

Einstein-Cartan Gravity and Higgs Inflation

Sebastian Zell

Based on work¹ with

M. Shaposhnikov, A. Shkerin and I. Timiryasov

École Polytechnique Fédérale de Lausanne

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¹ *Einstein-Cartan gravity, matter, and scale-invariant generalization*, JHEP **2020** no. 10, 177, arXiv:2007.16158;
Higgs inflation in Einstein-Cartan gravity, JCAP **2021** no. 02, 008, arXiv:2007.14978

How Well Do We Know Gravity?

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\Rightarrow Equivalent in pure gravity

Metric vs. Palatini Gravity

- ▶ Coupling to scalar field h

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- ▶ Phenomenology depends on formulation of gravity

Open question

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Einstein-Cartan gravity

- ▷ Generalizes metric and Palatini formulations
- ▷ Gauge theory of Poincaré group²

- ① From Palatini to Einstein-Cartan
- ② Higgs Inflation
- ③ Conclusions

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- ▶ Difference mapped to kinetic term

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Extend action

- ▶ Holst term

$$S \supset \bar{\gamma}^{-1} \frac{M_p^2}{4} \int d^4x \sqrt{-g} R_{\mu\nu\rho\sigma} \epsilon^{\mu\nu\rho\sigma}$$

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- ▶ New coupling constants

Analyze Theory

- ▶ Example: Nieh-Yan

$$\begin{aligned} S = & \int d^4x \sqrt{-g} \left\{ \frac{M_P^2}{2} \Omega^2 R - \frac{1}{2} (\partial_\mu h)^2 - U \right\} \\ & - \frac{1}{4} \int d^4x \partial_\mu (\sqrt{-g} \epsilon^{\mu\nu\rho\sigma} T_{\nu\rho\sigma}) \xi_\eta h^2 \end{aligned}$$

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- ▶ $\xi_\eta = 0$: Palatini

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- ▶ $\xi_\eta = 0$: Palatini
- ▶ $\xi_\eta = \xi$: metric

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Dimensionless couplings $\xi_\eta, \dots \Leftrightarrow$ Torsion-free theory:
energy scales $\frac{M_P}{\xi_\eta}, \dots$

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- ▶ Metric and Palatini formulations as special cases

1 From Palatini to Einstein-Cartan

2 Higgs Inflation

3 Conclusions

Metric and Palatini Higgs Inflation

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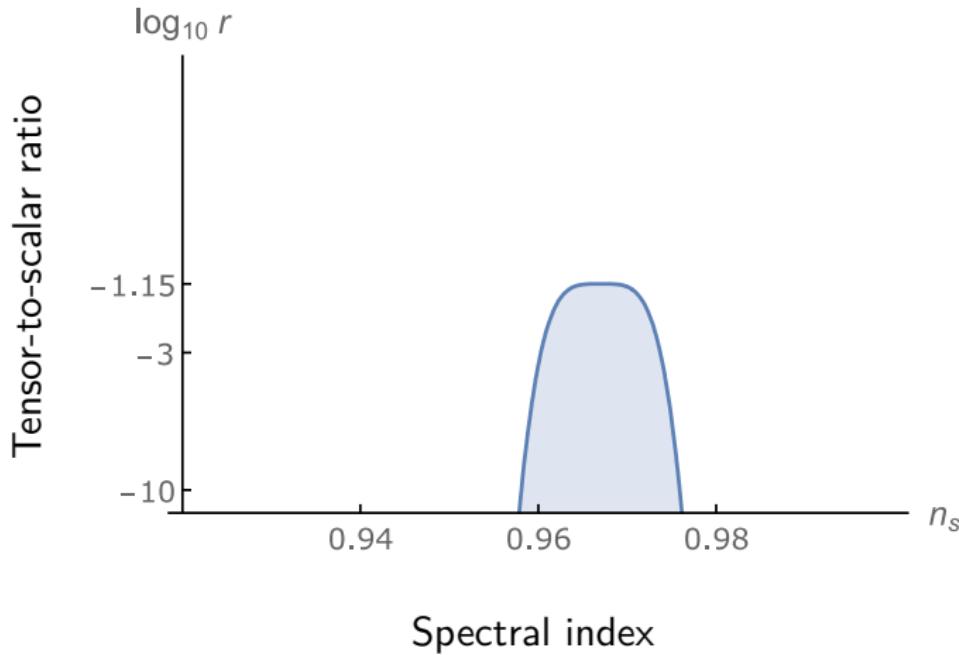
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- ▶ Match amplitude of perturbations:
metric: $\xi \sim 10^3$ Palatini: $\xi \sim 10^7$

