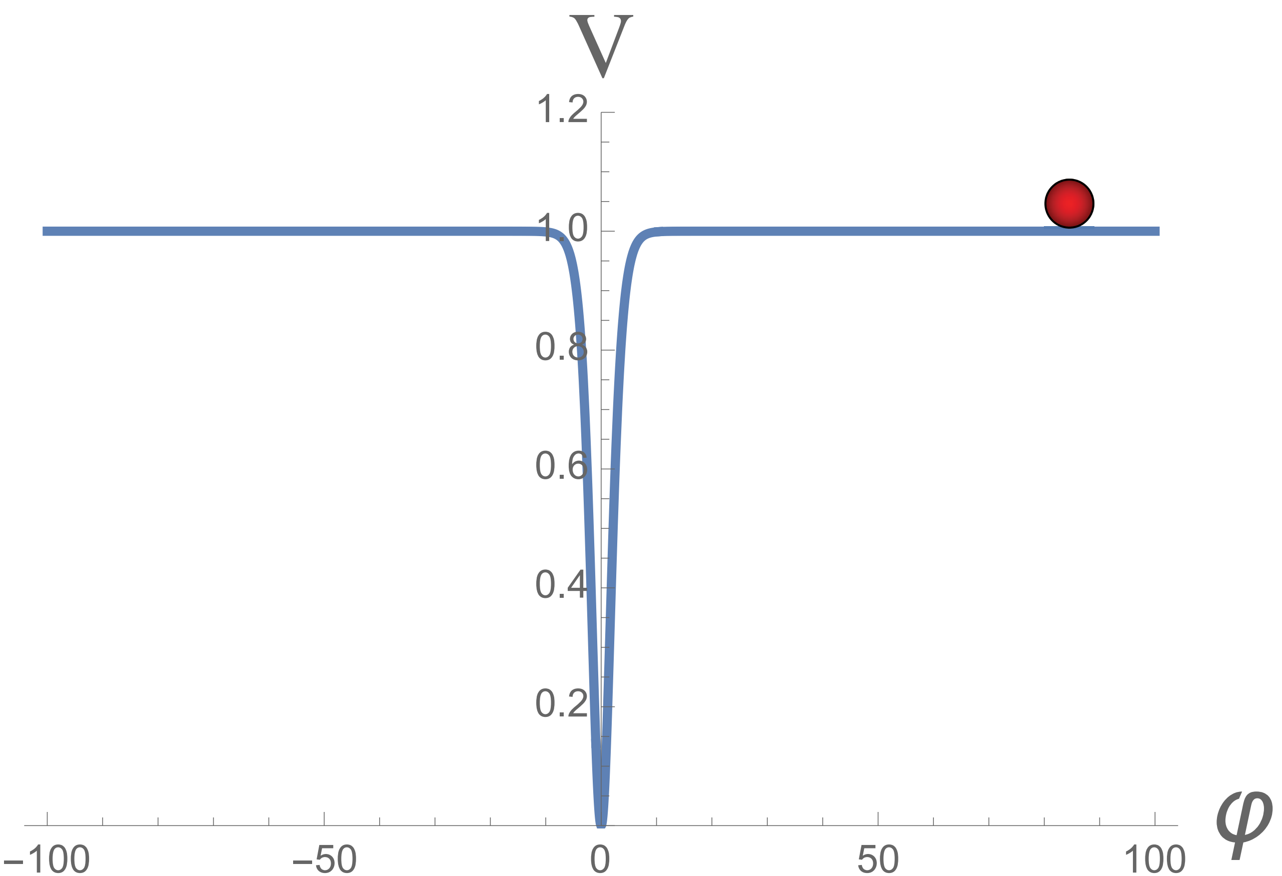
East, Kleban, AL, Senatore 1511.05143
Kleban, Senatore 1602.53520
Clough, Lim, DiNunno, Fischler, Flauger, Paban 1608.04408

Consider a smallest possible universe starting in the Planck size domain with Planck density and with a sufficiently flat potential (either plateau or power law). Suppose that the universe is flat or open, but compact (e.g. a torus). It can be grossly inhomogeneous, with inhomogeneities much greater than the value of an inflationary potential. If the universe begins with the field far away from the minimum of the potential, and if it does not immediately collapse as a whole within the Planck time, i.e. if the universe is not just a quantum fluctuation, then it continues expanding until inflation begins and makes the universe flat and homogeneous.

