Spinor-helicity formalism in AdS and higher spins

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Quarks 2021

Based on 1811.08438,
1912.07494,
2004.07989,
2104.02770
with B. Nagaraj

Motivation

Spinor-helicity formalism is an efficient tool to compute amplitudes in massless theories with spinning fields in 4d. It allows to present amplitudes in an extremely compact form

$$A_n[1^-, 2^-, 3^+, \dots, n^+] = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle \dots \langle n1\rangle}$$

There are many reasons to study amplitudes in AdS space, e g holography. It would be nice to achieve the same simplicity when dealing with massless spinning fields in AdS as in flat space

Similar but different approach - [Maldacena, Pimentel '11]

HS Motivation

Flat space

AdS

Chiral HS theory

Formulated in light-cone/spinor-helicity approach

Involves <u>additional</u> vertices

Flat limit: 3pt cubic couplings match

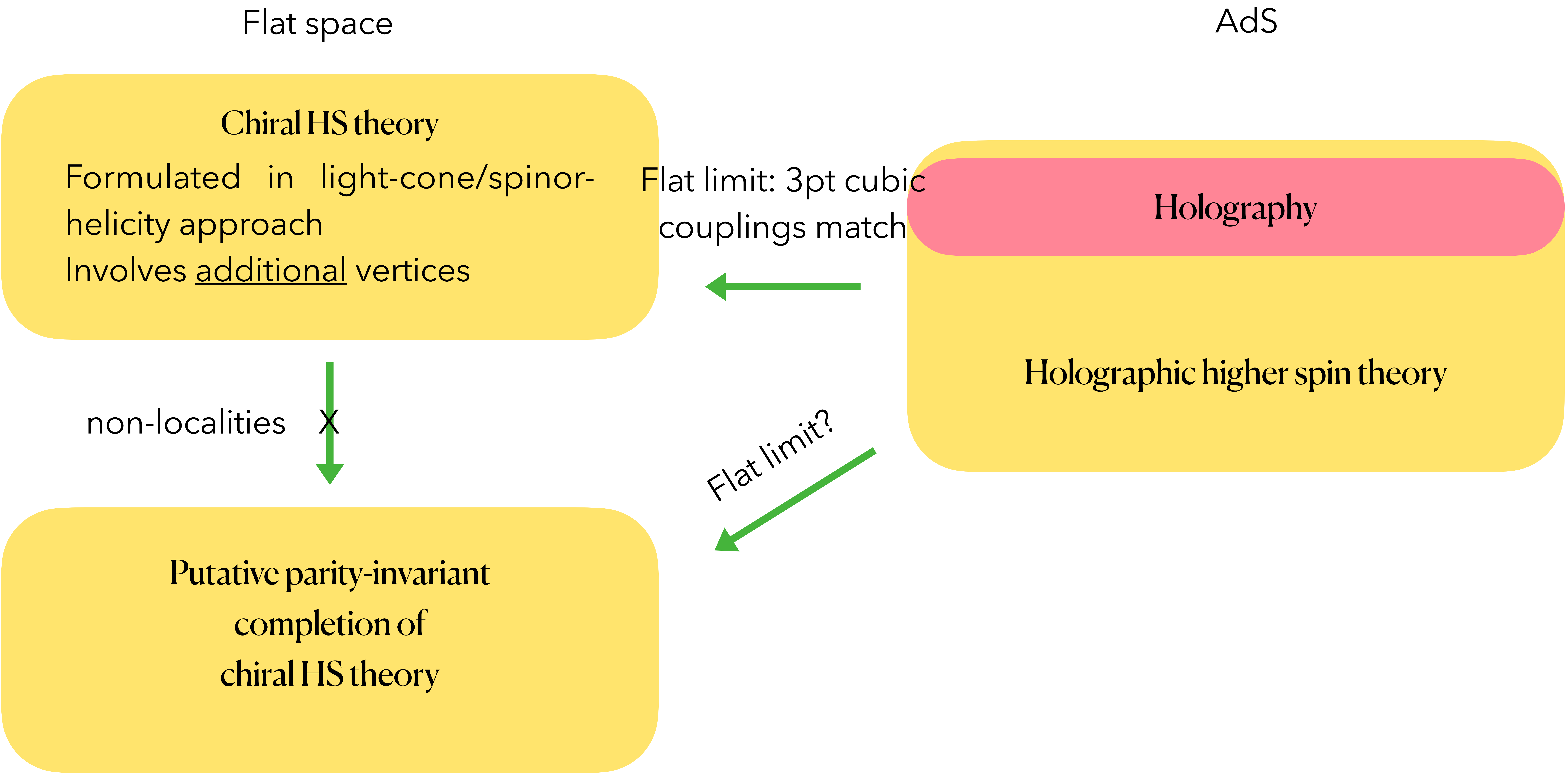
Holography

Holographic higher spin theory

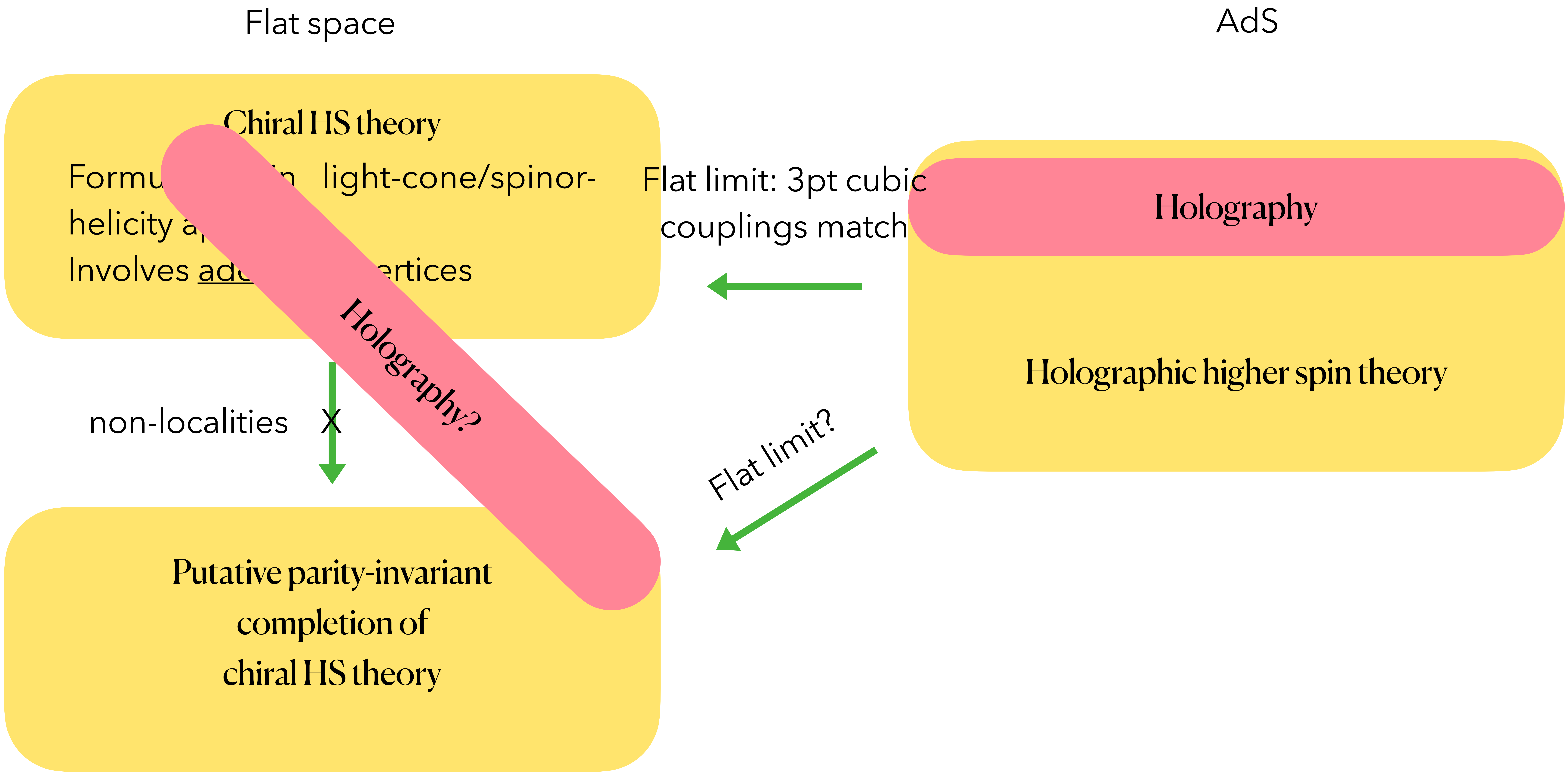
non-localities ~~X~~

Putative parity-invariant completion of chiral HS theory

HS Motivation



HS Motivation



Questions

1) Are these extra amplitudes also present in AdS? (New structures for 3-pt correlators of conserved currents)

