Unfolding Conformal Geometry

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Weyl Invariant Densities

- Type-B Anomaly: Weyl invariant densities [Bonora et al, Deser&Schwimmer, … , Boulanger]
- 8d Classification [Boulanger&Erdmenger]
  - Basis: Weyl-covariant derivatives of Weyl tensors
  - Weyl variation of each basis: ~ $\partial\sigma$
  - Make an ansatz in this basis / Ask the Weyl invariance
- Revisit the problem by unfolding conformal geometry [Vasiliev,…]

SO(2,d) Gauge Formulation

[Kaku&Townsend&Nieuwenhuizen, … , Fradkin&Tseytlin, … , Kuzenko&Ponds]

- Conformal Geometry, Q-curvature, Tractor calculus

  [Thomas, Fefferman, Graham, Gover, …]

- Cartan Geometry (Parabolic Geometry) [Cartan, … , Sharpe, …]

Gauge Field
$$\hat{A} = e^a\,\hat{P}_a + \frac{1}{2}\,\omega^{ab}\,\hat{J}_{ab} + f^a\,\hat{K}_a + b\,\hat{D}$$

Curvature
$$F^a_{\hat{P}} = (\mathrm{D}^L + b)\,e^a\,, \qquad F^{ab}_{\hat{J}} = R^{ab} - 2\,f^{[a} \wedge e^{b]}\,,$$
$$F^a_{\hat{K}} = (\mathrm{D}^L - b)\,f^a\,, \qquad F_{\hat{D}} = \mathrm{d}b + f^a \wedge e_a\,.$$

Constraints $F^a_{\hat P} \stackrel{!}{=} 0\,, \qquad i_a\, F^{ab}_{\hat J} \stackrel{!}{=} 0\,, \qquad (F_{\hat D} \stackrel{!}{=} 0)$

SOLVE

$$\omega_{ab,c} = E^\mu_{[b}\, E^\nu_{c]}\, \partial_\mu\, e_{a\nu} + E^\mu_{[c}\, E^\nu_{a]}\, \partial_\mu\, e_{b\nu} + E^\mu_{[b}\, E^\nu_{a]}\, \partial_\mu\, e_{c\nu} + 2\, b_{[a}\, \eta_{b]c}$$

$$f_{[a,b]} = \partial_{[a}\, b_{b]} \qquad f_{(a,b)} = \frac{1}{d-2}\left(R_{ab} - \frac{\eta_{ab}\, R}{2\,(d-1)}\right)$$

Schouten tensor

Gauge symmetry $\delta \hat A = \mathrm{d}\hat\Lambda + [\hat A, \hat\Lambda]\,, \quad \hat\Lambda = \epsilon^a\, \hat P_a + \frac{1}{2}\lambda^{ab}\, \hat J_{ab} + \kappa^a\, \hat K_a + \sigma\, \hat D$

FIX

$$\delta_\kappa\, b_a = \kappa_a \qquad \Rightarrow \qquad b_a = 0$$

$$\delta_\lambda e^a_\mu = \lambda^{ab}\, e_{b\mu} \qquad \qquad g_{\mu\nu} = \eta_{ab}\, e^a_\mu\, e^b_\nu$$

Under dilatation

$$\kappa_a = \partial_a \sigma$$

We recover Conformal Geometry based on metric tensor

What we have

$$\mathrm{D}^K e^a = 0\,,$$
$$\mathrm{D}^K \omega^{ab} - 2\,e^{[a} \wedge f^{b]} = \frac{1}{2}\,e_c \wedge e_d\,C^{ab,cd}\,,$$
$$\mathrm{D}^K b + e^a \wedge f_a = 0\,,$$
$$\mathrm{D}^K f^a = \frac{1}{2}\,e_b \wedge e_c\,C^{a,bc}\,,$$

$$C_{ab,cd} \sim \begin{array}{|c|c|}\hline a & c \\ \hline b & d \\ \hline\end{array}\,, \qquad C_{c,ab} \sim \begin{array}{|c|c|}\hline a & c \\ \hline b & \\ \cline{1-1}\end{array}\,.$$

