

## Measurement of exclusive hadronic cross sections with the BABAR detector and implications on the $g-2$ of the muon

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**On behalf of BaBar Collaboration**



# Outline

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- **Motivation**
- **Theory of  $(g-2)_\mu/2$**
- **Collider, detector and method**
- **BaBar ISR measurements**
- **Conclusion**

# (g-2)/2 of muon (Experiment)

Magnetic moment of Dirac particle:

$$\vec{\mu} = g \frac{e\hbar}{2mc} \vec{S}$$

- Gyromagnetic factor  $g$  for
  - Point-like fermions:  $g = 2$
  - Higher order contributions (QFT):  $g \neq 2$
- Muon anomaly
  - $a_\mu = (g-2)_\mu/2$



E821 Experiment @ BNL (1997-2001):

J. Muller et al., Annu. Rev. Nucl. Par S. Vo. 62(2012), 237

$$a_\mu = (11\,659\,208.9 \pm 6.3) \cdot 10^{-10} \text{ (0.54 ppm)}$$

E989 Experiment @ FNAL (2017-...):

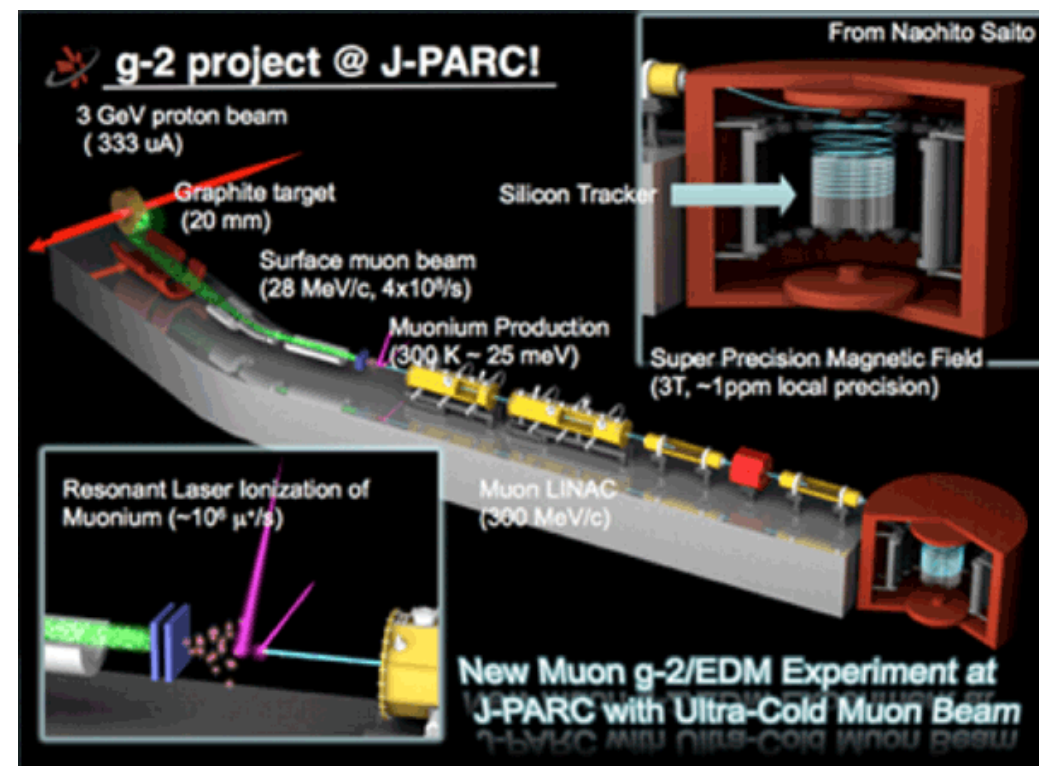
F. Gray et al., ArXiv 1510.003[physics.ins-det] (2015)

$$a_\mu = \dots \text{ (0.14 ppm)}$$

E34 Experiment @ J-PARC (????-...):

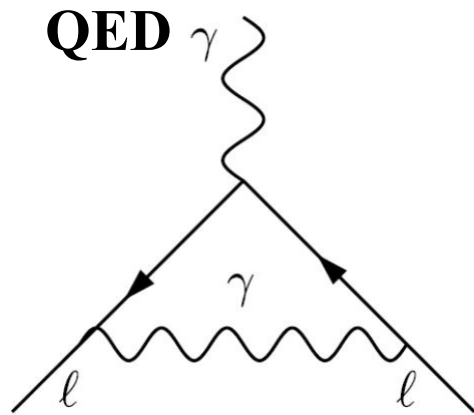
T. Mibe et al., Chin.Phys. C34 (2010) 745

$$a_\mu = \dots \text{ (0.1 ppm)}$$

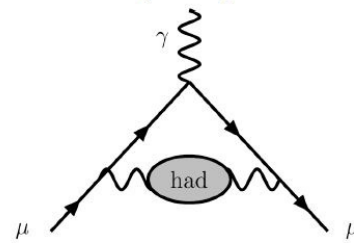




# (g-2)/2 of muon (Theory)

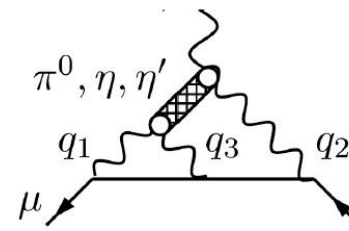


Hadronic Vacuum Polarisation (VP)

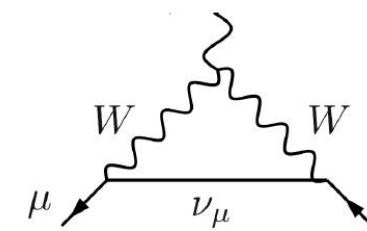


This talk

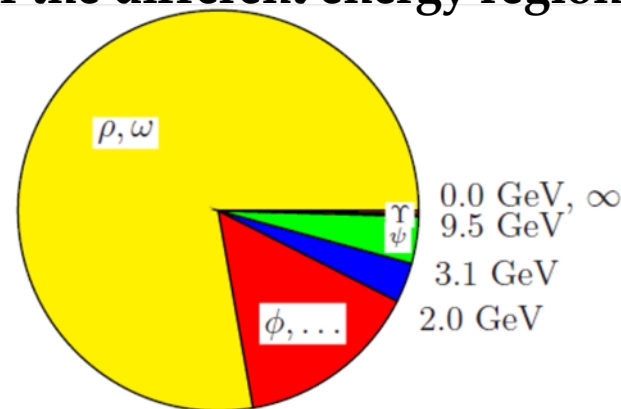
Hadronic light-by-light Scattering



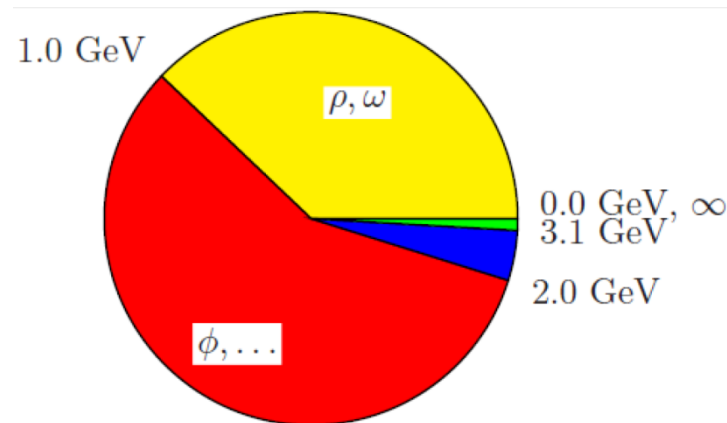
Weak Interactions



Contributions of the different energy regions to:



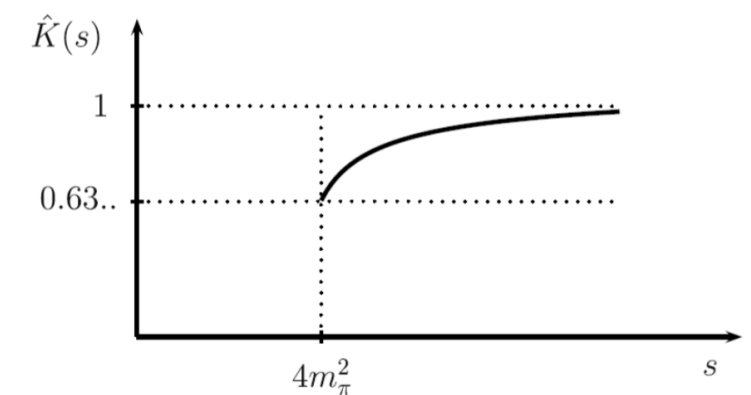
$a_\mu$  integral



$a_\mu$  uncertainty

$$R(s) = \frac{\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

$$a_\mu^{\text{had}} = \frac{\alpha^2}{3 \cdot \pi^2} \int_{4m_\pi^2}^{\infty} ds \cdot \frac{K(s)}{s} \cdot R(s)$$



$$a_\mu^{\text{theory(SM)}} = a_\mu^{\text{QED}} + a_\mu^{\text{had}} + a_\mu^{\text{weak}}$$

$$a_\mu^{\text{theo}} \times 10^{10} = 11\,659\,180.2 \pm 4.9$$

$$a_\mu^{\text{exp}} \times 10^{10} = 11\,659\,208.9 \pm 6.3$$

$$\Delta a_\mu \times 10^{10} = 28.7 \pm 8.0$$

$$a_\mu^{\text{EXP}} - a_\mu^{\text{SM}} = 3.6\sigma$$

(M. Davier et al., EPJC71(2011)1515)

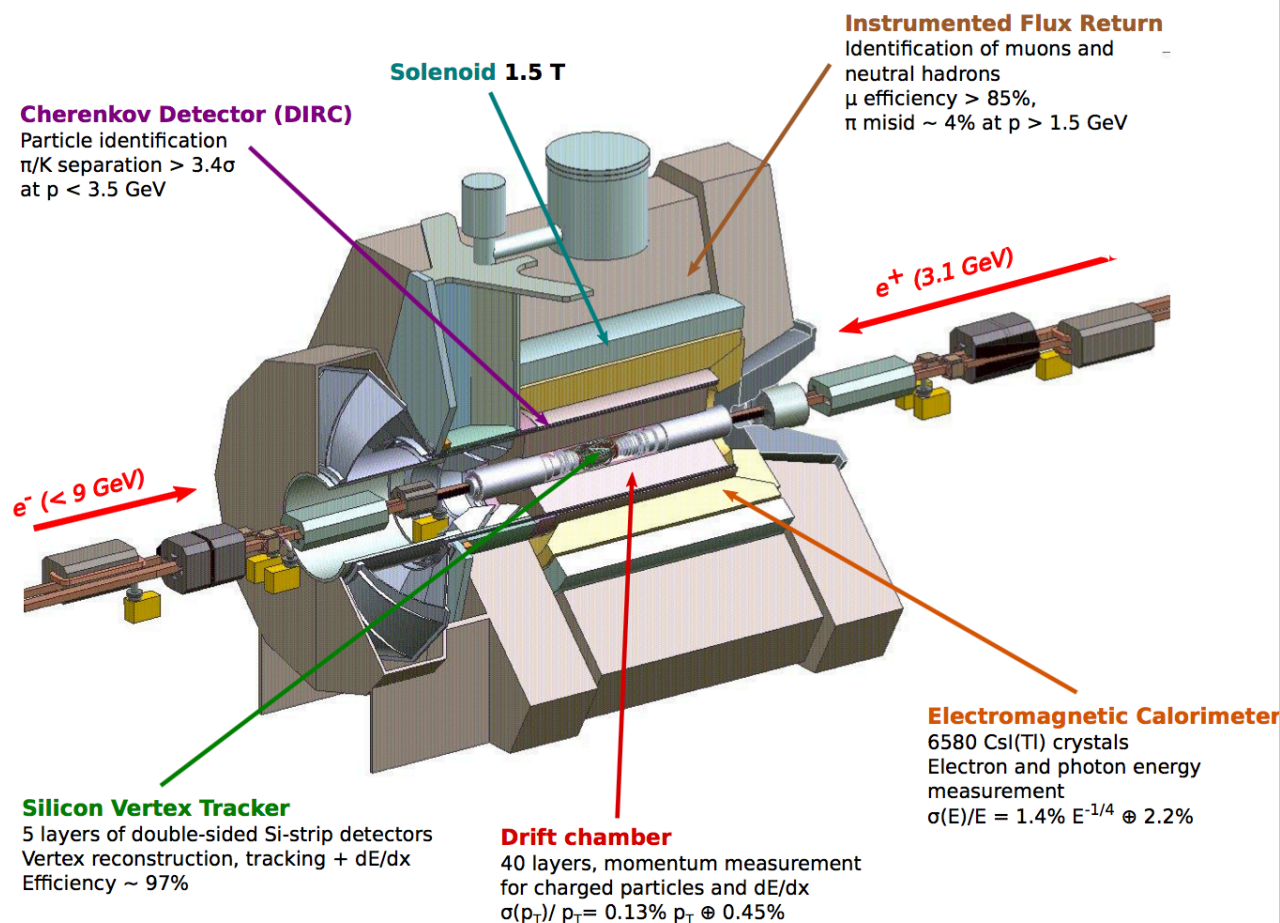


# ISR @ BaBar

## The *BABAR* experiment (1999-2008)

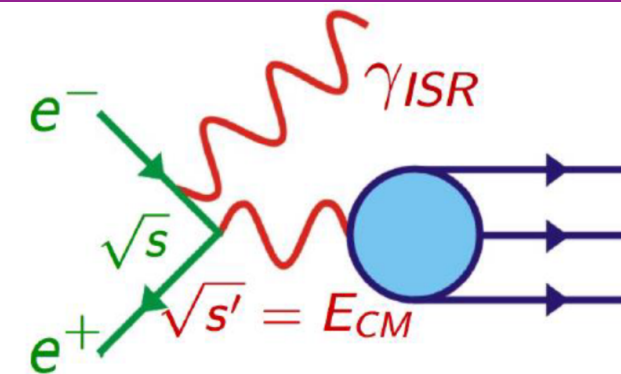
PEP-II:  $e^+e^-$  collider,  $3.1 \times 9 \text{ GeV}^2$

$\sqrt{s} = 10.58 \text{ GeV} [\Upsilon(4S)]$

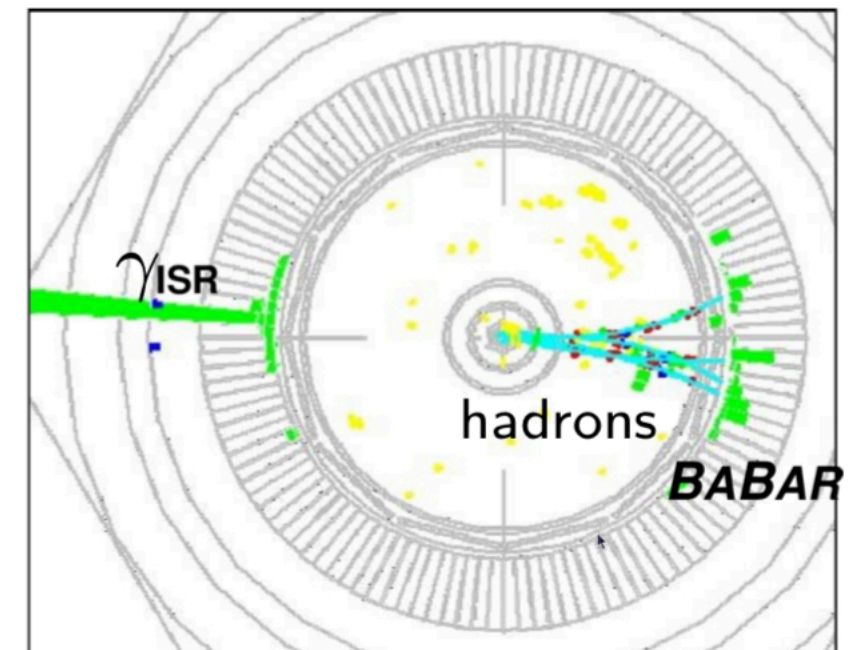


### ■ High luminosity

- ▶  $\mathcal{L}_{\text{peak}} = 12.069 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$
- ▶  $513.7(18) \text{ fb}^{-1}$  accumulated



