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The post-Newtonian limit of hybrid f(R)-gravity

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Prerequisites for the expansion of GR

- Dark matter
- Dark energy
- Incompatibility of gravity with quantum mechanics

f(R)-gravity

- Metric f(R)-gravity

$$R = g^{\mu\nu} R_{\mu\nu} \equiv g^{\mu\nu} \left(\Gamma^{\alpha}_{\mu\nu,\alpha} - \Gamma^{\alpha}_{\mu\alpha,\nu} + \Gamma^{\alpha}_{\alpha\lambda} \Gamma^{\lambda}_{\mu\nu} - \Gamma^{\alpha}_{\mu\lambda} \Gamma^{\lambda}_{\alpha\nu} \right)$$

- Palatini f(R)-gravity

$$\mathcal{R} \equiv g^{\mu\nu} \mathcal{R}_{\mu\nu} \equiv g^{\mu\nu} \left(\hat{\Gamma}^{\alpha}_{\mu\nu,\alpha} - \hat{\Gamma}^{\alpha}_{\mu\alpha,\nu} + \hat{\Gamma}^{\alpha}_{\alpha\lambda} \hat{\Gamma}^{\lambda}_{\mu\nu} - \hat{\Gamma}^{\alpha}_{\mu\lambda} \hat{\Gamma}^{\lambda}_{\alpha\nu} \right)$$

Hybrid f(R)-gravity

$$S = \frac{1}{2k^2}\int d^4x\sqrt{-g}\left[R + f(\mathfrak{R})\right] + S_m,$$

- g is the determinant of the metric
- R is the metric Ricci scalar
- $\mathfrak{R}$ is the Palatini curvature
- S_m is the standard matter action
- $k^2 = 8\pi G/c^4$
- G is the gravitational constant
- c is the speed of light

T. Harko, F.S.N. Lobo, G.J. Olmo, T.S. Koivisto "Metric-Palatini gravity unifying local constraints and late-time cosmic acceleration", Phys. Rev. D 85, 084016 (2012).

Field equations

$$\Re_{\mu\nu} = R_{\mu\nu} + \frac{3}{2}\frac{1}{F^2(\Re)}F(\Re)_{,\mu}F(\Re)_{,\nu}$$

$$- \frac{1}{F(\Re)}\nabla_\mu F(\Re)_{,\nu} - \frac{1}{2}\frac{1}{F(\Re)}g_{\mu\nu}\nabla_\alpha\nabla^\alpha F(\Re),$$

$$F(\Re) = \frac{df(\Re)}{d\Re}.$$

Scalar-tensor representation

$$S = \frac{1}{2k^2}\int d^4x\sqrt{-g}\left[\phi R + \frac{3}{2\phi}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right] + S_m$$

ϕ is the scalar field,

$V(\phi)$ is the scalar potential.

Field equations

$$R_{\mu\nu} = \frac{1}{1+\phi}\Big(k^2(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T)$$

$$+ \frac{1}{2}g_{\mu\nu}(V(\phi) + \nabla_\alpha\nabla^\alpha\phi) + \nabla_\mu\nabla_\nu\phi - \frac{3}{2\phi}\partial_\mu\phi\partial_\nu\phi\Big),$$

$$-\nabla_\mu\nabla^\mu\phi + \frac{1}{2\phi}\partial_\mu\phi\partial^\mu\phi + \frac{\phi[2V(\phi) - (1+\phi)V_\phi]}{3} = \frac{\phi k^2}{3}T,$$

$$V_\phi = \frac{dV(\phi)}{d\phi}.$$

PPN formalism

- Weak field limit,
- Asymptotically flat space-time background,
- Small velocities,
- Motion of matter would obey to the hydrodynamics equations for the perfect fluid.

C. M. Will, *Theory and Experiment in Gravitational Physics*, (Cambridge University Press, London, 1981);

PPN parameters	Solar system	GR	What means	Which experiment
γ	$1\pm2.3\times10^{-5}$	1	How much space-curvature produced by unit rest mass	Cassini tracking
β	$1\pm8\times10^{-5}$	1	How much “nonlinearity” in the superposition law for gravity?	Perihelion shift
ξ	$0\pm4\times10^{-9}$	0	Preferred-location effects	Gravimeter data
α_1	$0\pm4\times10^{-5}$	0	Preferred-frame effects	Lunar laser ranging
α_2	$0\pm2\times10^{-9}$	0		Sun axis' alignment with ecliptic
α_3	$0\pm4\times10^{-20}$	0		Pulsar spin-down statistics
ζ_1	0 ± 0.02	0	Violation of conservation of total momentum	Combined PPN bounds
ζ_2	$0\pm4\times10^{-5}$	0		Binary pulsar acceleration (PSR 1913+16)
ζ_3	0 ± 10^{-8}	0		Lunar acceleration
ζ_4	0 ± 0.006	0		Kreuzer experiment