$$\eta^{ab}\,C_{ab,cd} = 0\,, \qquad \eta^{ab}\,C_{a,bc} = 0\,.$$

Weyl tensor **Cotton tensor**

K = SO(1,1) x SO(1,d-1) covariant derivatives

$$\mathrm{D}^K W^{[\Delta]ab\cdots} = \mathrm{D}^L W^{[\Delta]ab\cdots} + (|W| - \Delta)\,b\,W^{[\Delta]ab\cdots}$$

$$\Delta_e = 0,\;\; \Delta_\omega = 1,\;\; \Delta_b = 1,\;\; \Delta_f = 2$$

$|W|$: differential form degree

Unfolding

[Vasiliev, …, Shaynkman&Tipunin&Vasiliev, …]

- Consider $C_{ab,cd}, C_{a,bc}$ as new fields (zero forms)

- Introduce "evolution" equation for $C_{ab,cd}, C_{a,bc}$

$$
\begin{aligned}
\mathrm{D}^K C^{ab,cd} &= (\mathrm{D}^L - 2\,b) C^{ab,cd} = e_f\, \boxed{C^{ab,cd,e}} + (\text{pre-existing fields})\,,\\
\mathrm{D}^K C^{a,bc} &= (\mathrm{D}^L - 3\,b) C^{a,bc} = e_d\, \boxed{C^{a,bc,d}} + (\text{pre-existing fields})\,.
\end{aligned}
$$

New zero form fields

Rule of the game

- New fields are completely determined by pre-existing fields
- No constraints on pre-existing fields
- Bianchi identity

$$
\begin{aligned}
&(\mathrm{D}^L - 2\,b)_{[k}\, C^{ab,}{}_{cd)} - 2\,\delta^{[a}_{[k}\, C^{b],}{}_{cd)} = 0\,,\\
&(\mathrm{D}^L - 3\,b)_{[k} C^{a,}{}_{cd)} - \; f_{b,[k}\, C^{ab,}{}_{cd)} = 0\,.
\end{aligned}
$$

- Infinitely many new zero forms

$$C^{[\Delta]a(m),b(n)} = C^{[\Delta]a_1\cdots a_m,b_1\cdots b_n}$$

$$C^{[2]a(2),b(2)} = C^{(a_1|b_1,|a_2)b_2}\,, \qquad C^{[3]a(2),b} = C^{(a_1,a_2)b}$$

- General form of the equations

$$\mathrm{D}^K C^{[\Delta]a(m),b(n)} = e_c \underbrace{\boxed{\mathcal{E}^{[\Delta+1]a(m),b(n),c}(C)}}_{\text{Total degree } \Delta+1} + f_c \underbrace{\boxed{\mathcal{F}^{[\Delta-1]a(m),b(n),c}(C)}}_{\text{Total degree } \Delta-1}$$

Polynomials of zero forms

$$\boxed{\begin{aligned}\mathcal{E}^{[\Delta+1]a(m),b(n),c}(C) &= \mathcal{E}^{[\Delta+1]a(m),b(n),c}_{d(p),e(q)}\,C^{[\Delta+1]d(p),e(q)} \\ &+ \sum_{\substack{\Delta_1,\Delta_2 \\ \Delta_1+\Delta_2=\Delta+1}} \mathcal{E}^{[\Delta_1,\Delta_2]a(m),b(n),c}_{d(p),e(q)|f(s),g(t)}\,C^{[\Delta_1]d(p),e(q)}\,C^{[\Delta_2]f(s),g(t)} + \cdots\end{aligned}}$$

First order

$$(\hat{P}^c\, C)^{[\Delta]a(m),b(n)} = -\mathcal{E}^{[\Delta+1]a(m),b(n),c}_{d(p),e(q)}\, C^{[\Delta+1]d(p),e(q)}$$

$$\boxed{\mathrm{D}^K C^{[\Delta]a(m),b(n)} + e^c\,(\hat{P}_c\, C)^{[\Delta]a(m),b(n)} + f^c\,(\hat{K}_c\, C)^{[\Delta]a(m),b(n)} = \mathcal{O}(C^2)}$$

