Constraints on multiparticle production in a scalar field theory from classical simulations

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Some known results on multiparticle production

Scalar field $\phi(\vec{x}, t)$ in $(3+1)$ dimensions

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4}\phi^4$$

Tree level calculations of $1 \to N$ scattering amplitude:

Cornwall '90, Goldberg '90, Voloshin '92, Brown '92

$$\mathcal{A}^{tree}_{1\to N}(E = Nm) = N! \left(\frac{\lambda}{8m^2}\right)^{\frac{N-2}{2}}$$

$$\mathcal{P}^{tree}_{1\to N} \sim N! \lambda^N e^{Nf(\mathcal{E})} \sim exp(\tfrac{1}{\lambda} F(\lambda N, \mathcal{E})), \quad \mathcal{E} = \frac{E - Nm}{N}$$

Semiclassical methods:

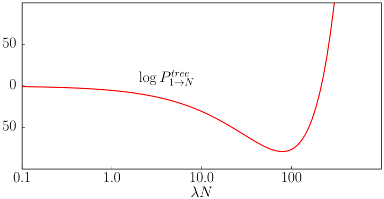
Rubakov, Tinyakov '92, Son '96

Bezrukov, Libanov, Troitsky '95

Loop corrections, $\lambda N \gtrsim 1$

Unitarity-base arguments, *Zakharov '91*

Relativistic regime $\mathcal{E} \gg 1$



Behaviour of $\mathcal{P}_{few \to N}$ is unknown at large $\lambda N \gtrsim 1$ and E

Going to classical transitions

V.Rubakov, D.T.Son '1994

C.Rebbi, R.Singleton '1995, S.D., Levkov '2011

- Let us study not *few* $\to N$ but $N_i \to N_f$ processes with large N_i and N_f – initial and final states are semiclassical
- $N_i, N_f \gg 1$ we have classical counterpart – classical scattering of waves – classical solutions which linearize at $t \to \pm\infty$
- Classical solution with N_i, N_f and E exists – classically allowed process – probability is exponentially unsuppressed
- Classical transition with N_i, N_f and E is forbidden – probability is exponentially suppressed

Setup

We rescale $x \to m^{-1}x$ and $\phi \to \sqrt{\frac{m^2}{\lambda}}\phi$

$$S[\phi] = \frac{1}{\lambda}\int d^4x \left[\frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}\phi^2 - \frac{1}{4}\phi^4\right].$$

We limit ourselves to spherically symmetric case:
$\phi(t,r) = \frac{1}{r}\chi(t,r), \quad \chi(t, r=0) = 0$

$$S = \frac{4\pi}{\lambda}\int dtdr \left[\frac{1}{2}\left(\frac{\partial\chi}{\partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial\chi}{\partial r}\right)^2 - \frac{\chi^2}{2} - \frac{\chi^4}{4r^2}\right]$$

We restrict the system to $r \in [0, R]$ with $\partial_r\chi(t, r=R) = 0$

$$\chi(t,r) = \sum_{n=0}^{\infty} c_n(t)\sqrt{\frac{2}{R}}\sin k_n r,$$

Equations of motion

$$\ddot{c}_n + \omega_n^2 c_n + I_n = 0\,, \quad I_n = \sqrt{\frac{2}{R}}\int_0^R dr\frac{\chi^3(t,r)}{r^2}\sin k_n r \quad n = 0, 1, \ldots$$

Setup

We consider solutions which linearize at initial and final times

$$c_n(t) \to \begin{cases} \frac{1}{\sqrt{2\omega_n}} \left(a_n \mathrm{e}^{-i\omega_n t} + a_n^* \mathrm{e}^{i\omega_n t} \right) & \text{as } t \to -\infty \\ \frac{1}{\sqrt{2\omega_n}} \left(b_n \mathrm{e}^{-i\omega_n t} + b_n^* \mathrm{e}^{i\omega_n t} \right) & \text{as } t \to +\infty \end{cases}$$