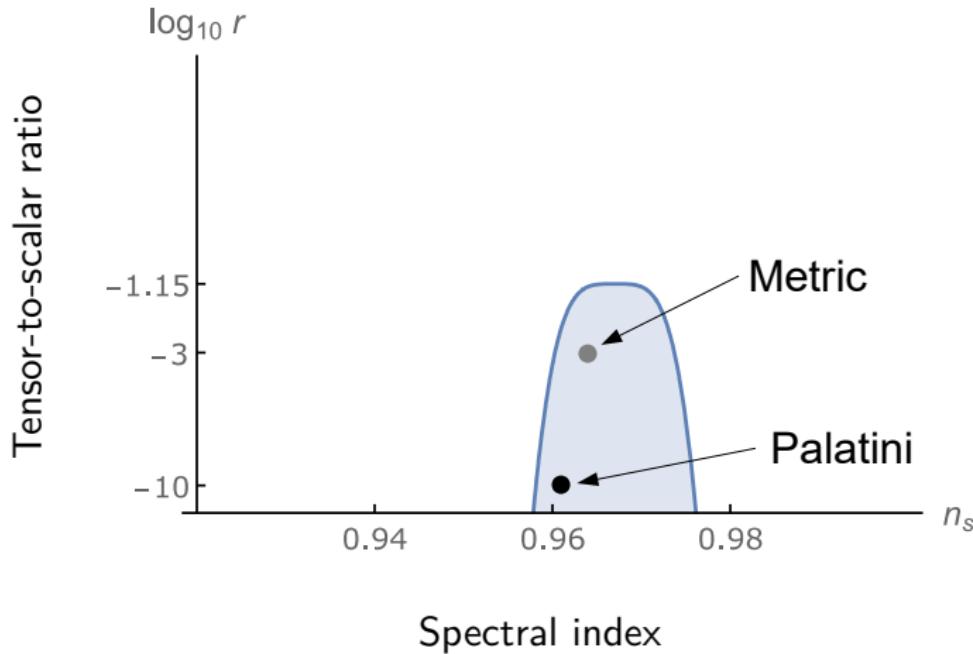
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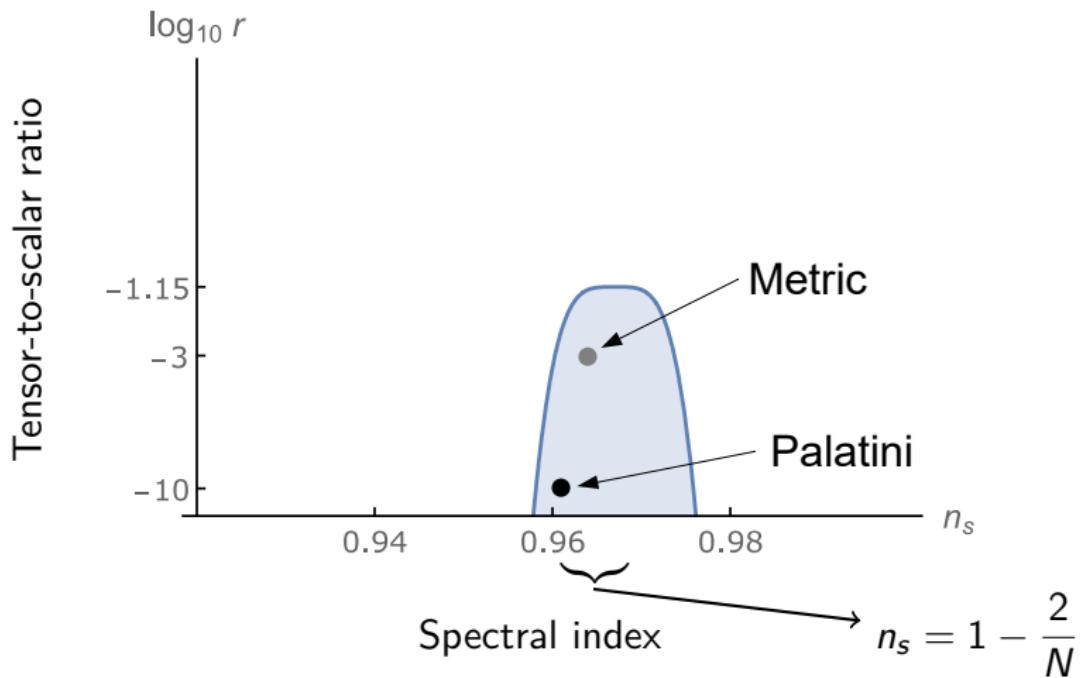
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Higgs Inflation in Einstein-Cartan³