- $\hat{P}_a$ and $\hat{K}_a$ map $C^{[\Delta+1]}$ and $C^{[\Delta-1]}$ to $C^{[\Delta]}$

- Bianchi identity identifies $\hat{P}_a$ and $\hat{K}_a$ with *SO*(2,d) generators

From Bianchi identity

$$(\hat{P}_{[c}\, \hat{P}_{d]}\, C)^{[\Delta]a(m),b(n)} = 0\,, \qquad (\hat{K}_{[c}\, \hat{K}_{d]}\, C)^{[\Delta]a(m),b(n)} = 0\,,$$

$$\Big(([\hat{K}_a, \hat{P}_b] + \hat{J}_{ab} - \eta_{ab}\, \hat{D})\, C\Big)^{[\Delta]a(m),b(n)} = 0\,,$$

$$\hat{P}^a\, C^{[\Delta](m,n)} = \left(\boxed{p^{\Delta,m,n}_{1+}}\mathcal{Y}^a_{1+} + \boxed{p^{\Delta,m,n}_{1-}}\mathcal{Y}^a_{1-} + \boxed{p^{\Delta,m,n}_{2+}}\mathcal{Y}^a_{2+} + \boxed{p^{\Delta,m,n}_{2-}}\mathcal{Y}^a_{2-}\right) C^{[\Delta](m,n)}$$

$$\hat{K}^a\, C^{[\Delta](m,n)} = \left(\boxed{k^{\Delta,m,n}_{1+}}\mathcal{Y}^a_{1+} + \boxed{k^{\Delta,m,n}_{1-}}\mathcal{Y}^a_{1-} + \boxed{k^{\Delta,m,n}_{2+}}\mathcal{Y}^a_{2+} + \boxed{k^{\Delta,m,n}_{2-}}\mathcal{Y}^a_{2-}\right) C^{[\Delta](m,n)}$$

Coefficients to determine from Bianchi

Cell Operators

[Ponomarev&Vasiliev]

$$\mathcal{Y}^a_{1+} = \Pi_{\mathbb{Y}}\, u^a\,, \qquad \mathcal{Y}^a_{1-} = \Pi_{\mathbb{Y}}\, \partial_{u_a}\,, \qquad \mathcal{Y}^a_{2+} = \Pi_{\mathbb{Y}}\, v^a\,, \qquad \mathcal{Y}^a_{2-} = \Pi_{\mathbb{Y}}\, \partial_{v_a}\,,$$

$$\mathcal{Y}^{a\ b}_{1+1+} = \Pi_{\mathbb{Y}}\, u^a u^b\,, \qquad \mathcal{Y}^{a\ b}_{1+2+} = \Pi_{\mathbb{Y}}\, u^a v^b\,, \qquad \mathcal{Y}^{a\ b}_{2+2+} = \Pi_{\mathbb{Y}}\, v^a v^b\,,$$

$$\mathcal{Y}^{a\ b}_{1-1-} = \Pi_{\mathbb{Y}}\, \partial_{u_a}\partial_{u_b}\,, \qquad \mathcal{Y}^{a\ b}_{1-2-} = \Pi_{\mathbb{Y}}\, \partial_{u_a}\partial_{v_b}\,, \qquad \mathcal{Y}^{a\ b}_{2-2-} = \Pi_{\mathbb{Y}}\, \partial_{v_a}\partial_{v_b}\,,$$