Energy

$$E = \frac{4\pi}{\lambda} \sum_n \omega_n \left| a_n \right|^2 = \frac{4\pi}{\lambda} \sum_n \omega_n \left| b_n \right|^2 \ .$$

Particle numbers

$$N_i = \frac{4\pi}{\lambda} \sum_n \left| a_n \right|^2 \ , \quad N_f = \frac{4\pi}{\lambda} \sum_n \left| b_n \right|^2$$

Notations

$$\tilde{E} = \frac{\lambda}{4\pi} E \ , \quad \tilde{N}_i = \frac{\lambda}{4\pi} N_i \ , \quad \tilde{N}_f = \frac{\lambda}{4\pi} N_f \ .$$

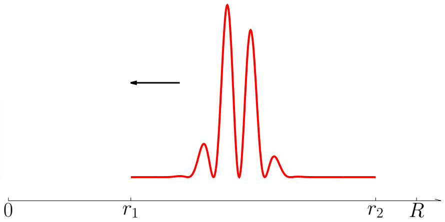
- Truncate Fourier expansion to $n = N_x = 400, 600$ with $R = 20, 30$
- Numerically solve equations, Bulirsch-Stoer method

Initial conditions

We take initial conditions which correspond to localized wavepackets in $I \equiv [r_1, r_2]$

$$\chi(r) = \begin{cases} \sum f_n \sin\left(\tilde{k}_n(r - r_1)\right), \ r \in I \\ 0, r \notin I \end{cases}$$

$\tilde{k}_n = \frac{\pi n}{r_2 - r_1}$, $\tilde{\omega}_n = \sqrt{\tilde{k}_n^2 + 1}$,



Initial wavepacket is defined f_n – initial Fourier amplitudes.
For wavepackets propagating to the left we expect

$$\chi_i(t, r) = \begin{cases} \sum_{n=1}^{i_2 - i_1} \tilde{f}_n \sin\left(\tilde{k}_n(r - r_1) + \tilde{\omega}_n(t - t_{in})\right), \quad r \in [r_1, r_2] \\ 0, r \notin [r_1, r_2] . \end{cases}$$

Going to the classical boundary

Our aim – absolute minimum (maximum) of $\tilde{N}_f$ at fixed $\tilde{N}_i$, $\tilde{E}$

Rebbi, Singleton '96

- Stochastic sampling technique
- Generate ensemble of classical solution with $\tilde{N}_i$ with a weight

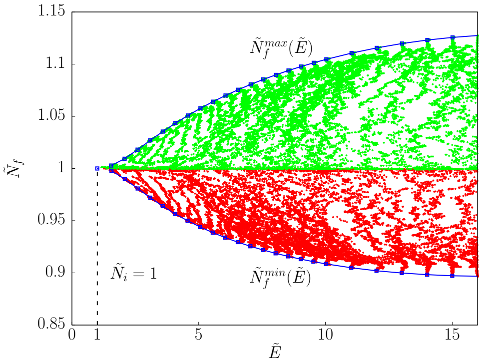
$$p \sim \mathrm{e}^{-F} \text{ , where } F = \beta\left(\tilde{N}_f + \xi(\tilde{E} - \tilde{E}_*)^2\right)$$

- At large positive β and ξ the ensemble will be dominated by solutions having small F, i.e. close to $\tilde{N}_f^{min}$ and E_*
- to reach $\tilde{N}_f^{max}$ one takes $\beta, \xi < 0$

To generate such ensemble we use Metropolis Monte Carlo:

1. generate f_n at random – a solution with $\tilde{N}_i =$ fixed, $\tilde{E}$, $\tilde{N}_f$
2. make a change $f_n \to f_n' = f_n + \Delta f_n$ – normalize to have the same $\tilde{N}_i$, evolve in time and find $\tilde{E}^{new}$, $\tilde{N}_f^{new}$
3. Compute ΔF; solution is accepted with $p_{acc} = \min\left(1, \mathrm{e}^{-\Delta F}\right)$
4. increase ξ, decrease "temperature" $\beta_i = \beta_0 \log(1+i)$