2) Is there a relation between chiral and holographic theories beyond higher derivative 3pt vertices?

3) Can consistent parity invariant theories in flat space be obtained by a flat space limit from holographic ones?

4) Is there any underlying flat space holographic description for higher theories?

Flat holography

Flat holography is a difficult problem. Despite significant effort no dual pairs are known. Higher spins is, probably, the simplest place to start

[see e.g. recent workshop “Flat Asymptotia”]

Relevance of spinor-helicity formalism

One may argue that if higher-spin holography exists in flat space, it can only be formulated in the spinor-helicity formalism, because in standard approaches the relevant higher-spin interactions are missing

What we will do

Goal 1: Make some general developments of the spinor-helicity formalism in AdS4: find how to label the states, how to compute simple amplitudes, study their analytic structure

Goal 2: Rephrase higher-spin holography in this language. In particular, apply spinor-helicity formalism to the boundary theory

Spinor-helicity in flat space, review

Review in flat space

Basic ideas:

1) massless states in 4d are labelled by

$$p, \quad p^2 = 0, \qquad h \in \frac{1}{2}\mathbb{Z}.$$

Other information is redundant. Removing redundant information from amplitudes makes them simpler.

2) One can employ so(3,1) = sl(2,C) to go on-shell

$$p_{\alpha\dot{\alpha}} \equiv p_\mu (\sigma^\mu)_{\alpha\dot{\alpha}}, \qquad p^2 = 0 \quad \Rightarrow \quad p_{\alpha\dot{\alpha}} = \lambda_\alpha \bar{\lambda}_{\dot{\alpha}}$$

3) Helicity can be encoded in the homogeneity degrees of wave functions on spinors

$$H = \frac{1}{2}\left(\bar{\lambda}_{\dot{\alpha}} \frac{\partial}{\partial \bar{\lambda}_{\dot{\alpha}}} - \lambda_\alpha \frac{\partial}{\partial \lambda_\alpha}\right)$$

Review in flat space

Field strength can be represented as

$$\bar{F}_{\dot{\alpha}_1\dots\dot{\alpha}_{2h}}(x,\lambda,\bar{\lambda}) = \bar{\lambda}_{\dot{\alpha}_1}\dots\bar{\lambda}_{\dot{\alpha}_{2h}} e^{-\frac{i}{2}x^{\alpha\dot{\alpha}}\lambda_\alpha\bar{\lambda}_{\dot{\alpha}}}$$

Potentials

$$\varphi_{a_1\dots a_h} = \varepsilon^+_{a_1}\dots\varepsilon^+_{a_h} e^{-\frac{i}{2}x^{\alpha\dot{\alpha}}\lambda_\alpha\bar{\lambda}_{\dot{\alpha}}}$$

where

$$\varepsilon^+_a \equiv -\frac{i}{2}\frac{(\sigma_a)^{\dot{\alpha}\alpha}\mu_\alpha\bar{\lambda}_{\dot{\alpha}}}{\mu^\beta\lambda_\beta}$$

Mu is an auxiliary spinor.

