String (In)Stability Issues with Broken Supersymmetry

Augusto Sagnotti
Scuola Normale Superiore and INFN – Pisa

Based mostly on recent work with J. MOURAD:
2102.06184, 2002.05372,1811.11448, and refs therein; & to appear

"Quarks" , dedicated to A.D. Sakharov centennial
Workshop on "Integrability, Holography, HS gravity and Strings"
Moscow (online), June 2, 2021

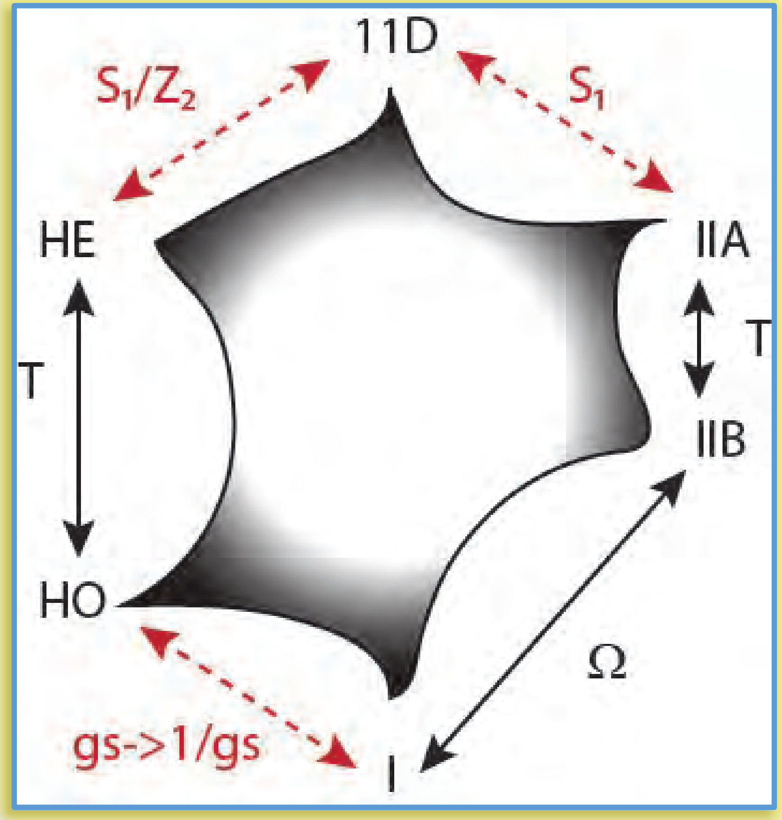
Broken SUSY in 10D with NO Tachyons

The (SUSY) 10D-11D Zoo

- **Highest point** of (SUSY) String Theory

- Exhibits **dramatically our limitarions**
- perturbative → **Solid arrows**
- 10&11D supergravity → **Dashed arrows**

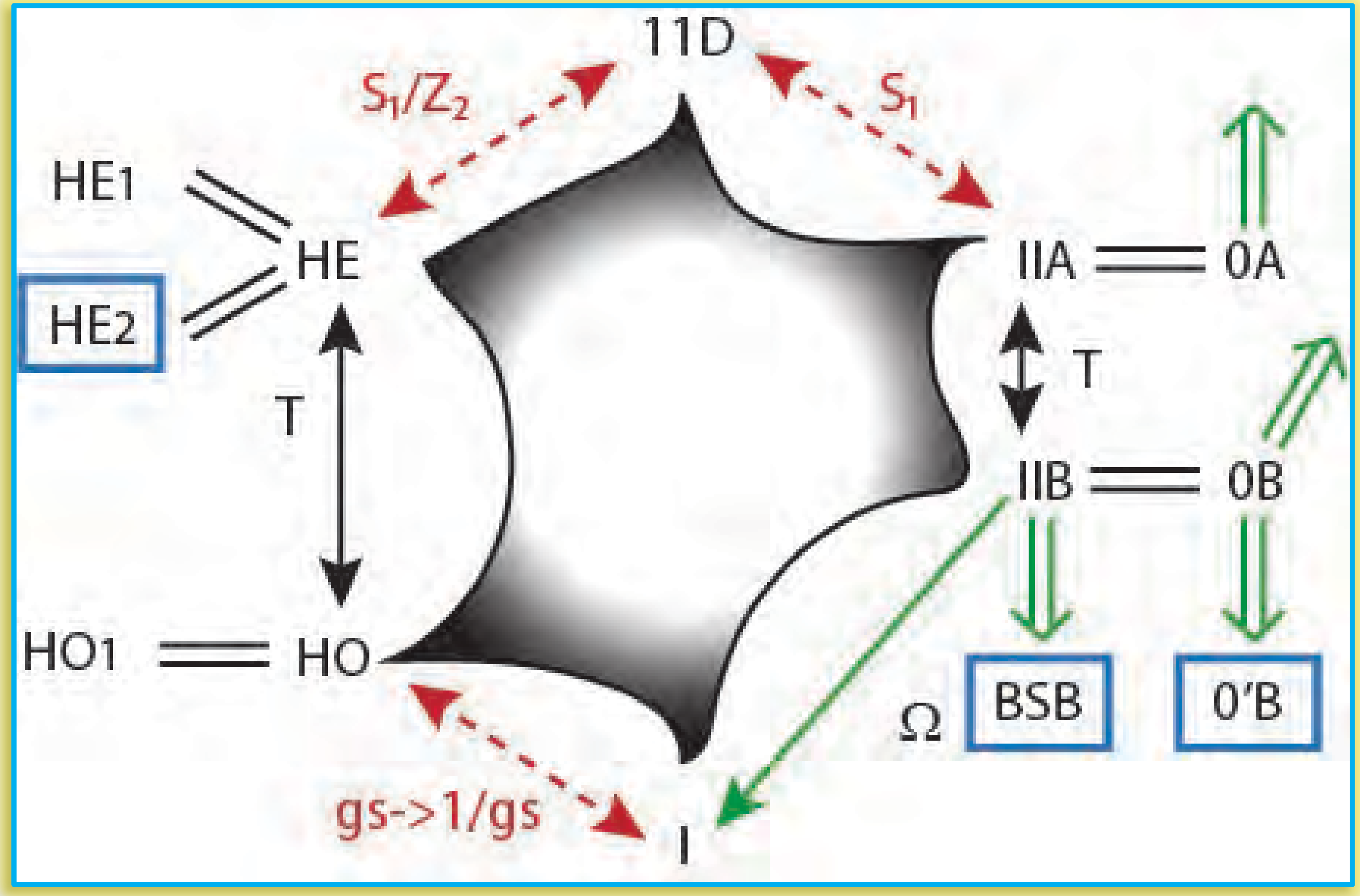
- **SUSY**: **stabilizes** these 10D Minkowski vacua