$\tilde{N}_i = 1$, classically allowed region; $R = 20$, $N_x = 400$

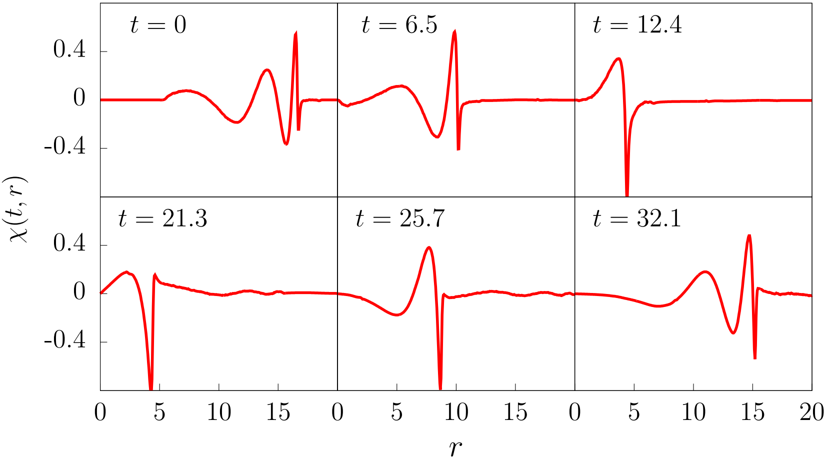


The picture is a combination of several runs with different $\frac{\tilde{E}_*}{\tilde{N}_i} = 1.5, 2.0, ..., 9.5, 10.0, 11.0, ..., 20.0$

The obtained region has a smooth envelopes, $\tilde{N}_f^{min}(\tilde{E})$ and $\tilde{N}_f^{max}(\tilde{E})$, which represent the boundary of the classically allowed region

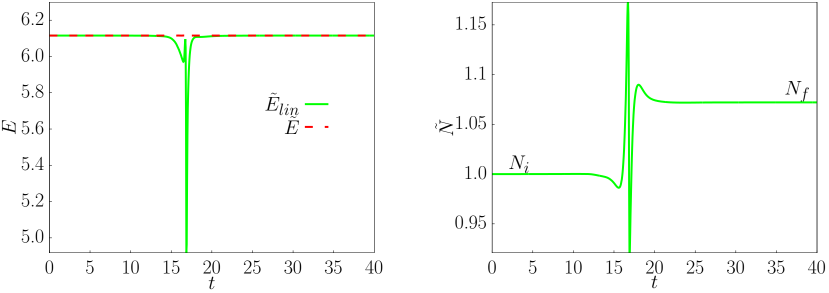
$\tilde{N}_i = 1$, example of solution

Typical time evolution (upper boundary solution at $\tilde{E} \approx 6.0$)



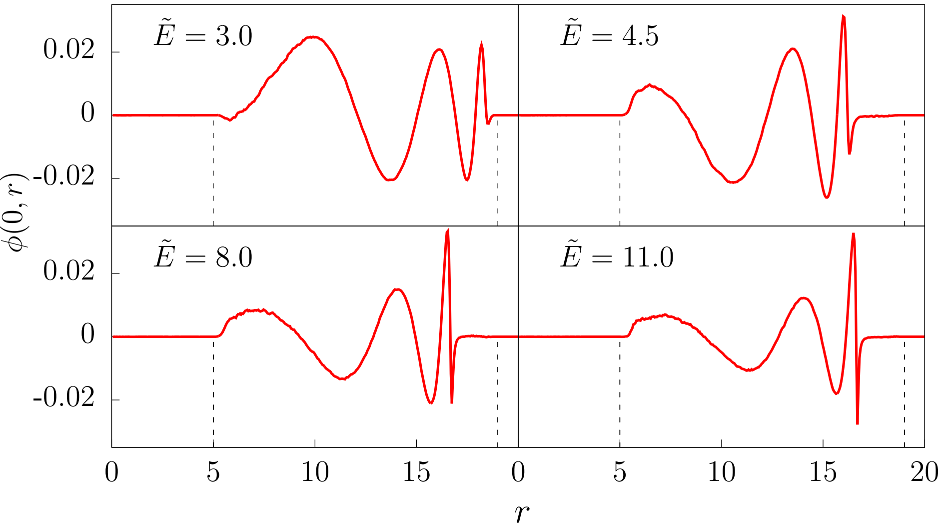
Initial wave packet has a sharp (spiky) part and soft oscillatory part

