Horava Gravity is Asymptotically Free in 2+1 Dimensions

A.O.Barvinsky

Theory Department, Lebedev Physics Institute, Moscow

with D. Blas
M. Herrero-Valea
S. Sibiryakov
and C. Steinwachs

Plan

Introduction:

towards local, unitary, perturbatively UV renormalizable QG

Horava-Lifshitz gravity

Problems with renormalization:

BPHZ renormalization and “regularity” of propagators
gauge invariance of UV counterterms (yesterday’s talk)

“Regular” propagators and gauge fixing conditions

Asymptotic freedom of (2+1)-dimensional Horava gravity

Conclusions

D. Blas, M. Herrero-Valea, S. Sibiryakov C. Steinwachs & A.B.,
Phys. Rev. D 93, 064022 (2016), arXiv:1512.02250;
arXiv:1705.03480; PRL 119,211301 (2017), arXiv:1706.06809

Einstein GR

$$S_{EH} = \frac{M_P^2}{2} \int dtd^dx \; R$$

nonrenormalizable

$$\Rightarrow \frac{M_P^2}{2} \int dtd^dx \; (h_{ij}\Box h_{ij} + h^2\Box h + \ldots)$$

Higher derivative gravity

Stelle (1977)

$$\int (M_P^2 R + R_{\mu\nu}R^{\mu\nu} + R^2)$$

$$\Rightarrow \int (M_P^2 h_{ij}\Box h_{ij} + h_{ij}\Box^2 h_{ij} + \ldots)$$

dominates at $k \gg M_P$

The theory is renormalizable and asymptotically free !

Fradkin, Tseytlin (1981)
Avramidy & A.B. (1985)

But has ghost poles $\Rightarrow$ no unitary interpretation

Saving unitarity

Horava (2009)

$$\underbrace{\int dt\, d^d x}_{\propto b^{-(z+d)}}\left(\dot{h}_{ij}\dot{h}_{ij} - h_{ij}(-\Delta)^z h_{ij} + \dots\right)$$

$$\mathbf{x} \mapsto b^{-1}\mathbf{x}\,, \qquad t \mapsto b^{-z} t \qquad \Longrightarrow \qquad h_{ij} \mapsto b^{(d-z)/2} h_{ij}$$

Critical theory in $z = d$

LI is necessarily broken. We want to preserve as many symmetries, as possible

$$x^i \mapsto \tilde{x}^i(\mathbf{x}, t) \qquad \Longrightarrow \qquad \gamma_{ij} \quad N^i, \qquad i = 1, \dots, d$$

$$t \mapsto \tilde{t}(t) \qquad \Longrightarrow \qquad N$$

Foliation preserving diffeomorphisms

$$x^i \mapsto \tilde{x}^i(\mathbf{x}, t) \ , \quad t \mapsto \tilde{t}(t)$$

ADM metric decomposition

$$ds^2 = N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \ , \quad i, j = 1, \ldots, d$$

space dimensionality

Anisotropic scaling transformations and scaling dimensions

$$x^i \to \lambda^{-1} x^i, \quad t \to \lambda^{-z} t, \quad N^i \to \lambda^{z-1} N^i, \quad \gamma_{ij} \to \gamma_{ij},$$

$$[x] = -1, \ [t] = -z, \ [N^i] = z - 1, \ [\gamma_{ij}] = 0, \qquad [K_{ij}] = z.$$

extrinsic curvature

$$K_{ij} = \frac{1}{2N}(\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i)$$

``Projectable" theory $N = \text{const} = 1$

Horava gravity action

$$S = \frac{1}{2G}\int dt\, d^dx \sqrt{\gamma} N \Big(\overbrace{K_{ij}K^{ij} - \lambda K^2}^{\text{kinetic term -- unitarity}} - \mathcal{V}(\gamma) \Big)$$

$$K_{ij} = \frac{1}{2N}(\dot\gamma_{ij} - \nabla_i N_j - \nabla_j N_i)$$

Potential term

$$\mathcal{V}(\gamma) = 2\Lambda - \eta R + \mu_1 R^2 + \mu_2 R_{ij}R^{ij} + \nu_1 R^3 + \nu_2 R R_{ij} R^{ij}$$
$$+\nu_3 R^i_j R^j_k R^k_i + \nu_4 \nabla_i R \nabla^i R + \nu_5 \nabla_i R_{jk} \nabla^i R^{jk} + \ldots$$