PPN and massive scalar-tensor theories

- Very massive scalar field $m_\phi r >> 1$

- Very light scalar field $m_\phi r << 1$

J. Alsing, E. Berti, C. M. Will, and H. Zaglauer, Phys. Rev. D **85**, 064041 (2012).

PPN approach

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\phi = \phi_0 + \varphi$$

$$R_{\mu\nu} = \frac{1}{1+\phi}\Big(k^2(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T)$$

$$+ \frac{1}{2}g_{\mu\nu}(V(\phi) + \nabla_\alpha\nabla^\alpha\phi) + \nabla_\mu\nabla_\nu\phi - \frac{3}{2\phi}\partial_\mu\phi\partial_\nu\phi\Big),$$

$$-\nabla_\mu\nabla^\mu\phi + \frac{1}{2\phi}\partial_\mu\phi\partial^\mu\phi + \frac{\phi[2V(\phi) - (1+\phi)V_\phi]}{3} = \frac{\phi k^2}{3}T.$$

General form of the PPN metric

$$
\begin{aligned}
g_{00} =& -1 + 2U - 2\beta U^2 - 2\xi\Phi_w + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 + \\
&+ 2\,(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\Phi_2 + 2(1 + \zeta_3)\Phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi)\Phi_4 - \\
&- (\zeta_1 - 2\xi)A - (\alpha_1 - \alpha_2 - \alpha_3)w^2 U - \alpha_2 w^i w^j U_{ij} + (2\alpha_3 - \alpha_1)w^i V_i, \\
g_{0i} =& -\frac{1}{2}\,(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i - \frac{1}{2}\,(1 + \alpha_2 - \zeta_1 + 2\xi)W_i - \\
&- \frac{1}{2}\,(\alpha_1 - 2\alpha_2)w^i U - \alpha_2 w^j U_{ij}, \\
g_{ij} =& (1 + 2\gamma U)\delta_{ij}.
\end{aligned}
$$

C. M. Will, *Theory and Experiment in Gravitational Physics*, (Cambridge University Press, London, 1981);

PPN parameter γ

$$\gamma = \frac{1+\phi_0 \exp(-m_\varphi r)/3}{1-\phi_0 \exp(-m_\varphi r)/3}^{*}$$

where
m_ϕ is scalar field mass,
ϕ_0 is the asymptotical value of the scalar field far away from the local system.

