

# The cosmological constant in supergravity and string theory

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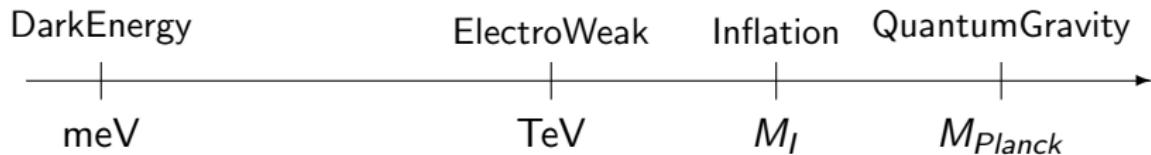
# Universe evolution: based on positive cosmological constant

- Dark Energy

simplest case: infinitesimal (tunable) +ve cosmological constant

- Inflation (approximate de Sitter)

describe possible accelerated expanding phase of our universe



# The cosmological constant in Supergravity

Highly constrained:  $\Lambda \geq -3m_{3/2}^2$

- equality  $\Rightarrow$  AdS (Anti de Sitter) supergravity  
 $m_{3/2} = W_0$  : constant superpotential
- inequality: dynamically by minimising the scalar potential  
 $\Rightarrow$  uplifting  $\Lambda$  and breaking supersymmetry
- $\Lambda$  is not an independent parameter for arbitrary breaking scale  $m_{3/2}$

What about breaking SUSY with a  $\langle D \rangle$  triggered by a constant FI-term?

standard supergravity: possible only for a gauged  $U(1)_R$  symmetry:

absence of matter  $\Rightarrow W_0 = 0 \rightarrow$  dS vacuum      Friedman '77

- exception: non-linear supersymmetry

# Non-linear SUSY in supergravity

I.A.-Dudas-Ferrara-Sagnotti '14

$$K = X\bar{X} \quad ; \quad W = fX + W_0$$

$X \equiv X_{NL}$  nilpotent goldstino superfield [6]

$$X_{NL}^2 = 0 \Rightarrow X_{NL}(y) = \frac{\chi^2}{2F} + \sqrt{2}\theta\chi + \theta^2 F$$

$$\Rightarrow V = |f|^2 - 3|W_0|^2 \quad ; \quad m_{3/2}^2 = |W_0|^2$$

- $V$  can have any sign contrary to global NL SUSY
- NL SUSY in flat space  $\Rightarrow f = \sqrt{3} m_{3/2} M_p$
- R-symmetry is broken by  $W_0$

gauge invariant at the Lagrangian level but non-local

becomes local and very simple in the unitary gauge

Global supersymmetry:

$$\mathcal{L}_{\text{FI}}^{\text{new}} = \xi_1 \int d^4\theta \frac{\mathcal{W}^2 \bar{\mathcal{W}}^2}{\mathcal{D}^2 \mathcal{W}^2 \bar{\mathcal{D}}^2 \bar{\mathcal{W}}^2} \mathcal{D}\mathcal{W} \stackrel{\text{gauge field-strength superfield}}{\swarrow} = -\xi_1 D + \text{fermions}$$

It makes sense only when  $\langle D \rangle \neq 0 \Rightarrow$  SUSY broken by a D-term

Supergravity generalisation: straightforward

unitary gauge: goldstino =  $U(1)$  gaugino = 0  $\Rightarrow$  standard sugra  $-\xi_1 D$

# New FI term in supergravity

Pure sugra + one vector multiplet  $\Rightarrow$  [4]

$$\mathcal{L} = R + \bar{\psi}_\mu \sigma^{\mu\nu\rho} D_\rho \psi_\nu + m_{3/2} \bar{\psi}_\mu \sigma^{\mu\nu} \psi_\nu - \frac{1}{4} F_{\mu\nu}^2 - \left( -3m_{3/2}^2 + \frac{1}{2}\xi_1^2 \right)$$

- $\xi_1 = 0 \Rightarrow$  AdS supergravity
- $\xi_1 \neq 0$  uplifts the vacuum energy and breaks SUSY
  - e.g.  $\xi_1 = \sqrt{6}m_{3/2} \Rightarrow$  massive gravitino in flat space

# The cosmological constant in Supergravity

I.A.-Chatrabhuti-Isono-Knoops '18

New FI-term introduces a cosmological constant in the absence of matter

However new FI-term in the presence of matter is not unique

Question: can one modify it to respect Kähler invariance?

