Two-Dimensional Gauge Theory with Adjoint Matter

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Talk at

Quarks-2021

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Based largely on the recent paper

Ross Dempsey, IRK, Silviu Pufu, “Exact Symmetries and Threshold States in Two-Dimensional Models for QCD,” arXiv:2101.05432

Introduction

- The problem of Color Confinement in QCD is among the deepest in modern Theoretical Physics.
- While there has been great progress in Lattice Gauge Theory, there is no quantitative analytic understanding yet of the mass gap and confinement, even in the large N pure glue theory in 2+1 or 3+1 dimensions.
- At the same time, there have been tantalizing experimental discoveries of exotic hadronic states that put new focus on the physics of strong interactions.
- For example, the charmonium state X(3872) has mass that is very close to the sum of D-meson and D*-meson masses. X appears to be a “molecular” threshold state.

Clay Millenium Problems

Yang-Mills and Mass Gap

Experiment and computer simulations suggest the existence of a "mass gap" in the solution to the quantum versions of the Yang-Mills equations. But no proof of this property is known.

Status: Unsolved

STRINGS MICHIGAN THEORY

July 10-15, 2000 University of Michigan Ann Arbor

"Millennium Madness"

Physics Problems for the Next Millennium

The best 10 problems were selected at the end of the conference by a selection panel consisting of:

- Michael Duff (University of Michigan)
- David Gross (Institute for Theoretical Physics, Santa Barbara)
- Edward Witten (Caltech & Institute for Advanced Studies)

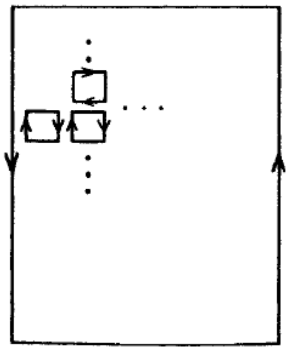
10. Can we quantitatively understand quark and gluon confinement in Quantum Chromodynamics and the existence of a mass gap?
    Igor Klebanov, Princeton University
    Oyvind Tafjord, McGill University

Lattice SU(N) Gauge Theory

- The gauge field kinetic term is encoded by the plaquette terms.

$$S = -(1/2g^2) \sum_{n,\,\mu\nu} \mathrm{tr}\, U_\mu(n) U_\nu(n+\mu)\, U_{-\mu}(n+\mu+\nu) U_{-\nu}(n+\nu) + \mathbf{h.c.}$$

- In the strong coupling expansion where these terms are treated as perturbations, the Area Law of the Wilson loop is obvious.



$$\left\langle \prod_C \exp[iB_\mu(n)] \right\rangle \approx \exp[-F(g^{-2})A]$$

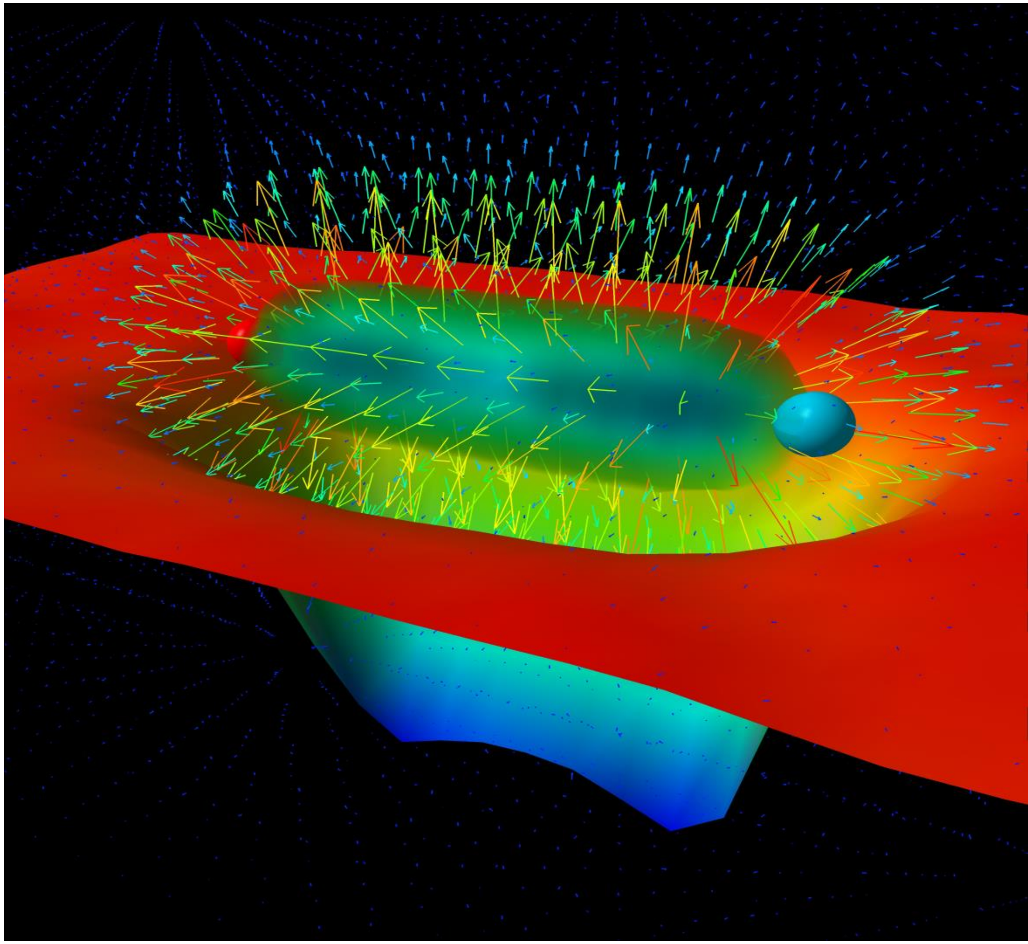
- To obtain the continuum limit, one needs to interpolate to the weak coupling limit on lattice scale due to Asymptotic Freedom

$$g^2(a) = \frac{g_0^2}{1 + (Cg_0^2/2\pi)\ln(a_0/a)}$$

- Can confinement disappear in this limit? Numerical simulations indicate that the answer is "No," but so far this is not proven analytically.

QCD and Strings

- At distances much smaller than 1 fm, the quark-antiquark potential is nearly Coulombic.
- At larger distances the potential should be linear (Wilson) due to formation of confining flux tubes. Their dynamics is approximately described by the Nambu-Goto area action. So, strings have been observed, at least in numerical simulations of Yang-Mills theory.



Large N Yang-Mills Theories

- Connection of gauge theory with string theory is strengthened in `t Hooft's generalization from 3 colors (SU(3) gauge group) to N colors (SU(N) gauge group).
- Make N large, while keeping the `t Hooft coupling fixed:

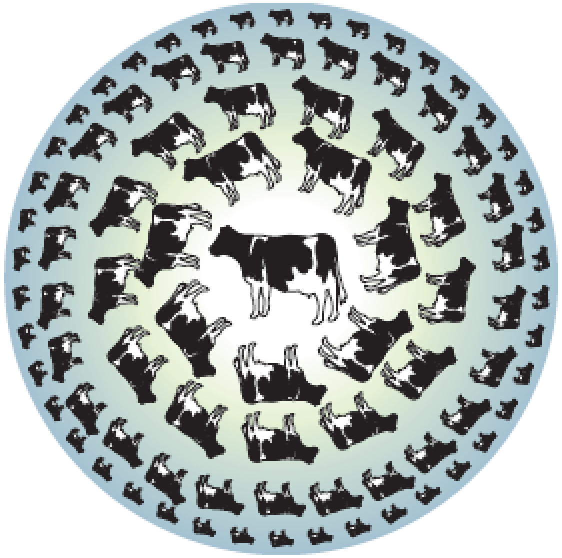
$$\lambda = g_{\mathrm{YM}}^2 N$$

- The probability of snapping a flux tube by quark-antiquark creation (meson decay) is 1/N. The string coupling is 1/N.