Many more versions: extra structures in non-projectable theory, reduction of structures for detailed balance case . . .

$$N \neq \text{const}, \quad a_i = \nabla_i \ln N$$

Divergences power counting

$$\text{Deg of div} \int \frac{d^{d+1}p}{\left(p^2\right)^N} = d + 1 - 2N = \text{physical dimensionality}$$

$$\Downarrow \qquad p = (\omega, \mathbf{k}), \quad p^2 \to \omega^2 + \mathbf{k}^{2z}$$

$$\text{Deg of div} \int \frac{d\omega\, d^d k}{\left(\omega^2 + \mathbf{k}^{2z}\right)^N} = z + d - 2zN = \text{scaling dimensionality}$$

$$\text{physical dimensionality} \neq \text{scaling dimensionality}$$

Counting degree of divergences and dimensionalities

$$\mathrm{Tr}\,\ln\Big(-\partial_t^2+(-\Delta)^z+\ldots\Big)\Big|_{\mathrm{div}}$$

$$= -\int_{\epsilon}^{\infty}\frac{ds}{s}\,e^{-s\,[-\partial_t^2+(-\Delta)^z+\ldots]}\Big|_{\mathrm{div}} = \int dt\,d^dx\,\gamma^{1/2}\sum\frac{\nabla^{2k}R^n\partial_t^rK^p}{\epsilon^D}$$

$$[\,s\,]=[\,\epsilon\,]=-2z$$

$$D=\frac{d+z-2(n+k)-(p+r)z}{2z}$$

degree of divergence

$$D\le 0,\;\; p\ge 2\;\Rightarrow z\ge d \qquad \boxed{z=d}$$

critical value

Log divergent potential terms

$$r+p=0,\;\; D=1-\frac{k+n}{d}=0\;\Rightarrow k+n=d,\;\;[\,\nabla^{2k}R^n\,]=2d$$

$$\mathcal{V}^{(d=2)}(\gamma)=2\Lambda-\eta R+\mu_1R^2$$

$$\mathcal{V}^{(d=3)}(\gamma)=2\Lambda-\eta R+\mu_1R^2+O(R^3,R\nabla^2R)$$

Things are not so simple:

Why power counting is not enough?

$$\int \prod_{l=1}^{L} d^{d+1}k^{(l)}\ \mathcal{F}_n(k) \prod_{m=1}^{M} \frac{1}{\left(P^{(m)}(k)\right)^2} \implies$$

$$\int \prod_{l=1}^{L} d\omega^{(l)} d^d k^{(l)}\ \mathcal{F}_n(\omega, \mathbf{k}) \prod_{m=1}^{M} \frac{1}{A_m \left(\Omega^{(m)}(\omega)\right)^2 + B_m \left(\mathbf{K}^{(m)}(\mathbf{k})\right)^{2z}}$$

Generalization of BPHZ renormalization theory (subtraction of subdivergences) works only for $\underbrace{A_m > 0 \ \text{and} \ B_m > 0}_{\text{depends on gauge fixing}}$

Toy model: d=2

$$S = \frac{1}{2G}\int dt\, d^2x\big(K_{ij}K^{ij} - \lambda K^2 - \mu R^2\big)$$

propagates a single scalar that is well-behaved at $\lambda < 1/2$ or $\lambda > 1$

