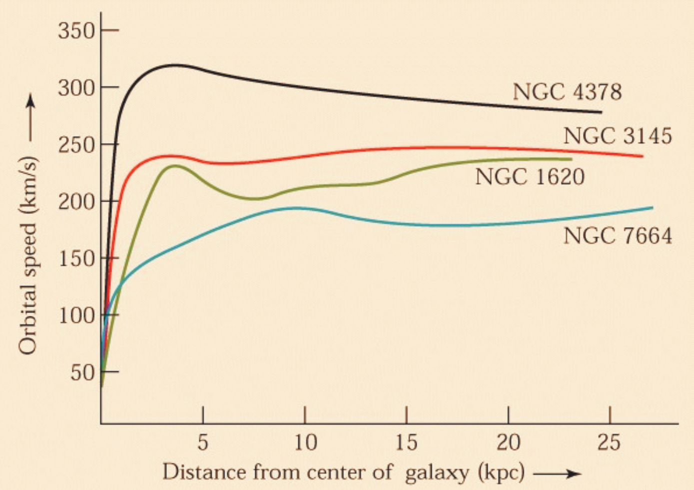
Difficulties of N-body cosmological simulations and the physics of dark matter

Anton N. Baushev

Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Moscow Region, Russia

Density profiles. Discovery.



Density profiles.

Isothermal profile

$$\rho \sim r^{-2}$$

Navarro-Frenk-White profile

$$\rho_{NFW} = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2}$$

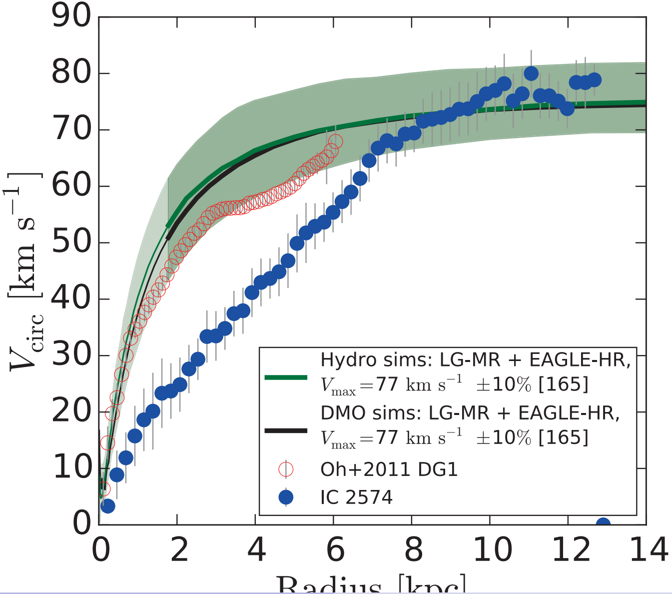
Einasto profile

$$\rho_{Ei} = \rho_s \exp\left\{-2n\left[\left(\frac{r}{r_s}\right)^{\frac{1}{n}} - 1\right]\right\}$$

Hernquist profile

$$\rho_H = \frac{Ma}{2\pi r(r + a)^3}$$

Simulations vs. observations (Oman et al. 2015)



Numerical effects

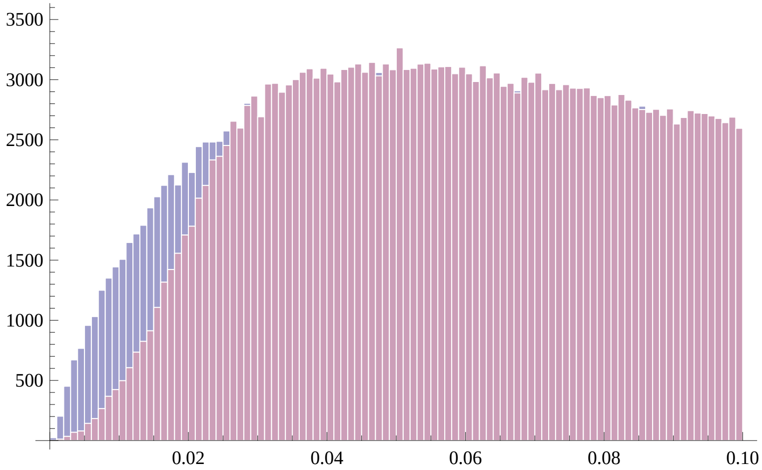
Relaxation time

$$\langle \Delta v \rangle \simeq 0 \qquad \langle \Delta v^2 \rangle \simeq \frac{8v^2 \ln \Lambda}{N(r)}$$