(Witten, 1995)

BROKEN SUSY ?

The 10D-11D Zoo



- 3 D=10 **non-SUSY non-tachyonic** strings
- SO(16)xSO(16) *(Dixon, Harvey, 1987) (Alvarez-Gaumé, Ginsparg, Moore, Vafa, 1987)*
- [BSB] *(Sugimoto, 1999, Antoniadis, Dudas, AS, 1999)*
- O'B *(AS, 1995)*

- **String consistency rules OK**
- **BUT: vacuum modified** (Tadpole potential)

- **QUESTIONS:** compactifications? Stability?

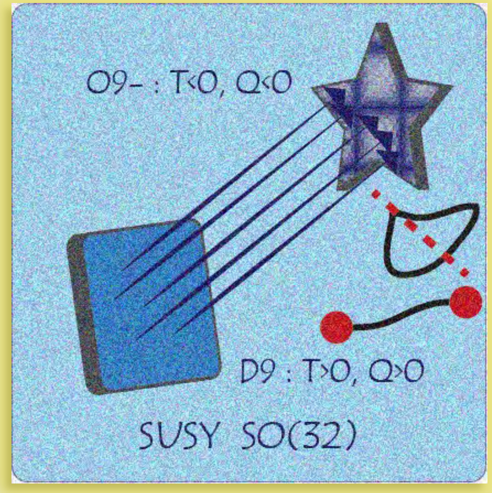
$$\mathcal{S} = \frac{1}{2\,k_{10}^2}\int d^{10}x\,\sqrt{-G}\left\{e^{-2\phi}\left[-R+4(\partial\phi)^2\right]-\frac{1}{12}\,\mathcal{H}_3^2-\frac{1}{4}\,e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2-T\,e^{-\phi}+\ldots\right\}$$

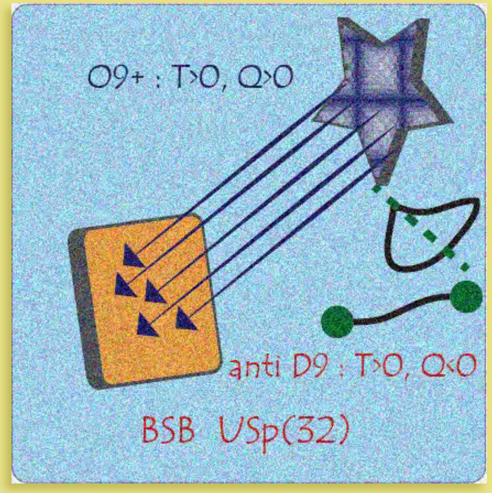
Brane SUSY Breaking

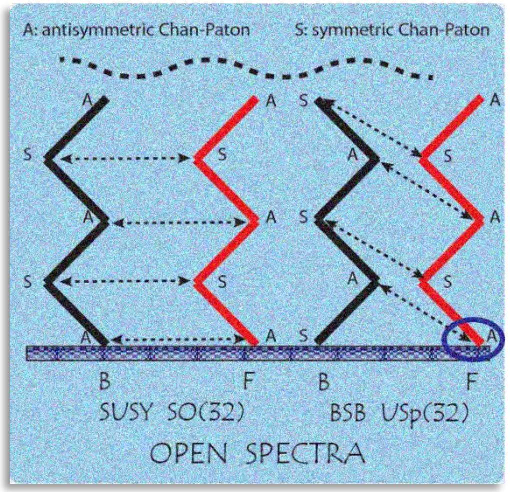
(Sugimoto, 1999)
(Antoniadis, Dudas, AS, 1999)
(Angelantonj, 1999)
(Aldazabal, Uranga, 1999)

