

# Gravitational Bose condensation of dark matter axions

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DL, A. Panin, I. Tkachev, arXiv:1804.05857

# Light Dark Matter

- Fits into the galaxy:

$$(mv)^{-1} \lesssim R \Rightarrow m \gtrsim 10^{-22} \text{ eV}$$

- Can be only bosonic:

$$f \sim \frac{\rho/m}{(mv)^3} \gtrsim 1 \Rightarrow m \lesssim 10^2 \text{ eV}$$

Highly occupied state!

- Forms condensate in equilibrium:

$$mv^2 < T_{BE} \sim \frac{\rho^{2/3}}{m^{5/3}} \Rightarrow m \lesssim 10^2 \text{ eV}$$



Interaction  $\Rightarrow$  Bose condensation!

## Fornax dwarf galaxy



Parameters are known!

$$\rho \sim M_\odot/\text{pc}^3$$

$$R \sim \text{kpc}$$

$$v \sim 10 \text{ km/s}$$

Can they interact strongly enough?

Answer: THEY DO, gravitationally!

# Candidates: Axion-Like Particles

$$\psi(t, x)$$

- (Pseudo)scalars
- Small mass  $m$
- Can form CDM by vacuum realignment

Pseudo-Goldstone bosons

Wise, Wilczek '83

## QCD axions

- Solve strong CP problem  
*Peccei, Quinn '77*
- CDM:  $m \approx 26 \mu\text{eV}$
- $\lambda_4 \sim 10^{-50}$   
*Klaer, Moore '17*

## String axions

- Appear in all string models
- $m \gtrsim 10^{-22} \text{ eV}$   
Fuzzy DM:  $m \sim 10^{-22} \text{ eV}$   
*e.g. Hu, Barkana, Gruzinov '00*
- $\lambda_4 \gtrsim 10^{-100}$   
*Arvanitaki et al '10*

Mass is small, self-interaction can be ignored!

# Field equations for light DM

- $f \gg 1$  — classical fields
- $v \ll 1$  — nonrelativistic approximation
- $\lambda_4 = 0$  — no self-interaction

$$\left. \begin{array}{l} \psi(t, x) \\ U(t, x) \end{array} \right\}$$

Scrödinger-Poisson system:

$$\begin{aligned} i\partial_t\psi &= -\Delta\psi/2m + mU\psi \\ \Delta U &= 4\pi G \underbrace{(m|\psi|^2)}_{\rho} - \langle\rho\rangle \end{aligned}$$

Solving these equations, we describe condensation!

# $t = 0$ : Random initial state

Maximally mixed (virialized) initial state

$$I_{coh} \sim (mv)^{-1} \ll R$$

$$(mv^2)^{-1} \ll \tau_{gr}$$

Kinetic regime!

⇒ Random initial field:

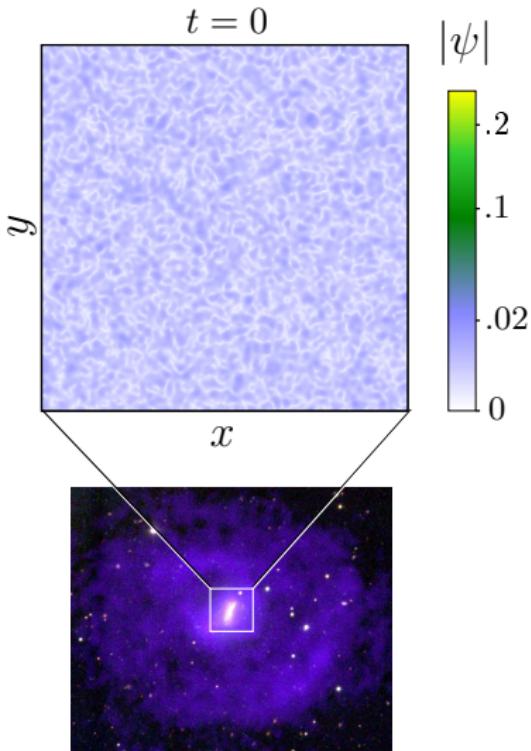
$$\psi_{\mathbf{p}} \propto \underbrace{e^{-\mathbf{p}^2/2(mv_0)^2}}_{\text{momentum distribution}} \times \underbrace{e^{iA_{\mathbf{p}}}}_{\text{random phases}}$$

$$\langle \psi(x) \psi(y) \rangle \propto e^{-\frac{(x-y)^2}{l_{coh}^2}}$$

$$I_{coh} = \frac{2}{mv_0}$$

Virialized!