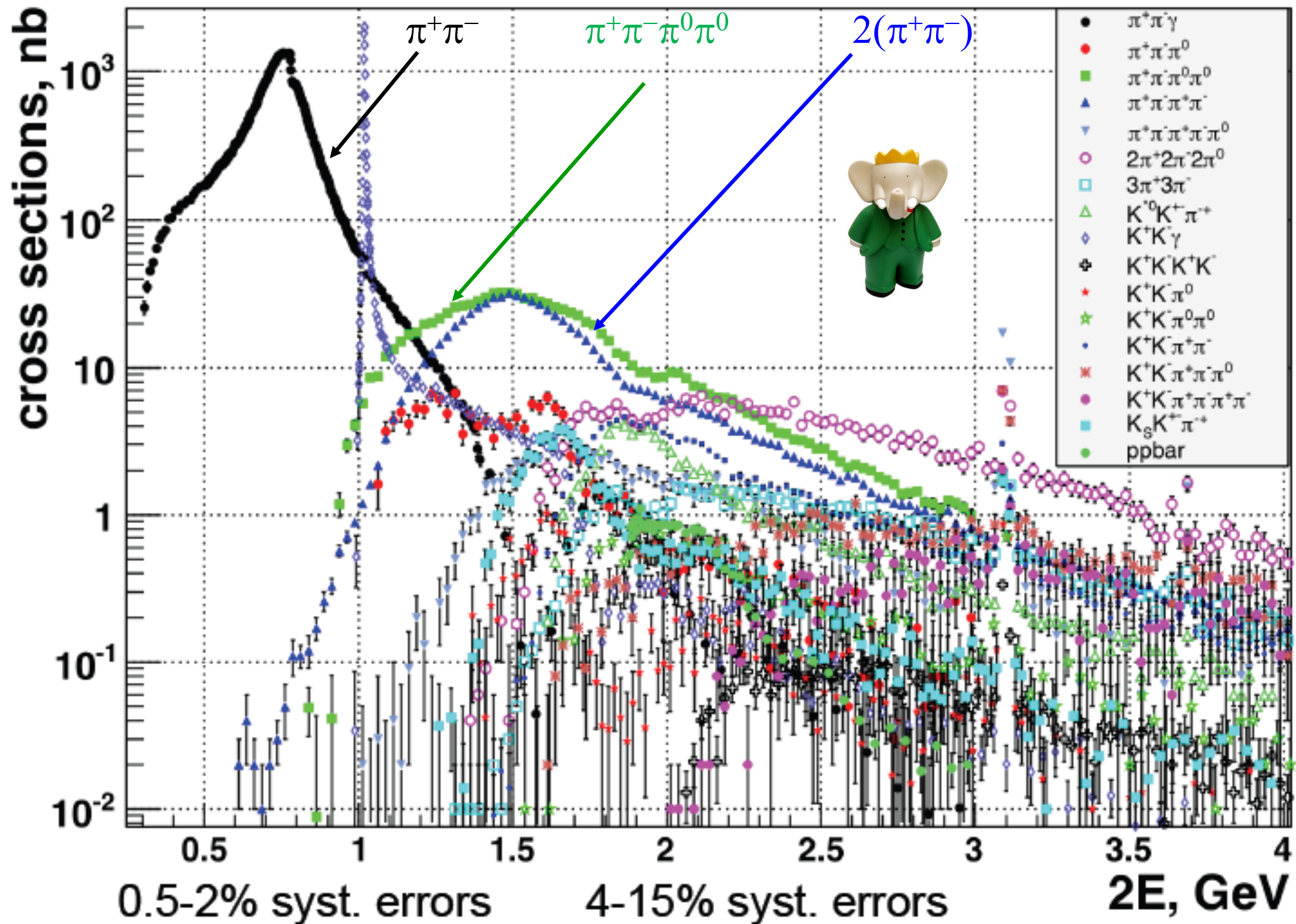
$$\frac{d\sigma[e^+e^- \rightarrow f\gamma]}{ds'}(s') = \frac{2m}{s} W(s, x) \sigma[e^+e^- \rightarrow f](s')$$



### ISR selection

- Detected high energy photon:  $E_\gamma > 3 \text{ GeV}$   
→ defines  $E_{CM}$  & provides strong background rejection
- Event topology:  $\gamma_{ISR}$  back-to-back to hadrons  
→ high acceptance
- Kinematic fit including  $\gamma_{ISR}$   
→ very good energy resolution (4 – 15 MeV)
- Continuous measurement from threshold to  $\sim 5 \text{ GeV}$   
→ provides common, consistent systematic uncertainties

# ISR @ BaBar



$$e^+e^- \rightarrow \pi^+\pi^-(\gamma)\gamma$$

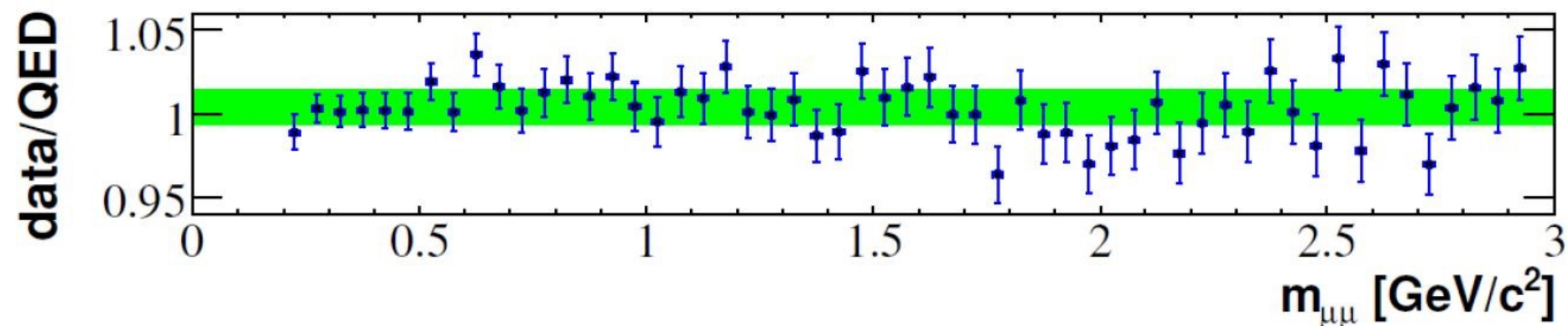
- Systematics mastered at the  $10^{-3}$  level for first time in BaBar
  - ISR  $\gamma$  in EMC (thus: at large angle)
  - Good quality tracks, particle identification (PID)
  - Kinematic Fit (using only direction of ISR  $\gamma$ )
    - Possibly including 1 additional  $\gamma$ : NLO!
  - All efficiencies (trigger, filter, tracking, PID, fit) from data.
- $\pi^+\pi^- / \mu^+\mu^-$  ratio:
- Cancellation of  $ee$  luminosity, additional ISR, VP, ISR  $\gamma$  efficiency
  - Correct for lowest order FSR in  $\mu^+\mu^-$  and for ISR + additional FSR, both calc. in QED and checked in data

$$R_{\text{exp}}(s') = \frac{\sigma_{[\pi\pi\gamma(\gamma)]}(s')}{\sigma_{[\mu\mu\gamma(\gamma)]}(s')} = \frac{\sigma_{[\pi\pi(\gamma)]}^0(s')}{(1 + \delta_{\text{FSR}}^{\mu\mu})\sigma_{[\mu\mu(\gamma)]}^0(s')} = \frac{R(s')}{(1 + \delta_{\text{FSR}}^{\mu\mu})(1 + \delta_{\text{add,FSR}}^{\mu\mu})}$$



# BaBar sanity check: comparison $\mu\mu$ spectra with QED

Phys.Rev.Lett. 103 (2009) 231801, Phys.Rev.D86 (2012) 032013

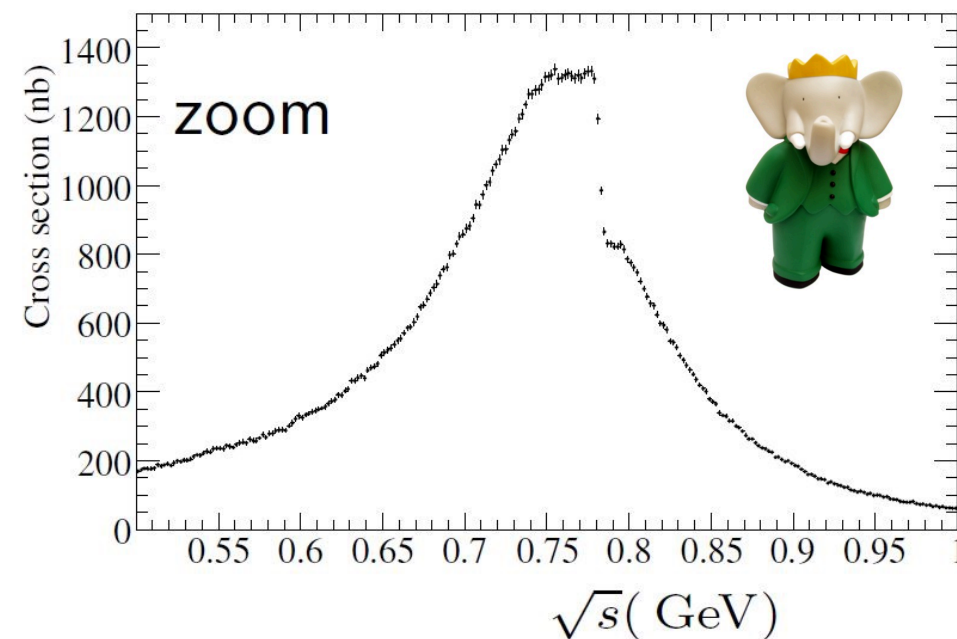
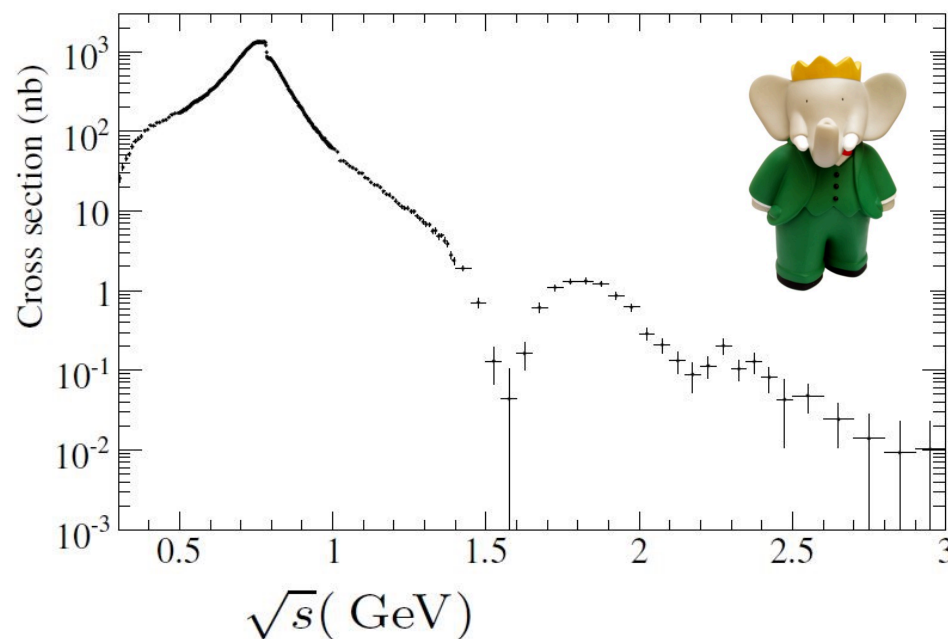


- Here the radiator function and the collider integrated luminosity are needed
- MC simulation corrected for all known MC/data differences.
  - ISR  $\gamma$  efficiency measured in data, from  $\mu\mu$ -only reco'ed evts.
  - MC corrected for known NLO deficiencies by comparing to PHOKHARA

**Good agreement within  $(0.4 \pm 1.1)\%$  ; Dominated by  $L_{ee}$  ( $\pm 0.9\%$ )**

# $e^+e^- \rightarrow \pi^+\pi^-(\gamma)\gamma$ Cross Section

Phys.Rev.Lett. 103 (2009) 231801, Phys.Rev.D86 (2012) 032013



**Bare (incl. additional FSR, VP removed) unfolded**  
 $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$  **232 fb<sup>-1</sup> @  $\sqrt{s} = 10.6$  GeV**  
**Excellent precision down to threshold:**

$$a_\mu^{\pi^+\pi^-}[2m_\pi, 1.8 \text{ GeV}] = (514.1 \pm 2.2 \pm 3.1) \times 10^{-10}$$

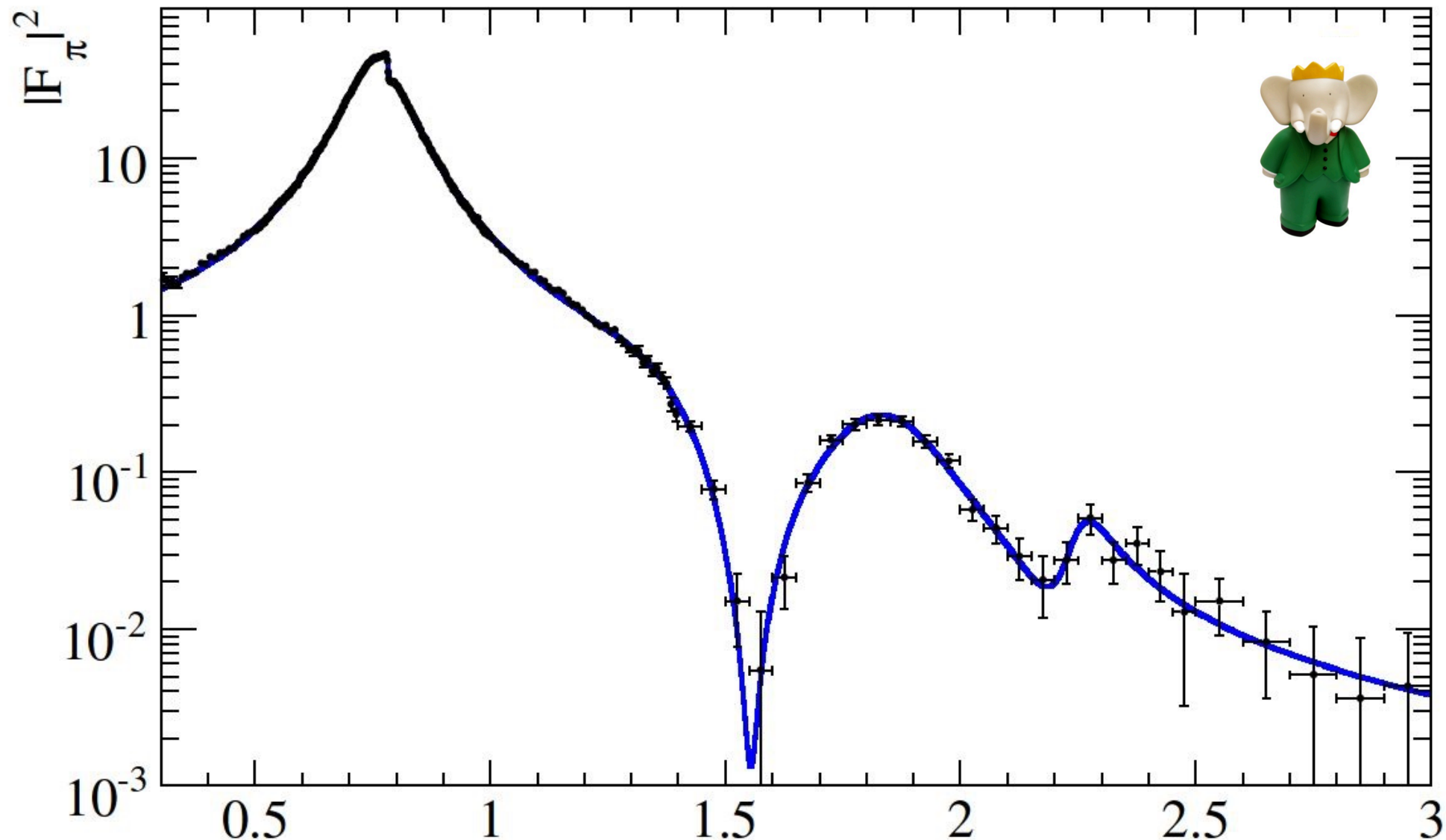
**Similar precision as combination of previous  $e^+e^-$  results:**

$$a_\mu^{\pi^+\pi^-}[2m_\pi, 1.8 \text{ GeV}] = (503.5 \pm 4.5) \times 10^{-10}$$