- ❖ Non-linear SUSY: ∃ goldstino!
- ❖ NO TACHYONS

(Dudas, Mourad, 2000)
(Pradisi, Riccioni, 2001)







$$\mathcal{S} = \frac{1}{2\,k_{10}^2}\int d^{10}x\,\sqrt{-G}\left\{e^{-2\phi}\left[-R+4(\partial\phi)^2\right]-\frac{1}{12}\,\mathcal{H}_3^2-\frac{1}{4}\,e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2 - T\,e^{-\phi} + \ldots\right\}$$

VACUUM ENERGY → POTENTIAL

NOTE:
- Expansion in powers of $\boxed{\alpha' R}$
- Expansion in powers of $\boxed{g_s = e^{\phi}}$

"Vacuum" Solutions with Broken SUSY

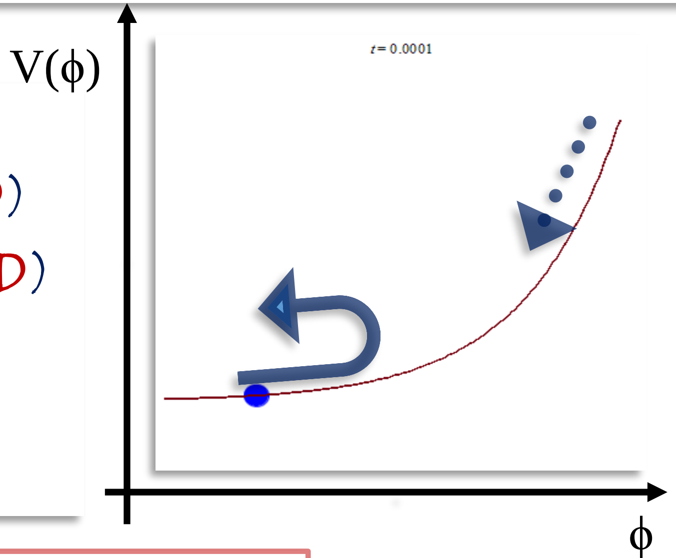
Cosmology with Tadpoles: The Climbing Scalar

$V = T\, e^{\gamma\phi}$ (Einstein frame)

(Halliwell, 1987;…, Dudas and Mourad, 1999; Russo, 2004)
(Dudas, Kitazawa, AS, 2010)

For $\gamma < 3/2$ [beware of different notation in earlier work]:

- "Climbing" solution (ϕ climbs, then descends) → $g_s = e^{\phi}$ BOUNDED)
- "Descending" solution (ϕ only descends) → $g_s = e^{\phi}$ UNBOUNDED)
- Limiting τ- speed (LM attractor) (Lucchin and Matarrese, 1985)



LM attractor & descending solution disappear for $\gamma \geq 3/2$!

CLIMBING : BSB (Usp(32)) and U(32) HAVE $\gamma = 3/2$!
[SO(16) x SO(16) has $\gamma = 5/2$] !

HINT of a fast roll – slow roll onset of inflation?

9D Dudas-Mourad Vacua

(Dudas, and Mourad, 2000, 2001)

$$S = \frac{1}{2\,k_{10}^2} \int d^{10}x \sqrt{-G} \left\{ e^{-2\phi} \left[-R + 4(\partial\phi)^2 \right] - \frac{1}{12}\, \mathcal{H}_3^2 - \frac{1}{4}\, e^{-\phi}\, \mathrm{tr}\, \mathcal{F}^2 - T\, e^{-\phi} + \ldots \right\}$$

9D solutions → T DRIVES the compactification

[Vacua for both Usp(32) and U(32), & similar for heterotic SO(16) x SO(16)]

- ❖ SPONTANEOUS COMPACTIFICATIONS: intervals of FINITE length $\sim \frac{1}{\sqrt{T}}$
- ❖ FINITE 9D Planck mass and gauge coupling
- g_s diverges at one end & curvature at the other
- NOT SYMMETRIC
- QUESTIONS:
  - Fermions?
  - String corrections: is large g_s NEEDED for these types of compactification ?
  - Stability ?