We need to fix the gauge:

linear combination of the fields

$$\mathcal{L}_{gf} = \frac{\sigma}{2G} F^i \mathcal{O}_{ij} F^j$$

invertible operator

$$\mathcal{O}_{ij} = \delta_{ij}, \quad \sigma \to \infty \quad \Rightarrow \quad F^i = N^i = 0$$

$$\Downarrow$$

$$\mathcal{P}_{\mathsf{s}}(\omega,p) = \frac{1}{\omega^2 + 4\mu\frac{1-\lambda}{1-2\lambda}\, p^4}$$

$$\langle h_{ij}(p) h_{kl}(-p)\rangle = \frac{4\kappa^2(1-\lambda)}{1-2l}\delta_{ij}\delta_{kl}\, P_s(p)$$

$$\sim \frac{1}{\omega^2}$$

$$+\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - 2\delta_{ij}\delta_{kl}\right)\frac{2\kappa^2}{\omega^2}$$

$$\frac{O(p^4)P_s(p)}{\omega^2} \sim \frac{1}{\omega^2}$$

$$+16\kappa^2\mu\left[\frac{1-\lambda}{1-2\lambda}\left(\delta_{ij}p_kp_l + p_ip_j\delta_{kl}\right)p^2 - p_ip_jp_kp_l\right]\frac{P_s(p)}{\omega^2}$$

$$\Rightarrow \quad (\ldots) \times |\, t \,|\, \delta^{(2)}(\mathbf{x})$$

Analogy: Coulomb gauge in QED and YM theory

What is the analogue of relativistic gauges?

Regular gauges for projectable HG

Hint from relativistic theory:

$$F^\mu \equiv \partial_\nu h^{\nu\mu} + \ldots \;\Rightarrow\; F^i = \dot{N}^i + c\partial_j \Delta h^{ji} + \ldots$$

extra derivatives to have homogeneity in scaling

$$[\, F^i \,] = 3 \;\Rightarrow\; [\, \mathcal{O}_{ij} \,] = -2, \qquad \mathcal{O}_{ij} = (\Delta \delta_{ij} + \xi \partial_i \partial_j)^{-1}$$

nonlocal in space

Localization by auxiliary fields:

$$\int dt\, dt^2 x\, F^i \mathcal{O}_{ij} F^j \;\mapsto\; \int dt\, d^2 x\, \left(-\frac{1}{2} \pi_i\, \mathcal{O}^{-1\, ij} \pi_j - i \pi_i F^i \right)$$

local

The choice

$$F^i = \dot{N}^i + \frac{1}{2\sigma}\mathcal{O}^{-1}_{ij}\partial_k h_{jk} - \frac{\lambda}{2\sigma}\mathcal{O}^{-1}_{ij}\partial_j h$$

decouples N^i from h_{ij} in the quadratic action

regular propagators for all fields (including Faddeev--Popov ghosts)

two free gf. parameters $\sigma,\ \xi$

Straightforward generalization to $d > 2$, e.g.

$$\mathcal{O}^{d=3}_{ij} = \Delta^{-1}\big(\delta_{ij}\Delta + \xi\partial_i\partial_j\big)^{-1}$$

Gauge invariance of counterterms

Background covariant gauge conditions + BRST structure of renormalization (yesterday's talk)

Background field method:

$$\gamma_{ij} = \bar\gamma_{ij} + h_{ij}, \;\; \partial_i \to \bar\nabla_i, \;\; \mathcal{O}^{-1\,ij} = \bar\Delta\,\bar\gamma^{ij} + \xi\bar\nabla^i\bar\nabla^j, \;\ldots$$

DeWitt, Tyutin, Voronov, Stelle, Batalin, Vilkovisky, Slavnov, Arefieva, Abbott…
Barnich, Henneaux, Grassi, Anselmi,…

Blas, Herrero-Valea, Sibiryakov, Steinwachs & A.B.
arXiv:1706.06809

Non-Projectable model

one more variable $N = 1 + \phi$ + one more equation

$\Rightarrow$ still TT + a single scalar

Good: $\omega_s^2 \propto +k^2$ at $k \to 0$

Bad: at $k \to \infty$

$$\langle \phi\phi \rangle = \text{regular} + \frac{1}{k^{2d}}$$

present even in $\sigma\xi$ - gauges
physical: shows up in the interaction of local sources

Blas, Pujolas, S.S. (2011)
Blas, S.S. (2011)

Asymptotic freedom in (2+1)-dimensions

$$S = \frac{1}{2G}\int dt\, d^2x\; N\sqrt{\gamma}\;\left(K_{ij}K^{ij} - \lambda K^2 + \mu R^2\right)$$

