

Metastable Q-balls.

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- 1 Classical solution
- 2 Saddle point approximation for path integral
- 3 Numerical results

Bose gas at $T = 0$

Ideal Bose gas can be localized in the harmonic trap:

$$\Psi \sim \exp(-\omega^2 x^2)$$

This system can be studied both experimentally and theoretically, as classical theory of the complex field Ψ with Lagrangian

$$i\Psi^* \frac{d}{dt} \Psi - \frac{1}{2m} |\nabla \Psi|^2 - U(x) |\Psi|^2 + \lambda_0 |\Psi|^4$$

Dimensionless combination $\lambda = m^2 \lambda_0$ is not small! Nonlinear term is just a correction to potential — dilute gas approximation.

Although Rb is a metal at usual temperature.

Field theory in semiclassical regime

In relativistic field theory one can obtain bag for the same field...
For the validity of semiclassical approximation one can use potential of the form

$$V = \frac{m^4}{g^2} U(g|\phi|/m),$$

then, after redefinition $\phi = g\phi$ we obtain Lagrangian without small parameters and overall factor $1/g^2$ before action.

In this case semiclassical method is a saddle point approximation for path integral.

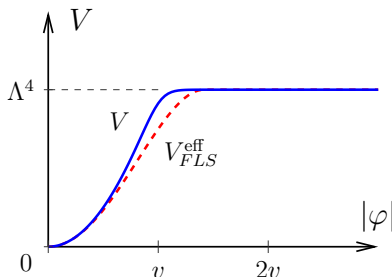
Soliton in classical theory is an analog of the bag — but interaction is crucial.

Choice of potential

Potential which admits analytical solution

$$V(|\phi|) = m^2 |\phi|^2 \theta \left(1 - \frac{|\phi|^2}{v^2} \right) + m^2 v^2 \theta \left(\frac{|\phi|^2}{v^2} - 1 \right)$$

is precisely what we need if we use smooth regularization and $g = m/v$, $\Lambda = m^2 v^2$. *(see also talk of A.Shkerin)*



Q-balls

Regularized potential admits Q-balls (general conditions on potential see Coleman'85) in single field theory with global $U(1)$ -invariance

$$\partial_\mu \phi^* \partial^\mu \phi - V(|\phi|)$$

We will assume some regularization, but it does not change classical solution seriously (we checked this statment numerically and will use numerical calculations for main result). Localized solution — stationary ansatz

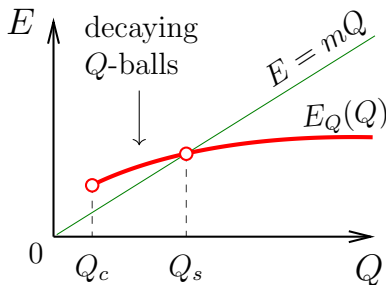
$$\phi = f(r)e^{i\omega t}$$

one can minimize energy E at fixed global charge

$$Q \sim \omega \int f^2(r) r^2 dr$$

Properties of Q-balls

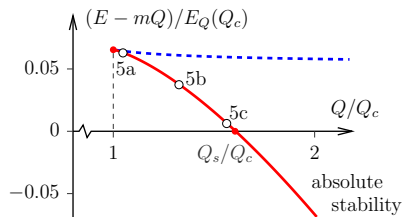
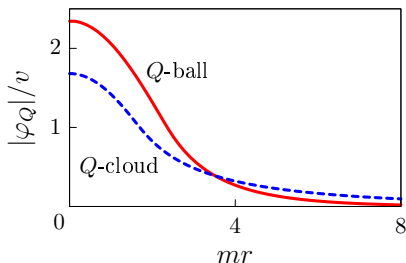
Energy can be compared with energy of free particles $E = mQ$ for the same charge



There is a region of charges where tunneling is kinematically possible... What about potential barrier?

Q-clouds

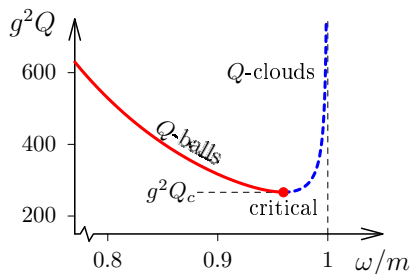
There is unstable branch of solutions
— Q-clouds (M. Alford'88).