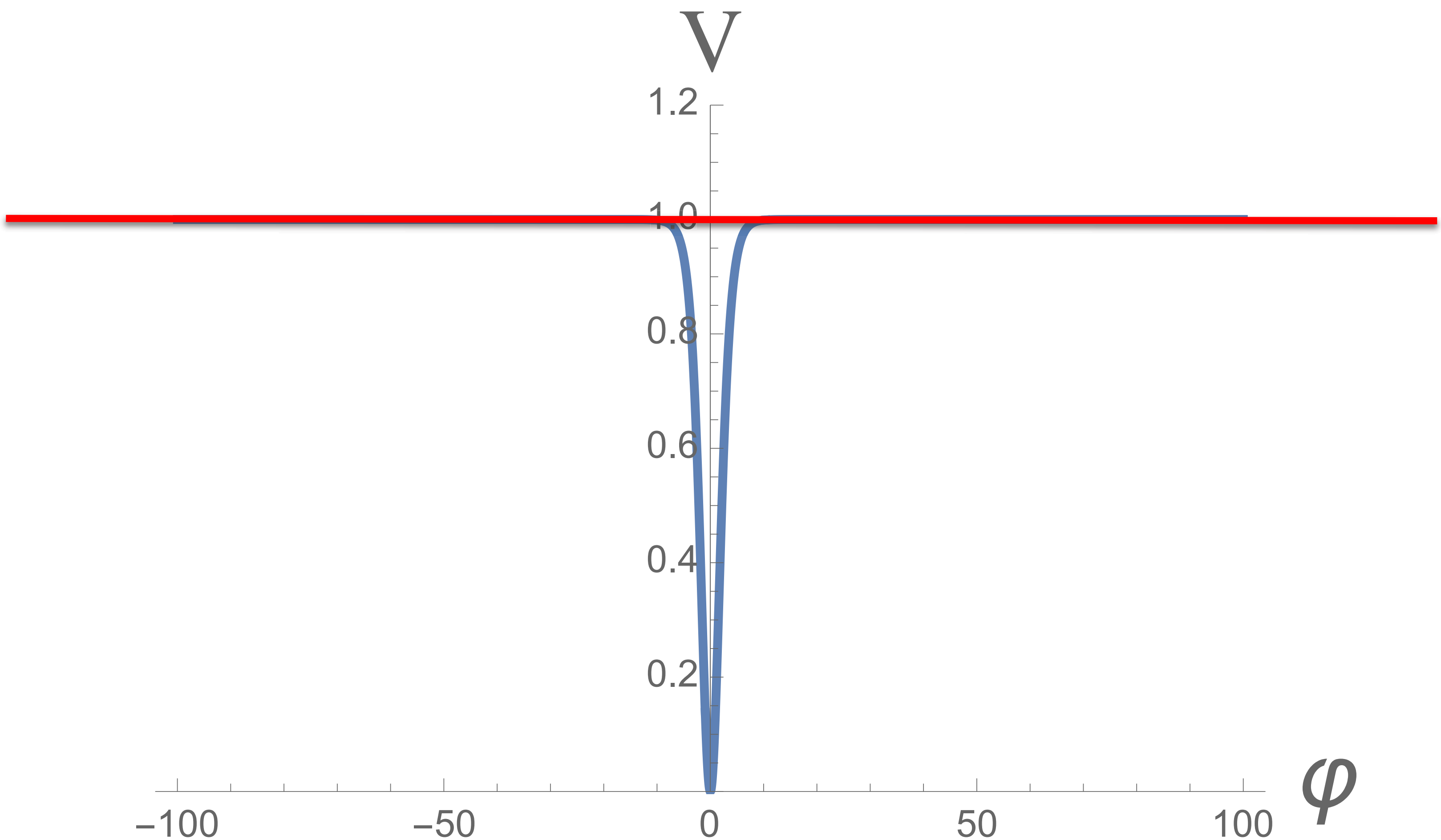
East, Kleban, AL, Senatore 1511.05143
AL 1710.04278

Consider α-attractors or D-brane inflation

Carrasco, Kallosh, AL 1506.00936
East, Kleban, AL, Senatore 1511.05143



This potential coincides with the cosmological constant almost everywhere.