**1.7  $\sigma$  larger than previous  $e^+e^-$  average:**  $\Delta = +(10.6 \pm 5.9) \times 10^{-10}$

# $e^+e^- \rightarrow \pi^+\pi^-(\gamma)$ : VDM Fit of $|F_\pi(s)|^2$

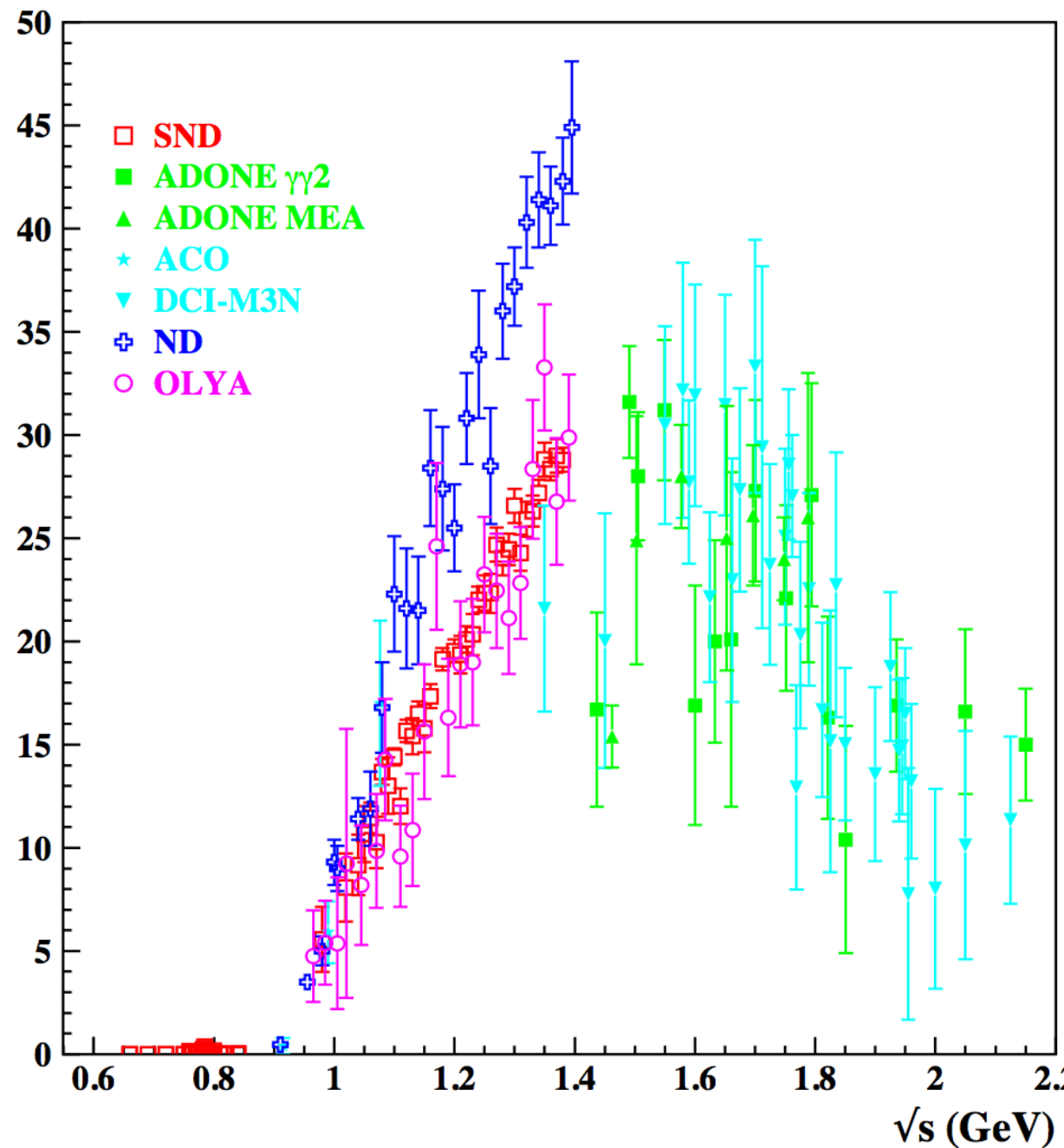
Phys.Rev.Lett. 103 (2009) 231801, Phys.Rev.D86 (2012) 032013



- $| \text{Form Factor} |^2$  fitted with a vector dominance model  $\rho, \rho', \rho'', \omega'$
- $\rho'$  s described by the Gounaris-Sakurai model  $\chi^2/\text{n.d.f} = 334/323$



# Process $\pi^+\pi^-\pi^0\pi^0$ (Before BABAR)



Before the BaBar measurement:

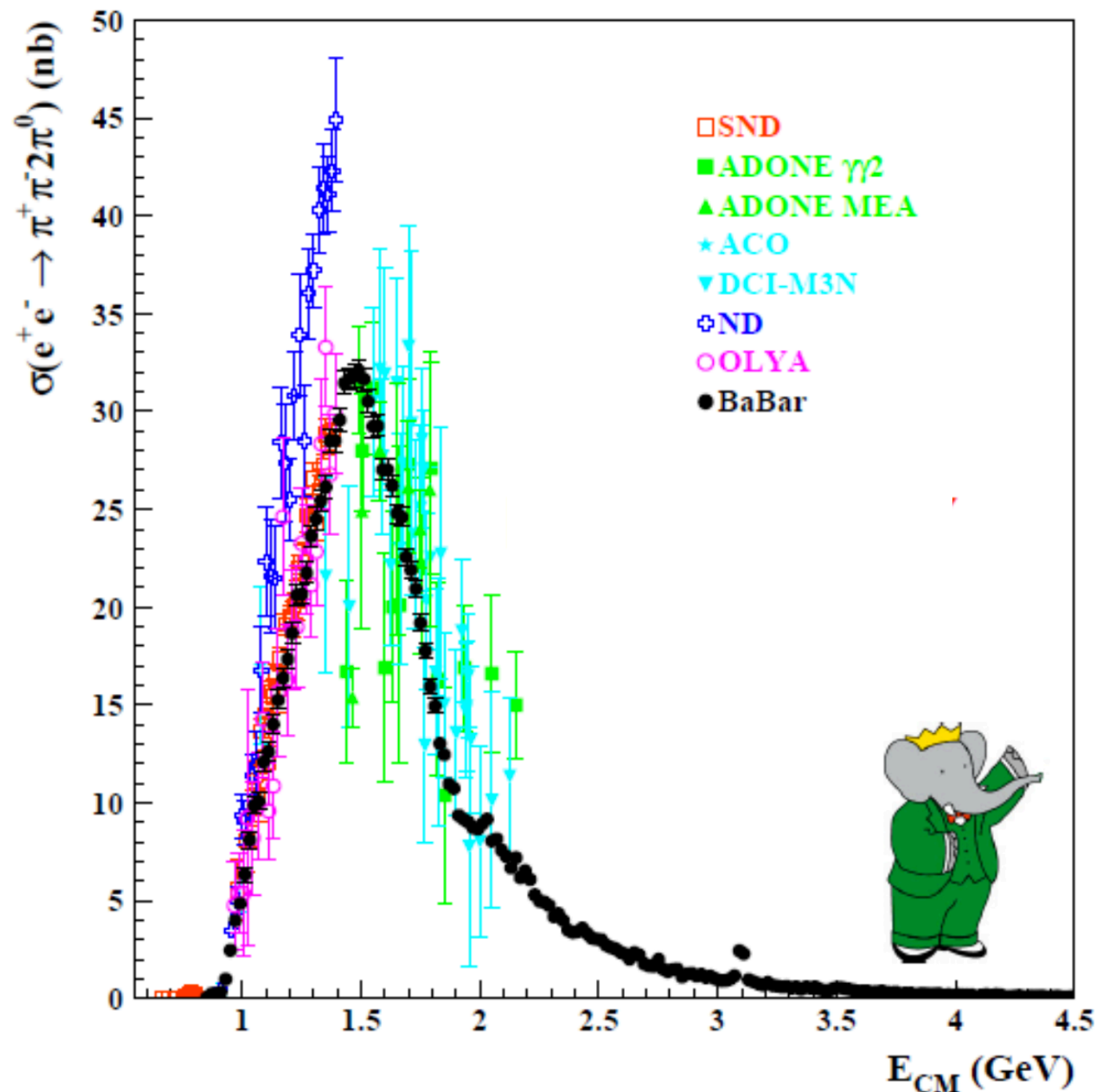
- Limited precision
- Big disagreement between experiments
- Small energy ranges

K.Hagiwara et al., arXiv:1105.3149 [hep-ph] (2011)

Channel	$a_\mu^{\text{had,LO}} [10^{-10}]$
$\pi^0\gamma$	$4.42 \pm 0.08 \pm 0.13 \pm 0.12$
$\eta\gamma$	$0.64 \pm 0.02 \pm 0.01 \pm 0.01$
$\pi^+\pi^-$	$507.80 \pm 1.22 \pm 2.50 \pm 0.56$
$\pi^+\pi^-\pi^0$	$46.00 \pm 0.42 \pm 1.03 \pm 0.98$
$2\pi^+2\pi^-$	$13.35 \pm 0.10 \pm 0.43 \pm 0.29$
$\pi^+\pi^-2\pi^0$	$18.01 \pm 0.14 \pm 1.17 \pm 0.40$
$2\pi^+2\pi^-\pi^0$ ( $\eta$ excl.)	$0.72 \pm 0.04 \pm 0.07 \pm 0.03$
$\pi^+\pi^-3\pi^0$ ( $\eta$ excl., from isospin)	$0.36 \pm 0.02 \pm 0.03 \pm 0.01$
$3\pi^+3\pi^-$	$0.12 \pm 0.01 \pm 0.01 \pm 0.00$
$2\pi^+2\pi^-2\pi^0$ ( $\eta$ excl.)	$0.70 \pm 0.05 \pm 0.04 \pm 0.09$
$\pi^+\pi^-4\pi^0$ ( $\eta$ excl., from isospin)	$0.11 \pm 0.01 \pm 0.11 \pm 0.00$
$\eta\pi^+\pi^-$	$1.15 \pm 0.06 \pm 0.08 \pm 0.03$
$\eta\omega$	$0.47 \pm 0.04 \pm 0.00 \pm 0.05$
$\eta 2\pi^+2\pi^-$	$0.02 \pm 0.01 \pm 0.00 \pm 0.00$
$\eta\pi^+\pi^-2\pi^0$ (estimated)	$0.02 \pm 0.01 \pm 0.01 \pm 0.00$
$\omega\pi^0$ ( $\omega \rightarrow \pi^0\gamma$ )	$0.89 \pm 0.02 \pm 0.06 \pm 0.02$
$\omega\pi^+\pi^-, \omega 2\pi^0$ ( $\omega \rightarrow \pi^0\gamma$ )	$0.08 \pm 0.00 \pm 0.01 \pm 0.00$
$\omega$ (non- $3\pi, \pi\gamma, \eta\gamma$ )	$0.36 \pm 0.00 \pm 0.01 \pm 0.00$
$K^+K^-$	$21.63 \pm 0.27 \pm 0.58 \pm 0.36$
$K_S^0K_L^0$	$12.96 \pm 0.18 \pm 0.25 \pm 0.24$
$\phi$ (non- $K\bar{K}, 3\pi, \pi\gamma, \eta\gamma$ )	$0.05 \pm 0.00 \pm 0.00 \pm 0.00$
$K\bar{K}\pi$ (partly from isospin)	$2.39 \pm 0.07 \pm 0.12 \pm 0.08$
$K\bar{K}2\pi$ (partly from isospin)	$1.35 \pm 0.09 \pm 0.38 \pm 0.03$
$KK3\pi$ (partly from isospin)	$-0.03 \pm 0.01 \pm 0.02 \pm 0.00$
$\phi\eta$	$0.36 \pm 0.02 \pm 0.02 \pm 0.01$
$\omega K\bar{K}$ ( $\omega \rightarrow \pi^0\gamma$ )	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$