Symmetric (Flux) Vacua with Tadpoles

(Mourad, AS, 2016)

$$\mathcal{S} = \frac{1}{2\,k_{10}^2}\int d^{10}x\,\sqrt{-G}\,\left\{e^{-2\phi}\left[-R + 4(\partial\phi)^2\right] - \frac{1}{12}\,\mathcal{H}_3^2 - \frac{1}{4}\,e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2 - T\,e^{-\phi} + \ldots\right\}$$

- Dilaton Eq: constraint from positivity of T (orientifolds NEED H_3 fluxes, SO(16)xSO(16) H_7 fluxes)
- Eqs. determine AdS (in Poincaré coordinates or in other slicings)
- $AdS_3 \times S^7$ (orientifolds) ; $AdS_7 \times S^3$ (SO(16)xSO(16))

❖ WIDE REGIONS where the two couplings $\boxed{\alpha' R}$ and $\boxed{g_s = e^{\phi}}$ are SMALL

❖ THE (H_3 or H_7) FLUXES SUPPORT THESE SYMMETRIC COMPACTIFICATIONS

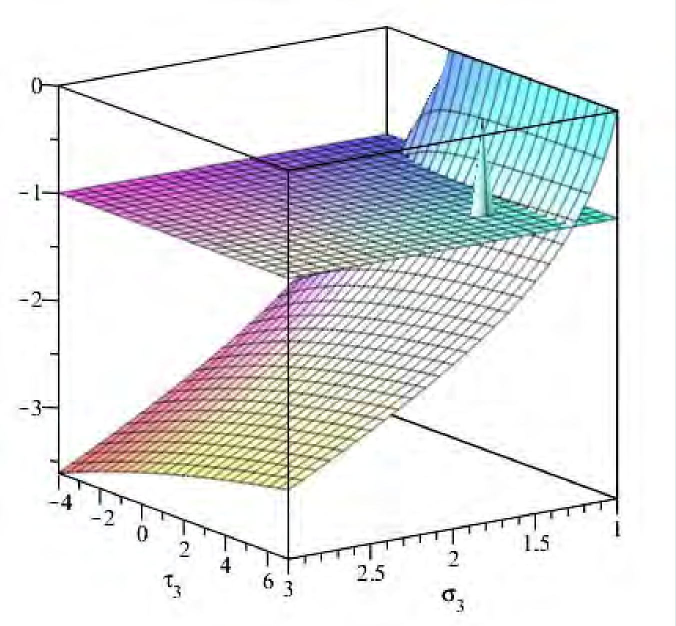
(In)Stability ?

$AdS_3 \times S^7$ ($\&$ $AdS_7 \times S^3$) Vacua

(Gubser, Mitra, 2001)
(Mourad ,AS, 2016)
(Basile, Mourad, AS, 2018)

❖ Orientifold & SO(16)xSO(16) vacua: WEAK coupling but UNSTABLE

- Violations of Breitenlohner-Freedman bounds for modes with INTERNAL EXCITATIONS (mixings)
- Wide NEARBY regions of stability. Quantum corrections?
- At least in SO(16)xSO(16): instabilities can be removed by internal projections on S^3



Dudas-Mourad Vacua

(Basile, Mourad, AS, 2018)

❖ Dudas-Mourad vacua: STRONG COUPLING but STABLE !

- E.g.: Scalar perturbations: $ds^2 = e^{2\Omega(z)}\left[(1+A)\,dx^\mu\,dx_\mu + (1-7A)\,dz^2\right]$,

$$A'' + A'\left(24\,\Omega' - \frac{2}{\phi'}\,e^{2\Omega}\,V_\phi\right) + A\left(m^2 - \frac{7}{4}\,e^{2\Omega}\,V - 14\,e^{2\Omega}\,\Omega'\,\frac{V_\phi}{\phi'}\right) = 0$$

❖ Schrödinger-like form

$$m^2\,\Psi = \left(b + \mathcal{A}^\dagger\,\mathcal{A}\right)\Psi$$

$$\mathcal{A} = \frac{d}{dr} - \alpha(r)\,, \qquad \mathcal{A}^\dagger = -\,\frac{d}{dr} - \alpha(r)\,, \qquad b = \frac{7}{2}\,e^{2\Omega}\,V\,\frac{1}{1 + \frac{9}{4}\,\alpha_O\,y^2} > 0$$

NO tachyons in 9D : PERTUBATIVE STABILITY

The Climbing Scalar

(Basile, Mourad, AS, 2018)

- **COSMOLOGY** : the issue is the time evolution of perturbations

- INITIALLY (large η) V is negligible: tensor perturbations evolve as

$$
\begin{aligned}
h''_{ij} &+ \frac{1}{\eta}\, h'_{ij} + \mathbf{k}^2\, h_{ij} = 0 \\
h_{ij} &\sim A_{ij}\, J_0(k\eta) + B_{ij}\, Y_0(k\eta) \qquad (\mathbf{k} \neq 0) \\
h_{ij} &\sim A_{ij} + B_{ij}\, \log\left(\frac{\eta}{\eta_0}\right) \qquad (\mathbf{k} = 0)
\end{aligned}
$$

- NOTICE: logarithmic growth for k=0 (instability of isotropy) !!

Dynamical origin of compactification ?