$\tilde{N}_i = 1$, energy and particle number

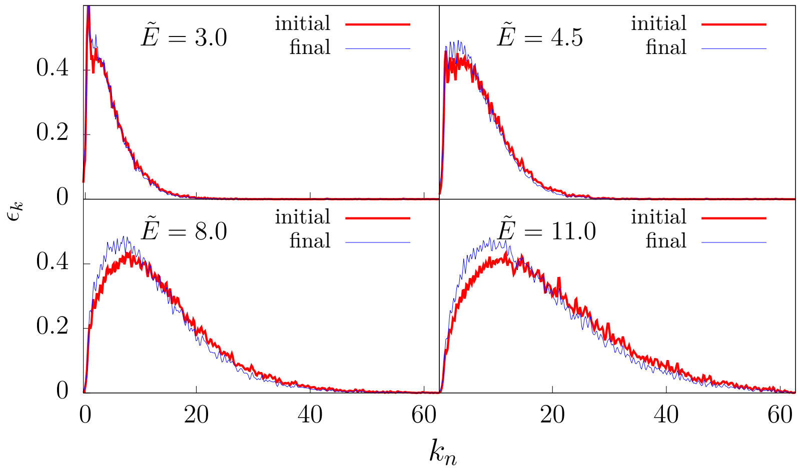


Actual change in $\tilde{N}(t)$ occurs when the sharpest part of the wavepacket reaches the interaction region

