Axion dark matter and cosmology

Ken'ichi Saikawa (MPP, Munich)

Based on

T. Hiramatsu, M. Kawasaki, KS, T. Sekiguchi, PRD85, 105020 (2012) [1202.5851]
T. Hiramatsu, M. Kawasaki, KS, T. Sekiguchi, JCAP01, 001 (2013) [1207.3166]
M. Kawasaki, KS, T. Sekiguchi, PRD91, 065014 (2015) [1412.0789]
A. Ringwald, KS, PRD93, 085031 (2016) [1512.06436]
S. Ho, KS, F. Takahashi (2018), work in progress

Outline

- Properties of the QCD axion
- Cosmological aspects of axion dark matter
  - Production in the early universe
  - Prediction for dark matter mass
- Axion and ALP dark matter

QCD axion

Strong CP problem and axion

- Strong CP problem
  - Quantum chromodynamics (QCD) allows a CP violating term:

$$\mathcal{L} \supset \frac{\alpha_s}{8\pi}\theta G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

Physical observable: $\bar{\theta} = \theta + \arg\,\det M_q$

  - Non-observation of neutron electric dipole moment implies

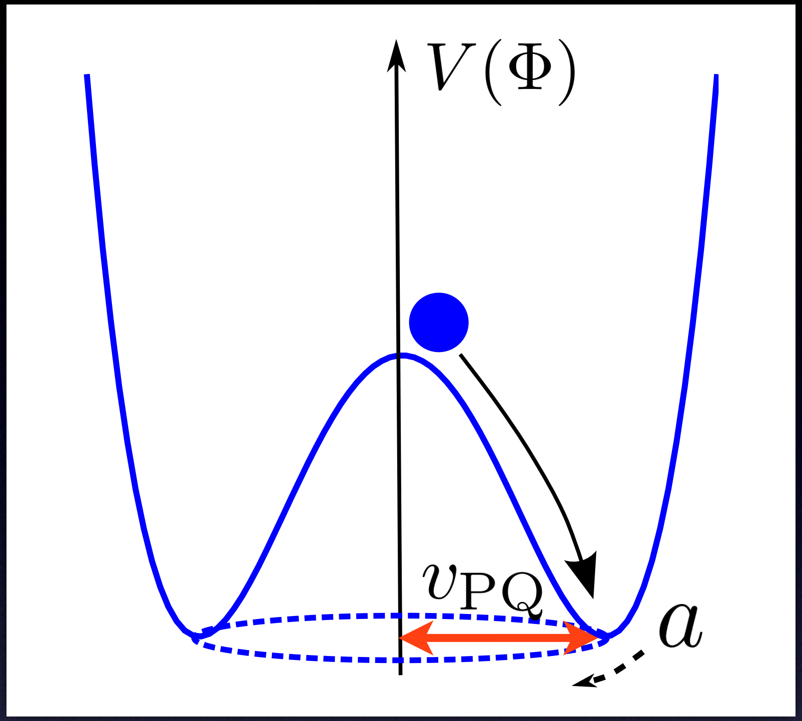
$$|\bar{\theta}| < \mathcal{O}(10^{-11})$$ "Why it is so small ?"

- Peccei-Quinn (PQ) mechanism Peccei and Quinn (1977)
  - Take $\bar{\theta}$ as a dynamical variable that explains its smallness, i.e. $\bar{\theta} \to \bar{\theta}_{\rm eff}(x) = a(x)/f_a$
  - Predicts the existence of light particle $a(x)$ = axion .

Weinberg(1978), Wilczek(1978)

Axion as a Nambu-Goldstone boson

- Axions can be identified as Nambu-Goldstone bosons arising from breaking of global symmetry. (Peccei-Quinn (PQ) symmetry)