Solutions with Flux and Tension

(Mourad, AS, to appear)

$$ds^2 = e^{2A(r)}dx^2 + e^{2B(r)}dr^2 + e^{2C(r)}dy^2$$

General Lesson:

a) SPATIAL PROFILES: (finite) intervals WITH strong coupling at one or both ends [ALSO: Scherk-Schwarz-like extensions in intervals]

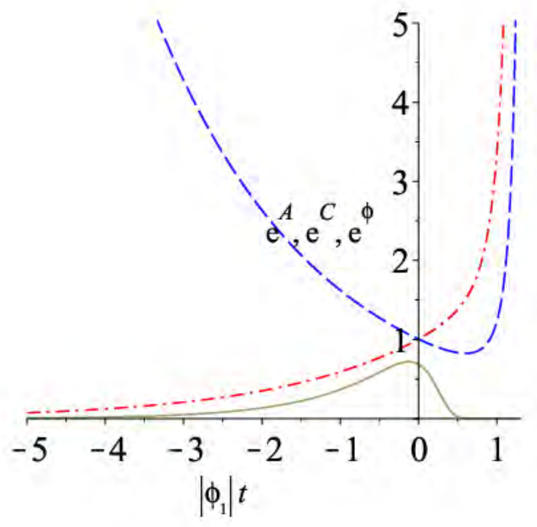
a) COSMOLOGY: better behavior (climbing), even with initial anisotropy

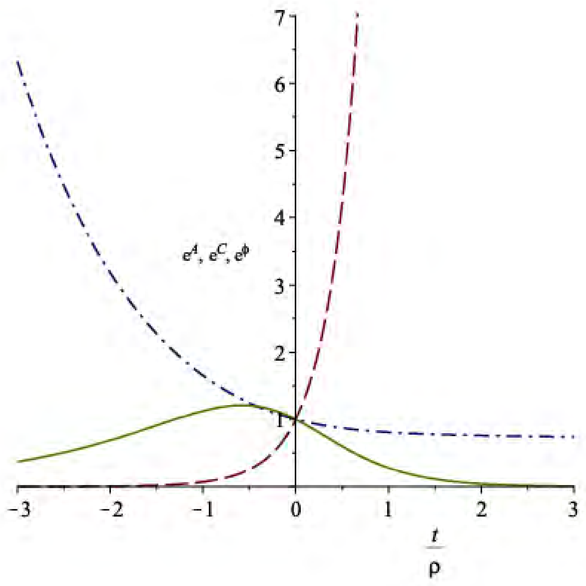
b) FLUXES: can remove or soften SOME singularities induced by the tension T

Anisotropic Cosmologies:

a) LEFT: $\gamma \leq \gamma_c$

b) RIGHT: $\gamma > \gamma_c$





More General Solutions

(Mourad, AS, to appear)

$$ds^2 = e^{2A(r)} dx^2 + e^{2B(r)} dr^2 + e^{2C(r)} dy^2$$

- With H_7-flux and Tadpole potential there is an instructive exact solution:

$$
\begin{aligned}
ds^2 &= e^{-\frac{T\, e^{2\,x_2}\, r^2}{8\,\alpha'} - \frac{\chi_1\, r}{6}} \left(\frac{dx \cdot dx}{\left[h\,\rho\,\sinh\left(\frac{r}{\rho}\right)\right]^{\frac{1}{4}}} + \left[h\,\rho\,\sinh\left(\frac{r}{\rho}\right)\right]^{\frac{3}{4}} dy \cdot dy \right) \\
&+ e^{-\frac{9\,T\, e^{2\,x_2}\, r^2}{8\,\alpha'} - \frac{3(\chi_1\, r + \chi_2)}{2} + 2\,x_2} \left[h\,\rho\,\sinh\left(\frac{r}{\rho}\right)\right]^{\frac{3}{4}} dr^2 \\
e^{\phi} &= \frac{e^{\frac{3\,T\, e^{2\,x_2}\, r^2}{4\,\alpha'} + \chi_1\, r + \chi_2}}{\left[h\,\rho\,\sinh\left(\frac{r}{\rho}\right)\right]^{\frac{1}{2}}} \\
\mathcal{H}_7 &= h\, \frac{\epsilon_6\, dr}{\left[h\,\rho\,\sinh\left(\frac{r}{\rho}\right)\right]^{2}}
\end{aligned}
$$

- LESSON: Tadpole wins over flux … and "closes" the interval

Boundary Conditions

(Mourad, AS, to appear)

NEED proper (symmetry preserving) BOUNDARY CONDITIONS

- Matter:

$$\delta\,\mathcal{S}_m \;=\; \int_{\mathcal{M}} d^Dx\; e\;\left[\delta e_M{}^A\,\mathcal{T}^M{}_A \;+\; \delta\omega_M{}^{AB}\;\mathcal{Y}^M{}_{AB}\right]$$

$$D_M\,e_N{}^A \;\equiv\; \partial_M\,e_N{}^A \;+\; \omega_M{}^{AB}\;e_{NB} \;-\; \Gamma^P{}_{MN}\,e_P{}^A \;=\; 0$$