For the cosmological constant, the question is opposite:

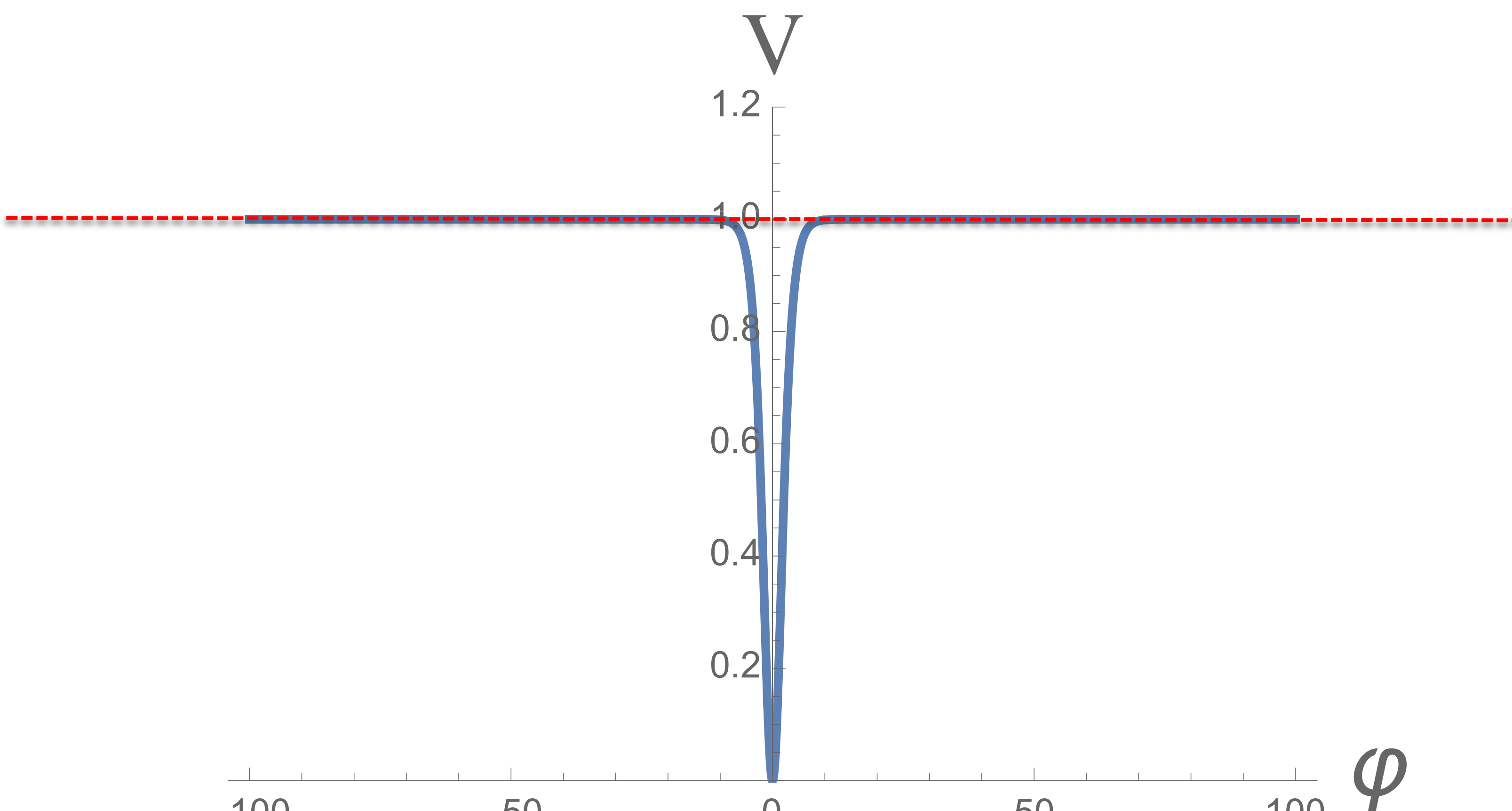
Start at the Planck density, in the universe dominated by inhomogeneities. Energy density of matter is diluted by the cosmological expansion as $1/t^2$. What could prevent exponential expansion of the universe, which becomes dominated by the cosmological constant Λ after the time $t = \Lambda^{-1/2}$?

Inflation does NOT happen in the expanding universe with $\Lambda = 10^{-10}$ only if if the whole universe collapses within 10^{-28} seconds after its birth.

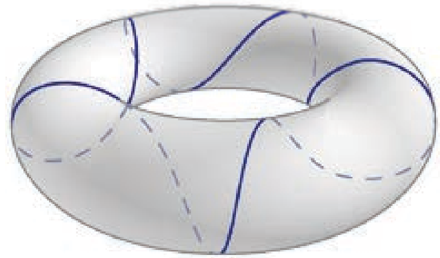
In other words, only instant global collapse could prevent an expanding universe to avoid exponential expansion dominated by the cosmological constant.

If the universe does not instantly collapse, it inflates.

This optimistic conclusion related to the cosmological constant applies to α-attractors and D-brane inflation as well, because their potential coincides with the cosmological constant almost everywhere.