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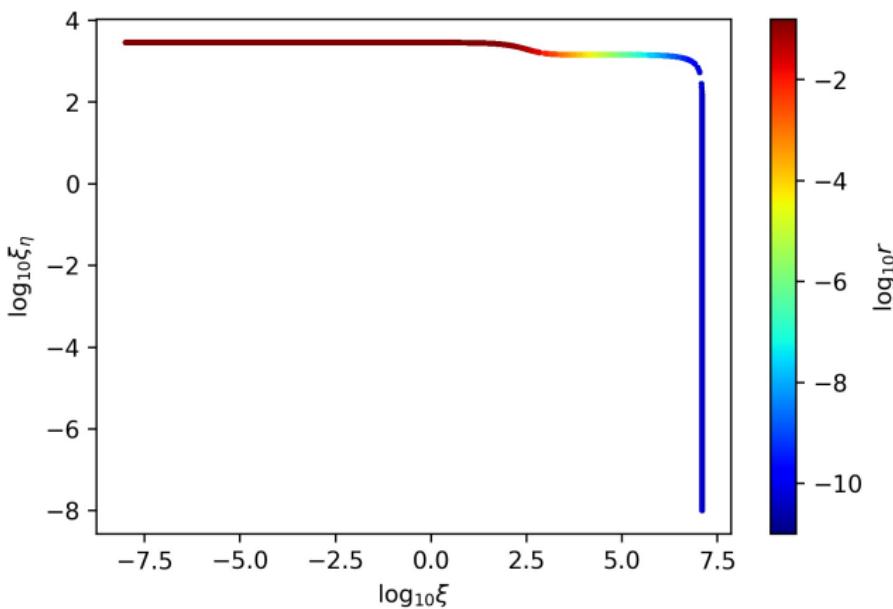
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- Generalizes Palatini ($\xi_\eta = 0$) and metric ($\xi_\eta = \xi$) scenarios