Answer: yes, constant FI-term + fermions as in the absence of matter

⇒ constant uplift of the potential,  $\Lambda$  free (+ve) parameter besides  $m_{3/2}$

In general  $\xi_1 \rightarrow \xi_1 f(m_{3/2}[\phi, \bar{\phi}])$

J.A.-Rondeau '99

It can also be written in  $N = 2$  supergravity

I.A.-Derendinger-Farakos-Tartaglino Mazzucchelli '19

# Swampland de Sitter conjecture

String theory: vacuum energy and inflation models  
related to the moduli stabilisation problem

Difficulties to find dS vacua led to a conjecture:

$$\frac{|\nabla V|}{V} \geq c \quad \text{or} \quad \min(\nabla_i \nabla_j V) \leq -c' \quad \text{in Planck units}$$

with  $c, c'$  positive order 1 constants

Ooguri-Palti-Shiu-Vafa '18

Dark energy: forbid dS minima but allow maxima

Inflation: forbid standard slow-roll conditions

Assumptions: heuristic arguments, no quantum corrections

→ here: explicit counter example

# Moduli stabilisation in type IIB

Compactification on a Calabi-Yau manifold  $\Rightarrow N = 2$  SUSY in 4 dims

Moduli: Complex structure in vector multiplets

Kähler class & dilaton in hypermultiplets

$\Rightarrow$  decoupled kinetic terms

turn on appropriate 3-form fluxes (primitive self-dual)  $\Rightarrow N = 1$  SUSY

+ orientifolds and D3/D7-branes

vectors and RR companions of geometric moduli are projected away  $\Rightarrow$

all moduli in  $N = 1$  chiral multiplets + superpotential for the

**complex structure & dilaton**  $\rightarrow$  fixed in a SUSY way Frey-Polchinski '02

Kähler moduli: no scale structure, vanishing potential (classical level)

# Stabilisation of Kähler moduli

Non perturbative superpotential from gaugino condensation on D-branes

⇒ stabilisation in an AdS vacuum

Derendinger-Ibanez-Nilles '85

Uplifting using anti-D3 branes

Kachru-Kallosh-Linde-Trivedi '03

or D-terms and perturbative string corrections to the Kähler potential

Large Volume Scenario

Conlon-Quevedo et al '05

Ongoing debate on the validity of these ingredients in full string theory

While perturbative stabilisation has the old Dine-Seiberg problem

put together 2 orders of perturbation theory violating the expansion

possible exception known from field theory:

logarithmic corrections → Coleman-Weinberg mechanism

# Log corrections in string theory:

localised couplings + closed string propagation in  $d \leq 2$

Effective propagation of massless bulk states in  $d \leq 2 \Rightarrow$  IR divergences [15]

$d = 1$ : linear,  $d = 2$ : logarithmic corrections for (brane) localised couplings  
on the size of the bulk due to local closed string tadpoles I.A.-Bachas '98

e.g. gauge coupling corrections, linear dilaton dependence on the 11th dim

Type II strings: correction to the Kähler potential  $\leftrightarrow$  Planck mass

I.A.-Ferrara-Minasian-Narain '97

Large volume limit: it corresponds to a 4d localised Einstein-Hilbert term  
in the 6d internal space I.A.-Minasian-Vanhove '02 [13] [14]

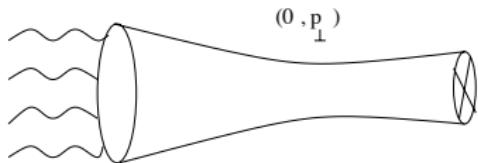
$$S_{\text{grav}}^{IIB} = \frac{1}{(2\pi)^7 \alpha'^4} \int_{M_4 \times \mathcal{X}_6} e^{-2\phi} \mathcal{R}_{(10)} + \frac{\chi}{(2\pi)^4 \alpha'} \int_{M_4} \left( 2\zeta(3)e^{-2\phi} + \frac{2\pi^2}{3} \right) \mathcal{R}_{(4)}$$

$\chi$ : Euler number  $= 4(n_H - n_V)$  [18] 4-loop  $\sigma$ -model  $\nearrow$  vanishes for orbifolds