Review in flat space

Plug external states in this form into the standard Feynman rules. You get something like

$$A_n[1^-, 2^-, 3^+, \ldots, n^+] = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle \ldots \langle n1\rangle}\delta(\sum p_i)$$

Features:

1) Momentum is conserved - translation invariance

2) Prefactor is expressed in terms of spinor products - Lorentz invariance

$$\langle ij\rangle \equiv \lambda^i_\alpha \lambda^{j\alpha}, \qquad [ij] \equiv \bar{\lambda}^i_{\dot{\alpha}} \bar{\lambda}^{j\dot{\alpha}}$$

3) Auxiliary spinors drop out - gauge invariance

4) Homogeneity degrees in spinors are fixed by helicities - correct representations selected

Review in flat space

These arguments are sufficient to fix 3-pt amplitudes up to an overall factor

$$A(-h_1, -h_2, -h_3) = \langle 12 \rangle^{d_{12,3}} \langle 23 \rangle^{d_{23,1}} \langle 31 \rangle^{d_{31,2}} \delta^4(p_1 + p_2 + p_3)$$

[Benincasa,Cachazo'07]

where

$$d_{12,3} \equiv h_1 + h_2 - h_3, \qquad d_{23,1} \equiv h_2 + h_3 - h_1, \qquad d_{31,2} \equiv h_3 + h_1 - h_2$$

Similarly for squared brackets

Review in flat space

Additional vertices

Derivative counting: each derivative in a vertex corresponds to a pair of spinors

Let us look at interactions of the form s-s-2. Then, there are the following options for amplitudes: A(+s,+s,+2), A(+s,+s,-2),A(+s,-s,+2) and cc.

These have 2s+2, 2s-2 and 2 derivatives respectively. <u>The amplitude with 2 derivatives is missing in the Fronsdal approach and corresponds to the minimal coupling to gravity</u>.

[Bengtsson'14][Conde,Joung,Mkrtchyan'16]

Review in flat space

Remarks

Combines well with other modern amplitude methods

1) On-shell methods
2) Double-copy
3) Scattering equation
4) Geometry underlying amplitudes

Closely related to the light-cone approach

[Ananth'12][Bengtsson'16][Ponomarev'16]

Spinor-helicity in AdS

Spinor-helicity in AdS

Deformation to AdS is based on the following representation for massless fields in 4d

$$J_{\alpha\beta} = i\left(\lambda_\alpha \frac{\partial}{\partial\lambda^\beta} + \lambda_\beta \frac{\partial}{\partial\lambda^\alpha}\right)$$

$$\bar{J}_{\dot\alpha\dot\beta} = i\left(\bar\lambda_{\dot\alpha} \frac{\partial}{\partial\bar\lambda^{\dot\beta}} + \bar\lambda_{\dot\beta} \frac{\partial}{\partial\bar\lambda^{\dot\alpha}}\right)$$

$$P_{\alpha\dot\alpha} = \lambda_\alpha\bar\lambda_{\dot\alpha} - \frac{1}{R^2}\frac{\partial}{\partial\lambda^\alpha}\frac{\partial}{\partial\bar\lambda^{\dot\alpha}}$$

Extensively used in [Vasiliev theory], [twistors literature]

Spinor-helicity in AdS

Field strengths (stereographic coordinates)

$$F_{\dot{\alpha}_1\dots\dot{\alpha}_{2h}} = \bar{\lambda}_{\dot{\alpha}_1}\dots\bar{\lambda}_{\dot{\alpha}_{2h}}\left(1-\frac{x^2}{4R^2}\right)^{1+h} e^{ipx}$$

We just get extra powers of conformal factors!