Algebra of Cell Operators

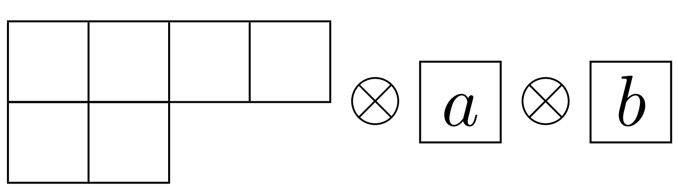
$$\mathcal{Y}^a_{1\pm}\,\mathcal{Y}^b_{1\pm} = \mathcal{Y}^b_{1\pm}\,\mathcal{Y}^a_{1\pm} = \mathcal{Y}^{a\ b}_{1\pm1\pm}\,, \qquad \mathcal{Y}^a_{2\pm}\,\mathcal{Y}^b_{2\pm} = \mathcal{Y}^b_{2\pm}\,\mathcal{Y}^a_{2\pm} = \mathcal{Y}^{a\ b}_{2\pm2\pm}\,,$$

$$\begin{aligned}
\mathcal{Y}^a_{1\pm}\,\mathcal{Y}^b_{2\pm} &= \mathcal{Y}^{a\ b}_{1\pm2\pm}\,, &\qquad \mathcal{Y}^b_{2\pm}\,\mathcal{Y}^a_{1\pm} &= \mathcal{Y}^{a\ b}_{1\pm2\pm} \mp \tfrac{1}{m-n+1}\,\mathcal{Y}^{b\ a}_{1\pm2\pm}\,,\\
\mathcal{Y}^a_{1+}\,\mathcal{Y}^b_{2-} &= \mathcal{Y}^{a\ b}_{1+2-}\,, &\qquad \mathcal{Y}^b_{2-}\,\mathcal{Y}^a_{1+} &= \mathcal{Y}^{a\ b}_{1+2-} - \tfrac{1}{d+m+n-3}\,\mathcal{Y}^{b\ a}_{1+2-}\,,\\
\mathcal{Y}^a_{2+}\,\mathcal{Y}^b_{1-} &= \mathcal{Y}^{a\ b}_{2+1-}\,, &\qquad \mathcal{Y}^b_{1-}\,\mathcal{Y}^a_{2+} &= \tfrac{(m-n-1)(m-n+1)}{(m-n)^2}\left(-\mathcal{Y}^{a\ b}_{2+1-} + \tfrac{1}{d+m+n-3}\,\mathcal{Y}^{b\ a}_{2+1-}\right)
\end{aligned}$$

$$\mathcal{Y}^a_{1+}\,\mathcal{Y}^b_{1-} = \mathcal{Y}^{a\ b}_{1+1-}\,, \qquad \mathcal{Y}^a_{2+}\,\mathcal{Y}^b_{2-} = \mathcal{Y}^{a\ b}_{2+2-}\,,$$

$$\begin{aligned}
\mathcal{Y}^a_{1-}\,\mathcal{Y}^b_{1+} = \eta^{ab} &+ \mathcal{Y}^{b\ a}_{1+1-} - \tfrac{2}{d+2m-2}\,\mathcal{Y}^{a\ b}_{1+1-}\\
&- \tfrac{m-n+3}{(m-n+2)(d+m+n-3)}\,\mathcal{Y}^{a\ b}_{2+2-} + \tfrac{1}{m-n+2}\,\mathcal{Y}^{b\ a}_{2+2-}\,,\\
\mathcal{Y}^a_{2-}\,\mathcal{Y}^b_{2+} = \eta^{ab} &+ \mathcal{Y}^{b\ a}_{2+2-} - \tfrac{2}{d+2n-4}\,\mathcal{Y}^{a\ b}_{2+2-}\\
&- \tfrac{m-n-1}{(m-n)(d+m+n-3)}\,\mathcal{Y}^{a\ b}_{1+1-} - \tfrac{1}{m-n}\,\mathcal{Y}^{b\ a}_{1+1-}\,,
\end{aligned}$$

$$\hat{J}^{ab} = \mathcal{Y}^{a\ b}_{1+1-} - \mathcal{Y}^{b\ a}_{1+1-} + \mathcal{Y}^{a\ b}_{2+2-} - \mathcal{Y}^{b\ a}_{2+2-}$$



$$\hat{P}^a\, C^{[\Delta](m,n)} = \left(\boxed{p_{1+}^{\Delta,m,n}}\mathcal{Y}_{1+}^a + \boxed{p_{1-}^{\Delta,m,n}}\mathcal{Y}_{1-}^a + \boxed{p_{2+}^{\Delta,m,n}}\mathcal{Y}_{2+}^a + \boxed{p_{2-}^{\Delta,m,n}}\mathcal{Y}_{2-}^a\right) C^{[\Delta](m,n)}$$