Q-cloud is more thick for the same
global charge

Instability of Q-clouds

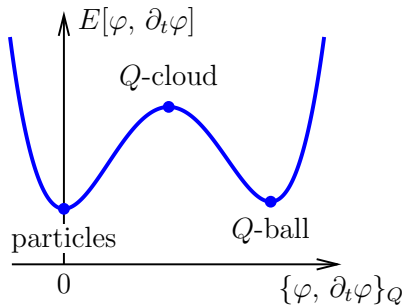
Vakhitov-Kolokolov criterion $\partial Q / \partial \omega > 0$



Only one decay mode for simple potential (EN, A. Shkerin, 2014).
Looks like sphaleron (critical bubble)?

Interpretation of Q-clouds

For compactified theory we have three solutions for the same charge. Height of the pass is finite and the tunneling is indeed possible.



Bounce solution

Let us compare the problem with ordinary false vacuum decay

- we already work in semiclassical regime
- we know initial configurations for different charges
- we have crosscheck for Euclidean solutions due to $U(1)$ -invariance
- additional check: kinematics

Objections:

- Euclidean continuation of stationary factors $e^{i\omega t}$ is not healthy for numerical calculations
- Euclidean solution is not $O(4)$ -invariant!

Euclidean theory

Saddle point solution \rightarrow minimum of euclidean action, $t = i\tau$.

$$S_E = \int_{-\beta}^{\beta} d\tau d^3\mathbf{x} (\partial_{\tau}\varphi\partial_{\tau}\bar{\varphi} + \nabla\varphi\nabla\bar{\varphi} + V(\varphi\bar{\varphi})) ,$$

Instead of being mutually conjugate, φ and $\bar{\varphi}$ should be considered as independent *real* functions of \mathbf{x} and τ . β will be sent to infinity in the end of the calculation.

$$(\partial_{\tau}^2 + \nabla^2)\varphi_{cl} = V'\varphi_{cl} , \quad (\partial_{\tau}^2 + \nabla^2)\bar{\varphi}_{cl} = V'\bar{\varphi}_{cl} ,$$

where V' is a derivative of $V(\bar{\varphi}\varphi)$ with respect to its argument.
Boundary conditions are nontrivial and $O(4)$ -invariance is broken.

Quantum state for Q-ball

One can define projector onto the states with charge Q

$$\hat{P}_Q = \int_0^{2\pi i} \frac{d\eta}{2\pi i} e^{\eta(\hat{Q}-Q)}, \quad \hat{P}_Q^2 = \hat{P}_Q,$$

then Q-ball state $|Q\rangle$ can be defined by limiting formula

$$e^{-\beta\hat{H}}\hat{P}_Q|i\rangle \rightarrow e^{-\beta E_Q}|Q\rangle\langle Q|i\rangle \quad \text{as} \quad \beta \rightarrow +\infty,$$

here $|i\rangle$ is arbitrary state.

Decay probability

$$\mathcal{P} = \sum_f \left| \langle f | e^{-i\hat{H}t_0} | Q \rangle \right|^2 = e^{2\beta E_Q} \sum_{i,f} \left| \langle f | e^{-i\hat{H}(t_0-i\beta)} \hat{P}_Q | i \rangle \right|^2$$

Technically it is easier to solve e.o.m. with parameter η assuming it will take saddle point value. This modify initial configuration at $\tau \rightarrow -\infty$

$$\varphi_{cl} = e^{-\omega\beta - \eta_0} \chi_Q(r), \quad \bar{\varphi}_{cl} = e^{\omega\beta + \eta_0} \chi_Q(r)$$

which can be formulated in terms of φ_{cl} , $\bar{\varphi}_{cl}$ and their derivatives only

$$\varphi_{cl} = e^{-\omega\beta - 2\eta_0} \bar{\varphi}_{cl}, \quad \partial_\tau \varphi_{cl} = -e^{\omega\beta - 2\eta_0} \partial_\tau \bar{\varphi}_{cl}$$

For the turning point $\tau = 0$ solution should be symmetric with respect to time reflections:

$$\varphi_{cl} = \bar{\varphi}_{cl}, \quad \partial_\tau \varphi_{cl} = -\partial_\tau \bar{\varphi}_{cl}$$

and can represent classical evolution after continuation to Minkowski time

Decay rate

Exponential suppression (saddle point approximation)

$$\Gamma_Q = A_Q \cdot e^{-F_Q} ,$$

where prefactor contains m — dimensional parameter and slow dependence on charge. We can also use our results for finite β to make some conclusions for the case $T \neq 0$.