$$\tau_r(r) = \frac{N(r)}{8 \ln \Lambda} \cdot \tau_d(r) \qquad \tau_d(r) \sim \frac{r}{v}$$

(Power et. al. 2003) $t_0 \le 1.7\tau_r$
(Hayashi et al. 2003; Klypin et al. 2013) $t_0 \le 30\tau_r$

Core formation



Fokker-Planck equation

$$\frac{\partial n(p)}{\partial t} = \frac{\partial}{\partial p}\left\{\tilde{A}n(p) + \frac{\partial}{\partial p}[Bn(p)]\right\}$$

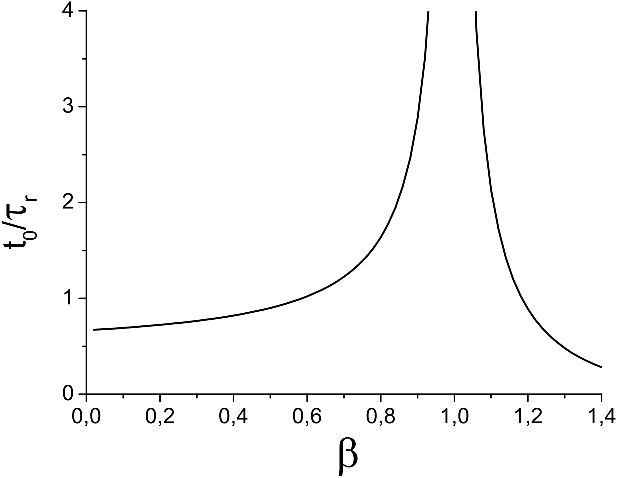
$$\tilde{A} = \frac{\langle \Delta p \rangle}{\delta t} = \mu\frac{\langle \Delta v \rangle}{\delta t} \qquad B = \frac{\langle \Delta p^2 \rangle}{2\delta t} = \frac{\mu^2}{2}\frac{\langle \Delta v^2 \rangle}{\delta t}$$

$$s = -\tilde{A}n(p) - \frac{\partial}{\partial p}[Bn(p)]$$

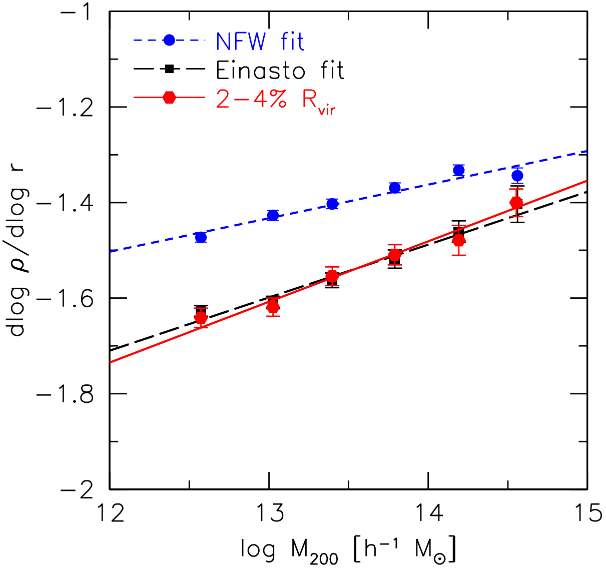
The Fokker-Planck equation has an attractor solution $\rho \propto r^{-\beta}$,

where $\beta \approx (1 - 4/3)$ (Evans & Collett 1997, Baushev 2015)