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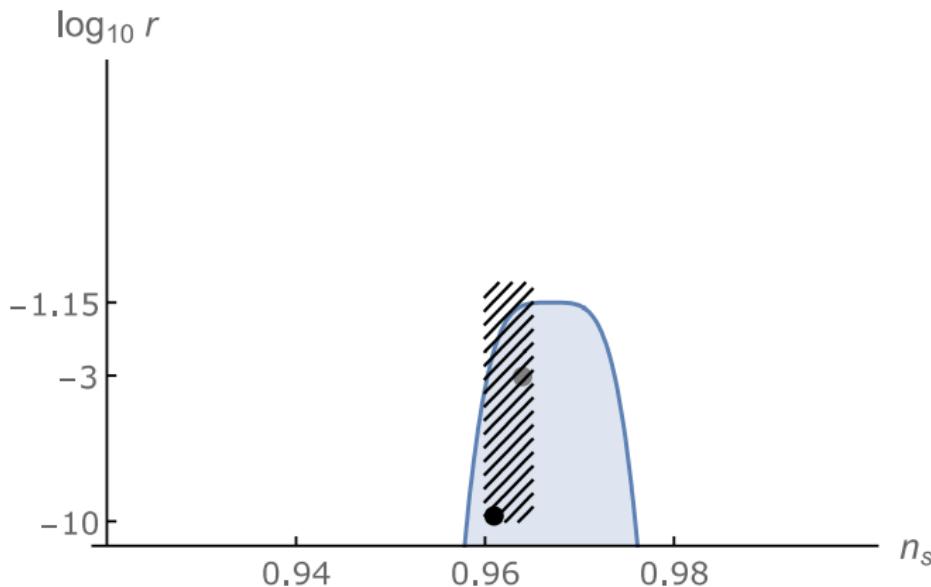
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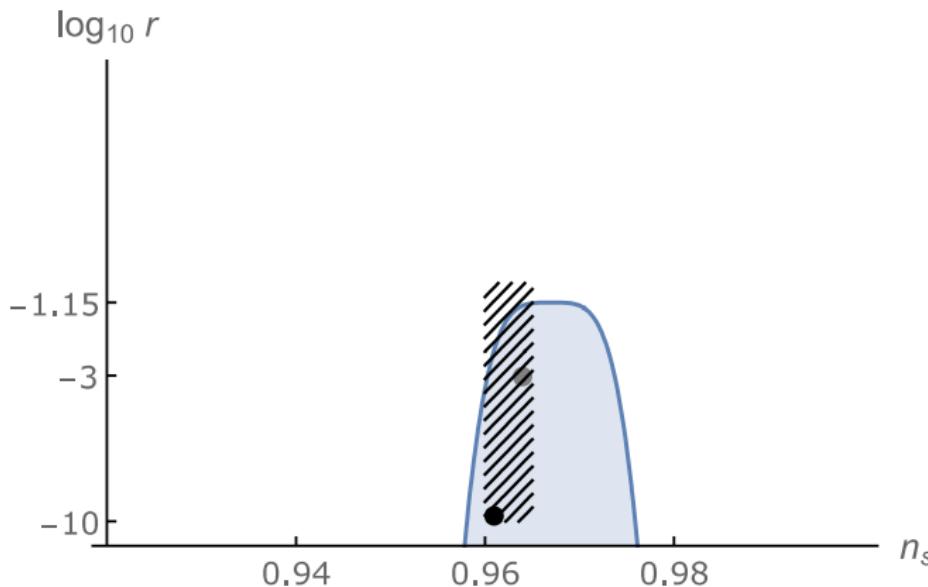


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Inflationary Predictions



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Detection of gravitational waves possible

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- additional material
- ▶ Predictions no longer unique

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Add Fermions

$$S = \frac{i}{2} \int d^4x \sqrt{-g} \left(\bar{\Psi} \gamma^\mu D_\mu \Psi - \text{h.c.} \right)$$

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- ▶ Non-minimal couplings⁴

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- ▶ Specific interactions for scalars and fermions additional material

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Further Applications

- ▶ Universal 4-fermion interaction:
new mechanism for production of fermionic dark matter⁵

See talk by I. Timiryasov in workshop on Dark Matter

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- ▶ Electroweak symmetry breaking by gravitational instanton⁶
See talk by M. Shaposhnikov on Monday

⁵ M. Shaposhnikov, A. Shkerin, I. Timiryasov, S. Z., *Einstein-Cartan Portal to Dark Matter*, arXiv:2008.11686.

⁶ M. Shaposhnikov, A. Shkerin, S. Z., *Standard Model Meets Gravity: Electroweak Symmetry Breaking and Inflation*, arXiv:2001.09088.

Summary

Einstein-Cartan theory

- ▶ Pure gravity: all formulations equivalent

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Einstein-Cartan theory

- ▶ Pure gravity: all formulations equivalent
- ▶ Include matter: additional couplings
- ▶ In equivalent torsion-free theory:
geometric criterion for selecting higher-dimensional operators
- ▶ Interesting phenomenology
 - ▷ Higgs inflation
 - ▷ Fermion dark matter production
 - ▷ Non-perturbative generation of Higgs mass
 - ▷ ...