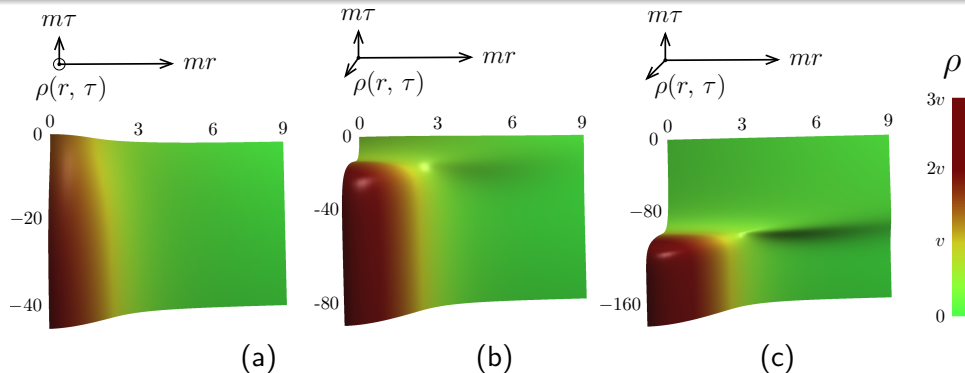


Figure: Semiclassical solutions $\rho(r, \tau) \equiv (\varphi \bar{\varphi})^{1/2}$ describing decay of Q -balls with $Q/Q_c \approx 1.05$ (a), 1.33 (b), and 1.56 (c).

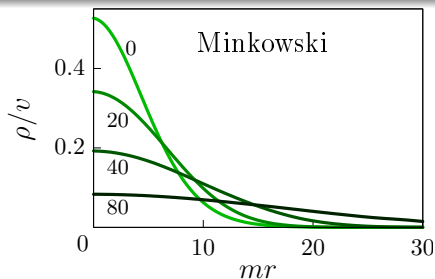
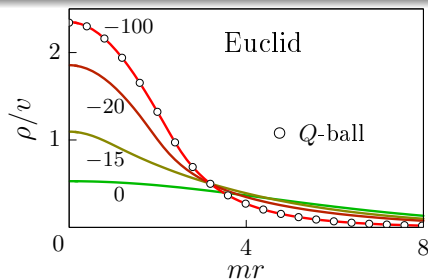
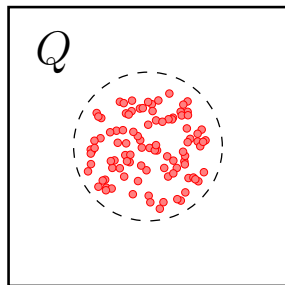


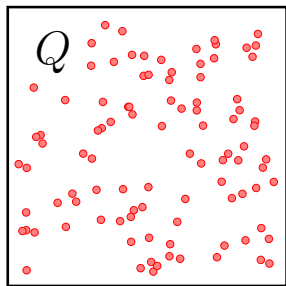
Figure: Left: Sections $\tau = \text{const}$ of the semiclassical solution in the centre of previous Figure (lines). Empty circles show Q -ball configuration with the same charge. The values of $m\tau$ are written near the graphs. On the right – the same semiclassical solution continued to Minkowski time $t = i\tau$ after the turning point. Graphs show field configurations $\rho(r, t) \equiv |\varphi|$ at different times mt (numbers near the graphs). The total disintegration of localized solution can be interpreted as collective tunneling to free particles.

Collective tunneling



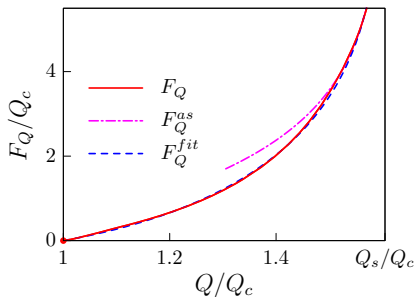
in: Q -ball

collective
tunneling



out: free particles

Suppression exponent



$$F_Q^{as} \rightarrow d_1 + d_2 \log(1 - Q/Q_s)$$

$$F_Q \approx (Q - Q_c) [c_1 + c_2 \log(1 - Q/Q_s)] ,$$

$$\text{where } c_1 = -0.28, c_2 = -2.6.$$

Conclusions

- The smallest classically stable Q -balls are, in fact, generically metastable: they decay into bunches of free warm particles via collective tunneling.
- Our method uses Euclidean field-theoretical solutions resembling the Coleman's bounce and avoid problems due to the stationary factor in Minkowski time.
- We have crosscheck for Euclidean solutions due to $U(1)$ -invariance.
- We obtain the fitting formula for F in the entire metastability window.