Off-shell extension is not unique:

$$\Gamma_{1-\mathrm{loop}} \to \Gamma_{1-\mathrm{loop}} + \int dt\, d^dx\, \Omega_{ij}\frac{\delta S}{\delta\gamma_{ij}}$$

$$\Gamma \to \Gamma + \varepsilon\int dt\, d^2x\;\sqrt{\gamma}\;\left[K_{ij}K^{ij} - \lambda K^2 - \mu R^2\right]$$

$$\delta G = -2G^2\varepsilon, \quad \delta\lambda = 0, \quad \delta\mu = -4G\mu\varepsilon$$

Essential coupling constants: $\lambda. \quad \mathcal{G} \equiv \dfrac{G}{\sqrt{\mu}}$

background split

$$\gamma_{ij} \to \gamma_{ij} + h_{ij}, \qquad N_i = 0 + n_i$$

background covariant gauge-fixing term
σ, ξ – free parameters

$$S_{\mathrm{gf}} = \frac{\sigma}{2G} \int dt\, d^2x\, \sqrt{\gamma}\, F_i\, \mathcal{O}^{ij} F_i$$

$$F_i = \partial_t n_i + \frac{1}{2\sigma} \mathcal{O}^{-1}_{ij} (\nabla^k h^j_k - \lambda \nabla^j h)$$

$$\mathcal{O}^{ij} = -[\gamma_{ij}\Delta + \xi \nabla_i \nabla_j]^{-1}$$

localization of the kinetic term by auxilairy field π

$$\frac{\sigma}{2G} \int dt\, dt^2 x \sqrt{\gamma}\; \partial_t n_i \frac{-1}{\gamma_{ij}\Delta + \xi \nabla_i \nabla_j} \partial_t n_j \mapsto$$

$$\frac{1}{2G} \int dt\, d^2x \sqrt{\gamma}\; \left(-\frac{1}{2\sigma} \pi^i\, \mathcal{O}^{-1}_{ij} \pi^j - i\pi^i \partial_t n_i \right)$$

action of ghosts:

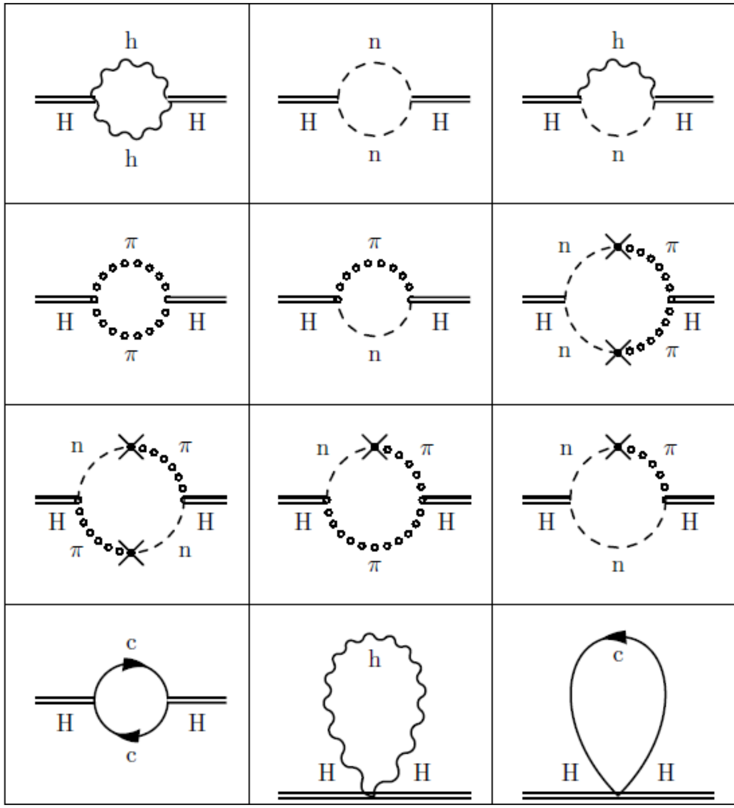
$$S_{\mathrm{gh}} = -\int dt\, d^2x \sqrt{\gamma}\, \bar{c}^i \Big[\partial_t \left(\gamma_{ij} \partial_t c^j \right) - \frac{1}{2\sigma} \Delta^2 (\gamma_{ij} c^j) - \frac{1}{2\sigma} \Delta \nabla_k \nabla_i c^k$$
$$+ \frac{\lambda}{\sigma} \Delta \nabla_i \nabla_j c^j - \frac{\xi}{2\sigma} \left(\nabla_i \nabla_j \Delta c^j + \nabla_i \nabla_j \nabla_k \nabla^j c^k - 2\lambda \nabla_i \Delta \nabla_j c^j \right) \Big]$$