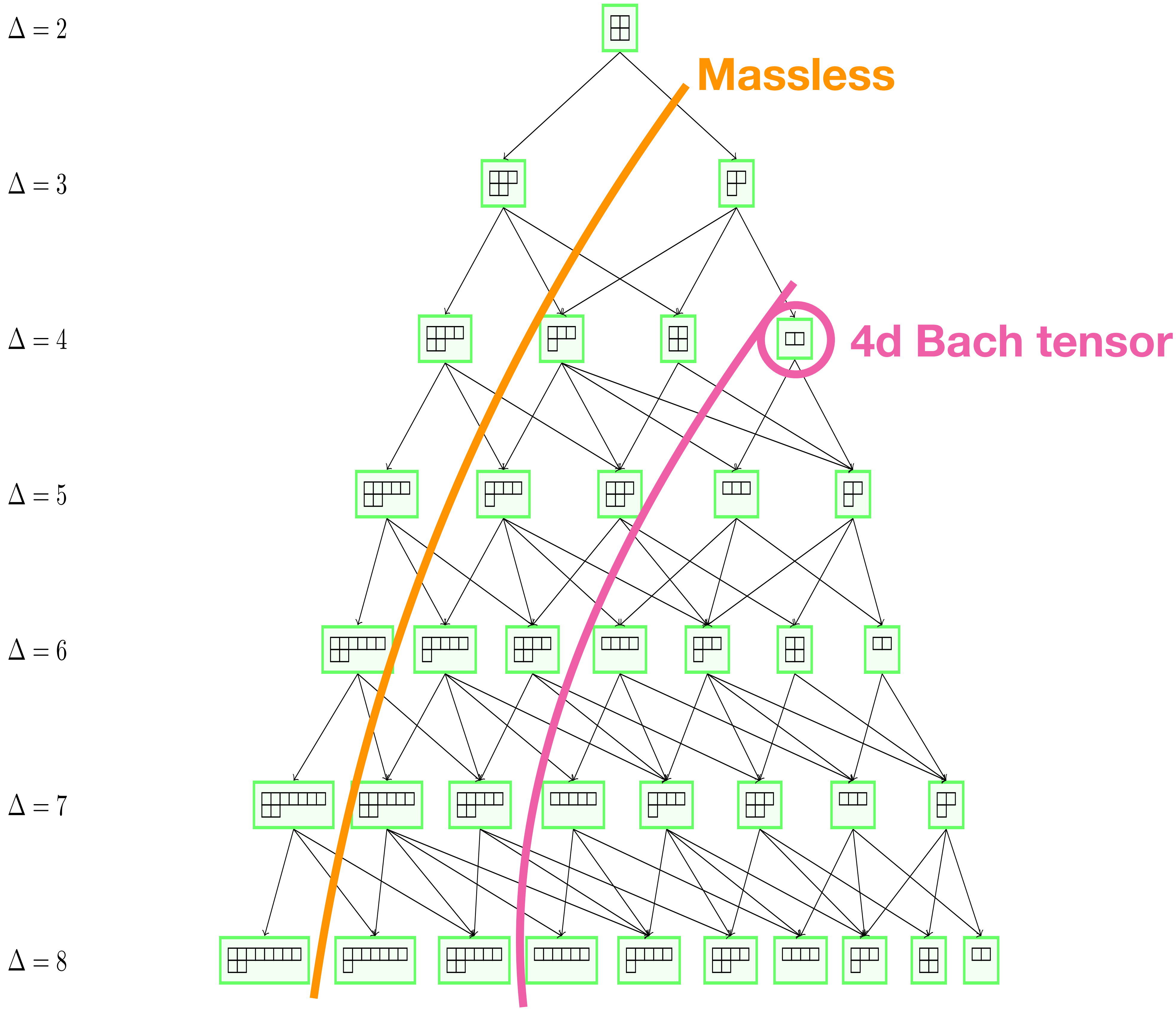
$$\hat{K}^a\, C^{[\Delta](m,n)} = \left(\boxed{k_{1+}^{\Delta,m,n}}\mathcal{Y}_{1+}^a + \boxed{k_{1-}^{\Delta,m,n}}\mathcal{Y}_{1-}^a + \boxed{k_{2+}^{\Delta,m,n}}\mathcal{Y}_{2+}^a + \boxed{k_{2-}^{\Delta,m,n}}\mathcal{Y}_{2-}^a\right) C^{[\Delta](m,n)}$$