The AdS/CFT Duality

Maldacena; Gubser, IRK, Polyakov; Witten

- Relates conformal gauge theory in 4 dimensions to string theory on 5-d Anti-de Sitter space times a 5-d compact space. For the $\mathcal{N}$=4 SYM theory this compact space is a 5-d sphere.
- The geometrical symmetry of the AdS_5 space realizes the conformal symmetry of the gauge theory.
- The AdS space-time is a generalized hyperboloid. It has negative curvature.



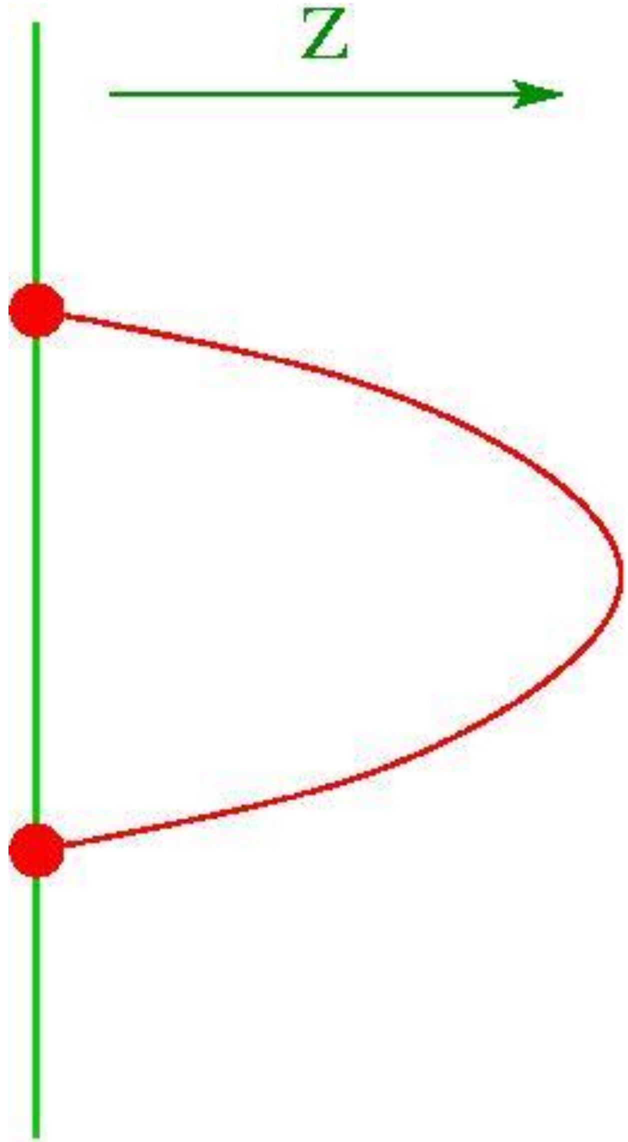
- When a gauge theory is strongly coupled, the radius of curvature of the dual AdS_5 and of the 5-d compact space becomes large: $$\frac{L^2}{\alpha'} \sim \sqrt{g_{\mathrm{YM}}^2 N}$$
- String theory in such a weakly curved background can be studied in the effective (super)-gravity approximation, which allows for a host of explicit calculations. Corrections to it proceed in powers of
$$\frac{\alpha'}{L^2} \sim \lambda^{-1/2}$$
- Feynman graphs instead develop a weak coupling expansion in powers of λ. At weak coupling the dual string theory becomes difficult.

The quark anti-quark potential

- The z-direction of AdS is dual to the energy scale of the gauge theory: small z is the UV; large z is the IR.

- The quark and anti-quark are placed at the boundary of Anti-de Sitter space (z=0), but the string connecting them bends into the interior (z>0). Due to the scaling symmetry of the AdS space, this gives Coulomb potential Maldacena; Rey, Yee

$$V(r) = -\frac{4\pi^2\sqrt{\lambda}}{\Gamma\left(\frac{1}{4}\right)^4 r}$$



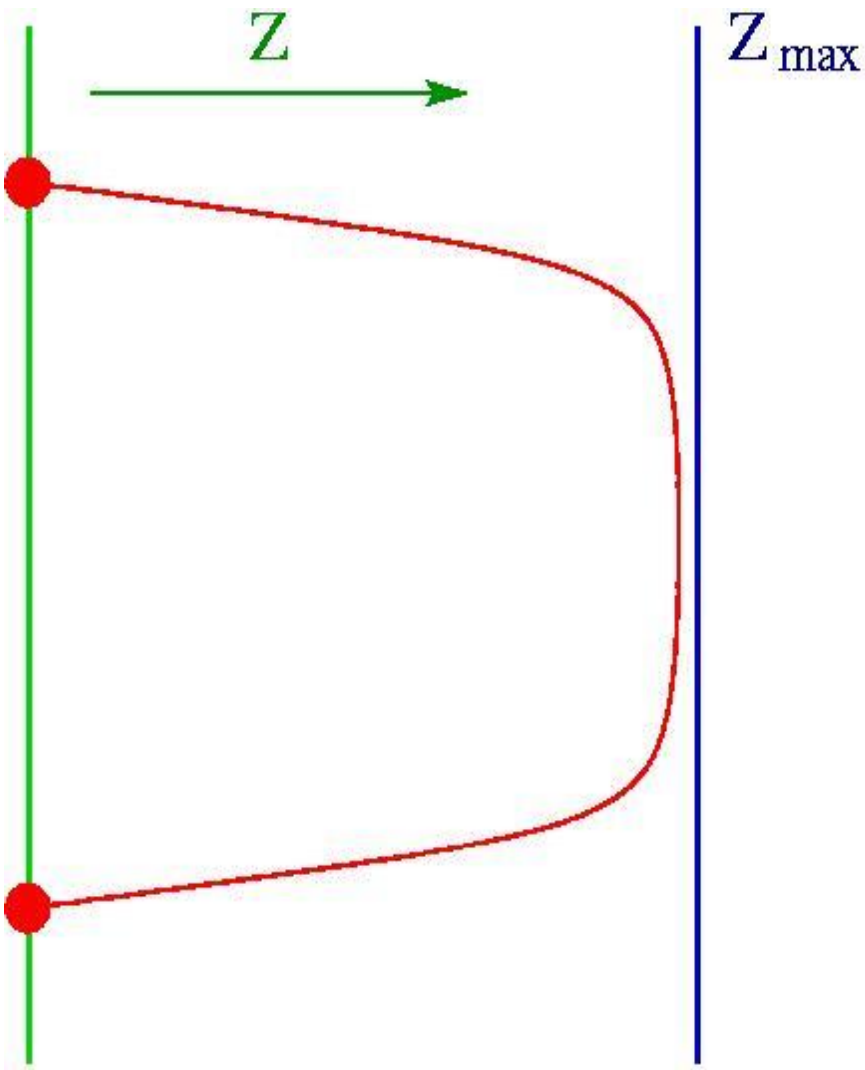
Color Confinement

- The quark anti-quark potential is linear at large distances but nearly Coulombic at small distances.
- The 5-d metric should have a warped form Polyakov