Also:  $R \gg (mv_0)^{-1}$  is assumed



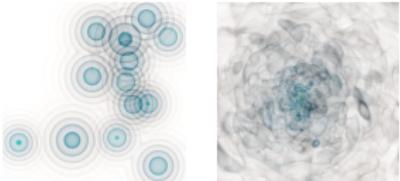
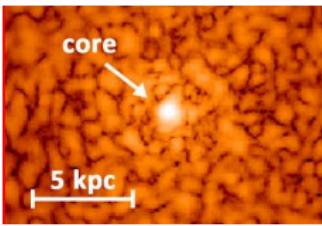
Virialized DM halo

# Different regime to previous simulations!

$$\boxed{\cancel{I_{coh} \sim (mv)^{-1} \ll R}}$$

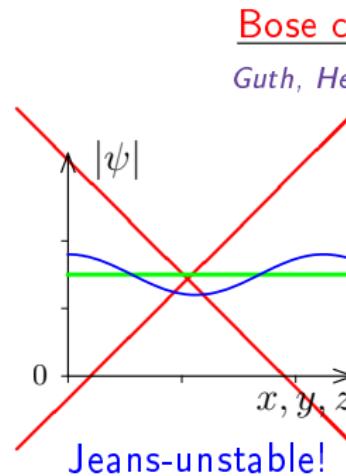
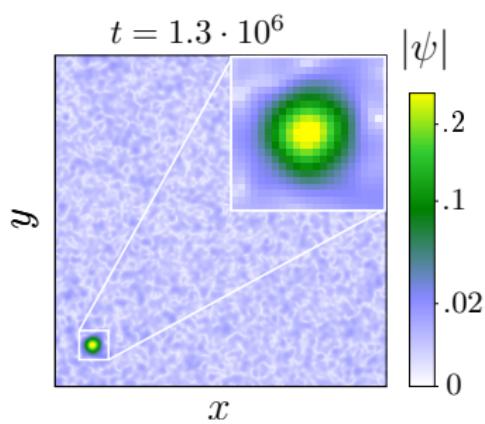
So far: **non-kinetic regime**

## Coherent initial states:

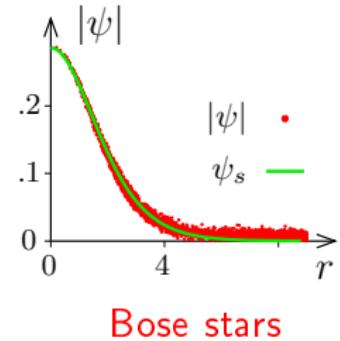
- Smooth wavepacket *Seidel, Suen '94*  
*Guzman, Urena-Lopez '06*
- Many Bose stars *Schive et al '14*  
*Schwabe, Niemeyer, Engels '16* → 
- Cosmological Bose condensate *Schive, Chiueh, Broadhurst '14* →  
*Veltmaat, Niemeyer, Schwabe '18* 
- Random fields in small box,  $R \sim (mv)^{-1}$  *Khlebnikov '99*

Start the first simulation in kinetic regime!