  - Torsion:

$$S^P{}_{MN} \;=\; \Gamma^P{}_{MN} \;-\; \Gamma^P{}_{NM}$$

- Local Lorentz:

$$\delta\,e_M{}^A \;=\; \epsilon^{AB}\,e_{MB}\,, \qquad \delta\,\omega_M{}^{AB} \;=\; -\;D_M\,\epsilon^{AB}$$

$$D_M\,\mathcal{Y}^M{}_{AB} \;-\; S^P{}_{PM}\,\mathcal{Y}^M{}_{AB} \;=\; \frac{1}{2}\left(\mathcal{T}_{AB} \;-\; \mathcal{T}_{BA}\right)$$

- Diffeomorphisms:

$$\delta\,e_M{}^A \;=\; D_M\,\xi^A \;-\; S^A{}_{MN}\,\xi^N\,, \qquad \delta\,\omega_M{}^{AB} \;=\; -\;R_{MN}{}^{AB}\,\xi^N$$

$$D_M\,\mathcal{T}^M{}_N \;+\; S^P{}_{MN}\,\mathcal{T}^M{}_P \;-\; S^P{}_{PM}\,\mathcal{T}^M{}_N \;=\; -\;R_{MN}{}^{AB}\;\mathcal{Y}^M{}_{AB}$$

- Gravity:

$$\delta\,\mathcal{S}_{EH} \;=\; -\;\frac{1}{k^2}\int_{\mathcal{M}} d^Dx\; e\left[\delta\,\omega_N{}^{AB}\,\Theta^N{}_{AB} \;+\; \delta\,e_M{}^A\,G^M{}_A\right]$$

$$D_M\,\Theta^M{}_{AB} \;-\; S^P{}_{PM}\,\Theta^M{}_{AB} \;=\; \frac{1}{2}\left(G_{AB} \;-\; G_{BA}\right)$$

$$D_M\,G^M{}_N \;+\; S^P{}_{MN}\,G^M{}_P \;-\; S^P{}_{PM}\,G^M{}_N \;=\; -\;\Theta^M{}_{AB}\,R_{MN}{}^{AB}$$

- Einstein eqs:

$$G^M{}_A \;=\; 2\,k^2\,\mathcal{T}^M{}_A \qquad\qquad \Theta^M{}_{AB} \;=\; 2\,k^2\,\mathcal{Y}^M{}_{AB}$$

Boundary Conditions

(Mourad, AS, 2020)

- **Killing vectors:**

$$\delta e_M{}^A \equiv D_M \zeta^A - S^A{}_{MN}\zeta^N + \theta^{AB} e_{NB} = 0\,,$$
$$\delta \omega_M{}^{AB} \equiv -R_{MN}{}^{AB}\zeta^N - D_M \theta^{AB} = 0$$

- **Killing equations:**

$$\theta^{AB} = D^A \zeta^B - S^{BA}{}_C\,\zeta^C\,,$$
$$D_M \zeta_N + D_N \zeta_M = (S_{MN}{}^P + S_{NM}{}^P)\,\zeta_P$$
$$D_M D_A \zeta_B = (D_M S_{BA}{}^N)\,\zeta_N + S_{BA}{}^N D_M \zeta_N - R_{MNAB}\,\zeta^N$$

❖ **Noether currents:**

$$\mathcal{J}^M = \mathcal{T}^M{}_N \zeta^N - \mathcal{Y}^M{}_{AB}\,\theta^{AB}$$

$$\mathcal{J}^M = \mathcal{T}^M{}_N \zeta^N - \mathcal{Y}^M{}_{AB}\,\theta^{AB}$$

❖ **Boundary conditions:**

$$\mathcal{J}^r\big|_{\partial\mathcal{M}} = 0$$

- **For:**

$$ds^2 = e^{2A(r)} g_{\mu\nu}(x)\,dx^\mu dx^\nu + e^{2B(r)}dr^2 + e^{2C(r)} g_{ij}(y)\,dy^i dy^j$$

$$\mathcal{T}^r{}_\mu\big|_{\partial\mathcal{M}} = 0\,, \qquad \mathcal{T}^r{}_i\big|_{\partial\mathcal{M}} = 0\,, \qquad \mathcal{Y}^r{}_{\mu\nu}\big|_{\partial\mathcal{M}} = 0$$

- **Fermi:**

$$(1-\Lambda)\,\lambda\big|_{\partial\mathcal{M}} = 0 \qquad \Lambda^2 = 1 \qquad \{\Lambda\,,\gamma^0\gamma^r\} = 0$$

Boundary Conditions: the matrix Λ

(Mourad, AS, 2020)

$$ds^2 = e^{2A(r)}\, g_{\mu\nu}(x)\, dx^\mu\, dx^\nu + e^{2B(r)} dr^2 + e^{2C(r)}\, g_{ij}(y)\, dy^i\, dy^j \qquad \Lambda^2 = 1 \qquad \{\Lambda\,,\, \gamma^0\gamma^r\} = 0$$