$$ds^2 = \frac{dz^2}{z^2} + a^2(z)\left(-(dx^0)^2 + (dx^i)^2\right)$$

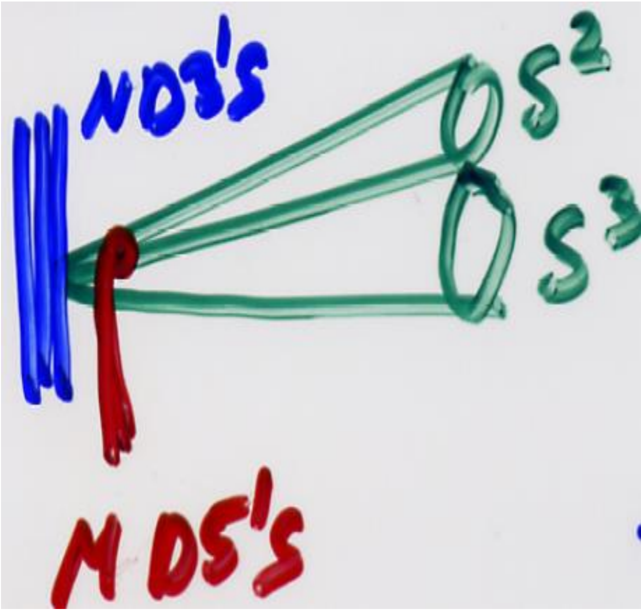
- The space ends at a maximum value of z where the warp factor is finite. Then the confining string tension is

$$\frac{a^2(z_{\max})}{2\pi\alpha'}$$



Confinement and Warped Throat

- To break conformal invariance, add to the N D3-branes M D5-branes wrapped over the sphere at the tip of the conifold.
- The 10-d geometry dual to the gauge theory on these branes is the warped deformed conifold IRK, Strassler (2000)



$$ds_{10}^2 = h^{-1/2}(y)\big(-(dx^0)^2 + (dx^i)^2\big) + h^{1/2}(y)ds_6^2$$

- ds_6^2 is the metric of the deformed conifold, a Calabi-Yau space defined by the following constraint on 4 complex variables:

$$\sum_{i=1}^{4} z_i^2 = \varepsilon^2$$

- The quark anti-quark potential is qualitatively similar to that found in numerical simulations of QCD (graph shows lattice QCD results by G. Bali et al with $r_0 \sim 0.5$ fm).

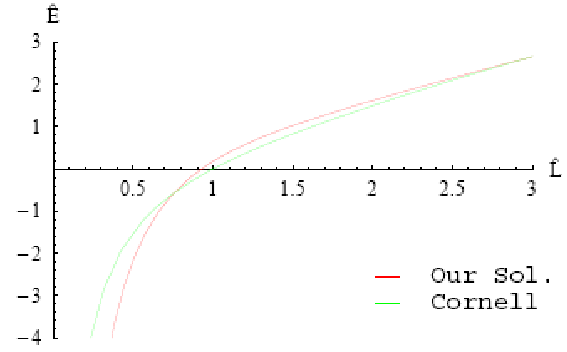
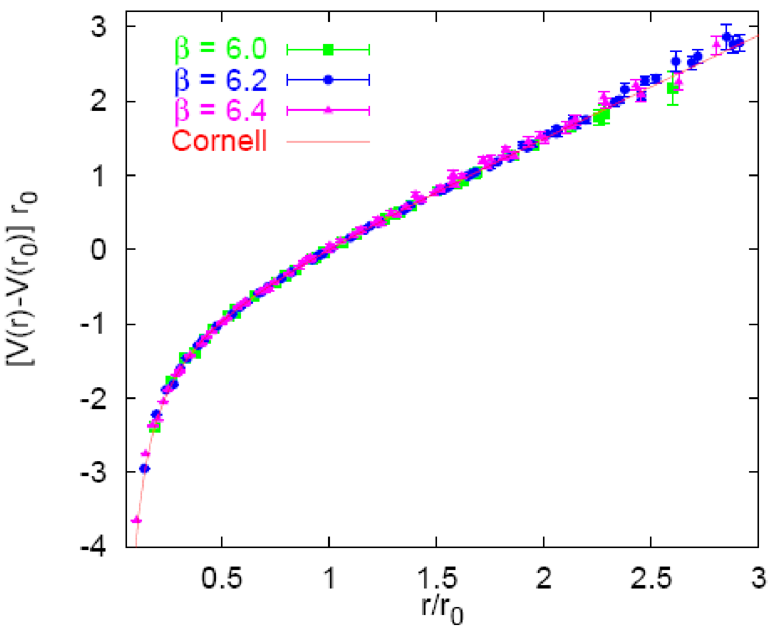


Figure 11: Comparison to the Cornell model

- Normal modes of the warped throat correspond to glueball-like bound states in the gauge theory.
- Their spectra have been calculated using standard methods of (super)gravity.
- The warped deformed conifold incorporates Dimensional Transmutation.