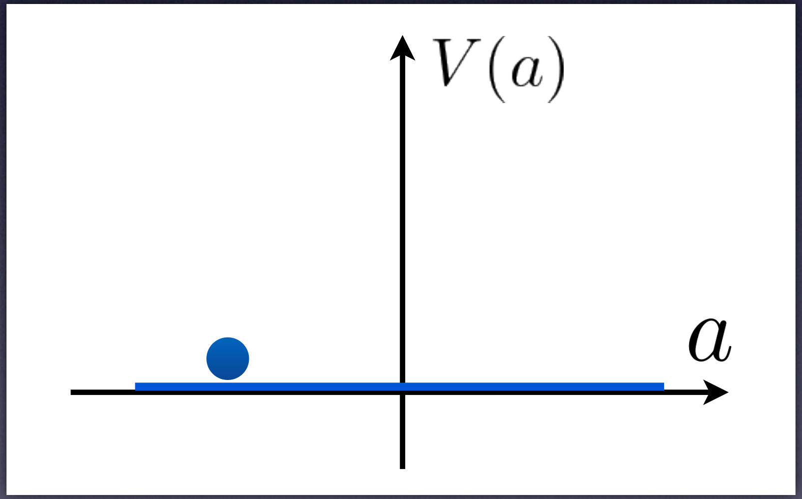


- Hidden scalar field:

$$\Phi(x) = \frac{1}{\sqrt{2}}\left[v_{\mathrm{PQ}} + \rho(x)\right] e^{ia(x)/v_{\mathrm{PQ}}}$$

Massive modulus, massless phase:

$$m_\rho \sim v_{\mathrm{PQ}}, \quad m_a = 0$$



- Interactions with standard model particles are suppressed by a large symmetry breaking scale.

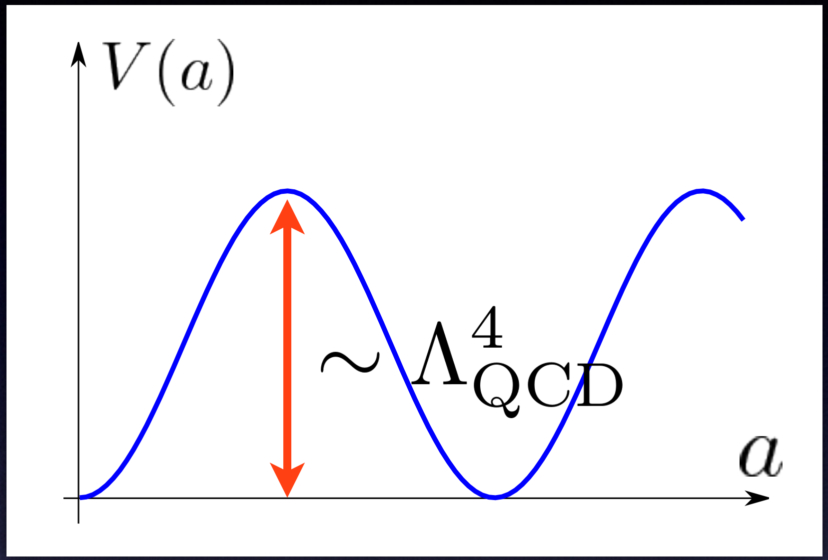
$$v_{\mathrm{PQ}} \gg v_{\mathrm{electroweak}} \approx \mathcal{O}(100)\,\mathrm{GeV}$$

Properties of the axion

- Axions can couple to gluons via

$$\mathcal{L} \supset -\frac{\alpha_s}{8\pi}\frac{a}{f_a} G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

$f_a \propto v_{\rm PQ}$: axion decay constant



- Below the QCD scale $\Lambda_{\rm QCD} \sim \mathcal{O}(100\,{\rm MeV})$, topological charge fluctuations in QCD vacuum induce the potential energy:

$$V(a) \sim \chi(T)\left(1 - \cos\frac{a}{f_a}\right), \quad \chi(T \to 0) \sim \Lambda_{\rm QCD}^4$$

➡ $\langle a \rangle = 0$ at the minimum, solving strong CP problem

- Mass of QCD axions $m_a \sim \Lambda_{\rm QCD}^2/F_a$:

$$m_a = \frac{m_\pi f_\pi}{f_a}\frac{\sqrt{z}}{1+z} \simeq 6\,\mu{\rm eV}\left(\frac{10^{12}\,{\rm GeV}}{f_a}\right) \qquad z = m_u/m_d = 0.48(3)$$

- Tiny coupling with matter + non-thermal production

  → good candidate of cold dark matter

Axion cosmology

Axion mass from cosmology

$$\Omega_a = \Omega_a(f_a), \quad m_a \simeq 6\,\mu\mathrm{eV}\left(\frac{10^{12}\,\mathrm{GeV}}{f_a}\right)$$

- Assuming that axions explain 100% of CDM abundance, we can estimate their "typical mass".
- Predictions strongly depend on the early history of the universe.
- Two possibilities:
  - PQ symmetry is never restored after inflation.
  - PQ symmetry is restored during/after inflation.