- Conditions: $\mathcal{T}^r{}_\mu|_{\partial\mathcal{M}} = 0\,, \quad \mathcal{T}^r{}_i|_{\partial\mathcal{M}} = 0\,, \quad \mathcal{Y}^r{}_{\mu\nu}|_{\partial\mathcal{M}} = 0$
- $\mathcal{Y}^r{}_{\mu\nu}|_{\partial\mathcal{M}} = 0$ demands: $[\Lambda, \gamma_{\mu\nu}] = 0$
- & for Weyl: $[\Lambda\,,\, \gamma_\chi] = 0$
- & for Majorana: $C^{-1}\,\Lambda^T\, C = -\,\gamma^0\,\Lambda\,\gamma^0$
- With D=11 Majorana (Horava-Witten) : $\Lambda = \gamma_{10}$
- In general, one can build the Λ's from the two lists:

$$\Lambda^{(n)} \;:\; (i)^{\frac{n(n+1)}{2}}\,\gamma^{r i_1,\dots i_n} \qquad \text{and} \qquad \Lambda^{(m)} \;:\; i(i)^{\frac{(m+d-1)(m+d)}{2}}\,\gamma^{01\dots(d-1) i_1,\dots i_m}$$

- Solutions of 2 constraints above: restrictions with Weyl, Majorana or Majorana-Weyl
- [More solutions exist combing the two gravitini in IIB:]

D=4 with Fluxes on T^5 x I

(Mourad, AS, to appear)

- Five-form flux in IIB → ϕ CONSTANT, SPATIAL INTERVAL of length ℓ

$$
\begin{aligned}
ds^2 &= \Lambda^2 \frac{\eta_{\mu\nu}\, dx^\mu\, dx^\nu}{[\sinh(r)]^{\frac{1}{2}}} + \ell^2 [\sinh(r)]^{\frac{1}{2}}\, e^{-\frac{5\,r}{\sqrt{10}}}\, dr^2 + \left(\sqrt{2}\,\Phi\,\ell\right)^{\frac{2}{5}} [\sinh(r)]^{\frac{1}{2}}\, e^{-\frac{r}{\sqrt{10}}}\, d\vec{y}^{\,2} \\
\mathcal{H}_5^{(0)} &= \frac{\Lambda^4}{\sqrt{2}}\, \frac{dx^0 \wedge \ldots \wedge dr}{[\sinh(r)]^2} + \Phi\, dy^1 \wedge \ldots \wedge dy^5 \\
q_3\,\Phi &= N \qquad\qquad q_3 = \sqrt{\pi}\, m_{Pl(10)}^4
\end{aligned}
$$

$$\boxed{\Lambda = i\,\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3\,\sigma_2}$$

- FINITE gs , BUT STILL CURVATURE SINGULARITY]

(Used extensively: Bergshoeff, Kallosh, Ortin, Roest, Van Proeyen, 2001)

- Split perturbations according to SO(1,3)xSO(5) [or SO(4) for internal excitations]
- SUSY BREAKING ~ 1/ℓ
- Tensor eqs:
- (+ Einstein eqs.)

$$
\begin{aligned}
\partial_{[\mu}\, b^{(2)}{}_{\nu]}{}^{lm} + \frac{1}{2}\,\epsilon^{lmnpq}\,\partial_n\, b_{\mu\nu pq} &= -\,\frac{e^{-4A-4C}}{2}\,\epsilon_{\mu\nu\rho\sigma}\,\partial_r\, b^{\rho\sigma lm}\,, \\
\partial_r\, b^{(2)}{}_\mu{}^{lm} &= e^{2A+6C}\left(\partial^{[l}\, b_\mu{}^{m]} + \frac{1}{2}\,\epsilon^{\alpha\beta\gamma}{}_\mu\,\partial_\alpha\, b_{\beta\gamma}{}^{lm}\right), \\
\partial_\mu\, b^m - \partial_n\, b^{(2)}{}_\mu{}^{mn} &= e^{-2C}\left[\frac{H_5}{2}\, h_\mu{}^m - e^{-6A}\,\partial_r\, b_\mu{}^m\right], \\
\partial_r\, b^m &= e^{-2C}\left[\frac{H_5}{2}\, h_r{}^m - e^{10C}\left(\partial^m b - \partial_\mu\, b^{\mu m}\right)\right], \\
\partial_p\, b^p &= \frac{H_5}{4}\left(-\,e^{-2A}\, h_\alpha{}^\alpha - e^{-2B}\, h_{rr} + e^{-2C}\, h_i{}^i\right) + e^{-8A}\,\partial_r\, b\,.
\end{aligned}
$$