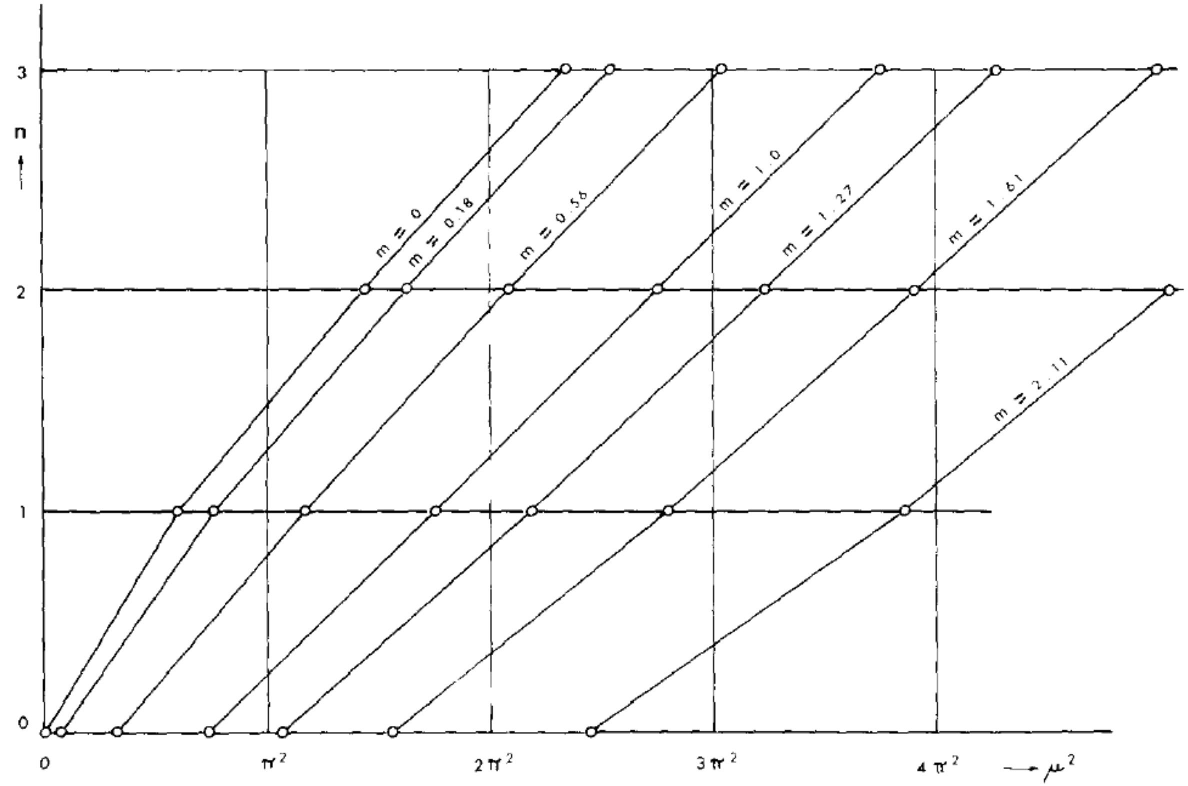


- The gauge/string duality has provided us with a “physicists’s proof of confinement” in some SU(N)xSU(N+M) minimally supersymmetric cascading gauge theories, which stay strongly coupled in the UV.
- Yet, we still don’t have a quantitative handle on the Asymptotically Free theories in 3+1 dimensions.
- Let us “drop back” to 1+1 dimensional gauge theories, which can hopefully give some intuition about aspects of the higher dimensional dynamics.

The 't Hooft Model

- 2d SU(N) gauge theory coupled to N_f fermions in the fundamental representation.
- Exactly solvable in the large N limit using the light-cone gauge: $A_- = 0$
- Find a single Regge trajectory of mesons whose masses are obtained by solving an integral equation.

$$\mu^2 \varphi(x) = \left(\frac{\alpha_1}{x} + \frac{\alpha_2}{1-x}\right) \varphi(x) - \mathrm{P}\int_0^1 \frac{\varphi(y)}{(y-x)^2}\, \mathrm{d}y$$



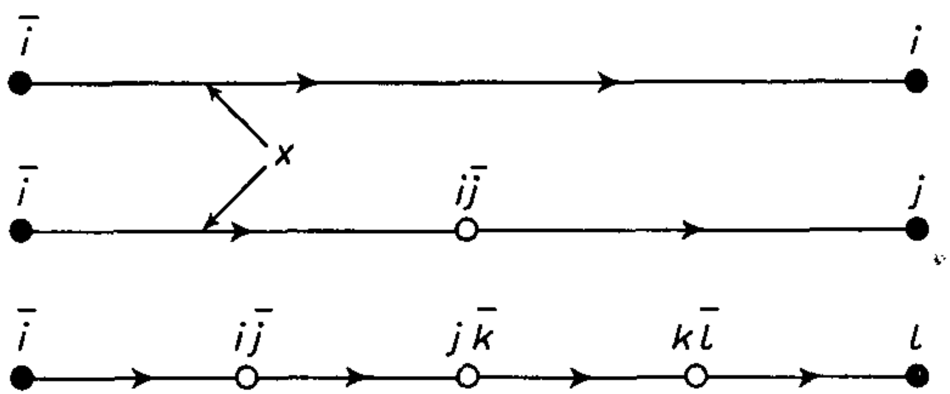
- Meson trajectories for different quark masses from ‘t Hooft’s 1974 paper.
- This beautiful toy model does not have any local adjoint degrees of freedom, which are crucial for higher-dimensional QCD.

2D QCD with Adjoint Matter

- Not exactly solvable at large N, but numerically tractable using Discretized Light-Cone Quantization (DLCQ). Dalley, IRK (1992)
- The model with an adjoint Majorana fermion (a toy gluino) has particularly nice properties. The mass is protected against renormalization by a discrete chiral Z_2 symmetry

$$S_{\mathrm{f}} = \int d^2x \, \mathrm{Tr}\left[i\Psi^T \gamma^0 \gamma^\alpha D_\alpha \Psi - m\Psi^T \gamma^0 \Psi - \frac{1}{4g^2} F_{\alpha\beta} F^{\alpha\beta} \right]$$

- That this SU(N) model has interesting topological structure has been known since the work of Witten in 1979.
- It has a discrete analogue of the 4-d theta angle, k=0,1, … N-1, where k is the number of probe quarks at spatial infinity and corresponding anti-quarks at minus infinity.



Mass Gap but No Confinement

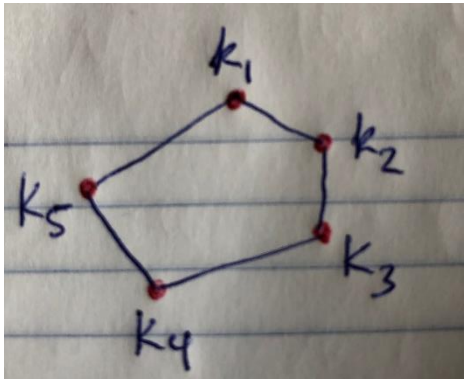
- For the m=0 theory, the infrared dynamics is governed by the gauged WZW (coset) model

$$\frac{\mathrm{SO}(N^2-1)_1}{\mathrm{SU}(N)_N}$$

- This model has vanishing central charge, which proves that the lightest bound state is massive.
- The model is in a “gapped topological phase.”
- The Wilson loop in the fundamental representation does not exhibit the area law. Gross, IRK, Matytsin, Smilga; Komargodski, Ohmori, Roumpedakis, Seifnashri; Dempsey, IRK, Pufu

- In the large N limit we can focus on the string-like single trace gluinoball states

$$|\Phi_{\mathrm{b}}(P^+)\rangle = \sum_{j=1}^{\infty} \int_0^{P^+} dk_1 \ldots dk_{2j}\, \delta\Big(\sum_{i=1}^{2j} k_i - P^+\Big)$$
$$f_{2j}(k_1, k_2, \ldots, k_{2j}) N^{-j}\, \mathrm{Tr}\,[b^\dagger(k_1) \ldots b^\dagger(k_{2j})]|0\rangle$$



- Z_2 symmetry implements string orientation reversal

$$\mathcal{C} b^\dagger_{ij}(k)\mathcal{C}^{-1} = b^\dagger_{ji}(k)$$

- Remarkably, for m=0 some of these states appear to be "threshold bound states" of other states. Gross, Hashimoto, IRK
- This is due to the current algebra module structure of the m=0 theory. Kutasov, Schwimmer; Dempsey, IRK, Pufu

DLCQ

- Make one of the light-cone directions compact. Brodsky, Hornbostel, Pauli
- Anti-periodic boundary conditions

$$\psi_{ij}(x^-) = -\psi_{ij}(x^- + 2\pi L) \qquad P^+ = K/(2L)$$

- K is an integer.

$$\psi_{ij}(x) = \frac{1}{\sqrt{2\pi L}} \sum_{\text{odd } n>0} \left(B_{ij}(n) e^{-in\frac{x}{2L}} + B^\dagger_{ji}(n) e^{in\frac{x}{2L}} \right)$$