# Final state



Bose condensate?  
Guth, Hertzberg, Prescod-Weinstein '15



## Stationary Bose star

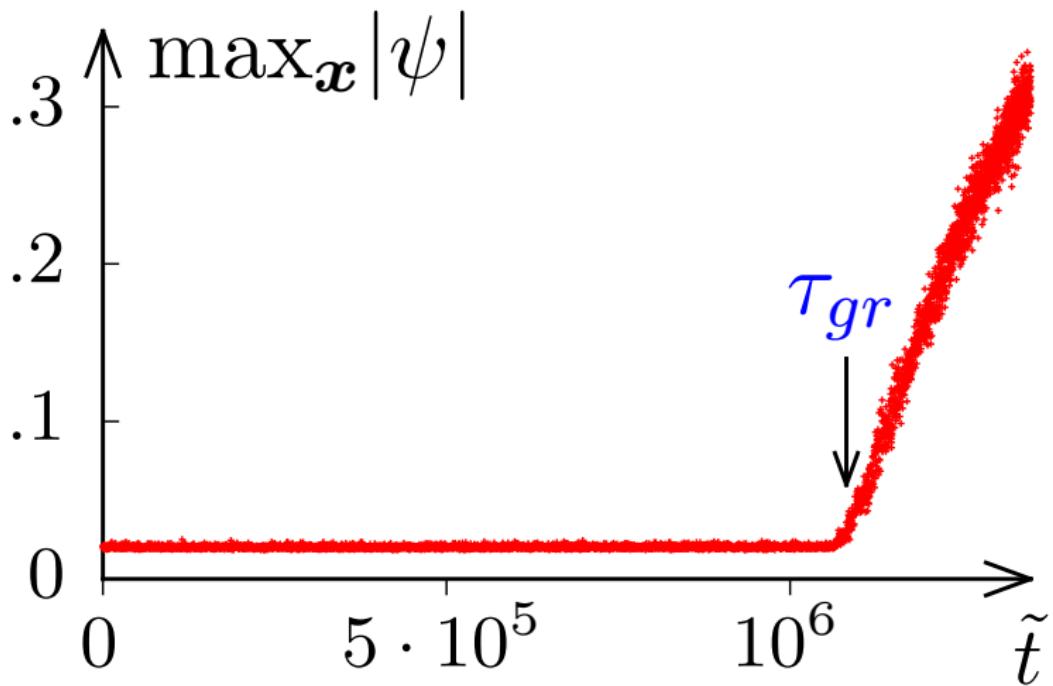
Bosons are at the lowest level in

$$U = U_s(r): \psi = \psi_s(r) e^{-i\omega t}$$

SP Equations  $\Rightarrow$  profile  $U_s(r)$ ,  $\psi_s(r)$ !

We observe formation of a Bose star at  $t = \tau_{gr}$  !

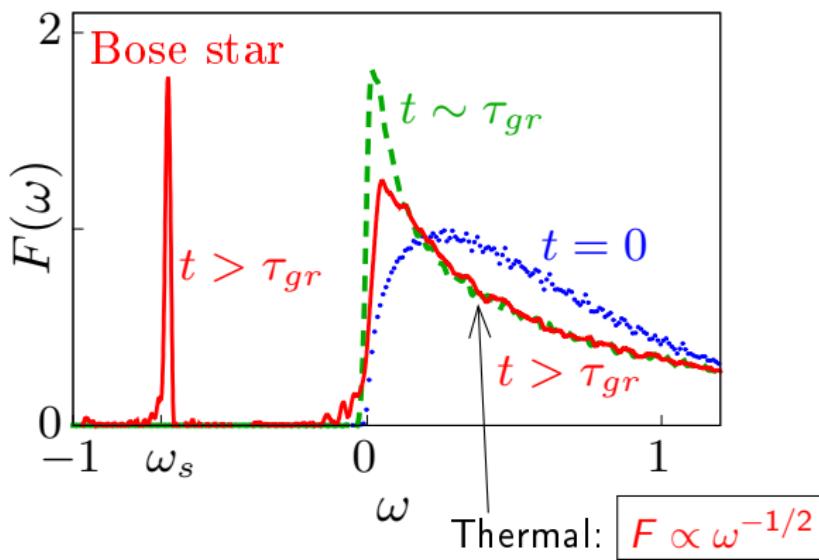
# No Bose star in the beginning!



# Energy distribution

What happens during evolution of a homogeneous gas?

$$F(\omega, t) \equiv \frac{dN}{d\omega} = \int d^3x \int \frac{dt_1}{2\pi} \psi^*(t, x) \psi(t + t_1, x) e^{i\omega t_1 - t_1^2/\tau_1^2}$$



## Landau equation — derivation

- Perturbative solution of Schrödinger-Poisson equation
- Kinetic approximations  $(mv)^{-1} \ll x, (mv^2)^{-1} \ll t$
- Compute Wigner distribution

$$f_{\mathbf{p}}(t, \mathbf{x}) = \int d^3\mathbf{y} e^{-i\mathbf{p}\mathbf{y}} \langle \psi(\mathbf{x} + \mathbf{y}/2) \psi^*(\mathbf{x} - \mathbf{y}/2) \rangle$$

random phase average

e.g. Zakharov, L'vov, Falkovich '92

$$\partial_t f_{\mathbf{p}} + \frac{\mathbf{p}}{m} \nabla_{\mathbf{x}} f_{\mathbf{p}} - m \nabla_{\mathbf{x}} \bar{U} \nabla_{\mathbf{p}} f_{\mathbf{p}} = \underset{\oplus}{\text{St}} f_{\mathbf{p}} \sim \frac{f_{\mathbf{p}}}{\tau_{kin}} \leftarrow \text{relaxation time}$$

$$f_{\mathbf{p}}^3 \leftarrow \text{Bose amplification}$$

cf. Landau, Lifshitz, X

Time to Bose star formation:  $\tau_{gr} = \frac{b}{\uparrow} \tau_{kin} = \frac{4\sqrt{2}b}{(\sigma_{gr} f_{\mathbf{p}}) v n}$