# Log corrections in string theory

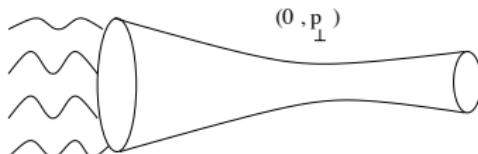
I.A.-Bachas '98

decompactification limit in the presence of branes



(a)

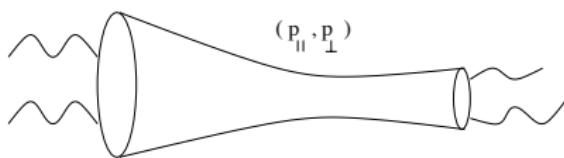
$$\mathcal{A} \sim \frac{1}{V_\perp} \sum_{|p_\perp| < M_s} \frac{1}{p_\perp^2} F(\vec{p}_\perp)$$



(b)

$$V_\perp = R^d \quad \vec{p}_\perp = \vec{n}/R$$

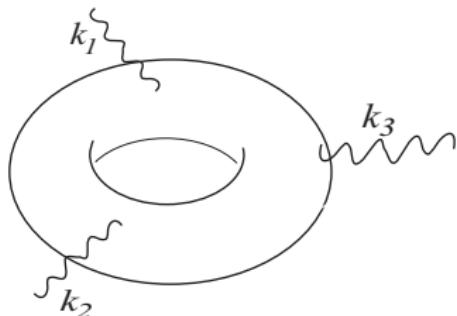
$$R \gg l_s \Rightarrow$$



(c)

$$\mathcal{A} \sim \begin{cases} \mathcal{O}(R) & \text{for } d=1 \\ \mathcal{O}(\log R) & \text{for } d=2 \\ \text{finite} & \text{for } d>2 \end{cases}$$

local tadpoles:  $F(\vec{p}_\perp) \sim \left( 2^{5-d} \prod_{i=1}^d (1 + (-)^{n_i}) - 2 \sum_{a=1}^{16} \cos(\vec{p}_\perp \vec{y}_a) \right)$



$$\sum_i k_i = 0, \quad k_1^2 = k_2^2 = 0, \quad k_3^2 = -q^2$$

$$\sim \chi e^{-q^2/2w^2} \quad Z_N \text{ orbifold: } \chi \sim N$$

compute  $w$  in the large  $N$  limit by saddle point analysis  
of the integral over the 2d torus modulus

$$\Rightarrow w \sim I_s / \sqrt{N} \sim I_p^{(4)} \text{ [11]}$$

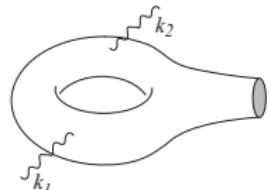
in agreement with general arguments of localised gravity

Dvali-Gabadadze-Porrati '00

# perturbative moduli stabilisation I.A.-Chen-Leontaris '18, '19

localised vertices from  $\mathcal{R}_{(4)}$  can emit massless closed strings

$\Rightarrow$  local tadpoles in the presence of distinct 7-brane sources

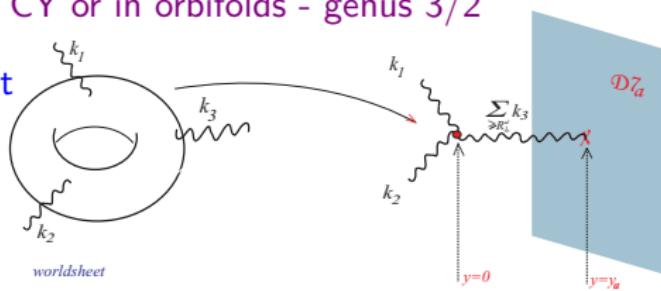


propagation in 2d transverse bulk  $\rightarrow \log R_\perp$  corrections

exact computation: difficult either in CY or in orbifolds - genus 3/2

computation in the degeneration limit

for  $Z_N$  orbifold



$$\sim - \sum_{q_\perp \neq 0} g_s^2 T N e^{-w^2 q_\perp^2 / 2} \frac{1}{q_\perp^2 R_\perp^2} = -N g_s^2 T \log(R_\perp/w) + \dots$$