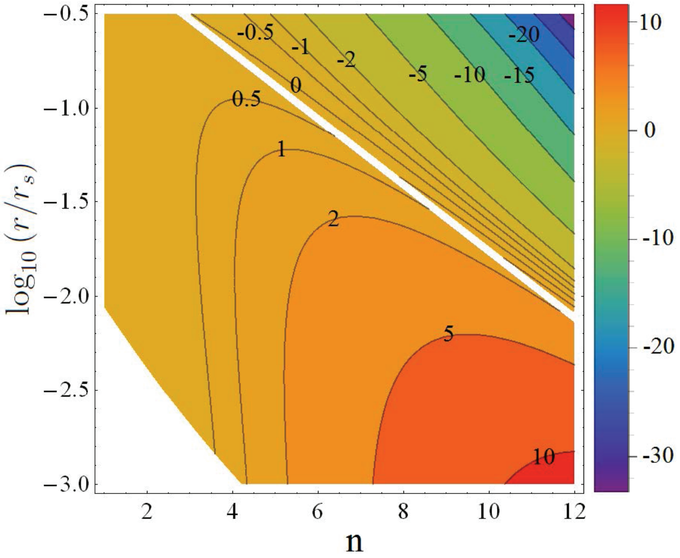
Power-law profiles



Einasto profile (Dutton & Macció 2010)



Einasto profile



Simulation details. Gadget-3.

$$\rho_H = \frac{Ma}{2\pi r(r+a)^3} \qquad \phi(r) = -\frac{GM}{r+a}$$

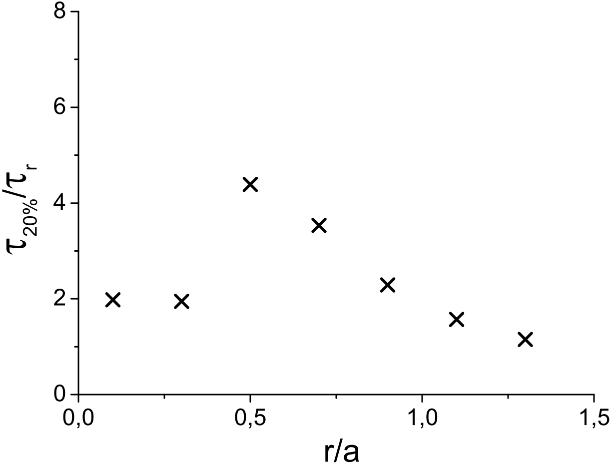
$M = 10^9 M_\odot$, $a = 100$ pc. We use $N = 10^6$ test bodies.

The relaxation time at $r = a$ is $\simeq 8.8 \cdot 10^{16}$s $\simeq 2.8 \cdot 10^9$ years. Therefore, we make 200 snapshots with the time interval $\Delta t = 10^{15}$ s, covering the time from 0 to $t_{max} = 2 \cdot 10^{17}$s $\simeq 6.5 \cdot 10^9$ years.

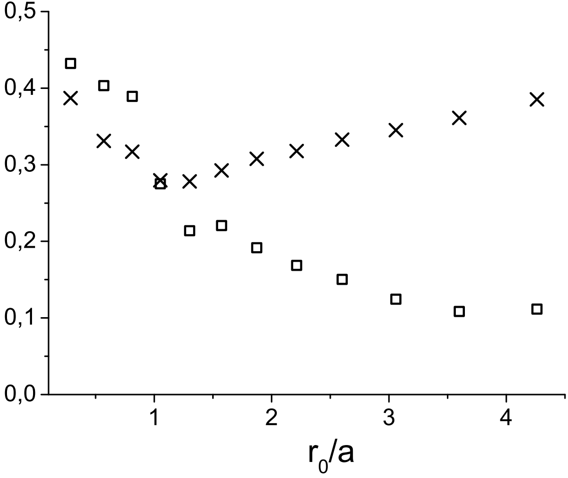
The integrals of motion $\epsilon = \phi(r) + v^2/2$, $\vec{K} = [\vec{v} \times \vec{r}]$, r_0:

$$\epsilon = \phi(r_0) + K^2/2r_0$$

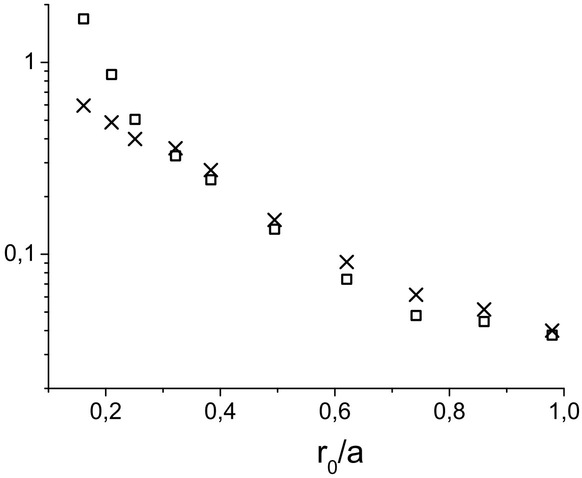
Core formation



$\langle \Delta K / K_{circ} \rangle$ (squares) and $\langle \Delta r_0 / r_0 \rangle$ (crosses)



The ratios $\frac{K_{circ}}{\tau_r}\left\langle\frac{\Delta K}{\Delta t}\right\rangle^{-1}$ (squares) and $\frac{1}{\tau_r}\left\langle\frac{\Delta r_0}{r_0\Delta t}\right\rangle^{-1}$ (crosses)



Kinetic equations

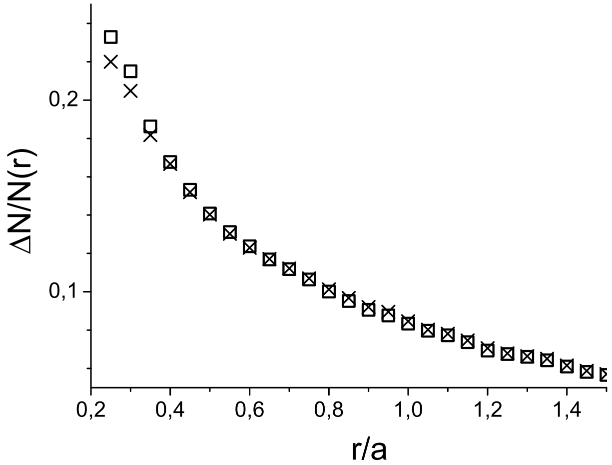
$$\frac{df}{dt} = \frac{\partial}{\partial p_\alpha}\left\{\tilde{A}_\alpha f + \frac{\partial}{\partial p_\beta}[B_{\alpha\beta} f]\right\}$$

where $\vec{q}$ is the momentum changing $\vec{p} \to \vec{p} - \vec{q}$ in a unit time.

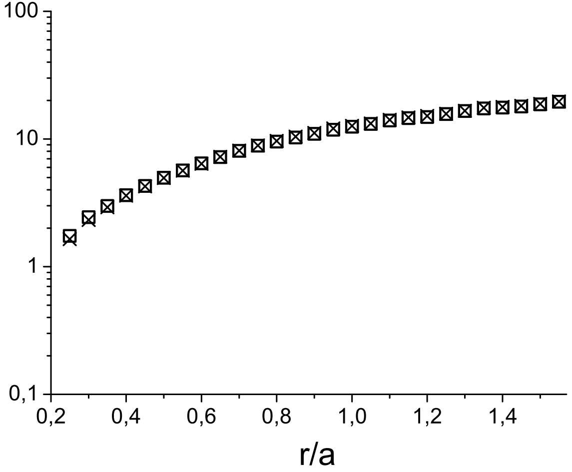
$$\tilde{A}_\alpha = \frac{\sum q_\alpha}{\delta t} \qquad B_{\alpha\beta} = \frac{\sum q_\alpha q_\beta}{2\delta t}$$

$$\frac{df}{dt} = 0 \qquad \text{vs} \qquad \frac{df}{dt} = \frac{\partial^2 [B_{\alpha\beta} f]}{\partial p_\alpha \partial p_\beta}$$