Additional Material

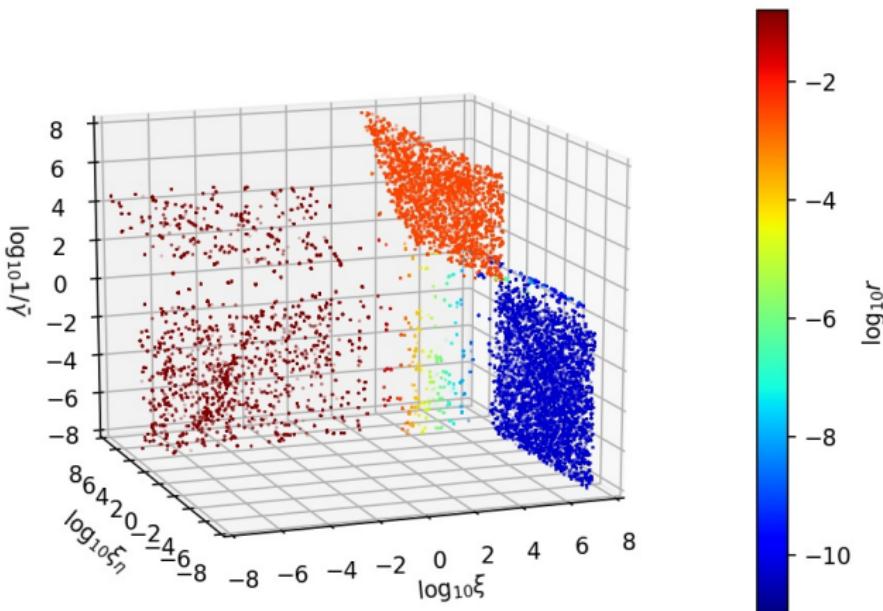
- 4 Einstein-Cartan Higgs Inflation
- 5 Full Action
- 6 Fermionic Dark Matter
- 7 Gravitational Instanton
- 8 Form Language

Generalization

Extend with Holst term: $\frac{1}{\gamma} \frac{M_p^2}{4} \int d^4x \sqrt{-g} R_{\mu\nu\rho\sigma} \epsilon^{\mu\nu\rho\sigma}$

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Full Action

Use form language additional material

$$\begin{aligned}
 S = & \frac{1}{4} \int (M_P^2 + \xi h^2) \epsilon_{IJKL} e^I e^J F^{KL} \\
 & + \frac{1}{2\bar{\gamma}} \int (M_P^2 + \xi_\gamma h^2) e^I e^J F_{IJ} + \frac{1}{2} \int \xi_\eta h^2 d(e^I e^J C_{IJ}) \\
 & + \int \frac{\epsilon_{IJKL} e^I e^J e^K e^L}{24} \left(-\frac{1}{2} (\partial_N h)^2 - U \right) \\
 & + \frac{i}{12} \int \epsilon_{IJKL} e^I e^J e^K \left(\bar{\Psi} (1 - i\alpha - i\beta\gamma^5) \gamma^L D\Psi - \text{h.c.} \right)
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 \end{aligned}$$

Conformal transformation

$$e^I \rightarrow \frac{e^I}{\Omega} \quad \omega^{IJ} \rightarrow \omega^{IJ} \quad \Omega^2 = 1 + \frac{\xi h^2}{M_P^2}$$

After Conformal Transformation

$$\begin{aligned}
 S = & \frac{M_P^2}{4} \int \left\{ \epsilon_{IJKL} e^I e^J F^{KL} + 2\gamma e^I e^J F_{IJ} + 2\eta d \left(\frac{e^I e^J C_{IJ}}{\Omega^2} \right) \right\} \\
 & + \int \frac{\epsilon_{IJKL} e^I e^J e^K e^L}{24} \left(-\frac{1}{2} \frac{1}{\Omega^2} (\partial_N h)^2 - \frac{U}{\Omega^4} \right) \\
 & + \frac{i}{12} \int \epsilon_{IJKL} e^I e^J e^K \left(\bar{\Psi} (1 - i\alpha - i\beta\gamma^5) \gamma^L D\Psi - \text{h.c.} \right) \\
 & + \frac{1}{8} \int \frac{d\Omega^2 \epsilon_{IJKL} e^I e^J e^K}{\Omega^2} (\alpha V^L + \beta A^L)
 \end{aligned}$$

$$\Omega^2 = 1 + \frac{\xi h^2}{M_P^2} \quad \gamma = \frac{1 + \frac{\xi_\gamma h^2}{M_P^2}}{\bar{\gamma} \Omega^2} \quad \eta = \frac{\xi_\eta h^2}{M_P^2}$$