$T = T_0/g_s$ : brane tension

Kähler potential:

$$\mathcal{K} = -2 \ln \left( \mathcal{V} + \xi + \eta \ln \frac{\mathcal{V}_\perp}{w^2} + \mathcal{O}\left(\frac{1}{\mathcal{V}}\right) \right) = -2 \ln (\mathcal{V} + \eta \ln \mu^2 \mathcal{V}_\perp)$$

$$\xi = -\frac{1}{4} \chi f(g_s); \quad f(g_s) = \begin{cases} \zeta(3) \simeq 1.2 & \text{smooth CY} \\ \frac{\pi^2}{3} g_s^2 & \text{orbifolds} \end{cases} \quad \eta = -\frac{1}{2} g_s T_0 \xi \quad [11]$$

Using 3 mutual orthogonal 7-brane stacks with D-terms (magnetic fluxes)  
and minimising with respect to transverse volume ratios

$$\Rightarrow V \simeq \frac{3\eta \mathcal{W}_0^2}{\mathcal{V}^3} (\ln \mu^6 \mathcal{V} - 4) + 3 \frac{d}{\mathcal{V}^2} \quad \mathcal{W}_0: \text{constant superpotential, } d: \text{D-term}$$

$$\text{dS minimum: } -0.007242 < \frac{d}{\eta \mathcal{W}_0^2 \mu^6} \equiv \rho < -0.006738 \text{ with } \mathcal{V} \simeq e^5 / \mu^6 \quad [17]$$

# FI D-terms

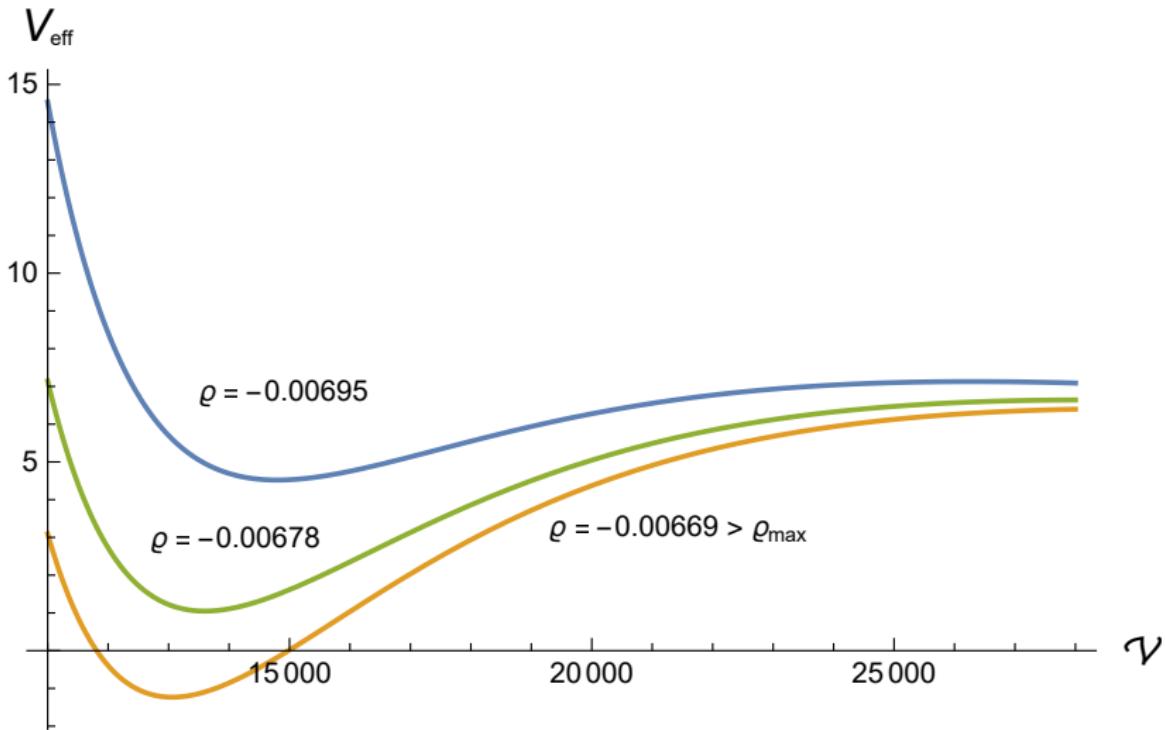
$$V_{D_i} = \frac{d_i}{\tau_i} \left( \frac{\partial K}{\partial \tau_i} \right)^2 = \frac{d_i}{\tau_i^3} + \mathcal{O}(\eta_j)$$