$$[\hat{P}_a, \hat{P}_b] = 0 \quad \Rightarrow \quad p\,p + p\,p = 0$$

$$[\hat{K}_a, \hat{K}_b] = 0 \quad \Rightarrow \quad k\,k + k\,k = 0$$

$$[\hat{K}_a, \hat{P}_b] = \eta_{ab}\,\hat{D} - \hat{J}_{ab} \quad \Rightarrow \quad p\,k + k\,p = \#$$

- Unique off-shell system up to field redefinition ambiguity
- Various on-shell systems
    - Conformal gravity (Bach flat geomtry)
    - Einstein gravity
    - Higher-depth analogues



$\Delta = 2$
$\Delta = 3$
$\Delta = 4$
$\Delta = 5$
$\Delta = 6$
$\Delta = 7$
$\Delta = 8$
Massless
4d Bach tensor

Representation

- On-shell Fradkin-Tseytlin module [Shaynkman&Tipunin&Vasiliev, Beccaria&Tseytlin&Bekaert]

$$\mathcal{D}(2,(s,s)) = \mathcal{S}(2-s,(s)) \ominus \mathcal{D}(s+d-2,(s))$$

Bach flat equation

- Off-shell Fradkin-Tseytlin module (shadow module)

$$\mathcal{S}(0,(2)) = \mathcal{V}(0,(2)) \ominus \mathcal{V}(-1,(1)) \oplus \mathcal{D}(-1,(1))$$

Metric Gauge Conformal Killing

$$\boxed{= \bigoplus_{m,n=0}^{\infty} [(2+n+2m,(n+2,2)) \oplus (3+n+2m,(n+2,1)) \oplus (4+n+2m,(n+2))]}$$

Match what we found!

Higher order (sketch)

Nonlinear equation

$$\mathrm{D}^K C^I = e_a\, \mathcal{E}^{I,a}(C) + f_a\, \mathcal{F}^{I,a}(C)$$

Bianchi identity

$$\begin{aligned}
0 = e_a \wedge e_b &\left(\frac{1}{4} C^{[2]ac,bd} (\hat{J}_{cd}\, C)^I - \frac{1}{2} C^{[3]ca,b} \mathcal{F}^I{}_c(C) + \mathcal{E}^{J,b}(C) \frac{\partial \mathcal{E}^{I,a}}{\partial C^J} \right) \\
&+ e_a \wedge f_b \left((\hat{J}^{ab}\, C)^I - \Delta_I\, \eta^{ab}\, C^I + \mathcal{F}^{J,b}(C) \frac{\partial \mathcal{E}^{I,a}}{\partial C^J} - \mathcal{E}^{J,a}(C) \frac{\partial \mathcal{F}^{I,b}}{\partial C^J} \right) \\
&+ f_a \wedge f_b\, \mathcal{F}^{J,b}(C) \frac{\partial \mathcal{F}^{I,a}}{\partial C^J} \,.
\end{aligned}$$