Solve for Torsion

Equation of motion

$$2C^{[I}_K B^{KJ]} + \left(\frac{d\eta}{\Omega^2} + d\gamma \right) e^I e^J = \frac{1}{M_P^2} \left(\frac{1}{2} e^I e^J e^P A_P + \frac{1}{6} \epsilon_{KLMN} e^K e^L e^M \delta^{N[I} (\alpha V^{J]} + \beta A^{J]}) \right)$$

$$B^{KJ} = \frac{1}{2} \epsilon^{KJLM} e_L e_M + \gamma e^K e^J$$

Solution

$$\begin{aligned} C^{IJ} &= -\frac{1}{2(\gamma^2 + 1)} \left(\epsilon^{IJKL} e_K \left(\frac{\partial_L \eta}{\Omega^2} + \partial_L \gamma \right) - 2\gamma e^{[I} \left(\frac{\partial^{J]}\eta}{\Omega^2} + \partial^{J]}\gamma \right) \right) \\ &\quad + \frac{1}{4M_P^2(\gamma^2 + 1)} \left(\epsilon^{IJKL} e_K (A_L + \gamma(\alpha V_L + \beta A_L)) \right. \\ &\quad \left. + 2e^{[I} (\alpha V^{J]} + (\beta - \gamma) A^{J]}) \right) \end{aligned}$$

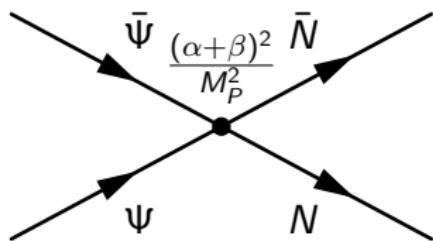
Equivalent Metric Theory

$$\begin{aligned}
 S^{\text{eff}} = & \int d^4x \sqrt{-g} \left\{ \frac{M_P^2}{2} \dot{R} + \frac{i}{2} \bar{\Psi} \gamma^\mu \dot{D}_\mu \Psi - \frac{i}{2} \bar{D}_\mu \Psi \gamma^\mu \Psi \right\} \\
 & - \int d^4x \sqrt{-g} \left\{ \frac{1}{2\Omega^2} (\partial_\mu h)^2 + \frac{U}{\Omega^4} \right\} \\
 & - \int d^4x \sqrt{-g} \frac{3M_P^2}{4(\gamma^2 + 1)} \left(\frac{\partial_\mu \eta}{\Omega^2} + \partial_\mu \gamma \right)^2 \\
 & + \int d^4x \sqrt{-g} \frac{3\alpha}{4} \left(\frac{\partial_\mu \Omega^2}{\Omega^2} + \frac{\gamma}{\gamma^2 + 1} \left(\frac{\partial_\mu \eta}{\Omega^2} + \partial_\mu \gamma \right) \right) V^\mu \\
 & + \int d^4x \sqrt{-g} \frac{3}{4} \left(\beta \frac{\partial_\mu \Omega^2}{\Omega^2} + \frac{1 + \gamma\beta}{\gamma^2 + 1} \left(\frac{\partial_\mu \eta}{\Omega^2} + \partial_\mu \gamma \right) \right) A^\mu \\
 & - \int d^4x \sqrt{-g} \frac{3}{16M_P^2(\gamma^2 + 1)} \left(\right. \\
 & \quad \left. \left(1 + 2\gamma\beta - \beta^2 \right) A_\mu^2 + 2\alpha (\gamma - \beta) A_\mu V^\mu - \alpha^2 V_\mu^2 \right)
 \end{aligned}$$