Perturbation theory around

flat ST: $\gamma_{ij} = \delta_{ij} + H_{ij}$

Diagrammatic technique in terms

of $H_{ij},\ n_i,\ \pi_i,\ c^i,\ \bar{c}^i$



$$\beta_G \qquad \beta_\lambda$$

$$S_{\mathsf{H}} = \frac{1}{2G}\int dt d^2x \Big\{ \tfrac{1}{4}\left(\dot{H}_{ij}\dot{H}^{ij} - \lambda \dot{H}\dot{H}\right)$$

$$-\mu\partial_b\partial_a H^{ab}(2\Delta H - \partial_j\partial_i H^{ij}) + \mu\Delta H\Delta H + O(H^3)\Big\}$$

$$\beta_\mu$$

Propagators $\propto \mathcal{P}_{\mathsf{S}}(\omega,p) = \dfrac{1}{\omega^2 + 4\mu\frac{1-\lambda}{1-2\lambda}\, p^4}$

$$\sigma = \frac{1-2\lambda}{8\mu(1-\lambda)}$$
$$\xi = -\frac{1-2\lambda}{2(1-\lambda)}$$

particular gauge parameters

Prototypical loop ntegrals

$$\int \frac{d\omega d^2q}{(2\pi)^3}\, \omega^{2a} q^{2b} \prod_I \mathcal{P}_s(\omega+\Omega_I,\, q+P_I)$$

Expansion in external momenta P and fequencies Ω

$$\rightarrow \sum_{a,b,A} \int \frac{d\omega d^2q}{(2\pi)^3}\, \omega^{2a} q^{2b}\, (\mathcal{P}_s(\omega,q))^A$$

$$= \sum_{a,b,A} \int_0^\infty \frac{ds\ s^{A-1}}{\Gamma(A)} \int \frac{d\omega d^2q}{(2\pi)^3}\, \omega^{2a} q^{2b}\, e^{-s(\mathcal{P}_s(\omega,q))^{-1}}$$

Logarithmic divergences and β-functions

$$\int d(\log s) \mapsto \log\left(\frac{\Lambda_{\mathsf{UV}}^4}{k_*^4}\right) \quad \Rightarrow \quad \beta_g \equiv k_* \frac{dg(k_*)}{dk_*}$$
$$g = G, \lambda, \mu$$

Mathematica package xAct

D. Brizuela, J. M. Martin-Garcia, and G. A. Mena Marugan, Gen. Rel. Grav. 41, 2415 (2009), arXiv:0807.0824

$$\beta_\lambda = \frac{15-14\lambda}{64\pi}\sqrt{\frac{1-2\lambda}{1-\lambda}}\,\mathcal{G}$$

$$\beta_{\mathcal{G}} = -\frac{(16-33\lambda+18\lambda^2)}{64\pi(1-\lambda)^2}\sqrt{\frac{1-\lambda}{1-2\lambda}}\,\mathcal{G}^2$$

Check in conformal gauge $h_{ij} = e^{2\phi}\gamma_{ij}, \quad \gamma_{ij} = \delta_{ij} + H_{ij}$