- Single-trace gluinoball states

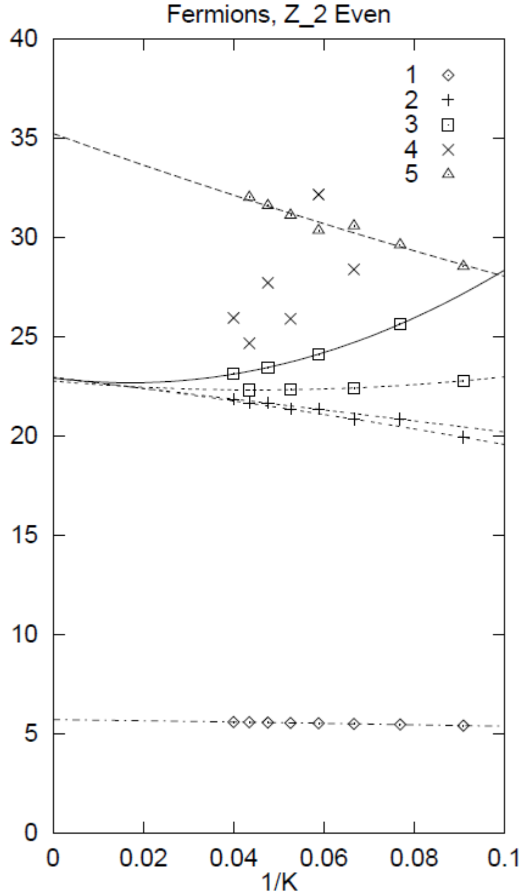
$$\frac{1}{N^{p/2}} \mathrm{tr}\left(B^\dagger(n_1) \cdots B^\dagger(n_p) \right) |0\rangle \qquad \sum_{i=1}^{p} n_i = K$$

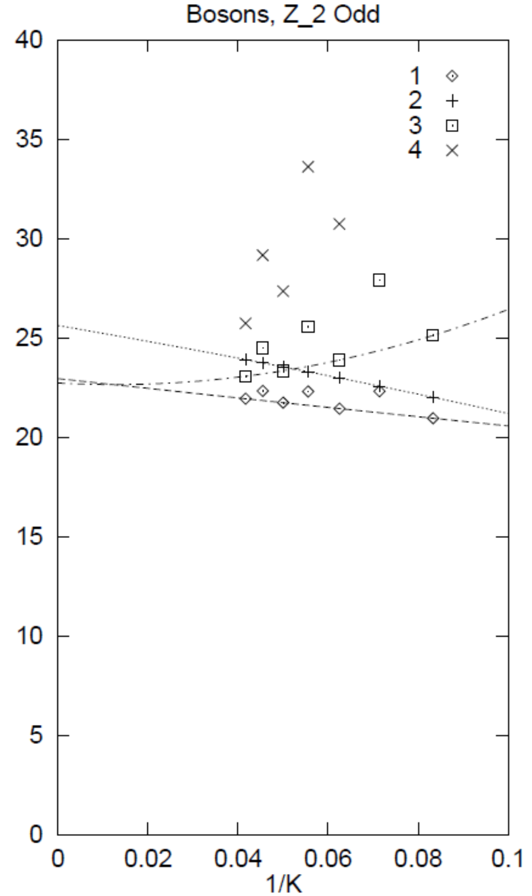
- For even K, these closed string-like states are bosons. For odd K they are fermions.
- In the large N limit, the light-cone Hamiltonian takes single-trace states into other single-trace states.
- We need to carry out "Exact Diagonalization"

K	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Dim	18	28	40	58	93	141	210	318	492	762	1169	1791	2786	4338	6712

- The continuum limit is that of large K, and the highest values we have reached are

K	35	36	37	38	39	40	41
Dim	5.9×10^5	9.3×10^5	1.5×10^6	2.3×10^6	3.6×10^6	5.7×10^6	9.0×10^6





$5.72g^2N/\pi$

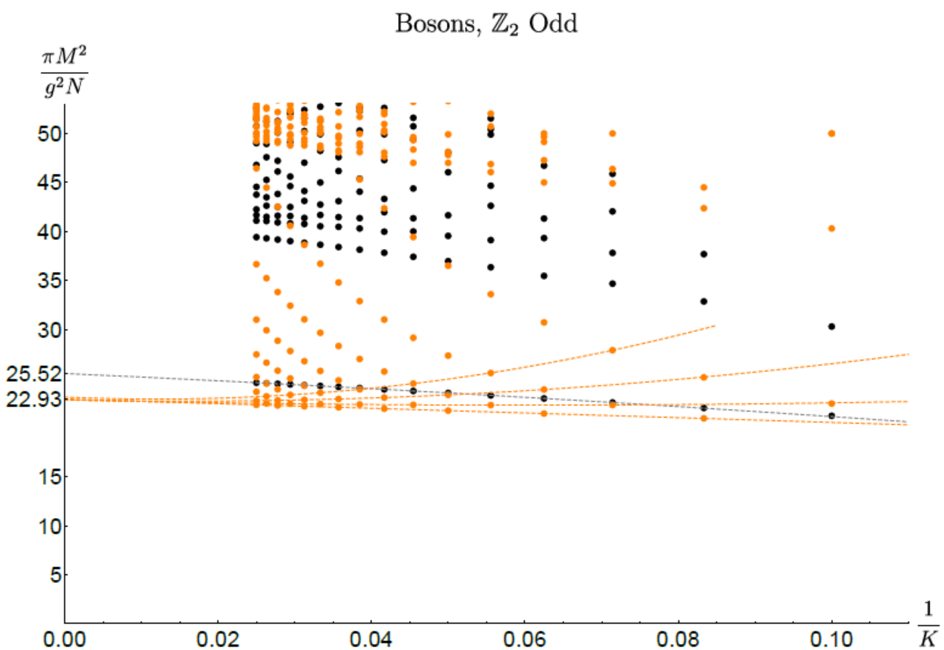
$$M^2 = K\left(\frac{M_{F1}^2(n)}{n} + \frac{M_{F1}^2(K-n)}{(K-n)}\right)$$

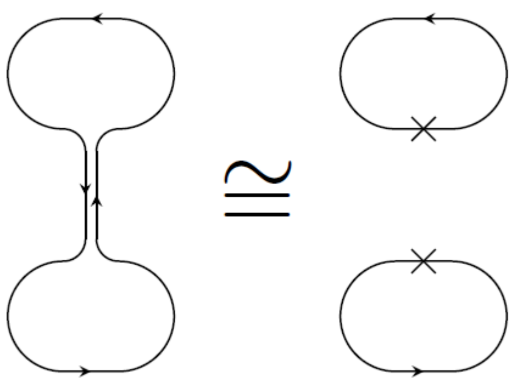
- First observation of threshold states. The threshold is at 4 times the lightest mass-squared.

  Gross, Hashimoto, IRK (1997)