The upward $\Delta N_+(r)/\Delta t$ (squares) and downward $\Delta N_-(r)/\Delta t$ (crosses) Fokker-Planck streams



$1.7\tau_r \frac{\Delta N_+(r)}{N(r)\Delta t}$ (squares) and $1.7\tau_r \frac{\Delta N_-(r)}{N(r)\Delta t}$ (crosses)



Conclusions

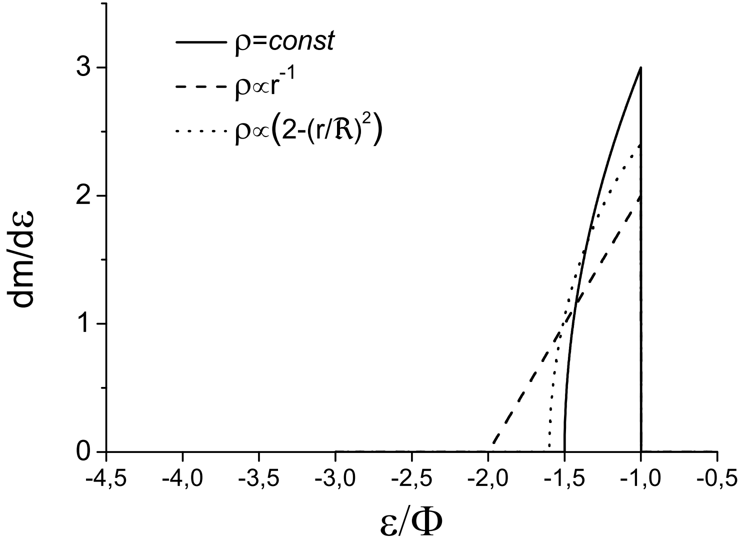
1) Though the cuspy profile is stable, all integrals of motion characterizing individual particles suffer strong unphysical variations along the whole halo, revealing an effective interaction between the test bodies.

2) This result casts doubts on the reliability of the velocity distribution function obtained in the simulations.

3) We find unphysical Fokker-Planck streams of particles in the cusp region. The same streams should appear in cosmological N-body simulations, being strong enough to change the shape of the cusp or even to create it.

4) A much better understanding of the N-body simulation convergency is necessary before a 'core-cusp problem' can properly be used to question the validity of the CDM model.

The initial energy profiles ($\epsilon = v^2/2 + \phi \qquad \phi_0 \equiv GM/R$)



Moderate relaxation

The final total specific energy ϵ_f *of most of the particles differs from the initial ones* ϵ_i *no more than by a factor* $c_{vir}/5$

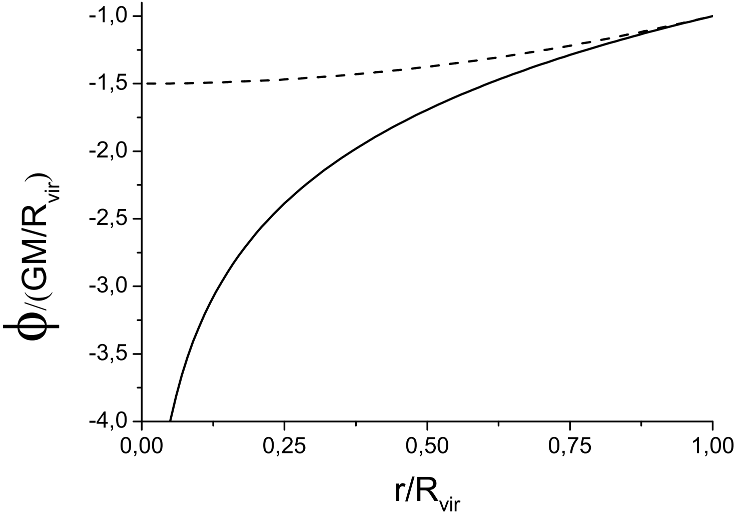
$$\frac{\epsilon_f}{\epsilon_i} \leq \frac{c_{vir}}{5}. \tag{1}$$

There can be particles that violate condition (1), but their total mass should be small with respect to the halo mass inside $r = \frac{2R_{vir}}{c_{vir}}$

$$M < \int\limits_0^{2R_{vir}/c_{vir}} dM_{halo}. \tag{2}$$

The reason for this limitation will be clear from the subsequent text.