$\tilde{N}_i = 1$, initial wavepackets at different energies

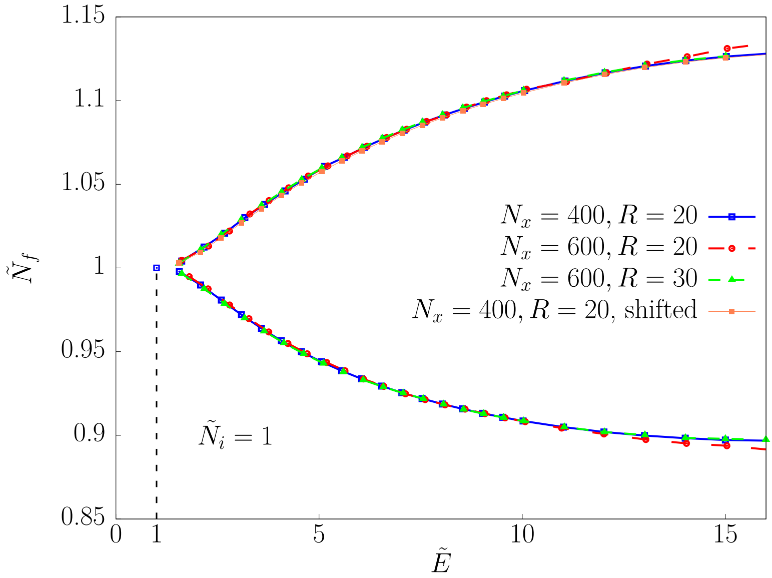


$\tilde{N}_i = 1$, energy distribution per wave number unit

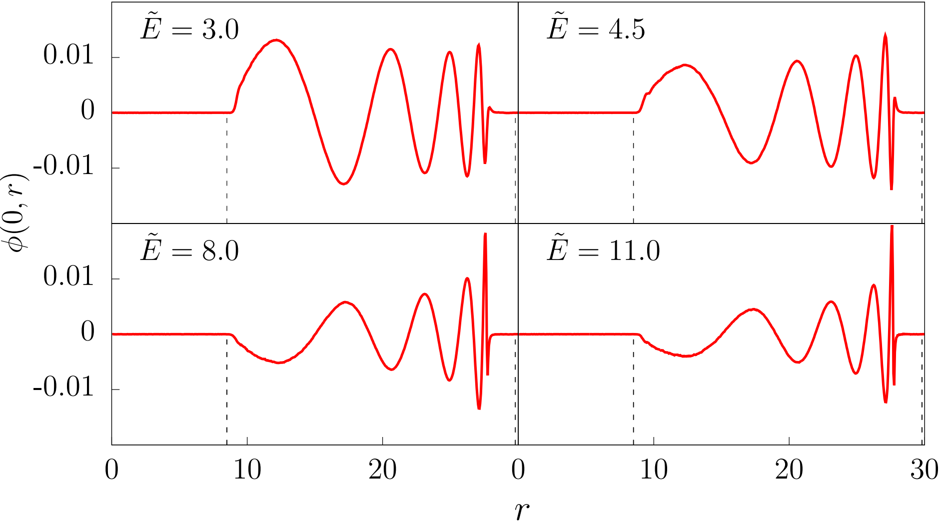


We observe expected softening of the energy distributions for the final wave packet as compared to the initial one