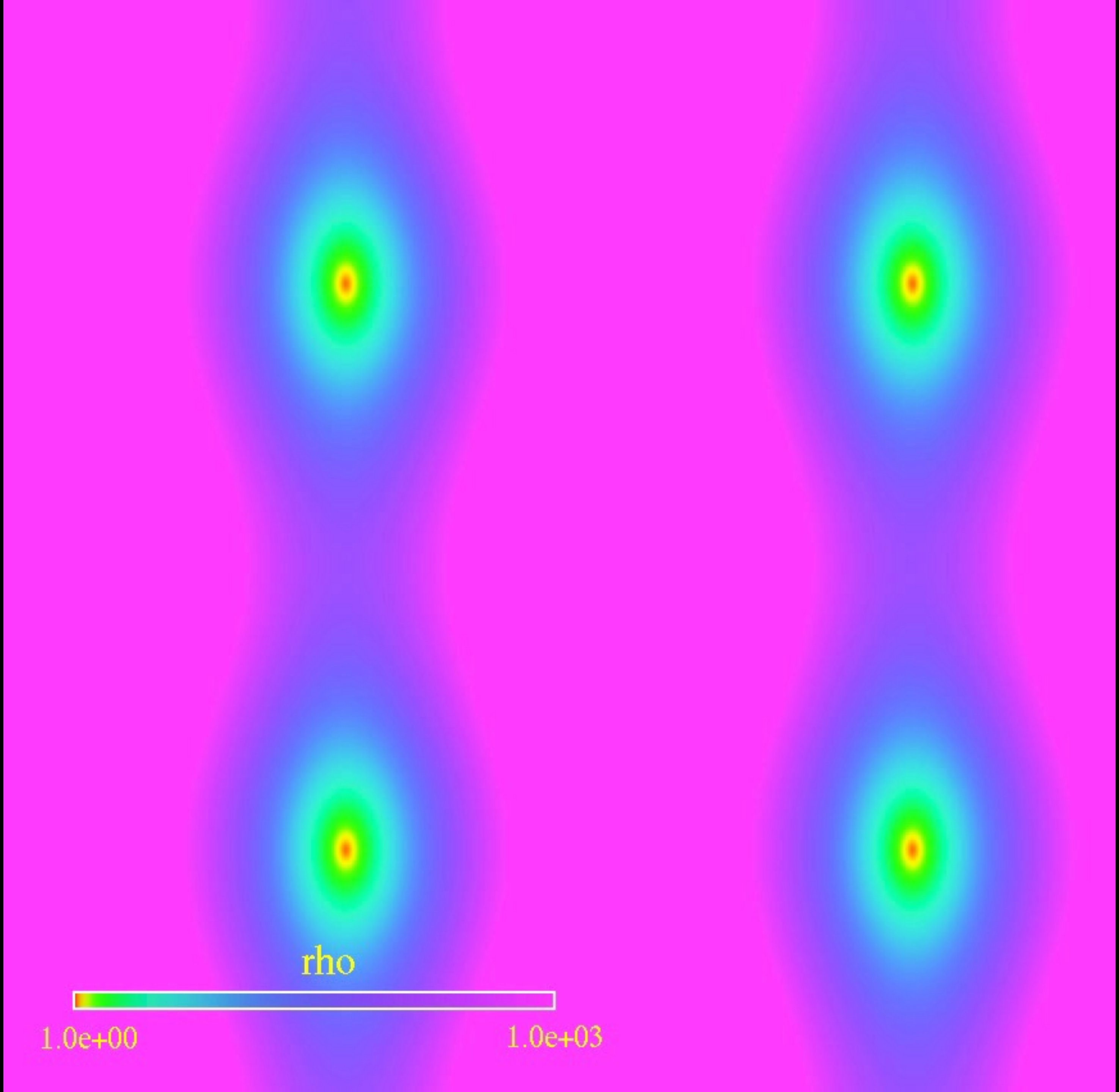
Yet another helpful consideration:



Consider a flat toroidal universe of Planck size O(1). The longest wavelength of the perturbation one can put there is O(1), so its momentum is O(1). After time t, the size of the universe grows slower than t, but the horizon size grows at t. This means that the original scalar fields have momenta $k > H$, they become ultra-relativistic and do not want to collapse. The best time for some parts of the universe to collapse is at the very beginning, i.e. at the Planck time.

Let us compare these arguments with the results of numerical analysis in a grossly inhomogeneous universe.

East, Kleban, AL, Senatore 1511.05143


rho
1.0e+00
1.0e+03

These conclusions should be valid for general large field inflation models

East, Kleban, AL, Senatore 1511.05143
AL 1710.04278
Clough, Lim, DiNunno, Fischler, Flauger, Paban 1608.04408
Joana, Clesse 2011.12190