[Hitchin'90][Bolotin,Vasiliev'00]

Spinor-helicity in AdS

Potentials (helicity +2)

$$h_{\alpha\dot\alpha,\beta\dot\beta} = -\left(1+\frac{b}{8R^2}-\frac{i}{2R^2}\frac{b}{a}\right)e^{-i\frac{a}{2}}\frac{\mu_\alpha\mu_\beta\bar\lambda_{\dot\alpha}\bar\lambda_{\dot\beta}}{\langle\mu\lambda\rangle^2} - \frac{i}{2aR^2}e^{-i\frac{a}{2}}\frac{\mu_\alpha\mu_\beta(\lambda^\gamma x_{\gamma\dot\alpha}\bar\lambda_{\dot\beta}+\lambda^\gamma x_{\gamma\dot\beta}\bar\lambda_{\dot\alpha})}{\langle\mu\lambda\rangle^3}\langle\mu x\lambda]$$

where

$$a \equiv \lambda_\alpha\bar\lambda_{\dot\alpha}x^{\alpha\dot\alpha}, \qquad b \equiv x_{\alpha\dot\alpha}x^{\dot\alpha\alpha}, \qquad \langle\lambda x\mu] \equiv \lambda^\alpha x_{\alpha\dot\alpha}\bar\mu^{\dot\alpha}$$

Computation of amplitudes gets somewhat more involved due to the presence of manifest x-dependence. However, in spirit it remains the same.

[DP,Nagaraj'18][DP,Nagaraj'19]

Spinor-helicity in AdS

For three-point amplitudes the result is

$$A(-h_1, -h_2, -h_3) = \langle 12 \rangle^{d_{12,3}} \langle 23 \rangle^{d_{23,1}} \langle 31 \rangle^{d_{31,2}} \left(1 + \frac{\Box_p}{4R^2}\right)^{\sum h - 1} \delta^4(p_1 + p_2 + p_3)$$

Achieved by computing amplitudes from the action and classifying them based on symmetries

[DP,Nagaraj'18][DP,Nagaraj'19]

So, the deformation boils down to adding powers of the conformal factor

Spinor-helicity in AdS

Remarks

The end result is in the same form as in twistor literature

[Adamo,Mason'12][Skinner'13]

Additional lower-derivative vertices do exist in AdS as well!

This result agrees with the analysis in the light-cone gauge

[Metsaev'18]

These amplitudes are nothing but Witten diagrams, with a different labelling of states on external lines, more transparent flat limit

Towards holography in spinor-helicity representation

Higher-spin holography recap

Higher spin theories are dual to O(N) vector models. Simplest of them is free O(N) vector model

$$S = \frac{1}{2}\int d^3x \phi_a \Box \phi^a$$

[Sezgin,Sundell'02][Klebanov,Polyakov'02]

It has an infinite set of higher-spin conserved currents of the form

$$J_{\mu_1 \dots \mu_s} = \phi_a \partial_{\mu_1} \dots \partial_{\mu_s} \phi^a + \dots .$$

Holography then implies

$$\langle J_{s_1} \dots J_{s_n} \rangle \qquad = \qquad A_n(s_1, \dots, s_n)$$

Higher-spin holography recap

Representation theory basis: Flato-Fronsdal theorem

(Boundary scalar)^2 = sum of massless higher spin fields in the bulk

[Flato,Fronsdal'78]

Boundary theory is free, hence, correlators are computed just from the Wick contractions

This can be used to define the bulk theory holographically

Higher-spin spinor-helicity holography

Major goal: We would like to revisit this analysis, within the spinor-helicity formalism and then explore the flat space case/limit