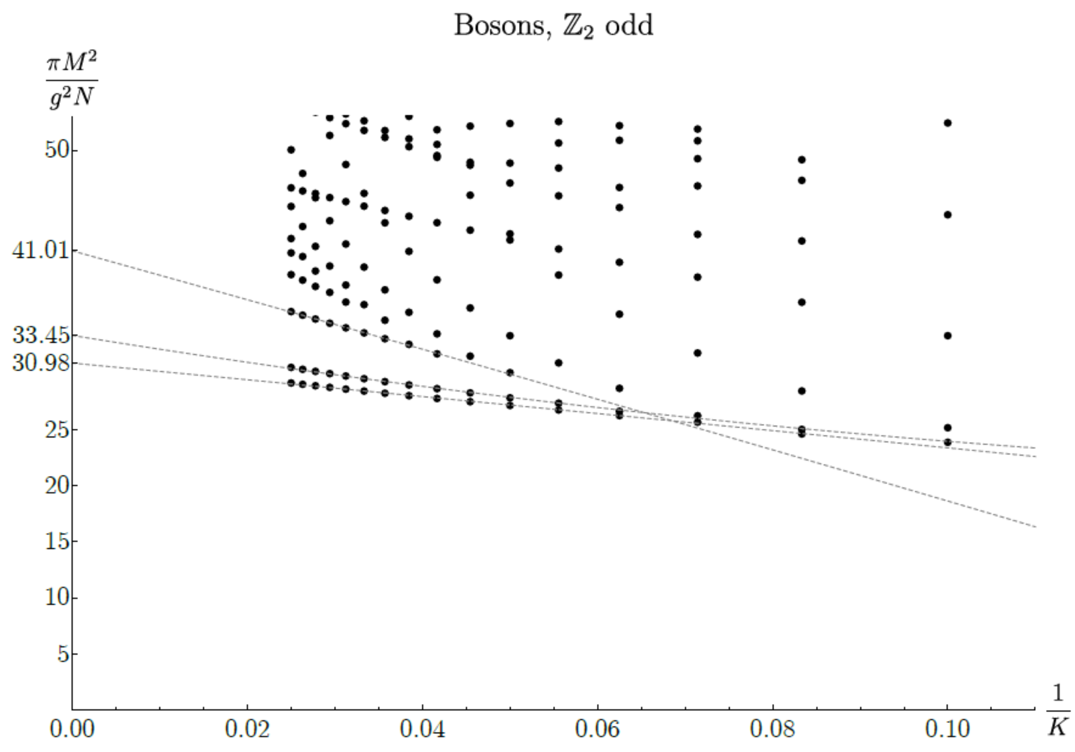
Exact Degeneracies

- In the work with Ross Dempsey and Silviu Pufu we obtained a better understanding of the exactly degenerate states marked with orange dots.





- Appearance of the continuous spectrum of single-trace states suggests that the massless adjoint model is not confining.
- Spectrum becomes discrete for m>0



Kac-Moody Algebra

$$[J_{ij}(n), J_{kl}(m)] = \delta_{kj} J_{il}(n+m) - \delta_{il} J_{kj}(n+m) + k_{\mathrm{KM}} \frac{n\,\delta_{n+m,0}}{2} \left(\delta_{il}\delta_{kj} - \frac{1}{N}\delta_{ij}\delta_{kl} \right)$$

- The Hamiltonian of massless models

$$P^- = \frac{g^2 L}{\pi} \sum_{\text{even } n \neq 0} \frac{\mathrm{tr}\,[J(-n)J(n)]}{n^2} = \frac{2g^2 L}{\pi} \sum_{\text{even } n > 0} \frac{\mathrm{tr}\,[J(-n)J(n)]}{n^2}$$

- Diagonalize within different current blocks

  Kutasov, Schwimmer

$$J_{i_1 j_1}(-n_1) J_{i_2 j_2}(-n_2) \cdots J_{i_p j_p}(-n_p) \, |\chi\rangle_I$$

- The primaries for the massless adjoint model are

$$
\begin{aligned}
\mathfrak{n} = 0: &\qquad |0\rangle\,, \\
\mathfrak{n} = 1: &\qquad B^\dagger_{ji}(1)|0\rangle\,, \\
\mathfrak{n} = 2: &\qquad \left(B^\dagger_{ji}(1)B^\dagger_{lk}(1) - \frac{1}{N}\delta_{kj}J_{il}(-2) + \frac{1}{N}\delta_{il}J_{kj}(-2)\right)|0\rangle \\
\mathfrak{n} = 3: &\qquad \left(B^\dagger_{ji}(1)B^\dagger_{lk}(1)B^\dagger_{nm}(1) - \text{traces}\right)|0\rangle\,,
\end{aligned}
$$

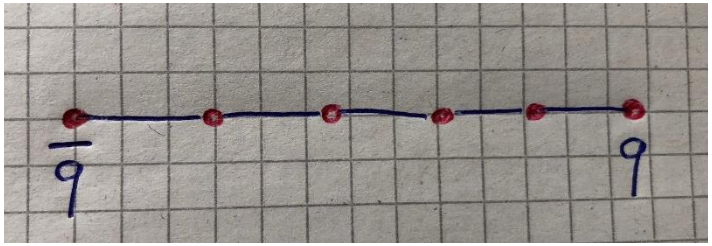
- After acting with raising operators, the descendants must be SU(N) singlets.
- This structure explains some exact degeneracies seen in the numerical diagonalizations. The P^- eigenvalues for n>1 sectors are sums of those in the n=1 sector.

A New 2D Model for Mesons

- If we add N_f fundamental Dirac fermions to the adjoint Majorana, we find a model which contains both gluinoballs and mesons:

$$S = \int d^2x \left[\mathrm{tr}\left(-\frac{1}{4g^2} F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\overline{\Psi}\not{D}\Psi - \frac{m_{\mathrm{adj}}}{2}\overline{\Psi}\Psi \right) + i\sum_{\alpha=1}^{N_f} \left(\overline{q}_\alpha \not{D} q_\alpha - m_{\mathrm{fund}}\overline{q}_\alpha q_\alpha \right) \right]$$

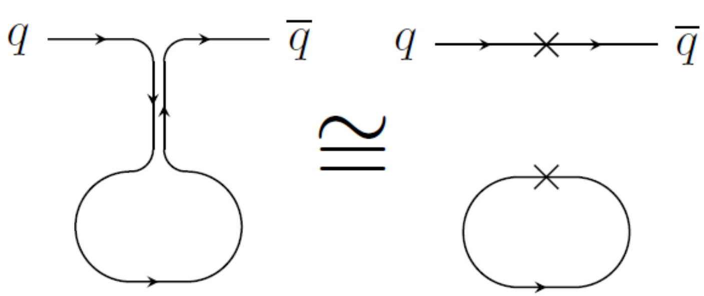
- The mesons are more complicated than in the 't Hooft model, since they also contain the adjoint quanta. There are now multiple Regge trajectories of mesons, which can be bosonic or fermionic.

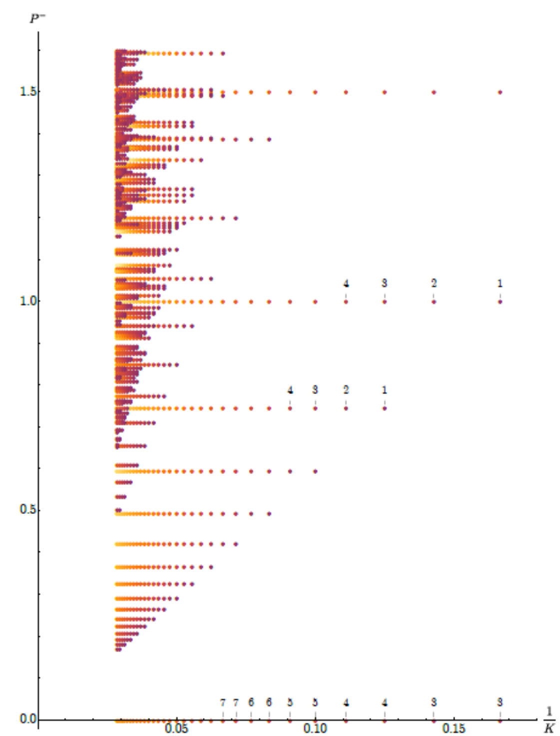


- The large N meson light-cone wave functions

$$|\{g\}_{\alpha\beta}; P^+\rangle = \frac{(P^+)^{(n-1)/2}}{N^{(n-1)/2}} \sum_n \int_0^1 dx_1 \cdots dx_n \, \delta\left(\sum_{i=1}^n x_i - 1\right)$$
$$\times\, g_n(x_1, \ldots, x_n) c_\alpha^\dagger(k_1) b^\dagger(k_2) \cdots b^\dagger(k_{n-1}) d_\beta^\dagger(k_n) \, |0\rangle$$

- The mesons exhibit interesting patterns of DLCQ degeneracies.