# Process $\pi^+\pi^-\pi^0\pi^0$ (After BABAR)

Phys.Rev. D96 (2017) no.9, 092009



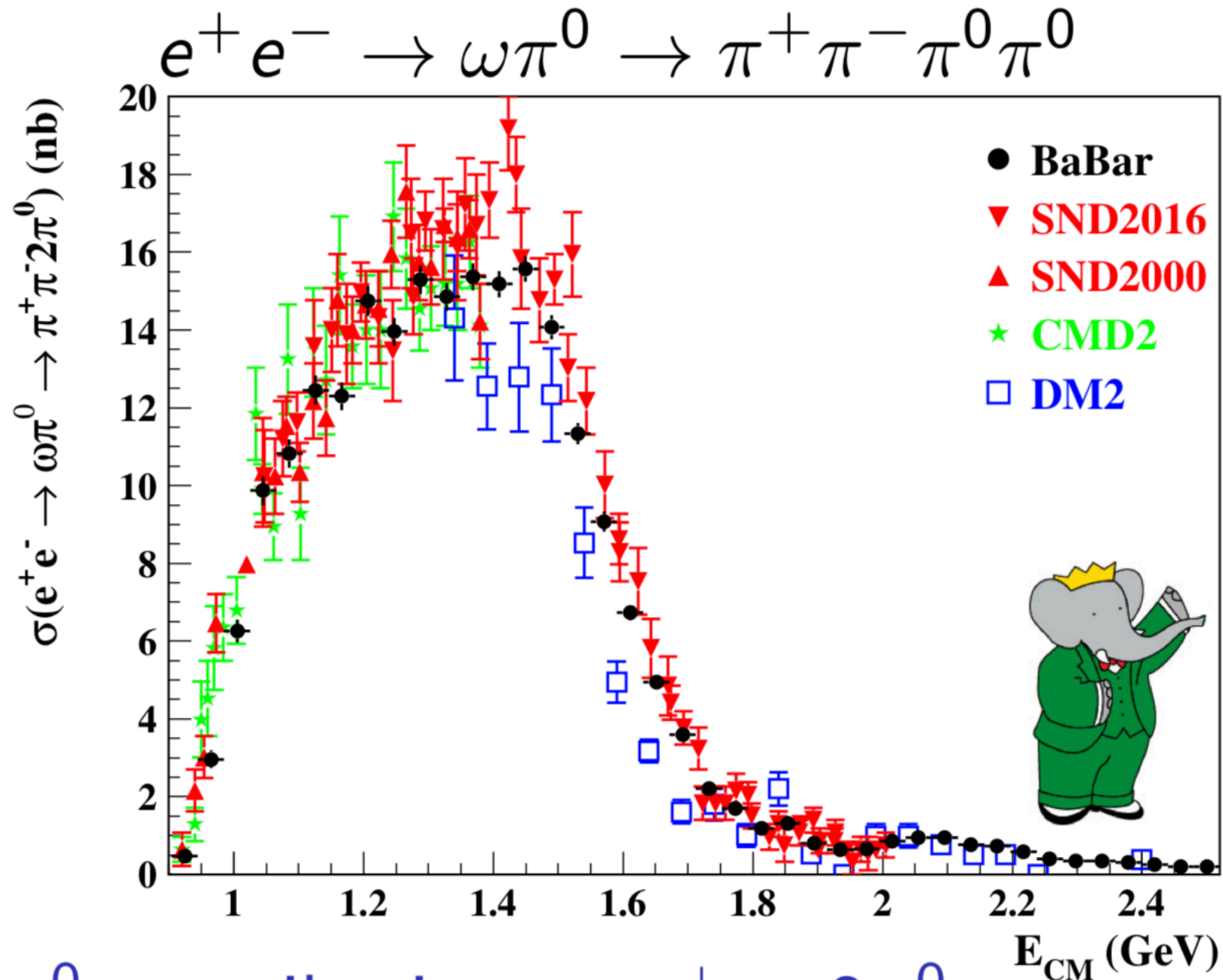
**BaBar measurement:**

- Much more precise
  - Larger energy range
- From 0.85 to 1.8 GeV:**
- Relative precision 3.3%
  - Improved by factor 2.5

$$a_\mu(\pi^+\pi^-2\pi^0) = (17.9 \pm 0.1 \pm 0.6) 10^{-10}$$

# Process $\omega\pi^0$

Phys.Rev. D96 (2017) no.9, 092009



$\omega\pi^0$  contribution to  $\pi^+\pi^-\pi^0$

$$(32.1 \pm 0.2_{\text{stat}} \pm 2.6_{\text{syst}}) \%$$

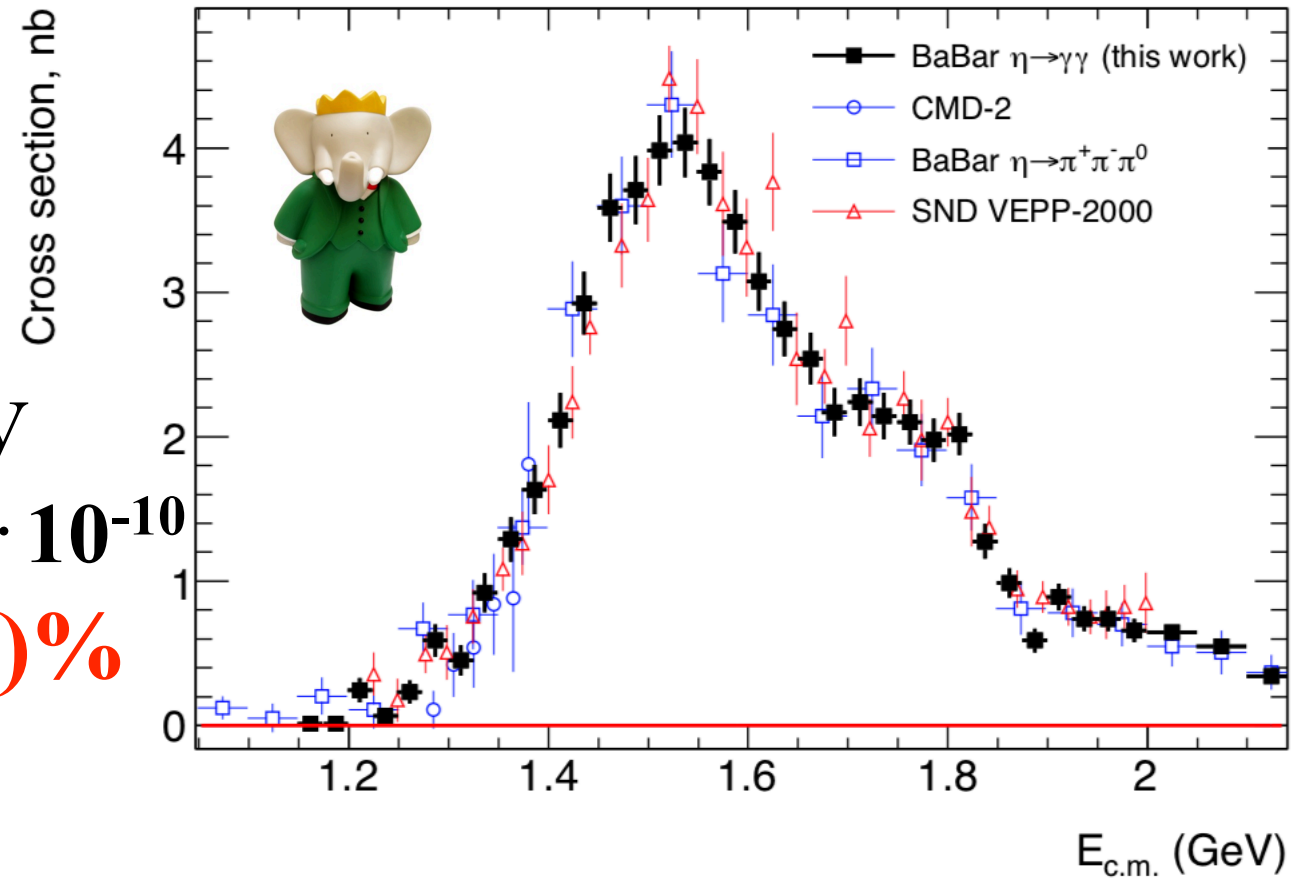
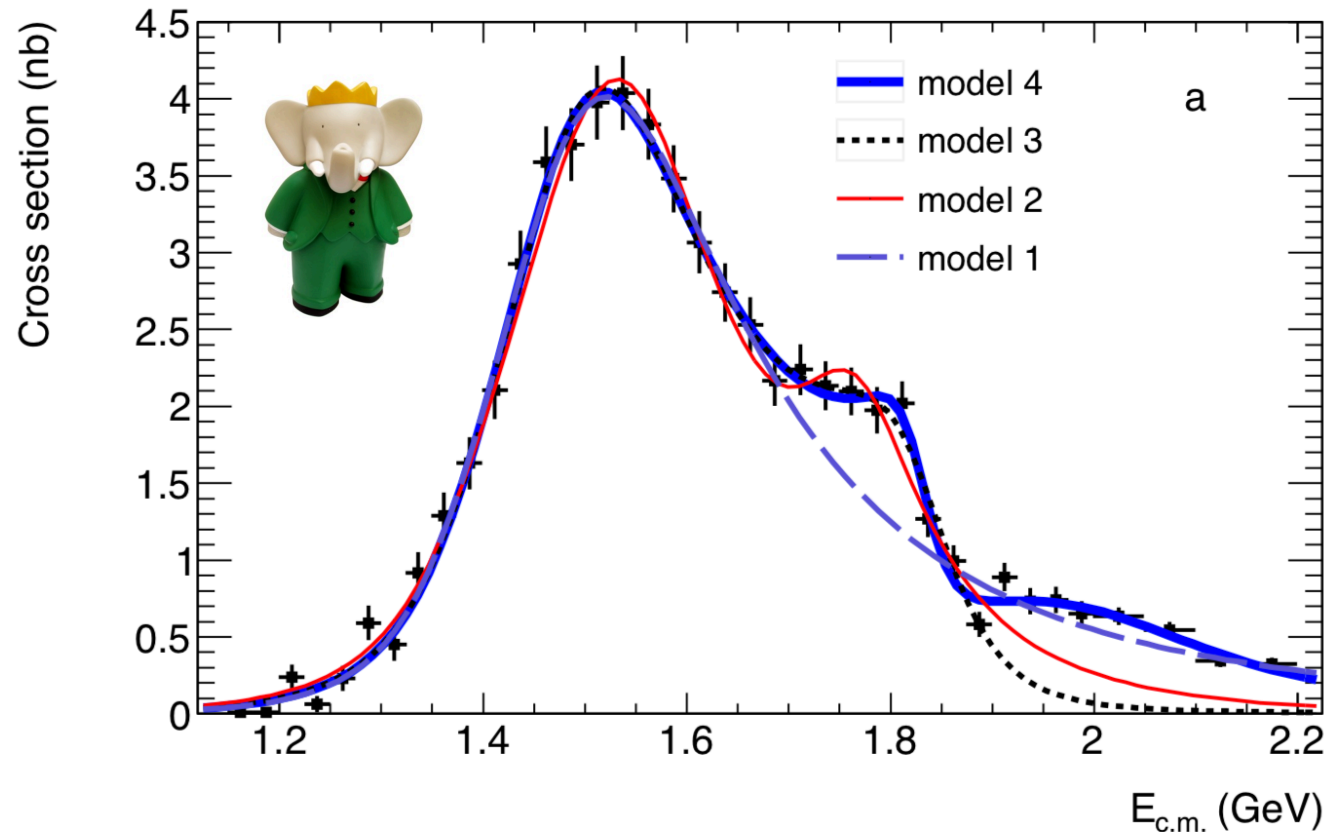


# Process $\pi^+\pi^-\eta$

Phys.Rev. D97 (2018) 052007

- $\eta \rightarrow \gamma\gamma$  decay is used
- The most precise measurement
- Extending energy range up 3.5 GeV
- $a_\mu^{\text{had LO}}(\sqrt{s} < 1.8 \text{ GeV}) = (1.18 \pm 0.06) \cdot 10^{-10}$

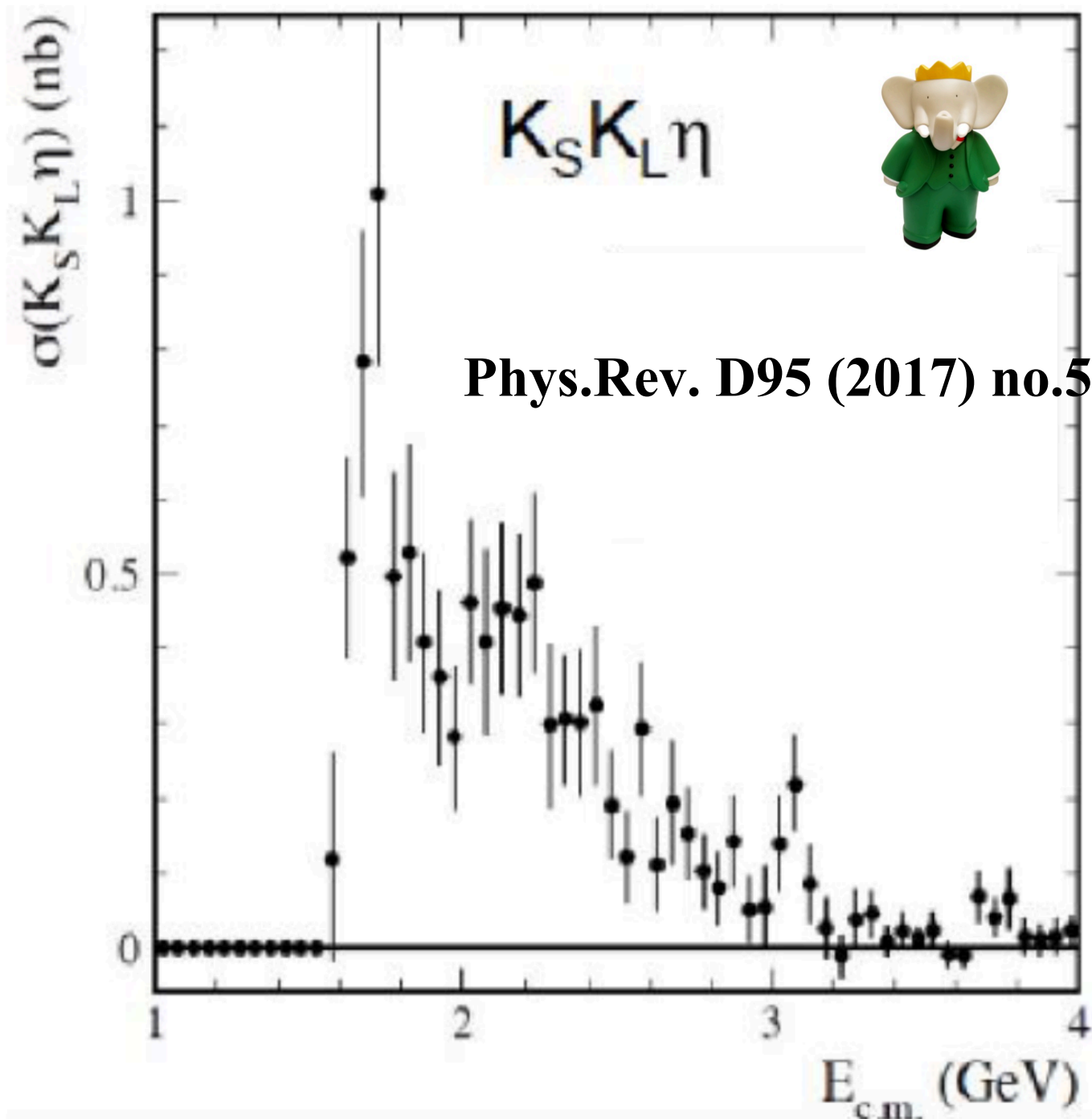
Systematic uncertainty is (4.5-12)%



model	Resonance model	Good fit for
0	$\rho(770) + \rho(1450)$	Doesn't fit
1	$\rho(770) - \rho(1450)$	$E_{\text{cm}} < 1.7 \text{ GeV}$
2	$\rho(770) - \rho(1450) - \rho(1700)$	$E_{\text{cm}} < 1.9 \text{ GeV}$
3	$\rho(770) - \rho(1450) + \rho(1700)$	$E_{\text{cm}} < 1.9 \text{ GeV}$
4	$\rho(770) - \rho(1450) + \rho(1700) + \rho(2150)$	$E_{\text{cm}} < 2.2 \text{ GeV}$