Stability with Fluxes on $T^5 \times I$

(Mourad, AS, to appear)

$$ds^2 = e^{2A(r)} dx^2 + e^{2B(r)} dr^2 + e^{2C(r)} dy^2$$

- NO instabilities for **k=0, BUT mixings** induce them for **k≠0**
- E.g:

$$\mathcal{M}\, Z = m^2 Z$$

$$\begin{pmatrix} \mathcal{K}^2 + (-\partial+\alpha)_z (\partial+\alpha)_z & \frac{k H_5}{\sqrt{2}} e^{2(A-3C)} \\ \frac{k H_5}{\sqrt{2}} e^{2(A-3C)} & \mathcal{K}^2 + (-\partial+\beta)_z (\partial+\beta)_z \end{pmatrix} \qquad \alpha = \frac{C-A}{2}, \qquad \beta = -\frac{5A+3C}{2}$$

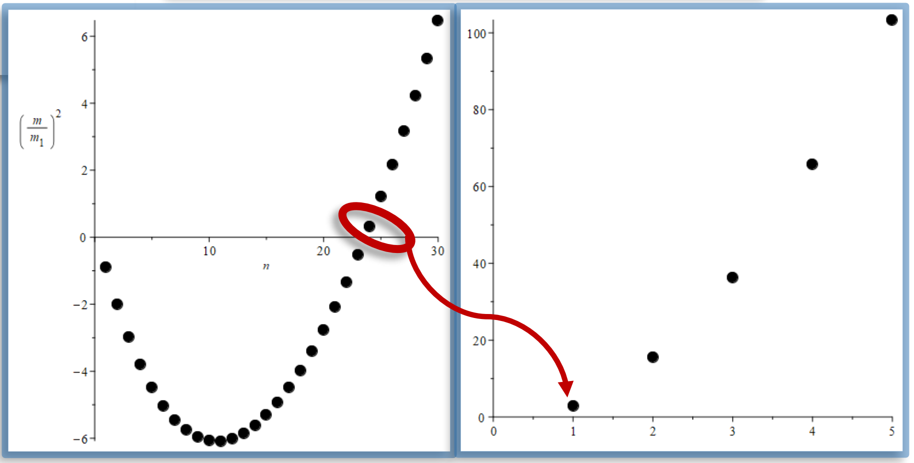
- PERTURBATIONS → Schrödinger-like systems

$$m^2 \Psi = \left(b + \mathcal{A}^\dagger \mathcal{A}\right) \Psi$$
$$\mathcal{A} = \frac{d}{dr} - \alpha(r)$$

  - Variational method: $$m_\Psi^2 = \frac{\langle \Psi | \widetilde{M} | \Psi \rangle}{\langle \Psi | \Psi \rangle} \geq m_0^2$$

- Mixings → unstable KK excitations
- BUT: NO link between 4D & internal scale

$$m_{Pl(10)}\, \ell > \mathcal{O}\left(10^2\right) N^{\frac{1}{4}}$$



Summary and Outlook

$$ds^2 \;=\; e^{2A(r)}dx^2 \;+\; e^{2B(r)}dr^2 \;+\; e^{2C(r)}dy^2$$

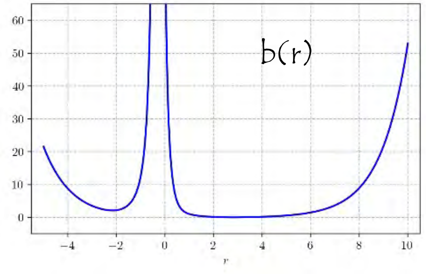
a) 10D → 4D Minkowski

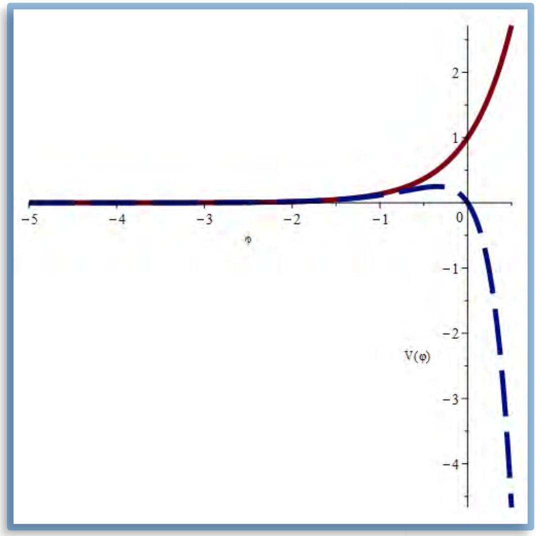
b) TADPOLE POTENTIAL: → NO (PERTURBATIVE) INSTABILITIES

- LEADING POTENTIAL → strong g_s
- (CORRECTED ?) [integrable dynamics] → can bound g_s
- CORRECTED V: unbounded from below
- YET: stable scalar perturbations

  (Fré, AS, Sorin, 2013) → *(Pelliconi, AS, 2021)*

- Non-perturbatively?
  - AdSxS Brane decay explored in *(Horowitz, Orgera, Polchinski, 2008)* *(Antonelli, Basile, (+Bombini), 2019)*





Drawing from the key Dudas-Mourad example (& keeping in mind the old Calabi-Yau/orbifold setup), even with broken SUSY in String Theory (or, better, in string-inspired Supergravity),

There seems to be some room for stable 4D Minkowski vacua …

Thank You