$\tau_i$ : world-volume modulus of D7<sub>i</sub>-brane stack with  $\mathcal{V} = (\tau_1 \tau_2 \tau_3)^{1/2}$

$$\eta_i \equiv \eta \Rightarrow V_{tot} = \frac{3\eta \mathcal{W}_0^2}{\mathcal{V}^3} (\ln(\mathcal{V}\mu^6) - 4) + \frac{d_1}{\tau_1^3} + \frac{d_2}{\tau_2^3} + \frac{d_3 \tau_1^3 \tau_2^3}{\mathcal{V}^6}$$

minimising with respect to  $\tau_1$  and  $\tau_2 \Rightarrow \frac{\tau_i}{\tau_j} = \left( \frac{d_i}{d_j} \right)^{1/3} \Rightarrow$

$$V_D = 3 \frac{d}{\mathcal{V}^2} \quad \text{with} \quad d = (d_1 d_2 d_3)^{1/3}$$



2 extrema min+max  $\rightarrow -0.007242 < \rho < -0.006738 \leftarrow +ve \text{ energy}$  [15]

$$\xi = -\frac{1}{4}\chi f(g_s); \quad f(g_s) = \begin{cases} \zeta(3) \simeq 1.2 & \text{smooth CY} \\ \frac{\pi^2}{3}g_s^2 & \text{orbifolds} \end{cases} \quad \eta = -\frac{1}{2}g_s T_0 \xi$$

dS minimum:  $-0.007242 < \frac{d}{\eta \mathcal{W}_0^2 \mu^6} \equiv \rho < -0.006738$  with  $\mathcal{V} \simeq e^5 / \mu^6$

exponentially large volume:

$$\mu = \frac{e^{\xi/6\eta}}{w} = \sqrt{|\chi|} e^{-\frac{1}{3g_s T_0}} \rightarrow 0 \quad \Rightarrow$$

weak coupling and

large  $\chi$  or/and  $\mathcal{W}_0$  from 3-form flux to keep  $\rho$  fixed

requirement: negative  $\chi$  ( $\eta < 0$ ) [11] and surplus of D7-branes ( $T_0 > 0$ )

- Inflaton: canonically normalised  $\phi = \sqrt{2/3} \ln \mathcal{V}$  (in Planck units)
- one relevant parameter:  $\rho$  or  $x = -\ln(-4\rho/3) - 16/3$