On the bulk side, we already learned how to compute amplitudes in a given form

Remains: spinor-helicity for the boundary side, matching bulk and boundary

More concretely:
realise the singleton representation
map the tensor product of singletons to higher spin currents
compute correlators by Wick contractions and extract higher spin amplitudes
see whether this can be generalised to the flat space case

sl(2,C) invariant description of conformal fields in 3d

sl(2,C) invariant description of conformal fields

First step: construct the singleton representation of so(3,2) within the spinor-helicity adapted approach

A bit expanded version: classify all short representations of so(3,2) within the spinor-helicity adapted approach

Def. Short representations = conformal fields on the boundary (not operators), satisfy some wave equation

Def. Spinor helicity = sl(2,C) symmetry made manifest by employing spinors

Representations of sl(2,C), recap

Representations of so(3,1) =sl(2,C) can be realised by homogeneous functions of spinors

$$f(\alpha\lambda, \bar{\alpha}, \bar{\lambda}) = \alpha^N \bar{\alpha}^{\bar{N}} f(\lambda, \bar{\lambda})$$

N and Nbar are <u>independent complex</u> numbers.

Lorentz transformations then act naturally

$$J_{\alpha\beta} = i\left(\lambda_\alpha \frac{\partial}{\partial \lambda^\beta} + \lambda_\beta \frac{\partial}{\partial \lambda^\alpha}\right), \qquad \bar{J}_{\dot{\alpha}\dot{\beta}} = i\left(\bar{\lambda}_{\dot{\alpha}} \frac{\partial}{\partial \bar{\lambda}^{\dot{\beta}}} + \bar{\lambda}_{\dot{\beta}} \frac{\partial}{\partial \bar{\lambda}^{\dot{\alpha}}}\right)$$

sl(2,C) invariant description of conformal fields

Deformed translations act as

$$P_{\alpha\dot{\alpha}} = \lambda_\alpha \bar{\lambda}_{\dot{\alpha}} A(N, \bar{N}) + \frac{\partial}{\partial \lambda^\alpha} \frac{\partial}{\partial \bar{\lambda}^{\dot{\alpha}}} B(N, \bar{N}) + \lambda_\alpha \frac{\partial}{\partial \bar{\lambda}^{\dot{\alpha}}} C(N, \bar{N}) + \bar{\lambda}_{\dot{\alpha}} \frac{\partial}{\partial \lambda^\alpha} D(N, \bar{N})$$

They change change sl(2,C) weights N and Nbar. We get the weight lattice

It remains to impose

$$[P_{\alpha\dot{\alpha}}, P_{\beta\dot{\beta}}] = -\frac{i}{R^2} \varepsilon_{\alpha\beta} \bar{J}_{\dot{\alpha}\dot{\beta}} - \frac{i}{R^2} \varepsilon_{\dot{\alpha}\dot{\beta}} J_{\alpha\beta}$$

This gives constraints on A, B, C and D. Solving them, one obtains a representation of so(3,2).

sl(2,C) invariant description of conformal fields

This approach should remind you of unfolding (see e.g. talk by Euihun). Mathematically, is also known as Harisch-Chandra decomposition

[Vasiliev'94][Iazeolla,Sundell'08]

For <u>bulk</u> fields of general mass and spin, see

[DP,Vasiliev'10][Khabarov,Zinoviev'19]

Important differences!