$O(1)$  correction

# Time to Bose star formation

$$\tau_{gr} = \frac{4\sqrt{2}b}{(\sigma_{gr} f_p) v n}$$

Rutherford cross section:  $\sigma_{gr} \approx 8\pi(mG)^2\Lambda/v^4$

$$\Lambda = \log(mvR)$$

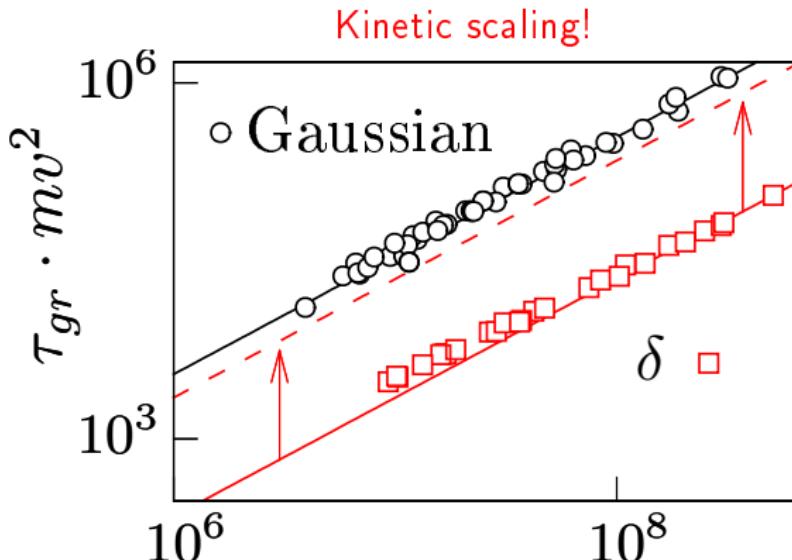
Coulomb logarithm

Average phase-space density:  $f_p = 6\pi^2 n/(mv)^3$

$$\tau_{gr} = \frac{b\sqrt{2}}{12\pi^3} \frac{mv^6}{G^2\Lambda n^2} = \frac{b\sqrt{2}\pi}{3G^2m^5\Lambda} f_p^{-2}$$

- Strongly depends on **local** quantities:  $n, v, f_p$
- Involves **global** logarithm  $\Lambda = \log(mvR)$

# Time to Bose star formation



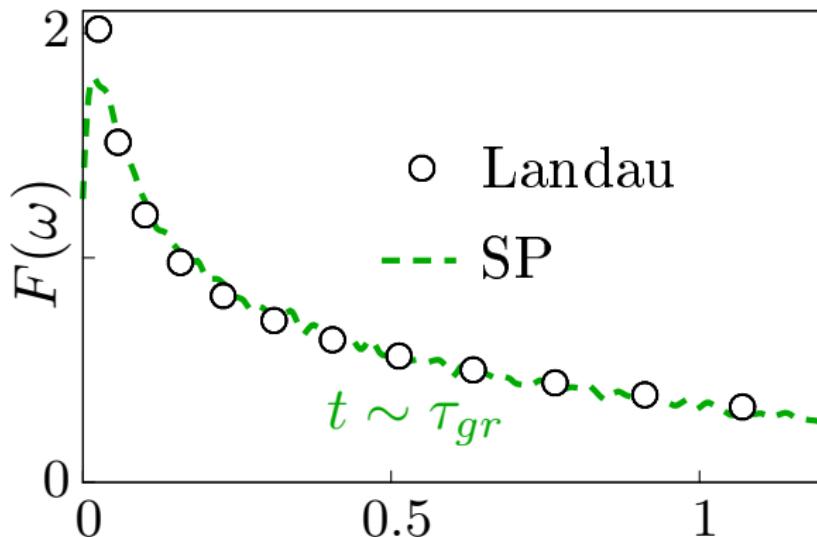
- Gaussian:  $f_p \propto |\psi_p|^2 \propto e^{-p^2/(mv_0^2)}$ ,  $b \approx 0.9$
- $\delta$ :  $f_p \propto \delta(|p - p_0|)$ ,  $b \approx 0.6$

# Comparison with Landau equation

Isotropic homogeneous gas:  $F(t, \omega) \propto p f_p$

$$\partial_t F = StF$$

Can be solved numerically!