Galaxy formation from an initial perturbation



Calculations

$$dm = f(r_0)\,\frac{2\mu}{\alpha^2}\exp\left(-\frac{\mu^2}{\alpha^2}\right)\,d\mu dr_0, \qquad \mu \equiv [\vec{v}\times\vec{r}]$$

$$\rho = \int\limits_{r_{in}}^{r_{out}}\int_0^{\mu_{max}} \frac{f(r_0) r_0 \mu \exp\left(-\mu^2/\alpha^2\right) d\mu dr_0}{2\pi r \alpha^2(r_0) T(r_0,\mu)\sqrt{r_0^2 - r^2}\sqrt{\mu_{max}^2 - \mu^2}}$$

$$\mu_{max}^2(r) = 2(\phi(r_0) - \phi(r))\left(\frac{1}{r^2} - \frac{1}{r_0^2}\right)^{-1}$$

Calculations

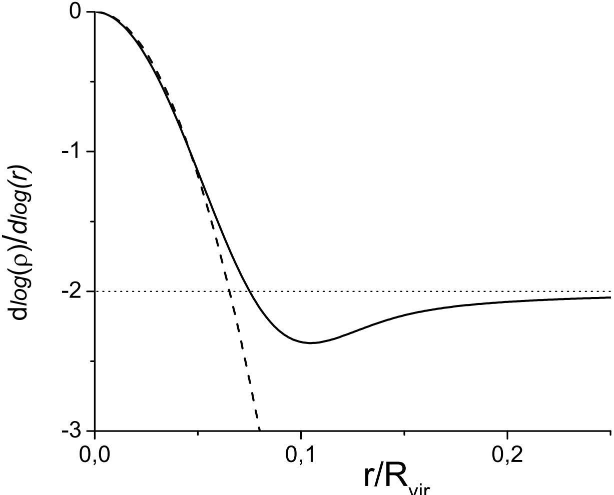
$$\rho = \int_0^\infty \frac{f(r_0)}{2\pi\alpha(r_0)T(r_0)r}\, D\left(r\frac{\sqrt{2(\phi(r_0)-\phi(0))}}{\alpha(r_0)}\right) dr_0$$

$$r_c = \left\langle \frac{\alpha(r_0)}{\sqrt{2(\phi(r_0)-\phi(0))}} \right\rangle \simeq \frac{\langle\alpha(r_0)\rangle}{\sqrt{2|\phi(0)|}}$$

$$D(x) \equiv e^{-x^2}\int_0^x e^{t^2}\,dt$$

$$\boxed{\rho = \rho_c \frac{r_c}{r}\, D\left(\frac{r}{r_c}\right)} \qquad \rho_c = \frac{1}{2\pi r_c}\int_0^\infty \frac{f(r_0)dr_0}{\alpha(r_0)T(r_0)}$$

The model density profile with $r_c = 0.05R_{vir}$ and Einasto profile with $n = 0.5$ and $r_s = 0.017R_{vir}$ (dashed line).



The $\rho_s r_s$ constancy.