We «unfold» boundary fields

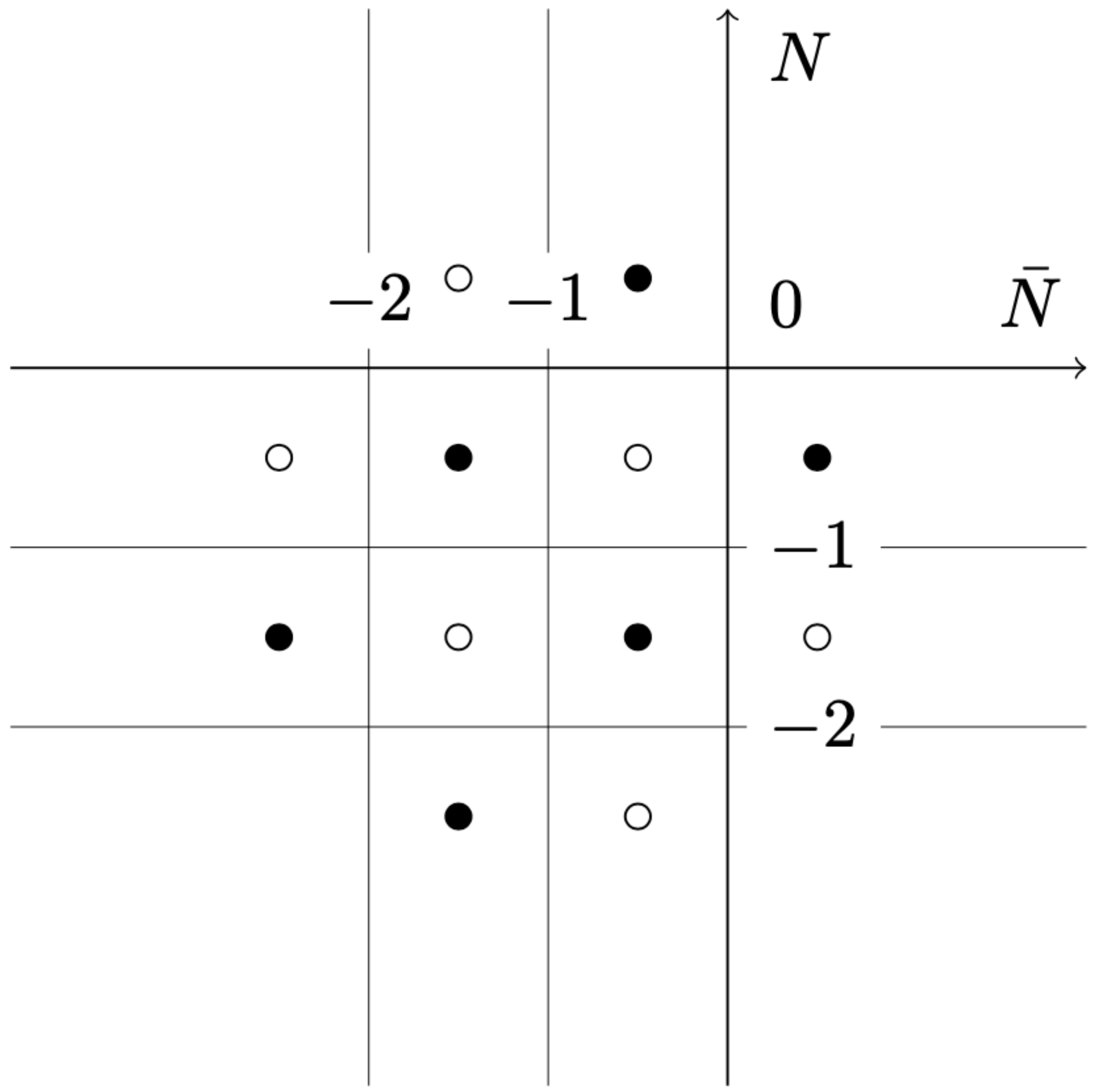
We still split representations of so(3,2) into irreps of so(3,1)=sl(2,C) as for the bulk fields. We do not split them into reps of so(2,1) (which is the Lorentz algebra on the boundary)

Relevant representations of sl(2,C) are infinite-dimensional (so it is not the derivative expansion)