One parameter to fit:  $\Lambda = \log(mvL) + a \leftarrow$  finite part

$$a \approx 5$$

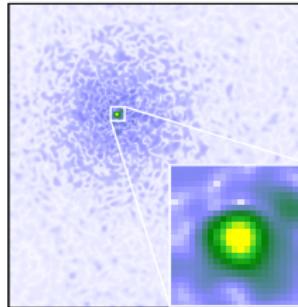
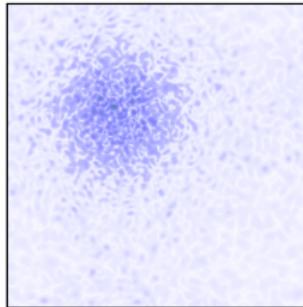
# Bose star formation in halo/minicluster

$t = 250$

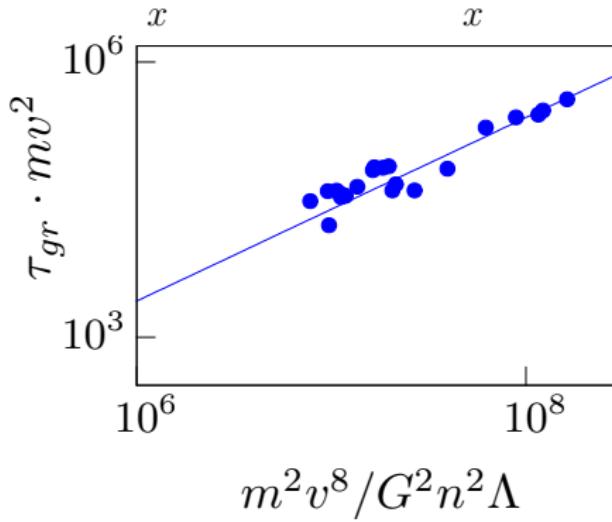
$t = 1250$

$|\psi|$

$y$



Large box  $\Rightarrow$  Jeans instability  
 $\Rightarrow$  minicluster



$$\tau_{gr} = \frac{b\sqrt{2}}{12\pi^3} \frac{mv^6}{G^2 \Lambda n}$$

Virial velocity:  $v^2 \sim 4\pi G m n R^2 / 3$

$$\tau_{gr} \sim \frac{0.05}{\Lambda} \frac{R}{v} (Rmv)^3$$

$\tau_{gr} \gg R/v \leftarrow$  free-fall time  
 $Rmv \sim 1$  — condense immediately!

# Applications to cosmology

## String axions

$$\tau_{gr} = \frac{b\sqrt{2}}{12\pi^3} \frac{m^3 v^6}{G^2 \Lambda \rho^2}$$

Dwarf galaxy:  $\rho \sim 0.1 M_\odot / pc^3$   
 $v \sim 30 km/s$

- $m \sim 10^{-22} eV \Rightarrow$  non-kinetic!  
 $\tau_{gr} \sim R/v$  — explains  
simulations  
cf. Schive, Chiueh, Broadhurst '14
- Experiment:  $m \gtrsim 2 \cdot 10^{-21} eV$   
Iršič et al '17  
Armengaud et al '17
- $m \sim 10^{-21} eV \Rightarrow \tau_{gr} \sim 10^{10} yr$

Condense faster in galaxy cores  
miniclusters

## QCD axions ( $m = 26 \mu eV$ )

Miniclusters:  $M \sim 10^{-13} M_\odot$

Kolb, Tkachev '93



$$M, \Phi = \rho_a / \bar{\rho}_a |_{RD} - 1$$

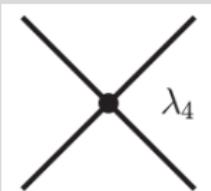
$$\tau_{gr} \sim \frac{10^9 \text{ yr}}{\Phi^4} \left( \frac{M_c}{10^{-13} M_\odot} \right)^2$$

- $\Phi \sim 1 \Rightarrow \tau_{gr} \sim 10^9 \text{ yr}$
- $\Phi \sim 10^3 \Rightarrow \tau_{gr} \sim \text{hr}$

Universe filled with Bose stars!

# Gravity vs self-interactions

Contact self-interaction:  $\lambda_4 = m^2/f_a^2$



$$\sigma_4 = \frac{m^2}{128\pi f_a^4}$$

Relaxation via contact scattering

Comparing kinetic rates:  $\frac{\tau_4}{\tau_{gr}} \sim \frac{\sigma_{gr}}{\sigma_4} \sim \left( \frac{10 f_a}{v M_{pl}} \right)^2$

- QCD axions in miniclusters:  $v \sim 10^{-9}, f_a = 10^{-8} M_{pl}$
  - Fuzzy axions in dwarfs:  $v \sim 10^{-4}, f_a = 10^{-2} M_{pl}$
- $\Rightarrow \tau_4 \gg \tau_{gr}$

Gravity is often more effective!

# Thanks for attention!