Polynomial Expansions

$$\mathcal{E}^{[\Delta],a}(C) = \sum_{k=1}^{[\frac{\Delta+1}{2}]} \mathcal{E}_k^{[\Delta],a}(C)\,, \qquad \mathcal{F}^{[\Delta],a}(C) = \sum_{k=1}^{[\frac{\Delta-1}{2}]} \mathcal{F}_k^{[\Delta],a}(C)$$

$$\frac{1}{4}\, C^{[2]ac,bd}\,(\hat{J}_{cd}\, C)^{[2]} + \boldsymbol{\mathcal{E}_2^{[3],[a}(C^{[2]}, C^{[2]})}\, \frac{\partial \mathcal{E}_1^{[2],b]}(C^{[3]})}{\partial C^{[3]}} = 0$$

$$\frac{1}{4}\, C^{[2]ac,bd}\,(\hat{J}_{cd}\, C)^{[3]} - \frac{1}{2}\, C^{[3]}{}_c{}^{[a,b]}\, \mathcal{F}_1^{[3],c}(C^{[2]}) + \mathcal{E}_1^{[2],[a}(C^{[3]})\, \frac{\partial \mathcal{E}_2^{[3],b]}(C^{[2]}, C^{[2]})}{\partial C^{[2]}} + \boldsymbol{\mathcal{E}_2^{[4],[a}(C^{[2]}, C^{[3]})}\, \frac{\partial \mathcal{E}_1^{[3],b]}(C^{[4]})}{\partial C^{[4]}} = 0$$

$$\frac{1}{4}\, C^{[2]ac,bd}\,(\hat{J}_{cd}\, C)^{[4]} + \mathcal{E}_1^{[3],[a}(C^{[4]})\, \frac{\partial \mathcal{E}_2^{[4],b]}(C^{[2]}, C^{[3]})}{\partial C^{[3]}} + \boldsymbol{\mathcal{E}_2^{[5],[a}(C^{[2]}, C^{[4]})}\, \frac{\partial \mathcal{E}_1^{[4],b]}(C^{[5]})}{\partial C^{[5]}} = 0$$

$$-\frac{1}{2}\, C^{[3]}{}_c{}^{[a,b]}\, \mathcal{F}_1^{[4],c}(C^{[3]}) + \mathcal{E}_1^{[2],[a}(C^{[3]})\, \frac{\partial \mathcal{E}_2^{[4],b]}(C^{[2]}, C^{[3]})}{\partial C^{[2]}} + \boldsymbol{\mathcal{E}_2^{[5],[a}(C^{[3]}, C^{[3]})}\, \frac{\partial \mathcal{E}_1^{[4],b]}(C^{[5]})}{\partial C^{[5]}} = 0$$

$$\mathcal{F}_1^{[3],b}(C^{[2]})\, \frac{\partial \mathcal{E}_2^{[4],a}(C^{[2]}, C^{[3]})}{\partial C^{[3]}} - \mathcal{E}_2^{[3],b}(C^{[2]}, C^{[2]})\, \frac{\partial \mathcal{F}_1^{[4],a}(C^{[3]})}{\partial C^{[3]}} + \boldsymbol{\mathcal{F}_2^{[5],b}(C^{[2]}, C^{[2]})}\, \frac{\partial \mathcal{E}_1^{[4],a}(C^{[5]})}{\partial C^{[5]}} = 0$$

At each order, finite dimensional linear equations

Reduction

Impose algebraic constraints $\Phi(e, \omega, b, f, C) = 0$

- Einstein gravity $\Phi^a = f^a - \Lambda\, e_b$

- Other gravitational theories $\Phi^a = f^a - \ell^2\, C^{[4]ab}\, e_b$

  $\Phi^a = f^a - F(C)\, e_b$

- Conformal gravity $\Phi^{[d](2,0)} = C^{[d](2,0)} + \mathcal{O}(C^2)$

***K*-invariance of constraint**

$$\boxed{\delta_\kappa \Phi^{[d](2,0)}(C) = \kappa^a\, \mathcal{F}^{I,a}(C)\, \frac{\partial \Phi^{[d](2,0)}(C)}{\partial C^I} = 0}$$