# Process $K_L K_S \eta$

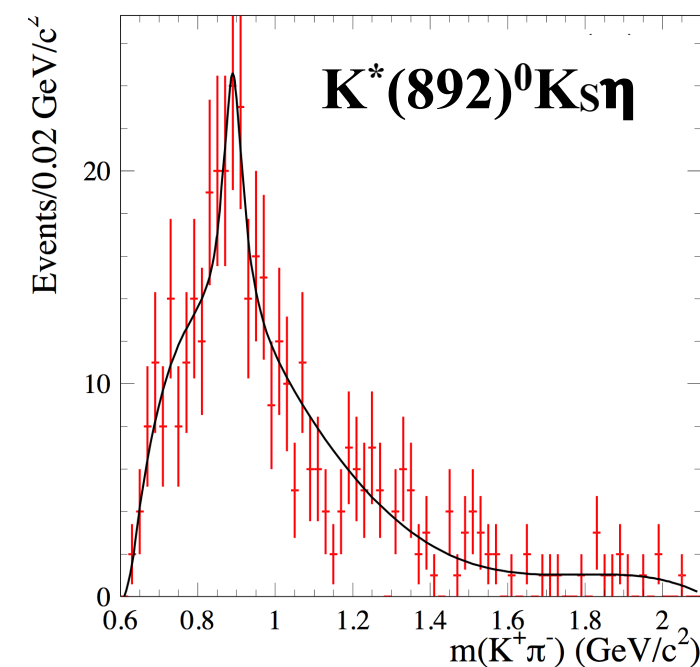
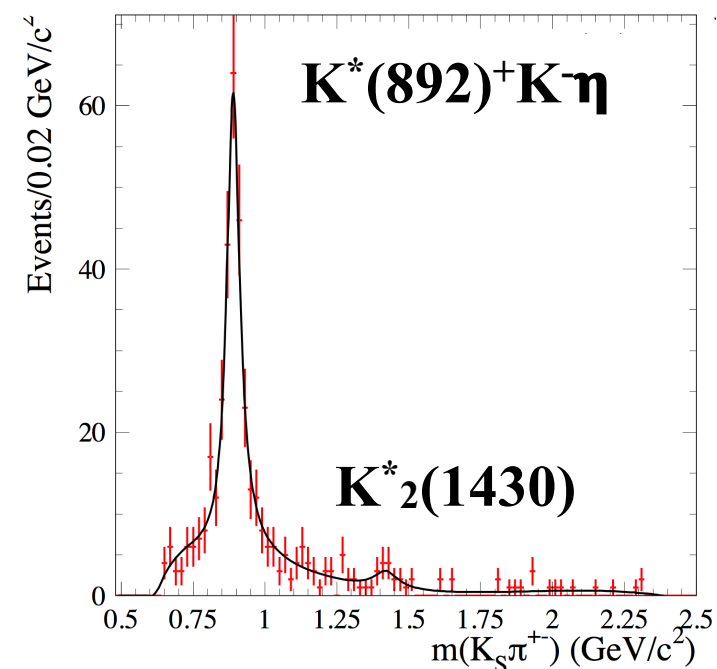
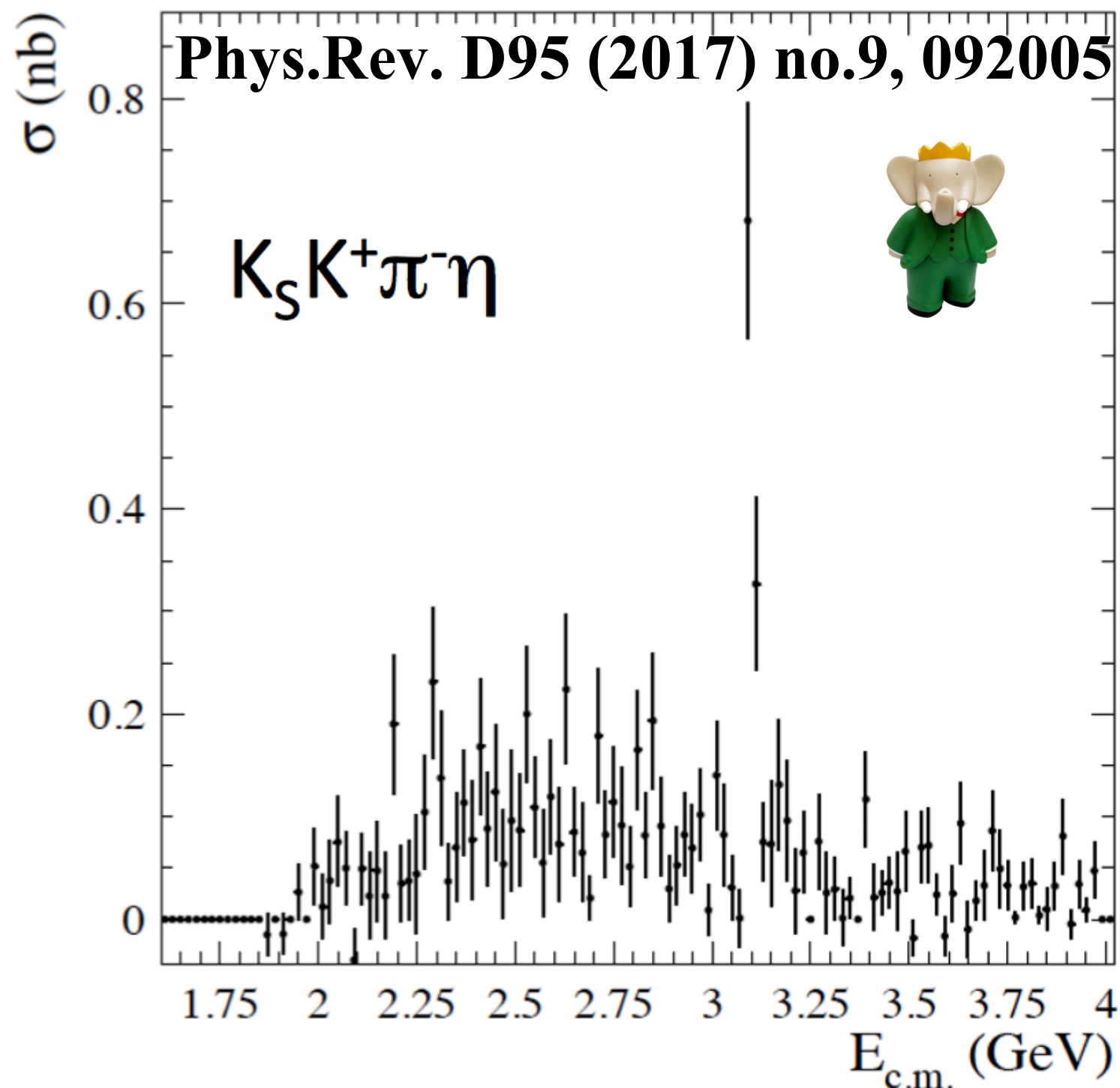
First measurement of this cross section