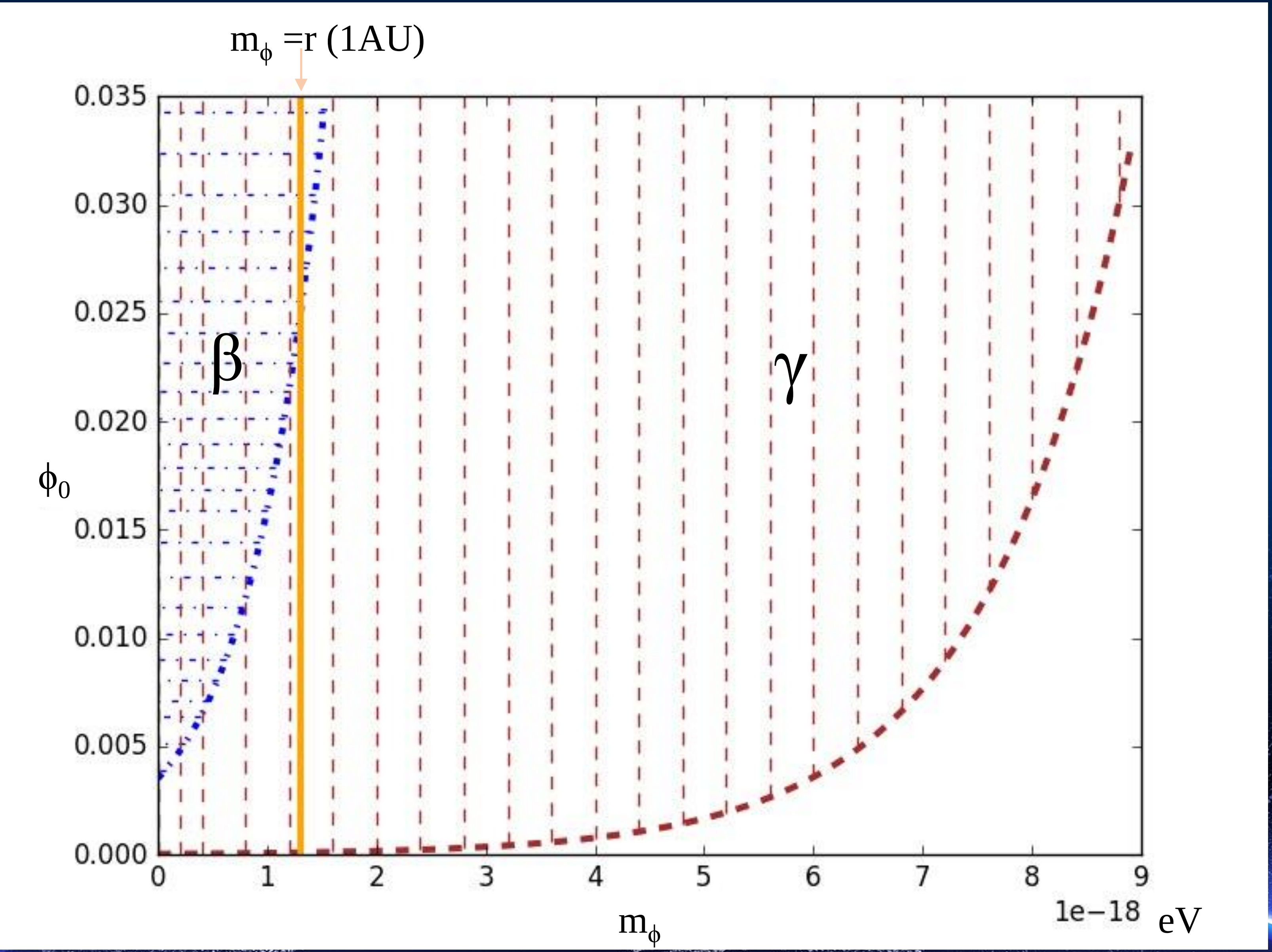
*Hybrid metric-Palatini gravity, S. Capozziello etc., Universe (2015)