Observations:
multiplication $\log(\rho_s r_s) = const \simeq 2.15 \pm 0.2$ $(M_\odot/\mathrm{pc}^2)$ for a wide variety of galaxies (Kormendy & Freeman, 2004), (Donato et.al., 2009)

$$\rho_s r_s = \frac{1}{2\pi} \int_0^\infty \frac{f(r_0) dr_0}{\alpha(r_0) T(r_0)}$$

$$\rho_s r_s \simeq \frac{1}{2\pi} \frac{M_{vir}}{\langle \alpha(r_0) \rangle \langle T(r_0) \rangle}$$

$$\langle \alpha(r_0) \rangle = \frac{1}{4} \sqrt{G M_{vir} R_{vir}} \qquad T(r_0) = \int_0^{r_0} \frac{dr}{v_r}$$

The $\rho_s r_s$ constancy.

$$\rho_s r_s \simeq 1.5 M_{vir}^{\frac{1}{3}} \bar{\rho}^{\frac{2}{3}}$$

ΛCDM-cosmology $H_0^{-1} = 4.2 \cdot 10^{17}$ s, $\Omega_M = 0.27$, $z_{eq} = 3100$

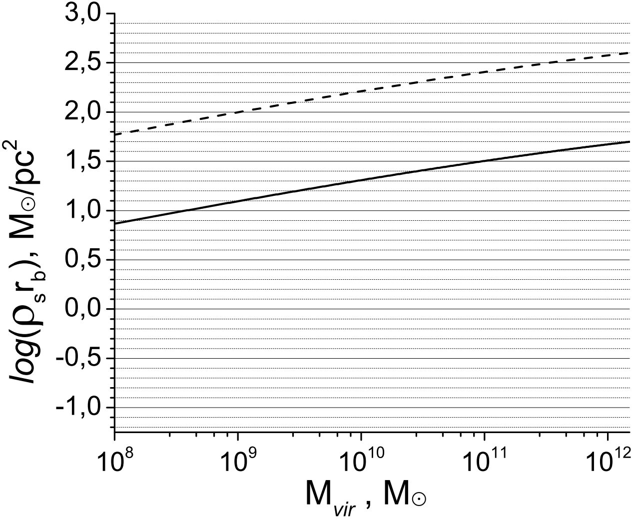
$$1 + z \simeq \frac{27}{2} \Phi(k)(1 + z_{eq}) \ln(0.2 k \eta_{eq})$$

where

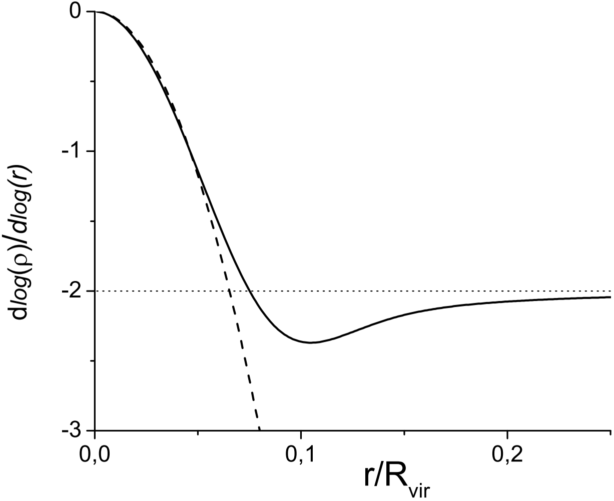
$$\eta_{eq} = \frac{2(\sqrt{2} - 1)}{a_0 H_0 \sqrt{\Omega_M} \sqrt{1 + z_{eq}}}$$

If we suppose the maximum possible angular momentum
$\langle \alpha(r_0) \rangle = \frac{1}{4} \sqrt{G M_{vir} R_{vir}}$

Dependence of $\log(\rho_s r_s)$ $(M_\odot/\mathrm{pc}^2)$ on $\log(M_{vir}/M_\odot)$



So if the relaxation is not very violent



Conclusion

1) The supposition that the energy exchange between the dark matter particles is moderate automatically leads to an Einasto-like density profile with $n \simeq 0.5$ in the halo center.

2) The supposition leads to approximative constancy of $\rho_s r_s$ multiplication, in accordance with observations.

3) The existence of a region with $\rho \propto r^{-2}$ in the density profiles of many galaxies finds a natural explanation.