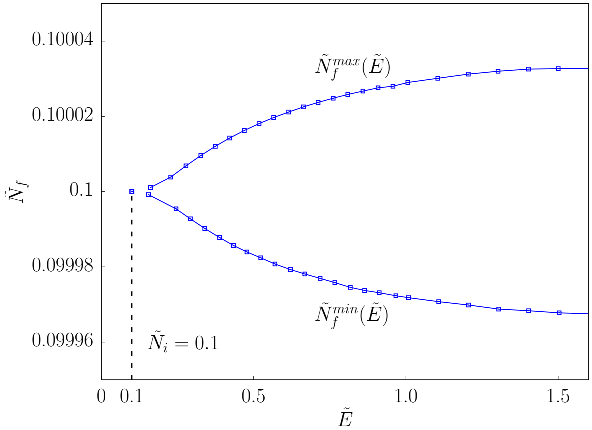
$\tilde{N}_i = 1$, lattice dependence



$\tilde{N}_i = 1$, examples (upper boundary), $L = 30$, $N_x = 600$

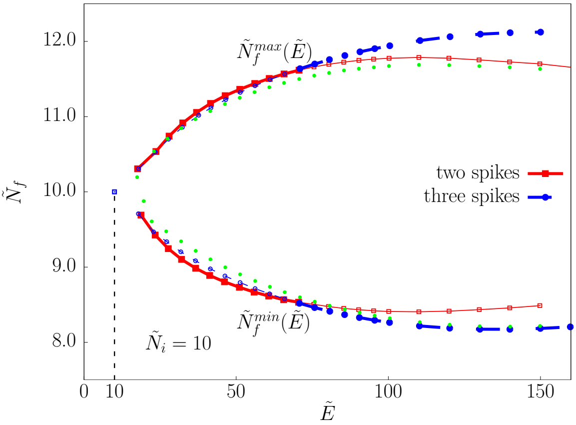


$\tilde{N}_i = 0.1$, classically allowed region, $L = 30$, $N_x = 600$



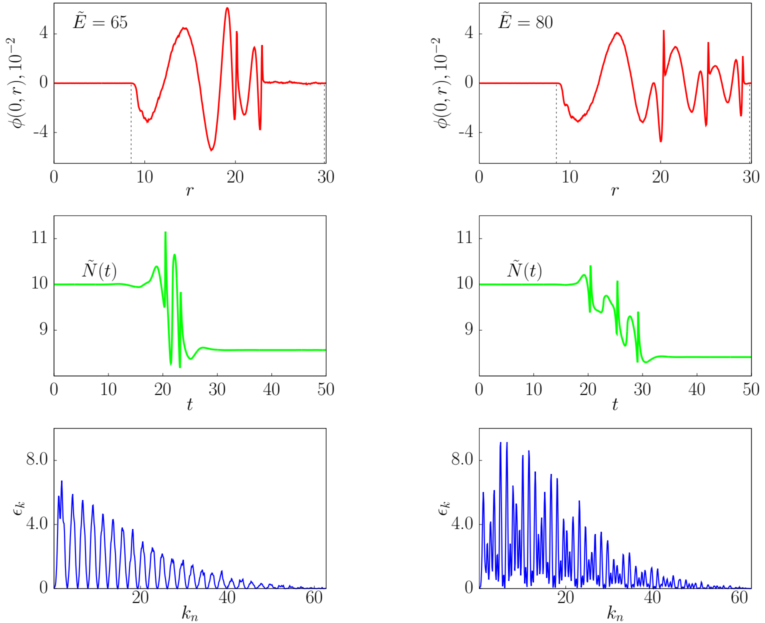
Maximal change in particle number is more than two order of magnitude smaller than for the case $\tilde{N}_i = 1$ for the same $\tilde{E}/\tilde{N}_i$. Form of the boundary solutions is the same (single spiky part and oscillating tail)

$\tilde{N}_i = 10.0$, classically allowed region, $L = 30$, $N_x = 600$

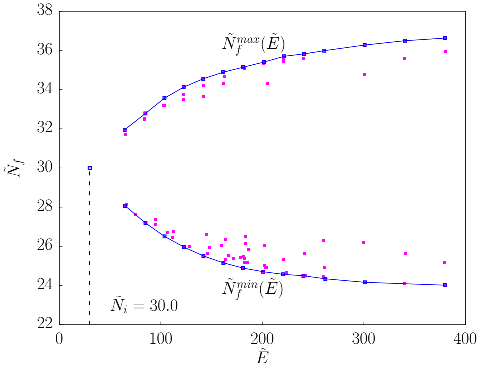


- The boundary consists of two different branches of classical solutions.
- Initial configuration consists of 2-3 similar but space shifted configurations.

$\tilde{N}_i = 10.0$, examples (lower boundary)

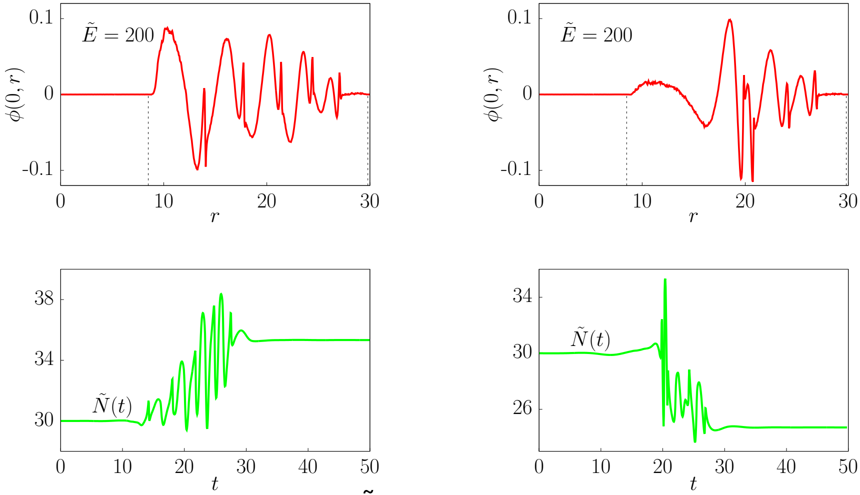


$\tilde{N}_i = 30.0$, classically allowed region



- Boundary solutions contain 4–7 spikes in the initial (and final) wavepackets.
- Task becomes complicated – many local minima, which correspond to solutions with larger distances between spikes in the initial wavepackets.