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Dark Matter Production⁷

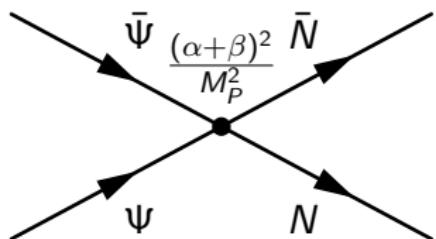
- ▶ Singlet fermion N in early Universe



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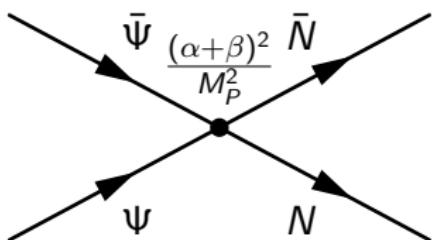


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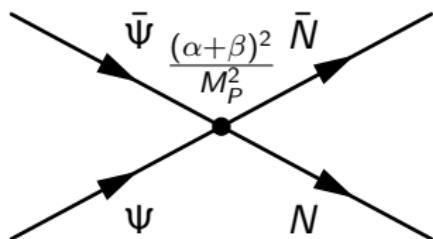
- ▶ Universal portal to dark matter
- ▶ Relative abundance

$$\frac{\Omega_N}{\Omega_{DM}} = 10^{-2} \frac{m_N}{10 \text{ keV}} (\alpha + \beta)^4 \frac{T^3}{M_P^3}$$

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$$\frac{\Omega_N}{\Omega_{DM}} = 10^{-2} \frac{m_N}{10 \text{ keV}} (\alpha + \beta)^4 \frac{T^3}{M_P^3}$$

- ▶ All of dark matter for

$$m_N = 10 \text{ keV}, \dots, 10^8 \text{ GeV}$$

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Example: Warm Dark Matter

- ▶ Cutoff in Palatini Higgs inflation

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- ▶ Characteristic momentum distribution [back](#)

Naturalness of Higgs mass m_H

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① Sensitivity to heavy particles

Disappears if no particles above weak scale

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- ➋ Why $m_H \ll M_P$?

Generation of hierarchy by gravitational instanton⁸

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More simple in Palatini⁹

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Einstein Frame Action

- ▶ Palatini action after conformal transformation

$$S = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{1}{2} \frac{1}{\Omega^2} (\partial_\mu h)^2 - \frac{\lambda}{4} \frac{h^4}{\Omega^4} + \frac{M_P^2}{2} \hat{R} \right\}$$

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- ▶ Canonical field

$$d\chi = \frac{dh}{\Omega} \quad \Rightarrow \quad h = \frac{M_P}{\sqrt{\xi}} \sinh \left(\frac{\sqrt{\xi}\chi}{M_P} \right)$$

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- ▶ Final action

$$S = \int d^4x \sqrt{-\hat{g}} \left\{ \frac{M_P^2}{2} \hat{R} - \frac{1}{2} (\partial_\mu \chi)^2 - \frac{\lambda M_P^4}{4\xi^2} \left(\tanh \left(\frac{\sqrt{\xi}\chi}{M_P} \right) \right)^4 \right\}$$

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- ▶ Expectation value

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- ▶ Effective source

$$S_{\text{eff}} = \int d^4x \left(-\frac{\sqrt{\xi}\chi(x)}{M_P} \delta^{(4)}(x) + \sqrt{\hat{g}_E} \mathcal{L}_E \right)$$

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- ▶ Regularize: $S = S_{\text{eff}} + S_{\text{reg}}$

$$S_{\text{reg}} = \int d^4x \sqrt{-\hat{g}} \frac{\delta}{M_P^8 \Omega^8} \left(1 + \frac{\delta}{\Omega^2} \right) (\partial_\mu h)^6$$