Symmetry of DLCQ at Large N

- Supergroup $\mathfrak{osp}(1|4)$ generated by

$$q^L_\pm = \frac{1}{\sqrt{2N}} \sum_{n_1,n_2} \Big(C^\dagger_j(n_1+n_2\pm 1) B_{ij}(n_1) C_i(n_2) + C^\dagger_j(n_1) B^\dagger_{ji}(n_2) C_i(n_1+n_2\mp 1) \Big) \,,$$

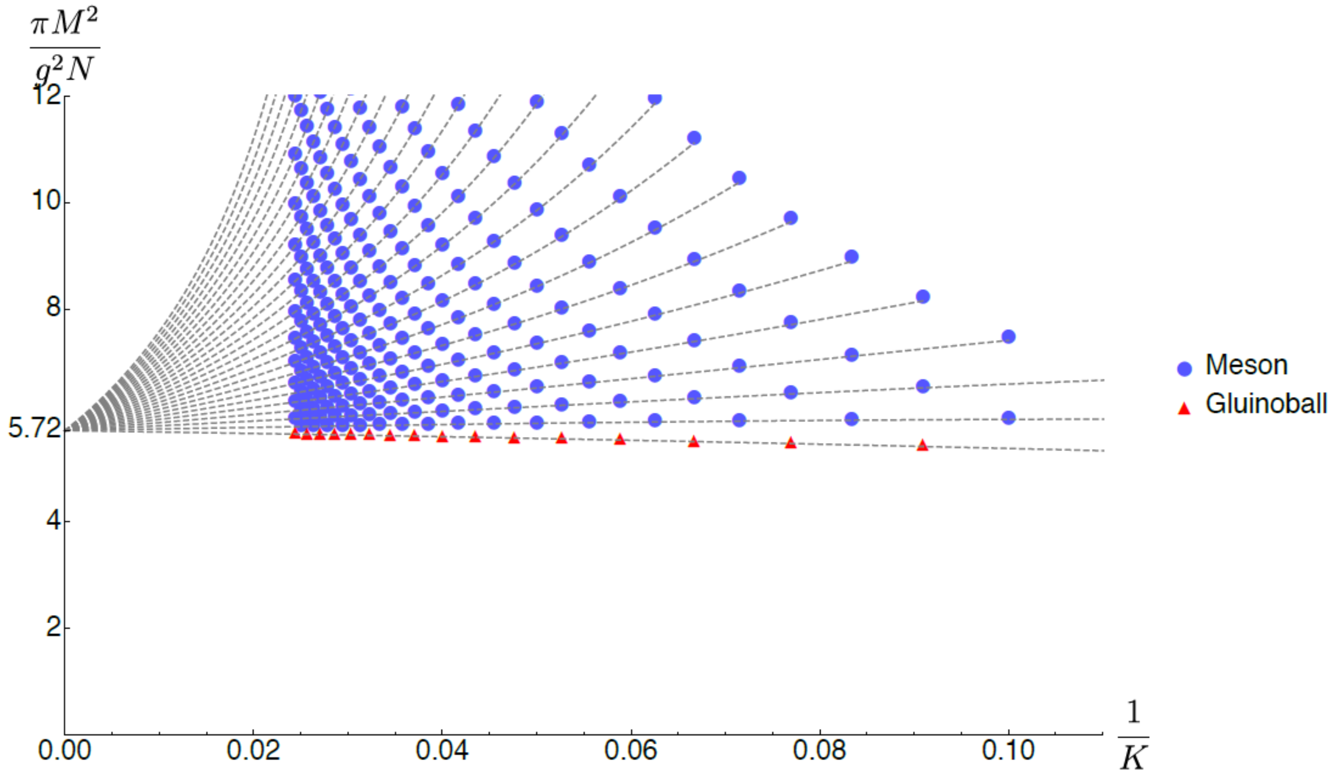
$$q^R_\pm = \frac{1}{\sqrt{2N}} \sum_{n_1,n_2} \Big(D^\dagger_i(n_1+n_2\pm 1) B_{ij}(n_1) D_j(n_2) + D^\dagger_i(n_1) B^\dagger_{ji}(n_2) D_j(n_1+n_2\mp 1) \Big)$$

- Commute with P^- but shift K by one unit:

$$[P^+, q^L_\pm] = \pm \frac{1}{2L} q^L_\pm \,, \qquad [P^+, q^R_\pm] = \pm \frac{1}{2L} q^R_\pm$$

$$[q^L_-, q^L_+] = [q^R_-, q^R_+] = \frac{1}{2}$$

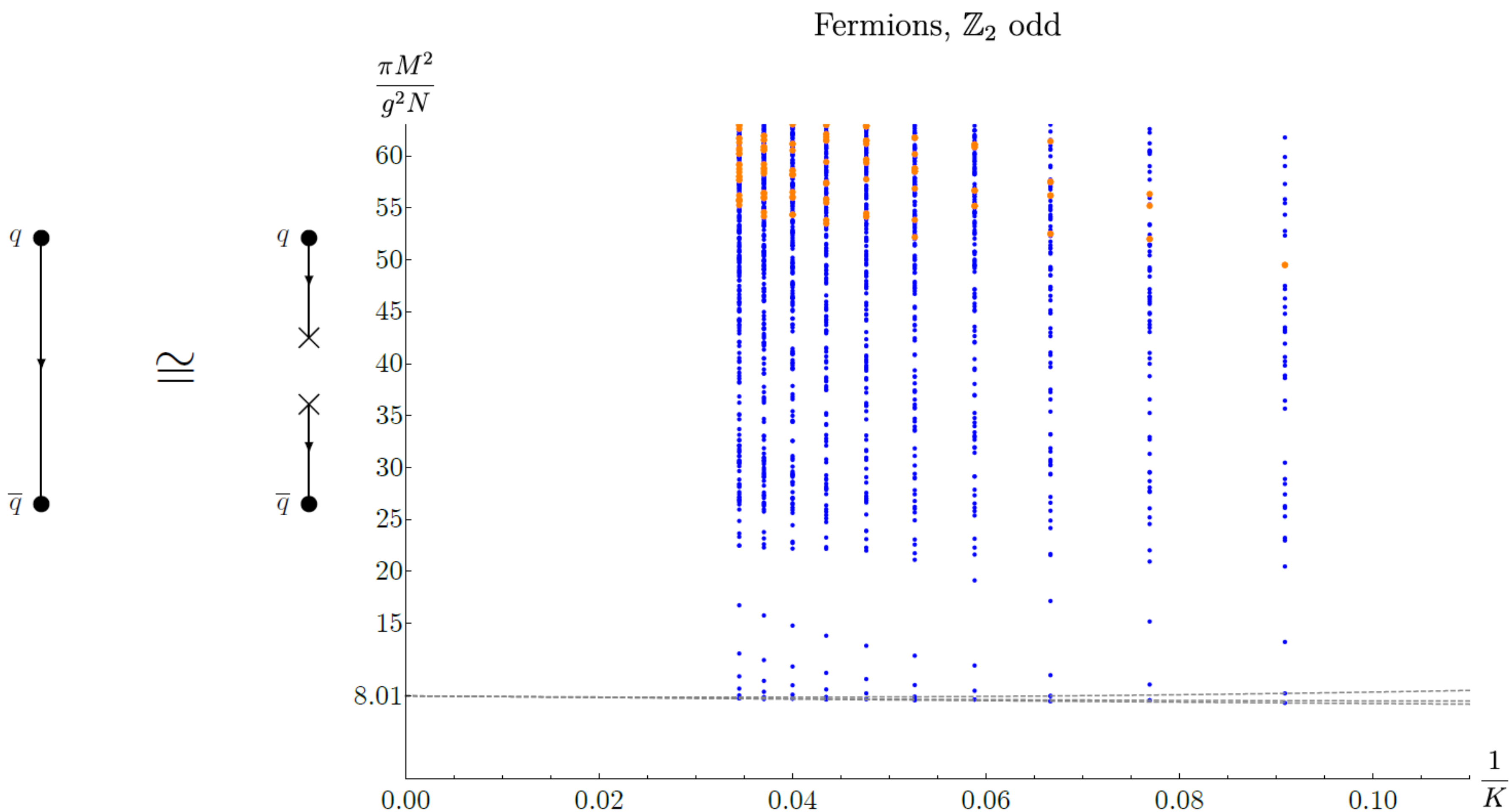
- In the massive meson spectrum, we see threshold states of massless mesons and massive gluinoballs:



No Confinement

- The WZW effective action depends only on the Kac-Moody level. It is the same in the SU(N) theory coupled to a massless Majorana adjoint as in the theory coupled to N fundamental Dirac fermions: $k_{\mathrm{KM}} = N$
- This implies that the massive spectra are the same in these two models. Kutasov, Schwimmer
- This was also used as an argument for screening: it is obvious in the second model. Gross, IRK, Matytsin, Smilga

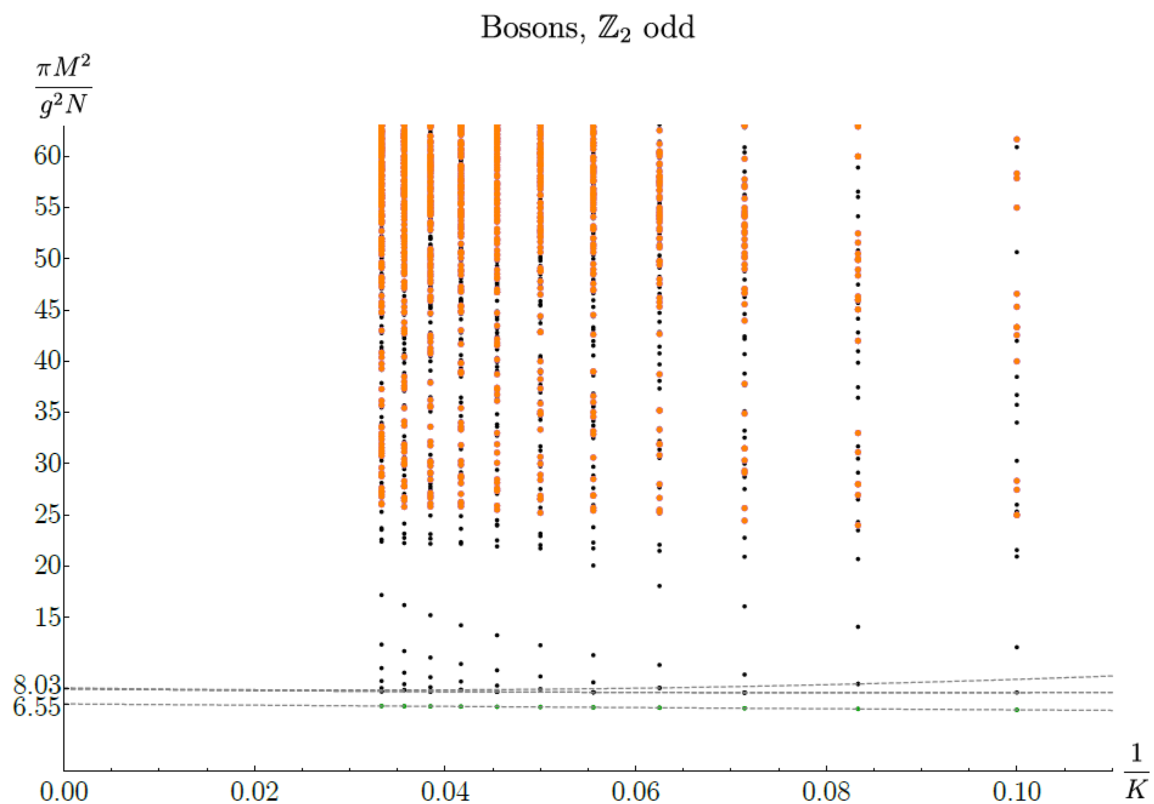
- This argument can be made quantitative by coupling both models to a massive quark.
- Some meson states in theory T (massive quark plus adjoints) are seen to “fall apart” into states in theory T’ (massive quark plus N massless ones).
- For fermionic mesons, the degeneracies are exact with pairs of states in theory T’ containing just one massive quark (the heavy-light states). Such mesons are shown with blue dots.



Fermions, $\mathbb{Z}_2$ odd
$\frac{\pi M^2}{g^2 N}$
$\frac{1}{K}$
q
$\overline{q}$
$\cong$
8.01

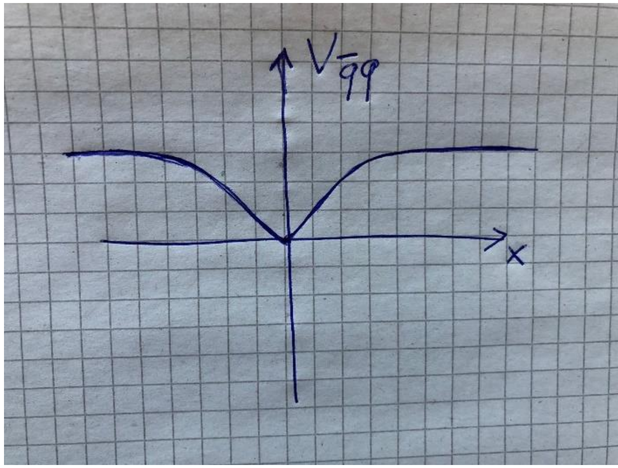
Bosonic Mesons

- Here the degeneracies are not exact, but appear in the continuum limit.
- There is also a bound state clearly seen below the threshold.



- The bound state indicates that the quark anti-quark potential has some attraction at short distances before it flattens at long distances
- The massless adjoints renormalize the string tension to zero.
- The non-vanishing of the short-distance q-qbar potential for N>2 can be seen in the Hamiltonian approach to the SU(N) gauge theory on a small spatial circle.

Smilga; Lenz, Shifman, Thies; Cherman, Jacobson, Tanizaki, Unsal



Conclusions

- Throughout its history, string theory has been intertwined with the theory of strong interactions.
- The Anti-de Sitter/Conformal Field Theory correspondence makes this connection precise. It makes many dynamical statements about strongly coupled conformal gauge theories.
- Extensions of AdS/CFT provide a new geometrical understanding of color confinement and other strong coupling phenomena.
- 2D gauge theories with adjoint matter are other interesting laboratories. They lead to appearance of threshold bound states in some limits.
- DLCQ gives a quantitative handle on the spectrum and demonstrates that the theory with massless adjoints is not confining.