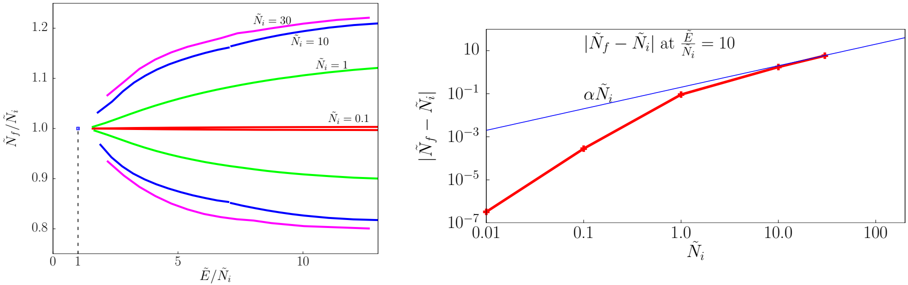
$\tilde{N}_i = 30.0$, examples (upper and lower boundaries)



Results indicate that at large $\tilde{N}_i$ boundary wavepacketes tend to consist of more and more space separated wavetrains.

Large $\tilde{N}_i$ limit

- If there are solutions with $(\tilde{N}_i^{(1)}, \tilde{N}_f^{(1)}, \tilde{E}^{(1)})$ and $(\tilde{N}_i^{(2)}, \tilde{N}_f^{(2)}, \tilde{E}^{(2)})$, than a solution with $(\tilde{N}_i^{(1)} + \tilde{N}_i^{(2)}, \tilde{N}_f^{(1)} + \tilde{N}_f^{(2)}, \tilde{E}^{(1)} + \tilde{E}^{(2)})$ exists
- Width of the classically allowed region should grow with increase of $\tilde{N}_i$ faster than linear function



An indication on existence of a limiting boundary at $\tilde{N}_i \to \infty$

Connections to $2 \to N_f$ scattering

1. Inclusive probability $(N_i, N_f, E \sim \frac{1}{\lambda})$

$$\mathcal{P}(N_i, N_f, E) = \sum_{i,f} \left| \langle i | \hat{P}_{N_i} \hat{S} \hat{P}_{N_f} \hat{P}_E | f \rangle \right|^2$$

Consider two subprocesses with different $\tilde{N}$ and $\tilde{E}$

$$\mathcal{P}(N_i^{(1)} + N_i^{(2)}, N_f^{(1)} + N_f^{(2)}, E^{(1)} + E^{(2)}) \geq \mathcal{P}(N_i^{(1)}, N_f^{(1)}, E^{(1)})\mathcal{P}(N_i^{(2)}, N_f^{(2)}, E^{(2)})$$

In particular, $\mathcal{P}(2 + N, 2N, E + E_0) \geq \mathcal{P}(2, N, E)\mathcal{P}(N, N, E_0)$
As $\mathcal{P}(N, 2N, E + E_0)$ is suppressed, $\mathcal{P}(2, N, E)$ is also suppressed

2. T–invariance:

$$\mathcal{P}(N_i, N_f, E) = \mathcal{P}(N_f, N_i, E)$$

As $\mathcal{P}(N, 2, E)$ is suppressed , then $\mathcal{P}(2, N, E)$ is also suppressed

Conclusions and plans

- We obtain classically allowed regions for processes describing $O(3)$-symmetric scattering of waves in unbroken scalar ϕ^4 theory and study properties of boundary solutions at different $\tilde{N}_i$
- Our results indicate on existence of limiting (at $\tilde{N}_i \to \infty$) boundary region of classically allowed transitions, which implies suppression of $2 \to N$ processes at any N (not only at small λN)
- We plan to calculate suppression exponent semiclassically starting with processes $\tilde{N}_i \to \tilde{N}_f$. The problem is reduced to solution of a corresponding semiclassical boundary value problem and by taking $\tilde{N}_i \to 0$. When approaching the boundary of the classically allowed region the suppression exponent should go to zero which can be used as a check of our procedure.

Thank you!