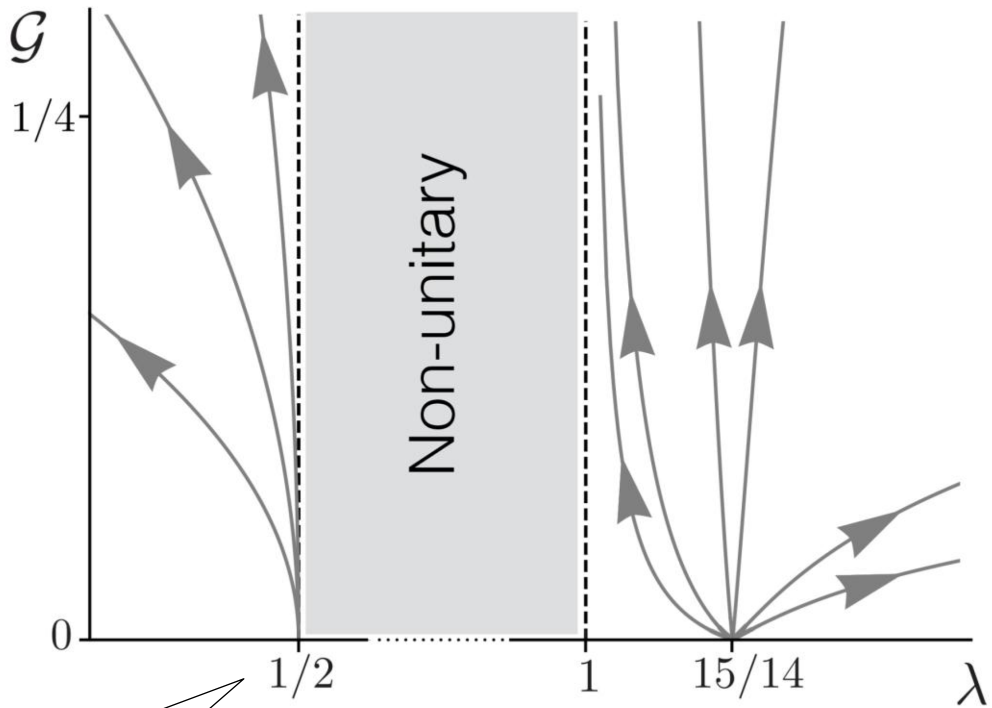
$$\left.\begin{aligned}\beta_\mu &= \frac{2-7\lambda+6\lambda^2}{32\pi(1-\lambda)^{3/2}\sqrt{1-2\lambda}}\,G\sqrt{\mu}\\ \beta_G &= -\frac{6\lambda-7}{32\pi\sqrt{(1-2\lambda)(1-\lambda)}}\,\frac{G^2}{\mu}\end{aligned}\right\} \Rightarrow \beta_{\mathcal{G}}$$

same

Compare to regular ``relativistic" gauge

$$\left.\begin{aligned}\beta_\mu &= \frac{30-73\lambda+42\lambda^2}{32\pi(1-\lambda)^{3/2}\sqrt{1-2\lambda}}\,G\sqrt{\mu}\\ \beta_G &= -\frac{30\lambda-23}{32\pi\sqrt{(1-2\lambda)(1-\lambda)}}\,\frac{G^2}{\mu}\end{aligned}\right\} \Rightarrow \beta_{\mathcal{G}}$$

Renormalization flows:



strongly coupled f. point

$$\mathcal{G} \to \tilde{\mathcal{G}} = \frac{G}{\sqrt{1-2\lambda}}$$

$$\beta_{\tilde{\mathcal{G}}} = -\frac{(1-2\lambda)^2}{64\pi(1-\lambda)^{3/2}}\,\tilde{\mathcal{G}}^2$$

AF UV fixed point

Cf. conformal truncation of (2+1)D HG, Benedetti and Guarneri, JHEP 03(2014)078: f. point λ=1/2, unitarity at G<0, λ>1

Conclusions

Projectable versions of HG are renormalizable -- “regular” propagators and gauge fixing conditions

(2+1)-dimensional Horava-Lifshitz gravity is unitary and asymptotically free – perturbatively UV-complete theory

In IR one can have $\lambda \to 1$ – GR limit? Need for nonperturbative analysis as $G \to \infty$

Towards consistent (3+1)-dimensional QG – other calculational methods (heat kernel technique)