$$0 < x < 0.072 \text{ for dS minimum}$$

- extrema  $V'(\phi_{\pm}) = 0$

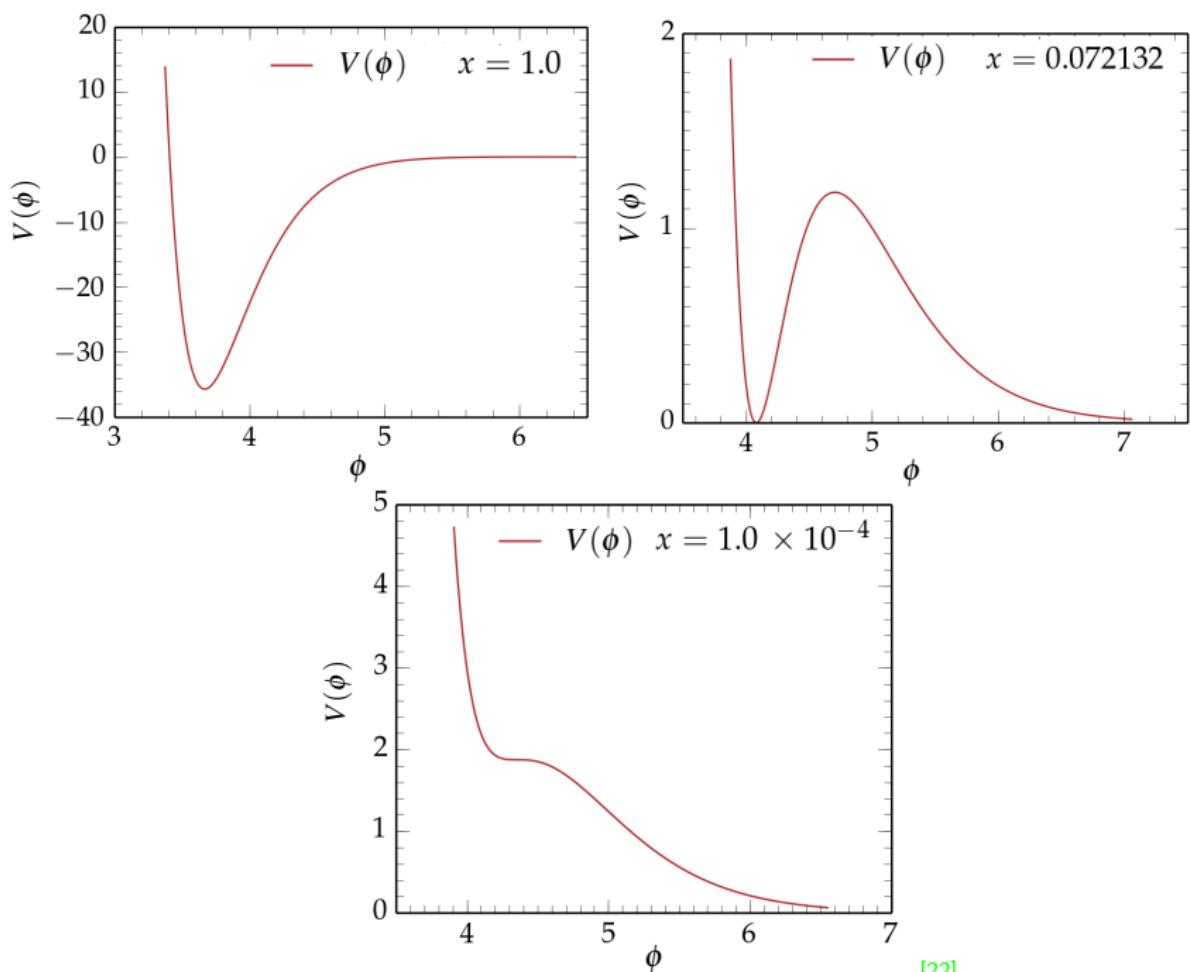
$$\phi_+ - \phi_- = \sqrt{2/3} (W_0(-e^{-x-1}) - W_{-1}(-e^{-x-1}))$$

$W_{0/-1}$ : Lambert functions satisfying  $W(xe^x) = x$

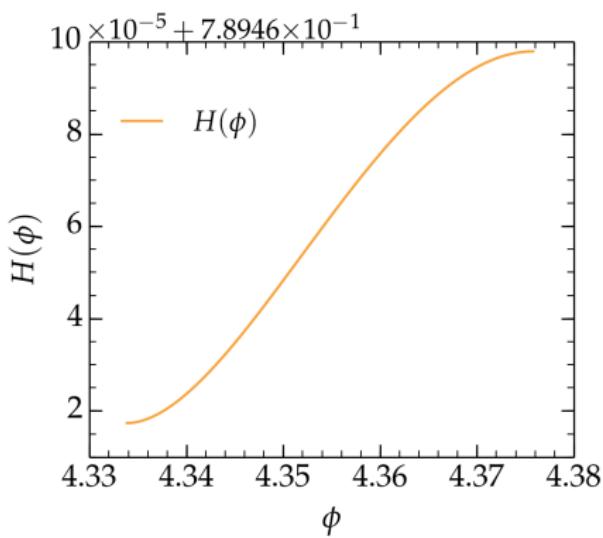
$$\frac{V(\phi_+)}{V(\phi_-)} = \frac{(W_0(-e^{-x-1}))^3 (2 + 3W_{-1}(-e^{-x-1}))}{(W_{-1}(-e^{-x-1}))^3 (2 + 3W_0(-e^{-x-1}))}$$

- slow roll parameter  $\eta(\phi_{-/+}) = \frac{V''(\phi_{-/+})}{V(\phi_{-/+})} = -9 \frac{1 + W_{0/-1}(-e^{-x-1})}{\frac{2}{3} + W_{0/-1}(-e^{-x-1})}$

successful inflation possible around the minimum from the inflection point



[22]