Saddle Point

- ▶ On saddle point:

$$\langle h \rangle \sim \frac{M_P}{\sqrt{\xi}} e^{-s}$$

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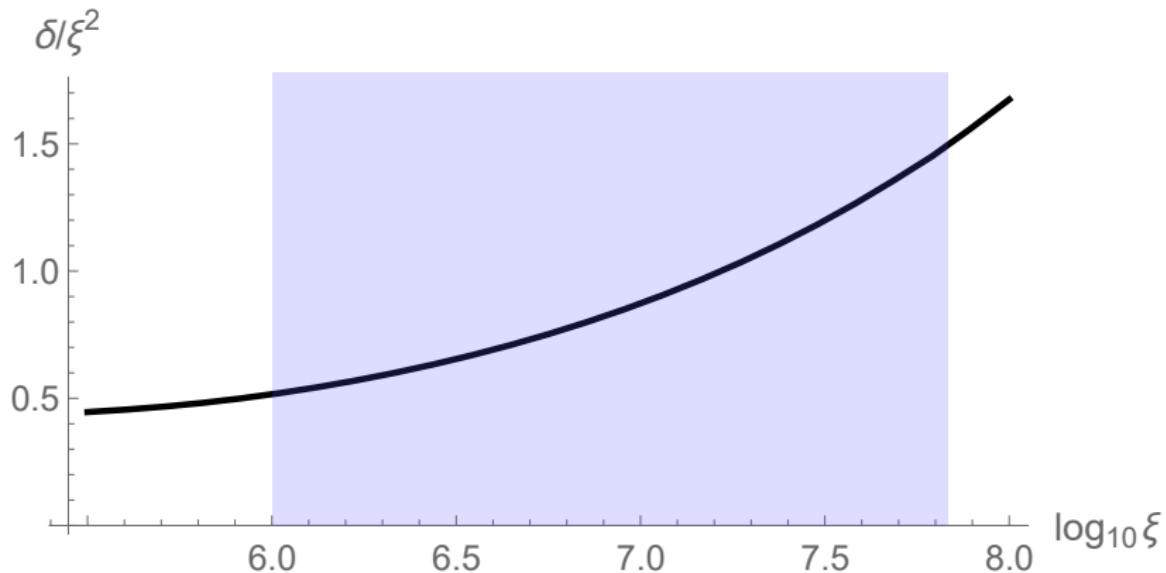
- ▶ E.o.m.

$$\partial_r \left(\frac{r^3 \chi'}{f} + \frac{6\delta r^3 \chi'^5 G(\chi)}{M_P^8 f^5} \right) - \frac{\delta r^3 \chi'^6 G'(\chi)}{M_P^8 f^5} - r^3 f U'(\chi) = -\frac{\sqrt{\xi}}{2\pi^2 M_P} \delta(r)$$

$$6 - 6f^2 + \frac{2r^2 f^2 U(\chi)}{M_P^2} - \frac{r^2 \chi'^2}{M_P^2} - \frac{10\delta r^2 \chi'^6 G(\chi)}{M_P^{10} f^4} = 0$$

Define $G(\chi) = 1 + \delta / \cosh^2(\sqrt{\xi} \chi / M_P)$

Solution



[back](#)

Spin connection

► Tetrad

$$\vec{e}_I = e_I{}^\mu \vec{\partial}_\mu \quad \vec{e}_I \cdot \vec{e}_J = \eta_{IJ}$$

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► Recover Christoffel

$$\Gamma_{\mu\nu}^\alpha = e_I{}^\alpha \left(e^I{}_{\mu,\nu} + \omega_\mu{}^I{}_K e^K{}_\nu \right)$$

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► Antisymmetry of spin connection

$$\nabla_{\mu} \eta_{IJ} = 0 \quad \Rightarrow \quad \omega_{\mu IJ} = -\omega_{\mu JI}$$

Introduce Forms

- ▶ 1-forms:

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- ▶ Vary with respect to connection

$$\frac{\delta S_{\text{EH}}}{\delta \omega_{IJ}} \propto D \left(\epsilon_{IJKL} e^I \wedge e^J \right) = 0$$

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$$\Rightarrow C_M^K \wedge \epsilon^{ML}{}^{IJ} e_I \wedge e_J = 0 \quad \Rightarrow C_M^K = 0$$