Systematic uncertainty is (15-100)%

# Process $K_S K^\pm \pi^\mp \eta$

First measurement of this cross section



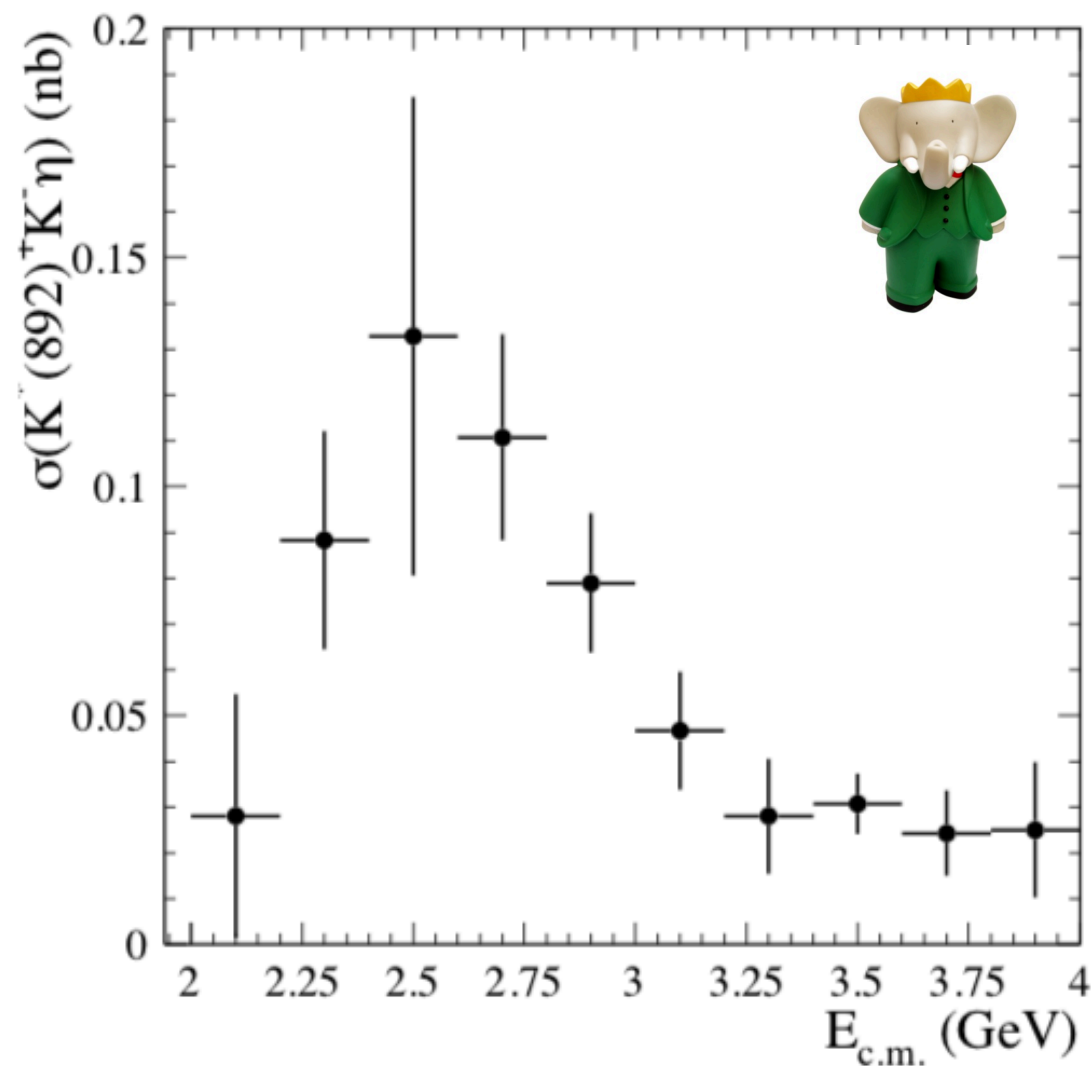
Systematic uncertainty is (12-19)%



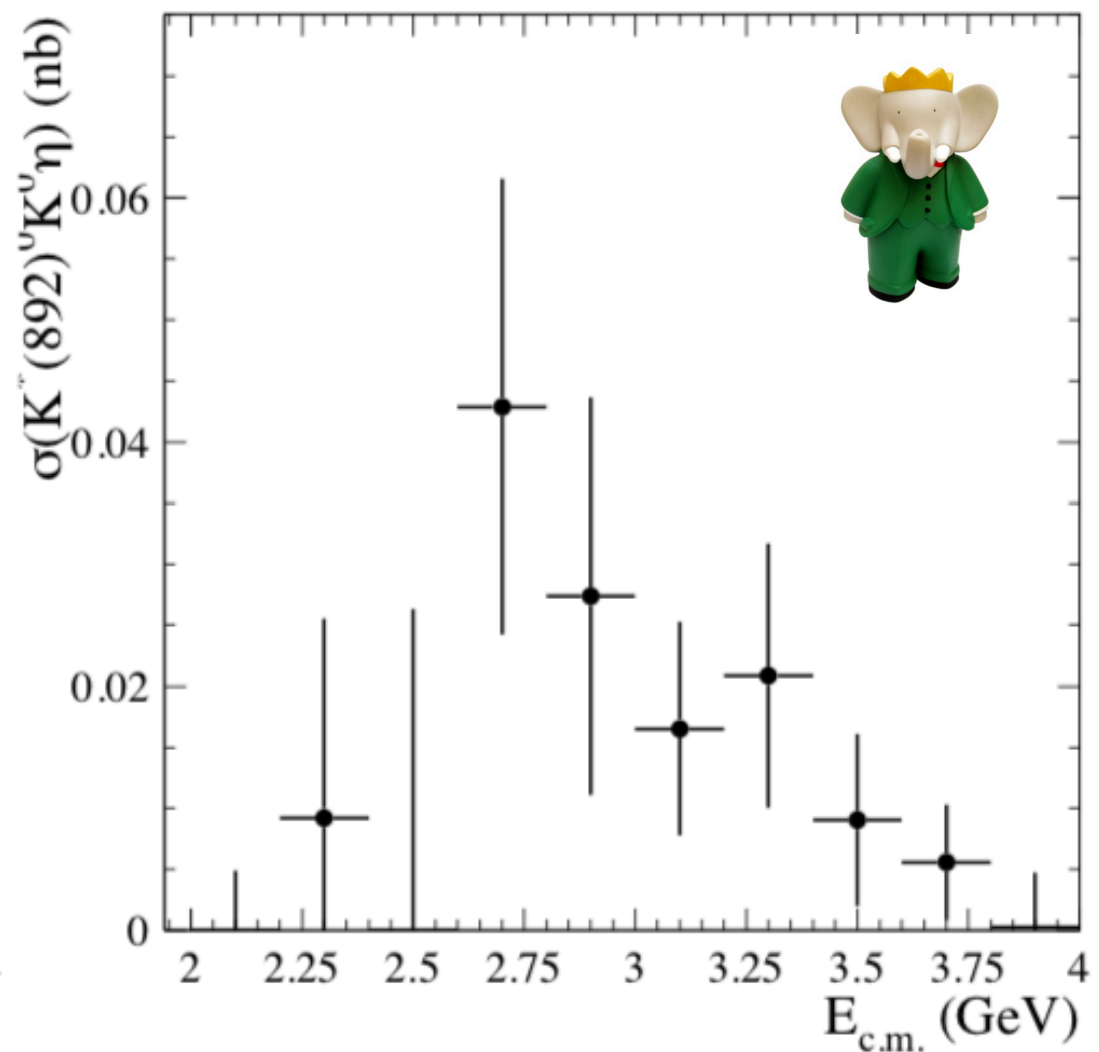
# Processes $K^*(892)K\eta$

Phys.Rev. D95 (2017) no.9, 092005

## Charged $K^*(892)$



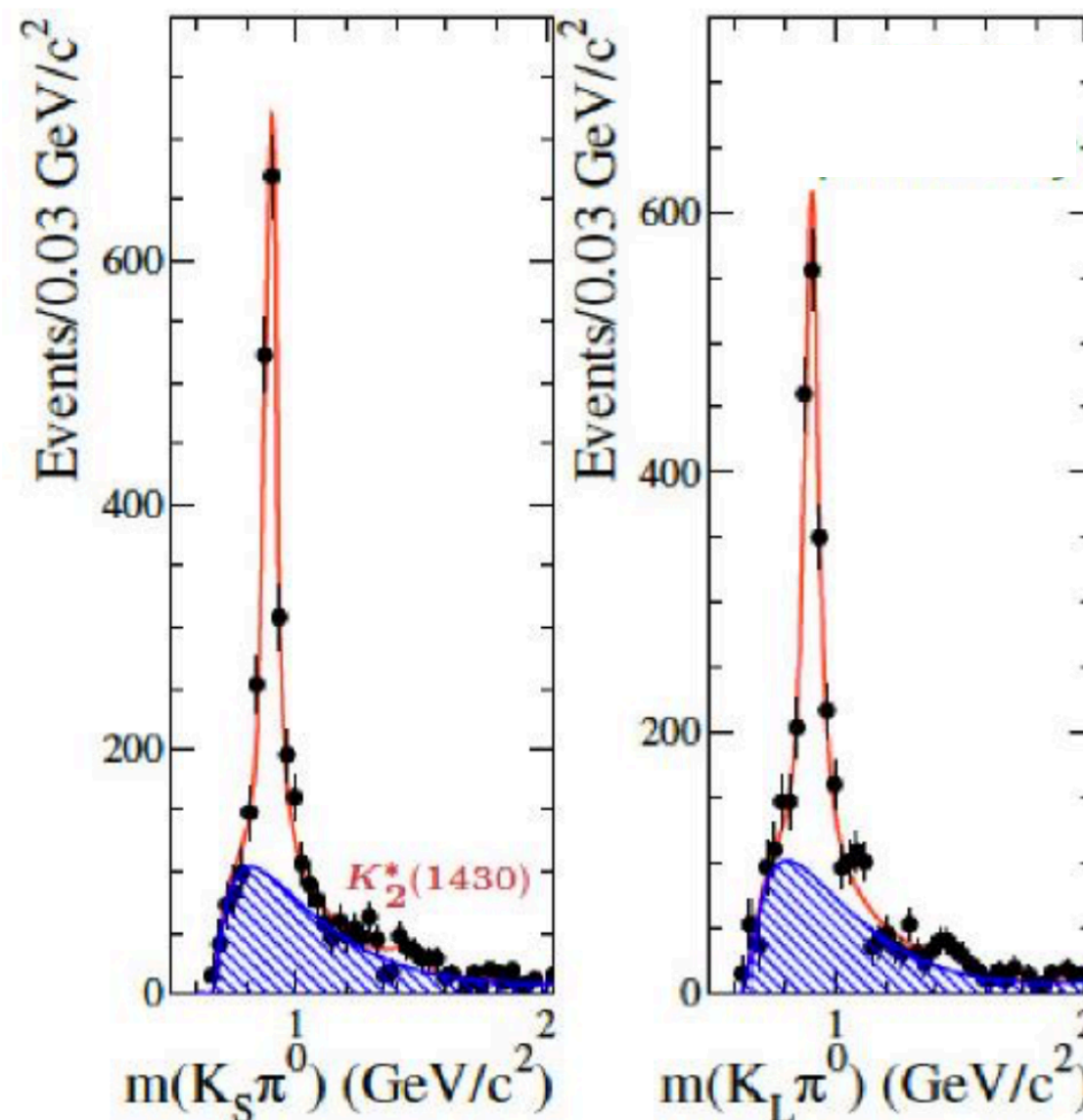
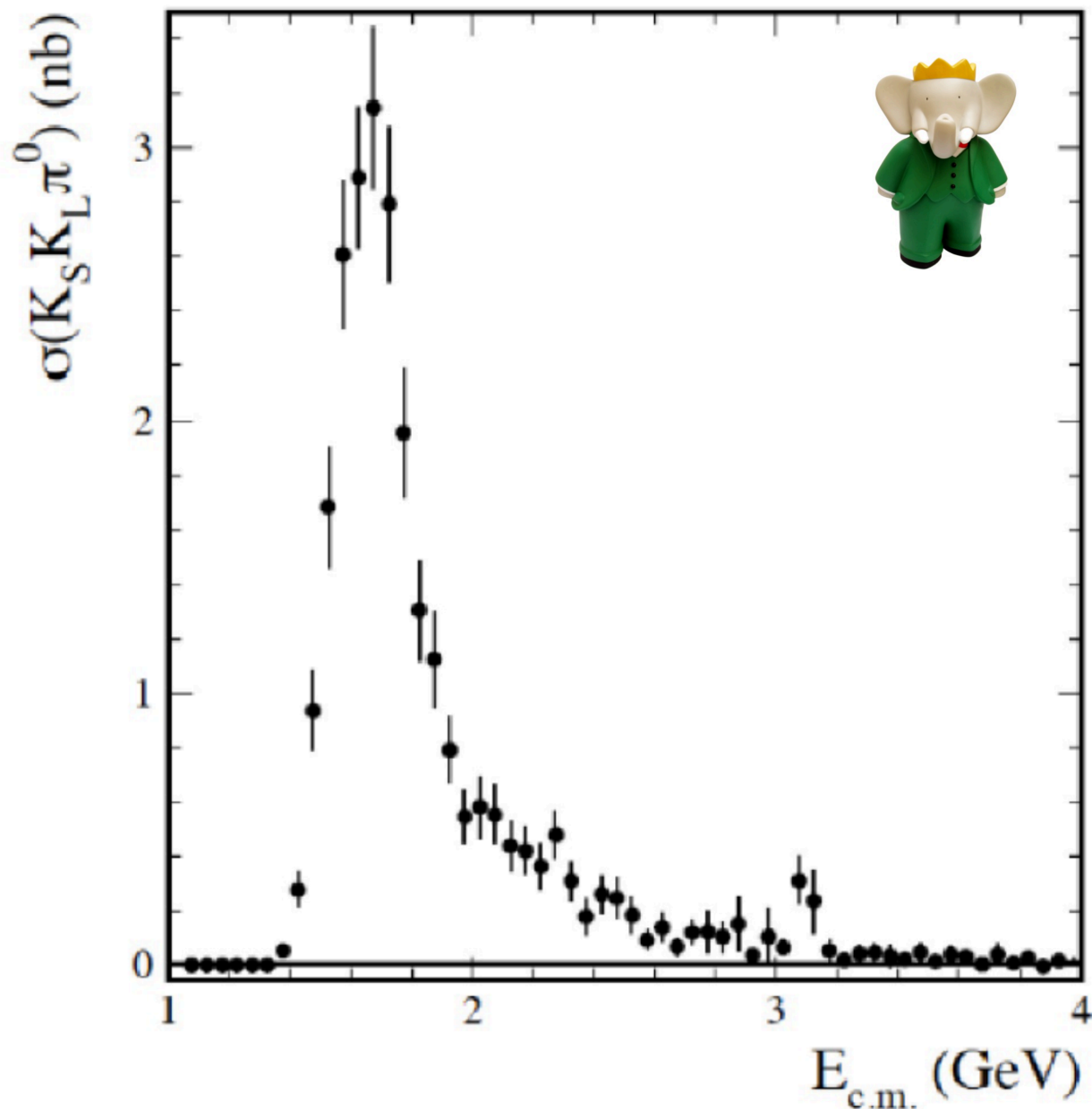
## Neutral $K^*(892)$