We need a finite number of irreps of sl(2,C)

sl(2,C) invariant description of conformal fields

Classification of short modules obtained this way



These are labelled by

$$i \in \mathbb{N}, \quad j \in \mathbb{N}, \qquad i + j \in 2\mathbb{N} + 1$$

Lowest weight state has

$$E_0 = \frac{3 - i}{2}, \qquad s_0 = \frac{j - 1}{2}$$

The associated Lagrangian (even spin)

$$\kappa \equiv \frac{i}{2}$$

$$\mathcal{L} = \frac{1}{2} \sum_{n=0}^{N} \frac{2^n (\kappa + 1 - n)_n}{n!(s-n)! \left(\kappa + s + \frac{1}{2} - n\right)_n} \partial^{b_1} \dots \partial^{b_n} \phi_{b_1 \dots b_n a_1 \dots a_{s_0 - n}} \Box^{\kappa - n} \partial^{c_1} \dots \partial^{c_n} \phi_{c_1 \dots c_n}{}^{a_1 \dots a_{s_0 - n}}$$

Matches bosonic part of classification in [Vasiliev'09]

sl(2,C) invariant description of scalar singleton

Scalar singleton: two sl(2,C) weight spaces

$$(N,\bar{N}) = \left(-\frac{3}{2},-\frac{3}{2}\right) \cup \left(-\frac{1}{2},-\frac{1}{2}\right) \qquad f(\lambda,\bar{\lambda}) = f^{\left(-\frac{3}{2},-\frac{3}{2}\right)}(\lambda,\bar{\lambda}) + f^{\left(-\frac{1}{2},-\frac{1}{2}\right)}(\lambda,\bar{\lambda})$$

Deformed translations act

$$P_{\alpha\dot{\alpha}} f(\lambda,\bar{\lambda}) = A\lambda_\alpha \bar{\lambda}_{\dot{\alpha}} f^{\left(-\frac{3}{2},-\frac{3}{2}\right)}(\lambda,\bar{\lambda}) + B\frac{\partial^2}{\partial\lambda^\alpha \partial\bar{\lambda}^{\dot{\alpha}}} f^{\left(-\frac{1}{2},-\frac{1}{2}\right)}(\lambda,\bar{\lambda})$$

where

$$AB = -\frac{4}{R^2}$$

Flat space limit of singletons

Flat space limit

$$P_{\alpha\dot{\alpha}}P_{\beta\dot{\beta}} \propto AB = -\frac{4}{R^2} \to 0$$

So, in the flat space limit translations are inevitably nilpotent.

[Flato,Fronsdal'78]

<u>This prevents flat space Flato-Fronsdal theorem from working</u>: nilpotent translations for singletons result into nilpotent translations for the tensor product. The latter is not the case for massless higher spin fields

General issue for flat holography:

More generally, there are no known UIRs of iso(3,1) with GK=2 (3d theory with one equations of motion imposed) and non-trivially realized translations. So, flat space amplitudes cannot have an underlying field-theoretic description on the boundary

Ways out for flat holography?

1) Construct representations irrespectively from contraction from so(3,2)? We did not do a systematic analysis, though it is not quite clear how it may work – see e.g. Wigner classification

2) Going to complex momenta? (Helps making 3pt scattering of massless particles non-trivial. May be helpful in regularising flat space singleton degeneracy)

[Atanasov,Ball,Melton,Riclariu,Strominger'21]

3) Replace tensor product with something else?

[Iazeolla,Sundell'08]

Results and Conclusions

1) Spinor-helicity formalism allows to construct additional 3-pt amplitudes for hs fields. This includes AdS space

2) AdS spinor-helicity formalism allows to deal efficiently with massless spinning particles. Somewhat complicated with the explicit x-dependence (momentum is not conserved)

3) First steps towards rephrasing AdS holography in this language were made, e. g. singletons were formulated in a suitable way. More generally, 3d conformal fields in this approach were classified

Future Directions

Proceed with AdS higher spin holography in a given form:

1) extract all cubic couplings in holographic theories and compare them with chiral theories
2) study four-point amplitudes and their properties in a given form
3) study flat limit at the level of amplitudes

This picture feels incomplete without flat singletons. Is there any natural deformation of the problem that brings suitable singleton representations into flat space?

Thank you!