PPN parameter β

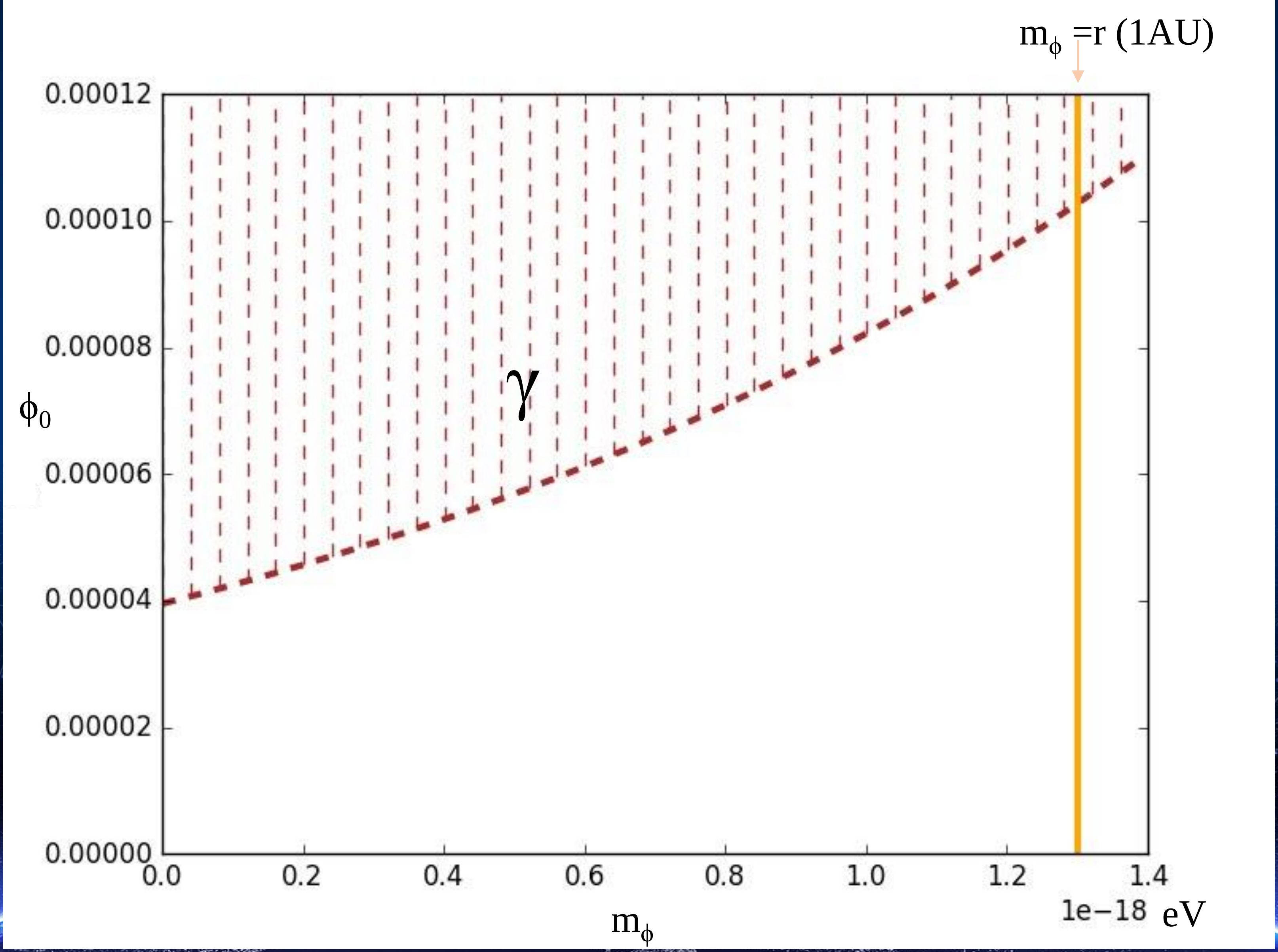
$$\beta = 1 - \frac{\phi_0(1+\phi_0)}{18}\frac{\exp(-2m_\varphi r)}{(1-\phi_0\exp(-m_\varphi r)/3)^2}$$

Other PPN parameters ξ, α_1, α_2, α_3, ζ_1, ζ_1, ζ_1, ζ_1 equal to zero in hybrid gravity in considered approximation $m_\phi \ll 1/r$.

Constraints on ϕ_0 and m_ϕ



Constraints on ϕ_0 and m_ϕ



Conclusions

- PPN formalism is applied to hybrid f(R)-gravity in the case $m_\phi\, r \ll 1$
- restrictions on the m_ϕ and ϕ_0 were obtained in the weak-field limit
- the theory is valid in the Solar system in the case of light scalar field

Thank you for your attention!

PPN potentials

$$U = \int \frac{\rho'}{|\mathbf{x} - \mathbf{x}'|} \, d^3x', \quad U_{ij} = \int \frac{\rho'(x - x')_i (x - x')_j}{|\mathbf{x} - \mathbf{x}'|^3} \, d^3x',$$

$$\Phi_w = \int \frac{\rho' \rho'' (\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} \left(\frac{\mathbf{x}' - \mathbf{x}''}{|\mathbf{x} - \mathbf{x}''|} - \frac{\mathbf{x} - \mathbf{x}''}{|\mathbf{x}' - \mathbf{x}''|} \right) d^3x' d^3x'',$$

$$A = \int \frac{\rho' (\mathbf{v}'(\mathbf{x} - \mathbf{x}'))^2}{|\mathbf{x}\text{-}\mathbf{x}'|^3} \, d^3x',$$

$$\Phi_1 = \int \frac{\rho' v'^2}{|\mathbf{x} - \mathbf{x}'|} \, d^3x', \quad \Phi_2 = \int \frac{\rho' U'}{|\mathbf{x} - \mathbf{x}'|} \, d^3x',$$

$$\Phi_3 = \int \frac{\rho' \Pi'}{|\mathbf{x} - \mathbf{x}'|} \, d^3x', \quad \Phi_4 = \int \frac{p'}{|\mathbf{x} - \mathbf{x}'|} \, d^3x',$$

$$V_i = \int \frac{\rho' v_i'}{|\mathbf{x} - \mathbf{x}'|} \, d^3x', \quad W_i = \int \frac{\rho' \mathbf{v}' (\mathbf{x} - \mathbf{x}')(x - x')_i}{|\mathbf{x} - \mathbf{x}'|^3} \, d^3x'.$$