# Process $K_L K_S \pi^0$

First measurement of  
this cross section

Phys.Rev. D95 (2017) no.5, 052001



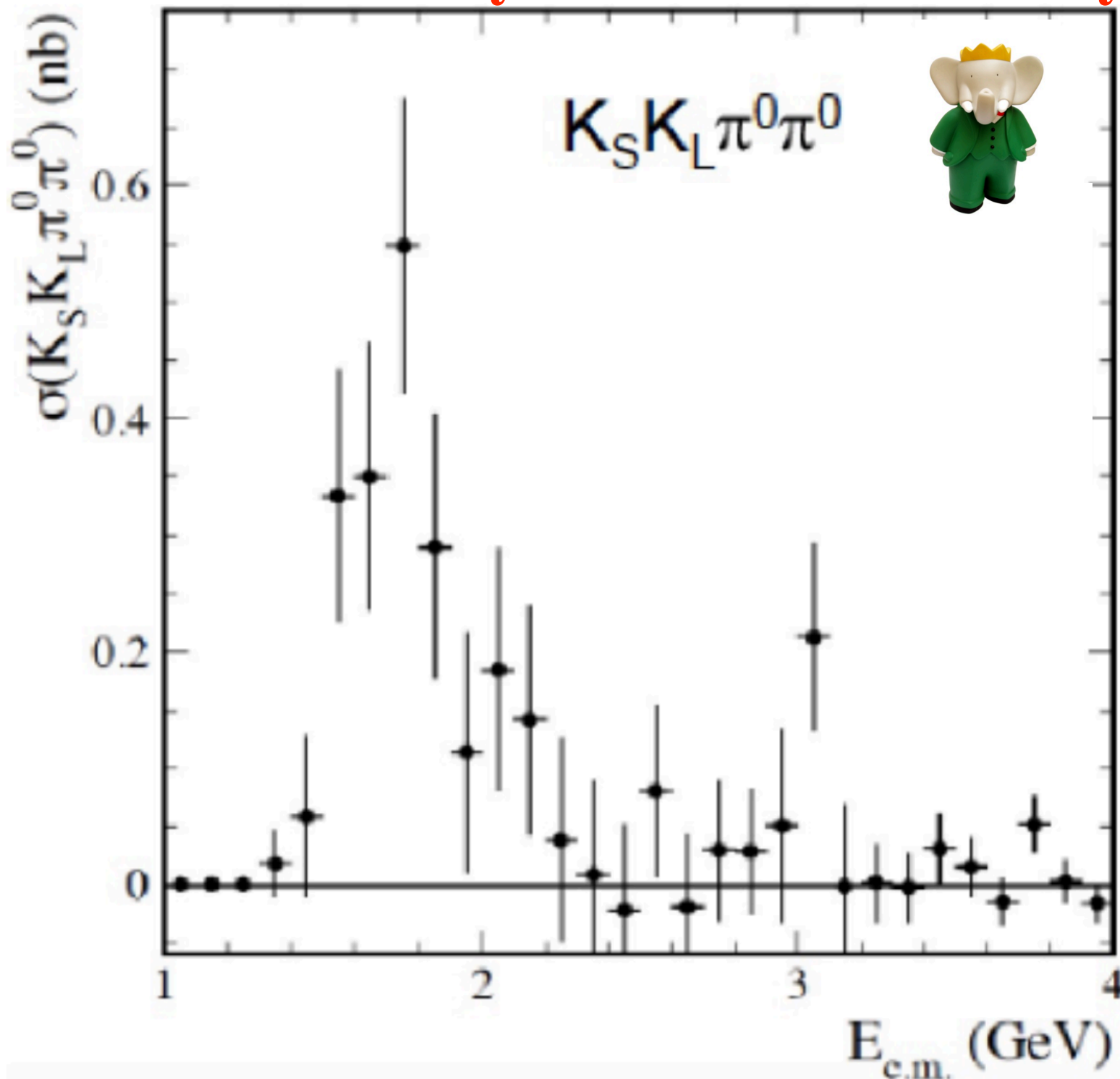
Dominant  $K^*(892)^0 \bar{K}^0$

Small  $K^*(1430)^0 \bar{K}^0, \phi \pi^0$

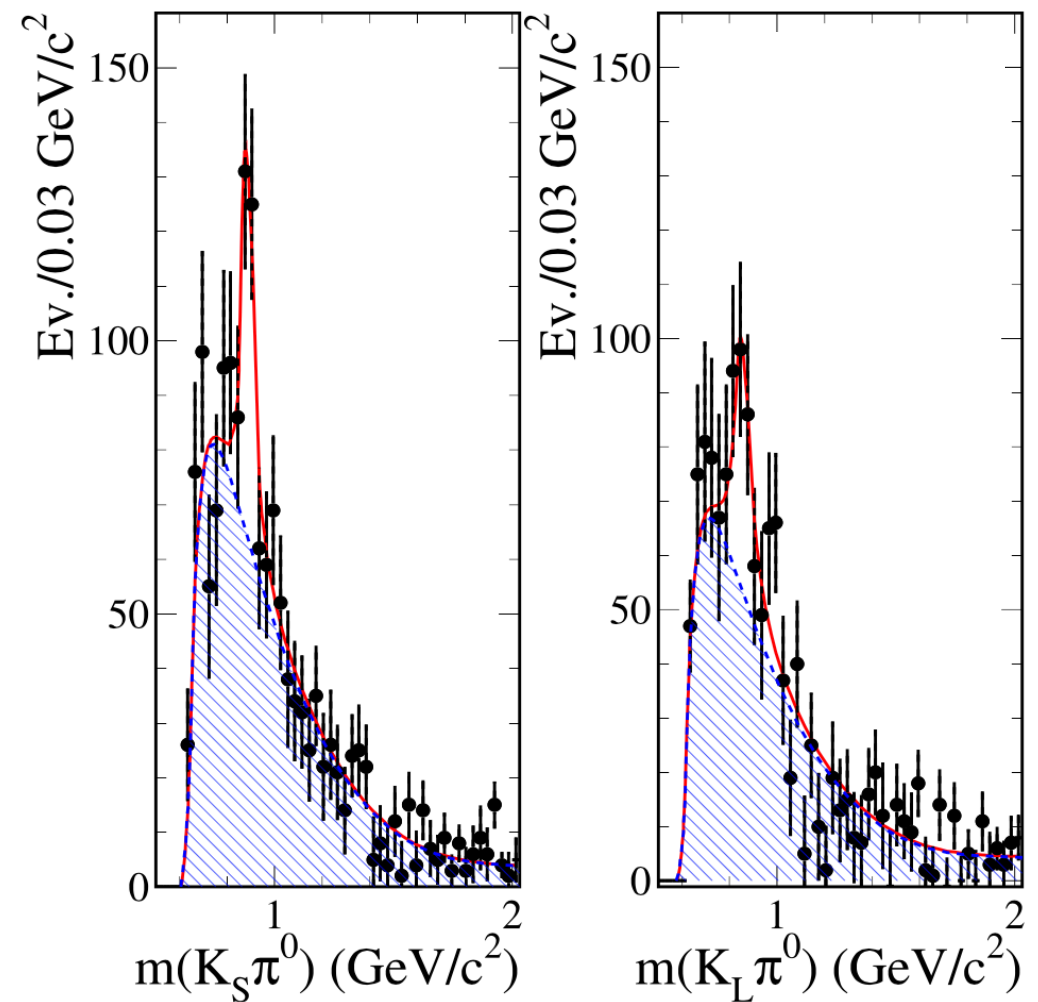
Systematic uncertainty is (10-30)%

# Process $K_L K_S \pi^0 \pi^0$

First measurement of this cross section  
Systematic uncertainty is (25-100)%



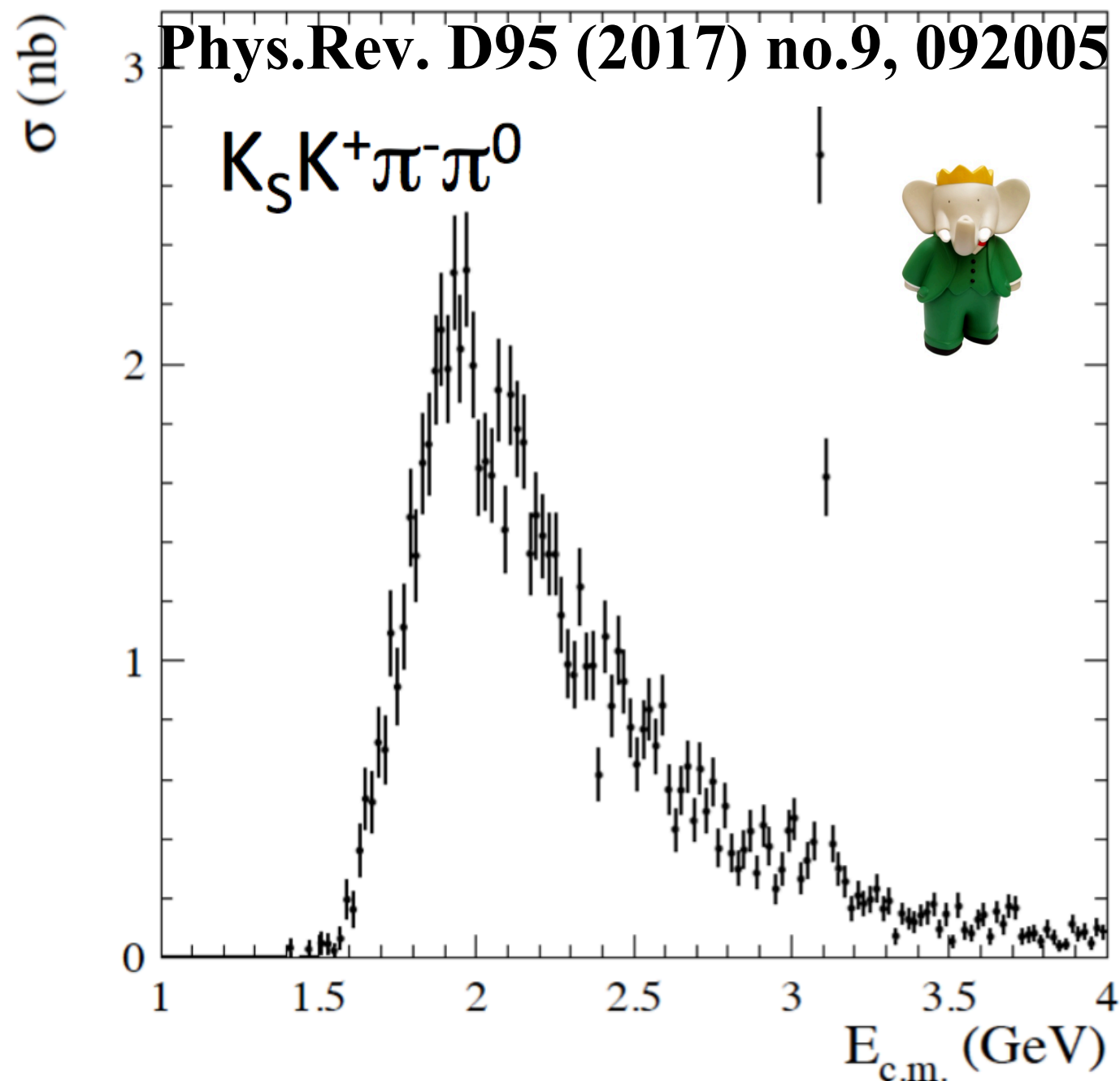
Phys.Rev. D95 (2017) no.5, 052001



# Process $K_S K^\pm \pi^\mp \pi^0$

First measurement of  
this cross section

More than 10 intermediate states



Intermediate state

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$$K^{*0} K_S^0 \pi^0$$

$$K^{*0} K^\pm \pi^\mp$$

$$K_2^*(1430)^0 K_S^0 \pi^0$$

$$K_2^*(1430)^0 K^\pm \pi^\mp$$

$$K^*(892)^\pm K_S^0 \pi^\mp$$

$$K^*(892)^\pm K^\mp \pi^0$$

$$K_2^*(1430)^\pm K_S^0 \pi^\mp$$

$$K_2^*(1430)^\pm K^\mp \pi^0$$

$$K^{*0} \bar{K}^{*0}$$

$$K^*(892)^+ K^*(892)^-$$

$$K_S^0 K^\pm \rho(770)^\mp$$

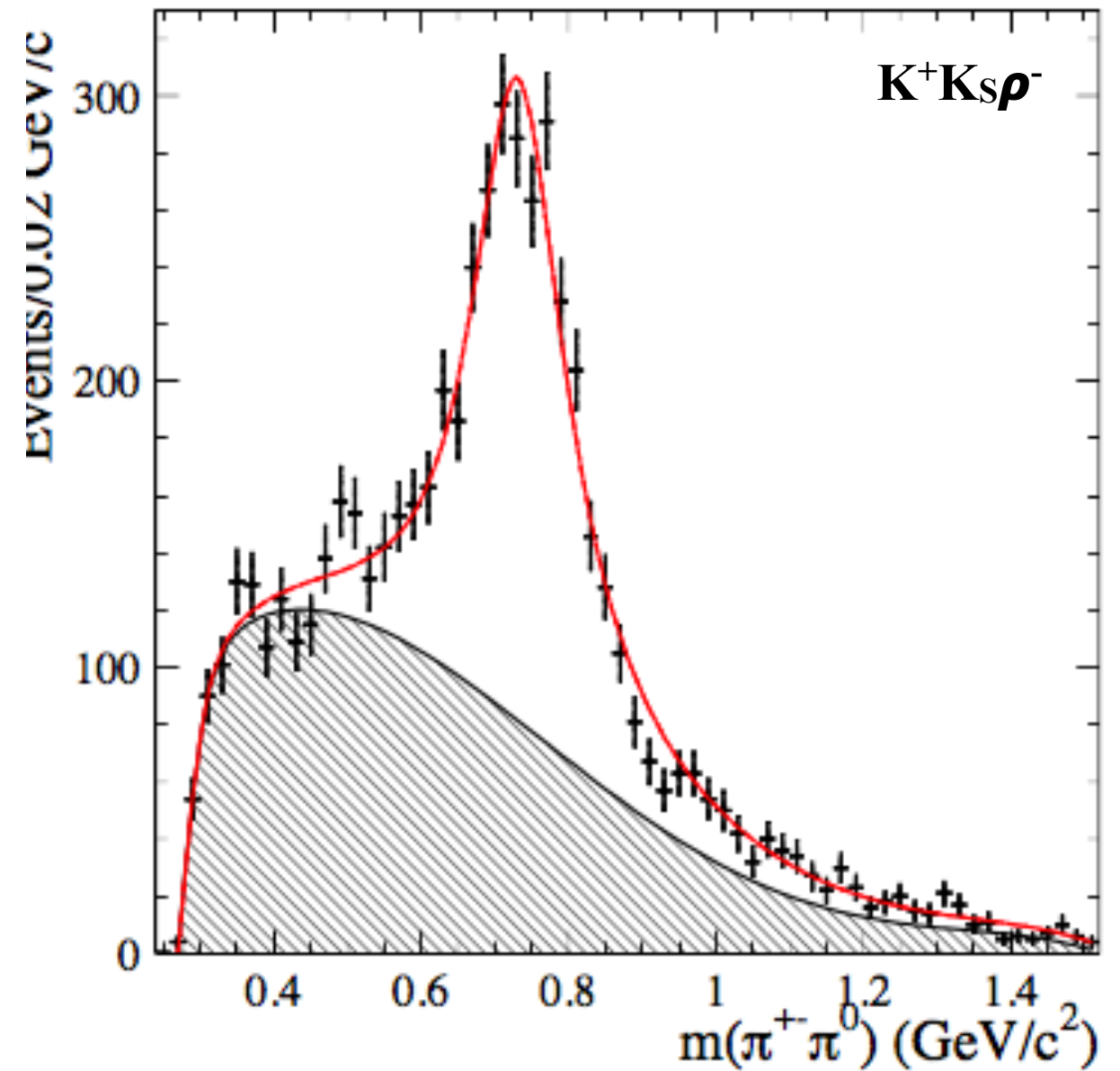
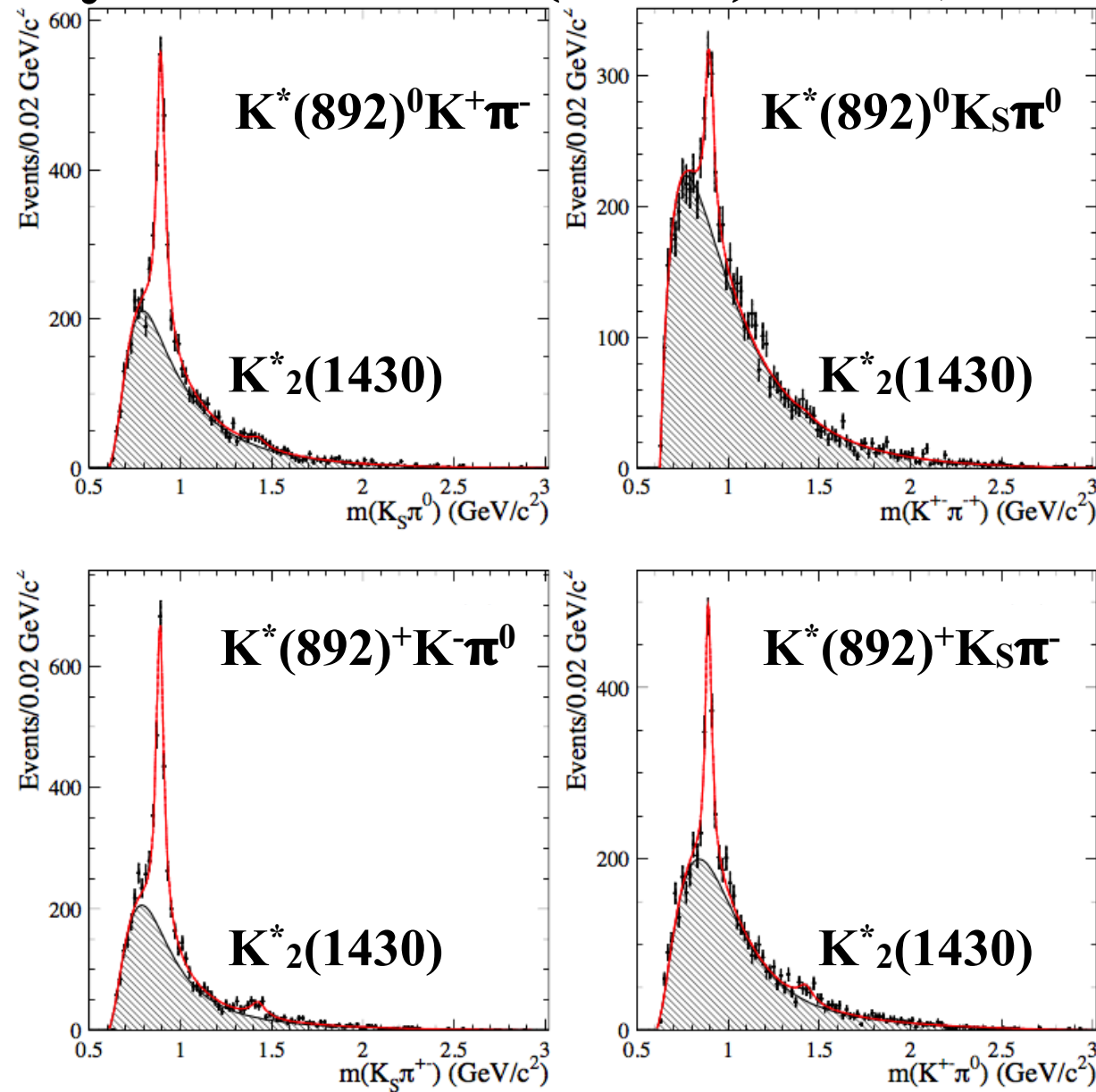

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Systematic uncertainty is (6-12)%