Weyl invariants

- Ansatz for strictly Weyl invariant d-forms

$$I_d = \epsilon_{a_1\cdots a_d}\, e^{a_1} \wedge \cdots \wedge e^{a_d}\, \mathcal{I}_d(C)$$

- Gauge variation factorizes in even d

$$\delta I_d \;=\; \epsilon_{a_1\cdots a_d}\,(\varepsilon^{a_1}\, e^{a_2} \wedge \cdots \wedge e^{a_d} \wedge f_c + \kappa_c\, e^{a_1} \wedge \cdots \wedge e^{a_d})\, \mathcal{F}^{I,c}(C)\, \frac{\partial \mathcal{I}_d(C)}{\partial C^I}$$

***K*-invariance of ansatz** $\mathcal{I}_d(C) = \mathcal{I}_d(C^{[2]}, C^{[3]}, \ldots, C^{[d-2]})$

$$\boxed{\delta_\kappa\, \mathcal{I}_d(C) = \kappa^a\, \mathcal{F}^{I,a}(C)\, \frac{\partial \mathcal{I}_d(C)}{\partial C^I} = 0}$$

❖ Essential same as [Boulanger&Erdmenger]

4d $\mathcal{I}_4(C) = c_1\, C^{[2]a(2),b(2)}\, C^{[2]}{}_{a(2),b(2)}$

Δ=4 scalar : 1

6d $\mathcal{I}_6(C) = c_1\, C^{[4]a(2),b(2)}\, C^{[2]}{}_{a(2),b(2)} + c_2\, C^{[3]a(3),b(2)}\, C^{[3]}{}_{a(3),b(2)} + c_3\, C^{[3]a(2),b}\, C^{[3]}{}_{a(2),b}$

(non-trivial) Δ=6 scalars : 3

$$\begin{aligned}\delta_\kappa \mathcal{I}_6(C) &= c_1\,(\kappa_c\, C^{[3]a(2)c,b(2)} + \kappa^b\, C^{[3]a(2),b})\, C^{[2]}_{a(2),b(2)} \\ &\quad + 2\,c_2\,\kappa^a\, C^{[2]a(2),b(2)}\, C^{[3]}{}_{a(3),b(2)} + 2\,c_3\,\kappa_b\, C^{[2]a(2),b(2)}\, C^{[3]}{}_{a(2),b} \\ &= (c_1 + 2\,c_2)\,\kappa^a \boxed{C^{[2]a(2),b(2)}\, C^{[3]}{}_{a(3),b(2)}} + (c_1 + 2\,c_3)\,\kappa^a \boxed{C^{[2]a(2),b(2)}\, C^{[3]}{}_{b(2),a}}\end{aligned}$$

Δ=5 vector : 2

In 6d, there is 3-2=1 non-trivial Weyl invariant

8d

	CC	CCC
Δ=8 scalars	7	11
Δ=7 vectors	8 (6)	7

❖ Boulanger, Erdmenger found 5=1(CC+CCC)+4(CCC)

10d

	CC	CCC	CCCC
Δ=10 scalars	12	62	83
Δ=9 vectors	19 (11)	85	46

Nonlinear Action of SO(2,d)

- Nonlinear action $\hat{P}^a\, C^I = \mathcal{E}^{I,a}(C) \qquad \hat{K}^a\, C^I = \mathcal{F}^{I,a}(C)$

  LIE ALGEBROID

- SO(2,d) naturally acts on **Space of functions of C**
  - Space of C: Hilbert space
  - Space of functions of C: Fock space?
- Lowest states ➔ **Lowest functions**
  - Weyl invariant: Lowest scalar function
  - On-shell conformal gravity: Lowest tensor function

Future Plan

- Weyl invariants
- Add scalar field : Q-curvature
- Add conformal spin 3 field in 4d [Beccaria,Tseytlin,Grigoriev, Kuzenko,Ponds,…]
- Conformal higher spin gravity [Segal, Bekaert&EJ&Mourad, …]
- Nonlinear representation theory?!

Thank you for your attention!