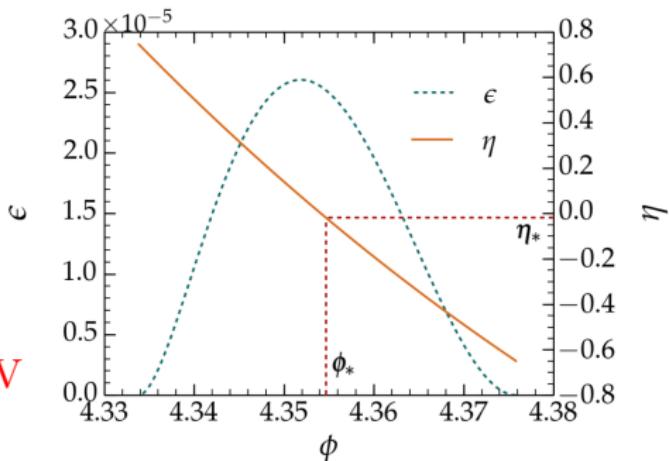
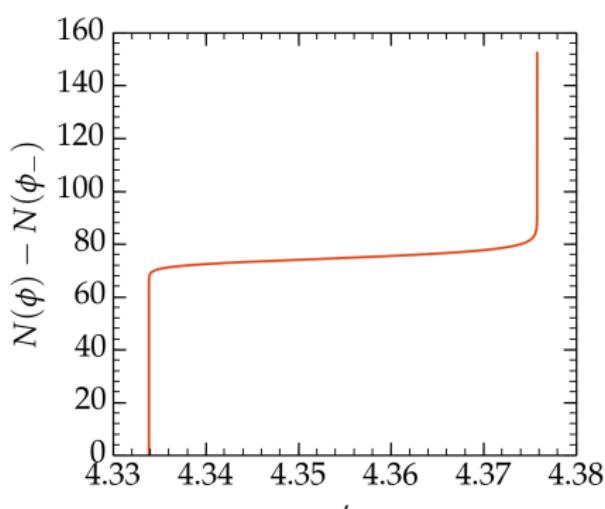
$$x = 3.3 \times 10^{-4}; \quad \eta(\phi_*) = -0.02$$

$\phi_*$  near the inflection point

$\Delta\phi \simeq 0.02$  : small field

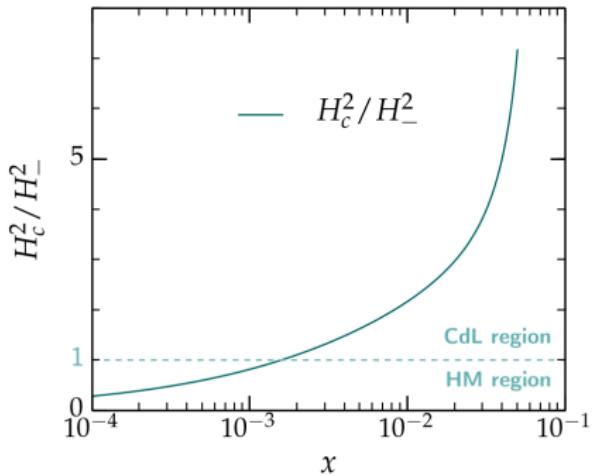
$$\Rightarrow r \simeq 4 \times 10^{-4} \text{ [23]}$$

$$H_* \simeq 5 \times 10^{12} \text{ GeV}$$



# dS vacuum metastability [20]

- through tunnelling       $H_c > H_-$       Coleman - de Luccia instanton
- over the barrier       $H_c < H_-$       Hawking - Moss transition



$$\frac{H_c^2}{H_-^2} \equiv -\frac{3V''(\phi_+)}{4V(\phi_-)}$$

KM region:  $\Gamma \sim e^{-B}$ ;  $B \simeq \frac{24\pi^2}{V} \frac{\Delta V}{V}$

$$\frac{\Delta V}{V} \simeq 24\sqrt{2}x^{3/2} \Rightarrow$$

$$B \simeq 3 \times 10^9 \text{ for } x \simeq 3 \times 10^{-4}$$

# Conclusions

Novel D-terms in supergravity that do not gauge the R-symmetry  
allow to write a positive cosmological constant even without matter fields  
**their implementation in string theory: open problem**

New mechanism of moduli stabilisation is string theory (type IIB)

- perturbative: weak coupling, large volume
- based on log corrections in the transverse volume of 7-branes  
due to local tadpoles induced by localised gravity kinetic terms  
arising only in 4 dimensions!
- can lead to de Sitter vacua in string theory  
explicit counter-example to dS swampland conjecture
- inflation possible around the minimum from the inflection point