# Substructures in $K_S K^\pm \pi^\mp \pi^0$

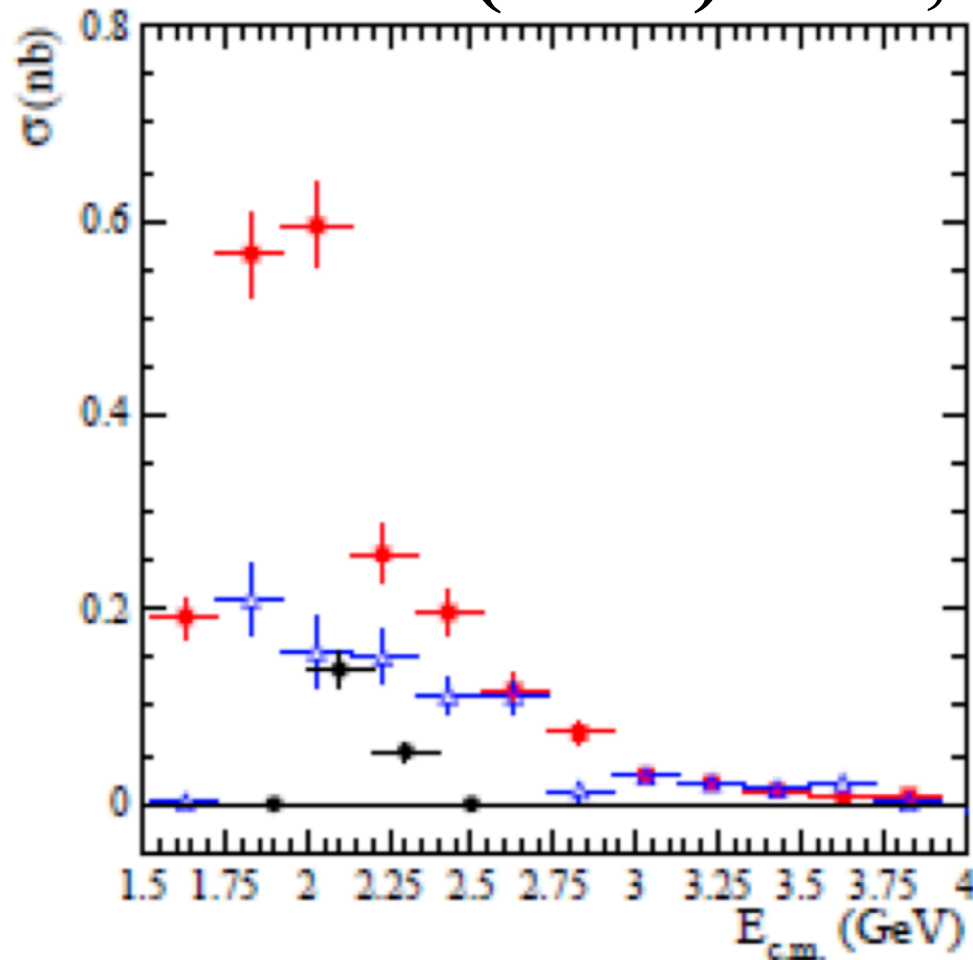
Phys.Rev. D95 (2017) no.9, 092005



All  $K^*(892)K\pi$  signals include also signals from  $K^*(892)K^*(892)$

# Processes $K^*(892)K_S\pi$ and $K^*(892)K\pi$

Phys.Rev. D95 (2017) no.9, 092005

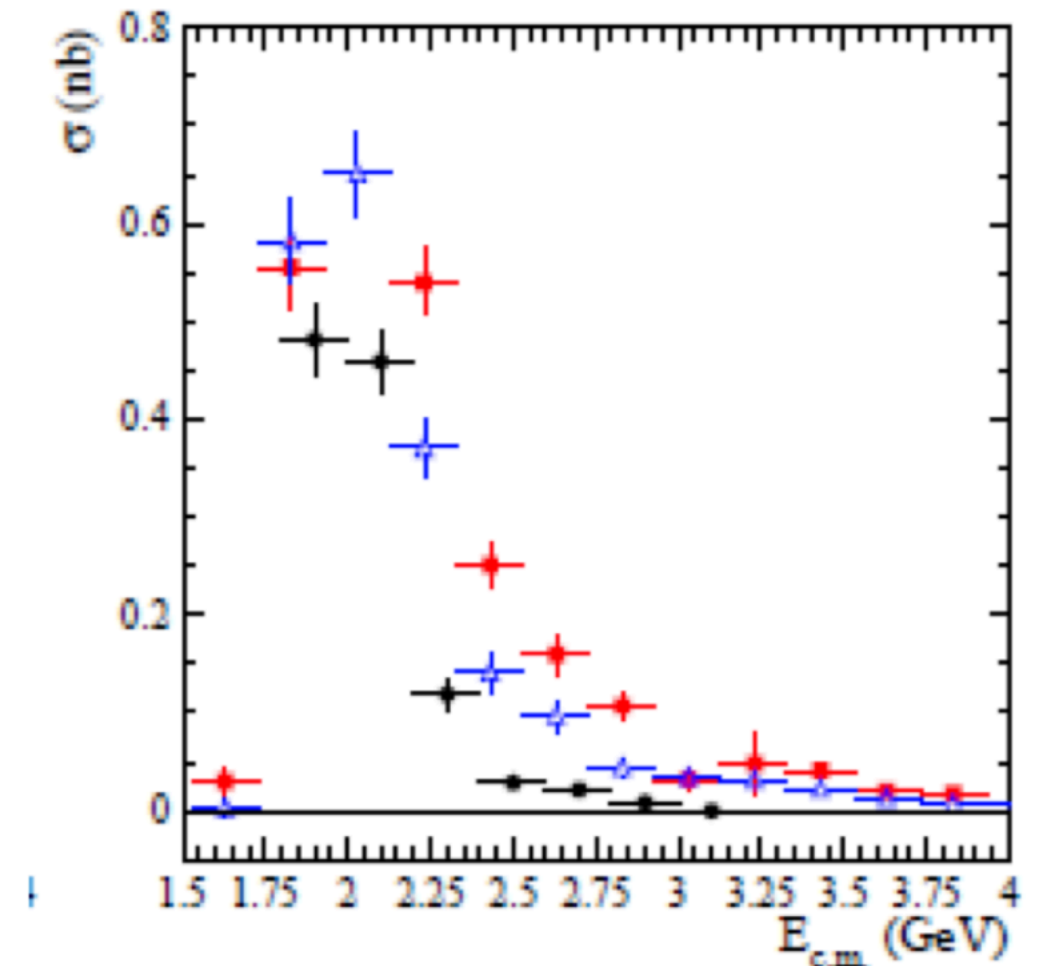


Neutral  $K^*(892)$

$$e^+e^- \rightarrow K^{*0}K^\pm\pi^\mp$$

$$e^+e^- \rightarrow K^{*0}K_S\pi^0$$

$$e^+e^- \rightarrow K^{*0}\bar{K}^{*0}$$



Charged  $K^*(892)$

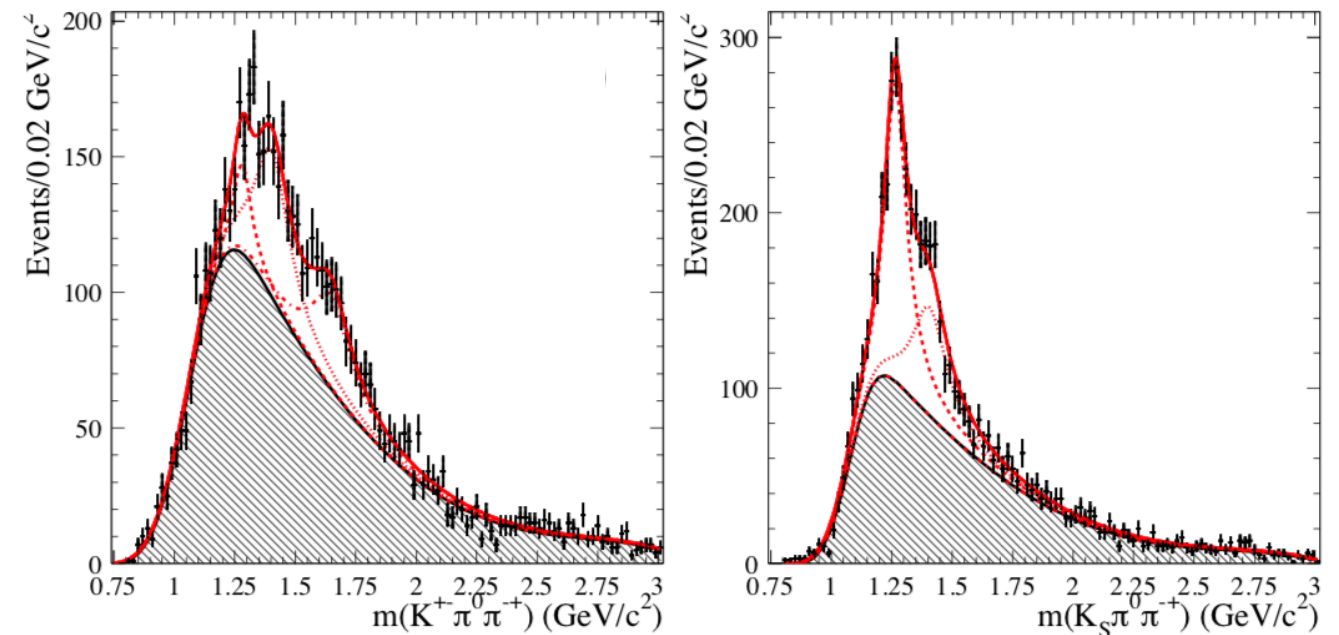
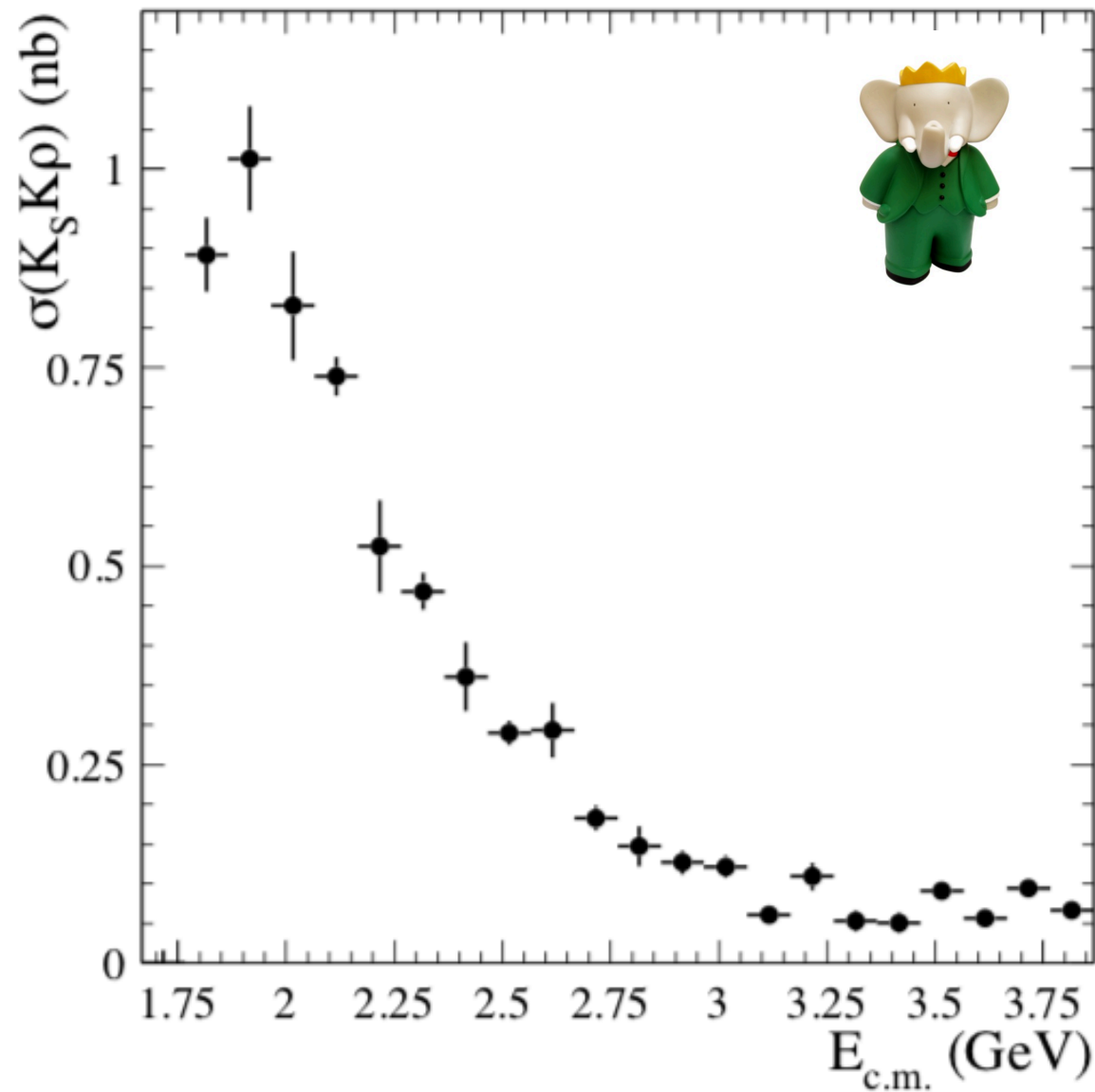
$$e^+e^- \rightarrow K^{*\pm}K_S\pi^\mp$$

$$e^+e^- \rightarrow K^{*\pm}K^\mp\pi^0$$

$$e^+e^- \rightarrow K^{*+}K^{*-}$$

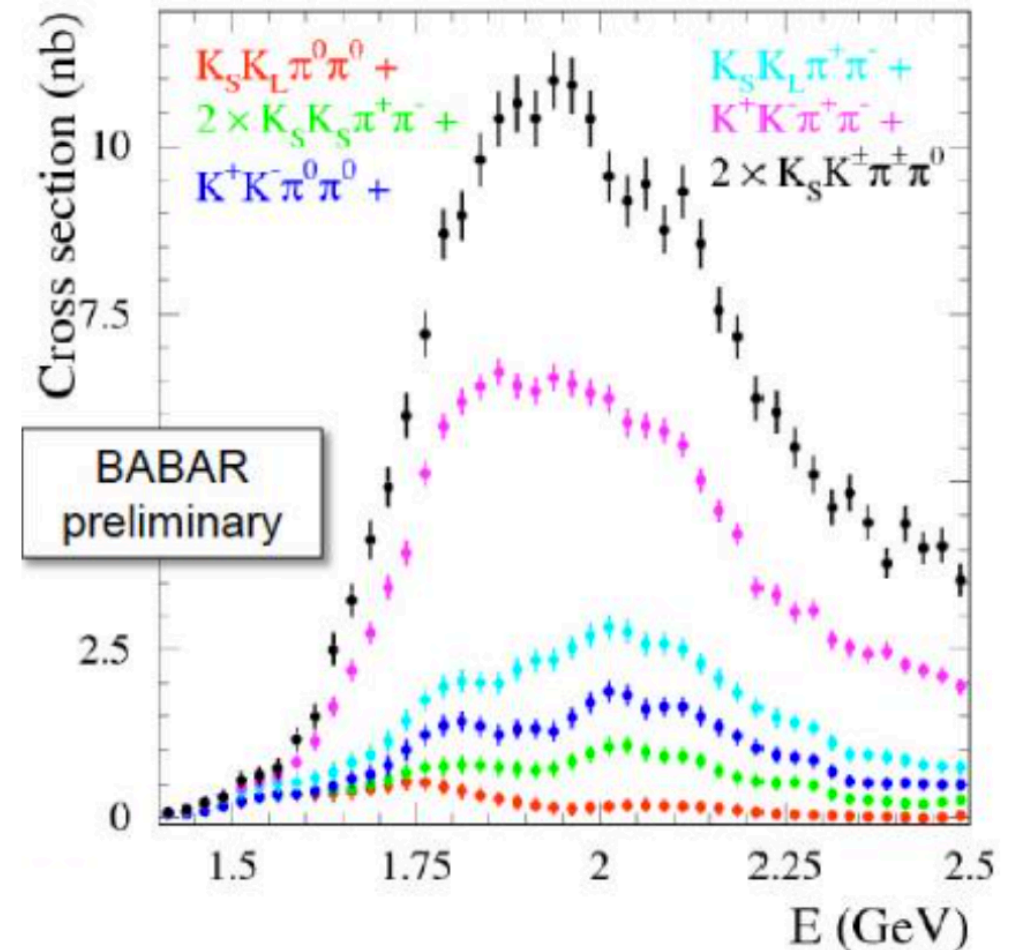
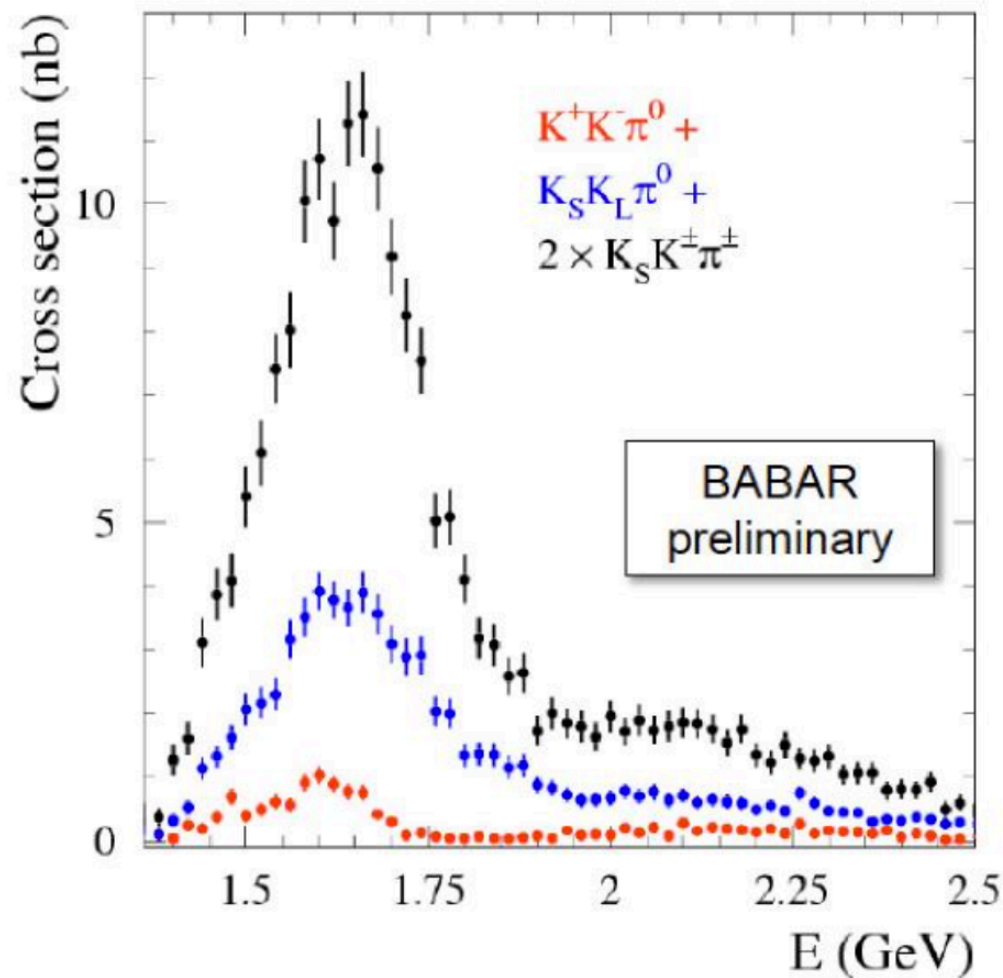
# Process $KK_S\rho$

Phys.Rev. D95 (2017) no.9, 092005



Cross section may contain contributions from  $K_1 \rightarrow K\rho$ , where  $K_1 = K_1(1270), K_1(1400), K_1(1650)$

# Total $KK\pi(\pi)$ cross sections



- All modes have now been measured by BABAR
- $KK\pi$  is about 12% of the total cross section for  $E_{\text{cm}} = 1.65$  GeV
- $KK\pi\pi$  is about 25% of the total cross section for  $E_{\text{cm}} = 2.0$  GeV
- Precision on  $(g-2)/2$  improved (no reliance on isospin)

$$a_\mu(KK\pi) = (2.45 \pm 0.15) 10^{-10}$$

$$a_\mu(KK\pi\pi) = (0.85 \pm 0.05) 10^{-10}$$



# Conclusion

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- Using ISR technique BABAR does precision studies of low energy  $e^+e^-$  annihilation.
- All  $KK\pi$  and  $KK\pi\pi$  modes now directly measured by BABAR. No isospin relations needed any more for cross sections and dispersion relations.
- Resonant substructures explored with  $\mathcal{O}(10^2-10^3)$  events.
- Contributions to  $a_\mu$ :  
$$a_{\mu(\pi^+\pi^-\pi^0\pi^0)} = (17.4 \pm 0.6) 10^{-10}$$
$$a_{\mu(KK\pi)} = (2.45 \pm 0.15) 10^{-10} \quad a_{\mu(KK\pi\pi)} = (0.85 \pm 0.05) 10^{-10}$$
- Improvement of the total  $a_\mu^{\text{had LO}}$  prediction:

$$\text{DHMZ 2011} \\ (692.3 \pm 4.2) 10^{-10}$$

$$\text{Tau2016 Conference} \\ (692.8 \pm 3.3) 10^{-10}$$