Pre-inflationary PQ symmetry breaking scenario

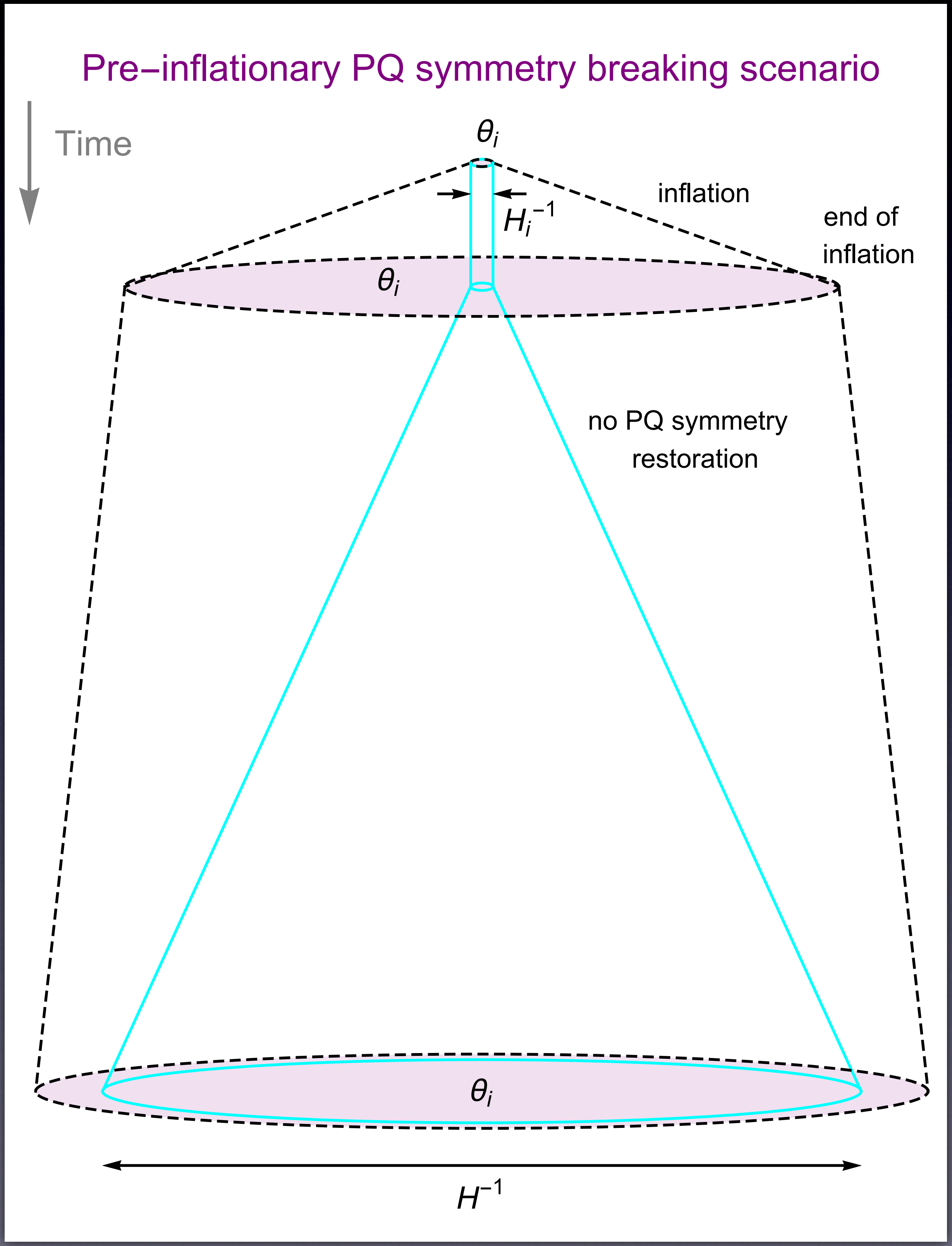
- How the spatial distribution of an angular field

$$\theta(x) = \frac{a(x)}{f_a}$$

  evolves over time ?

- The size of the patch of universe in which θ takes a certain value θ_i can be much larger than the Hubble radius at the present time.

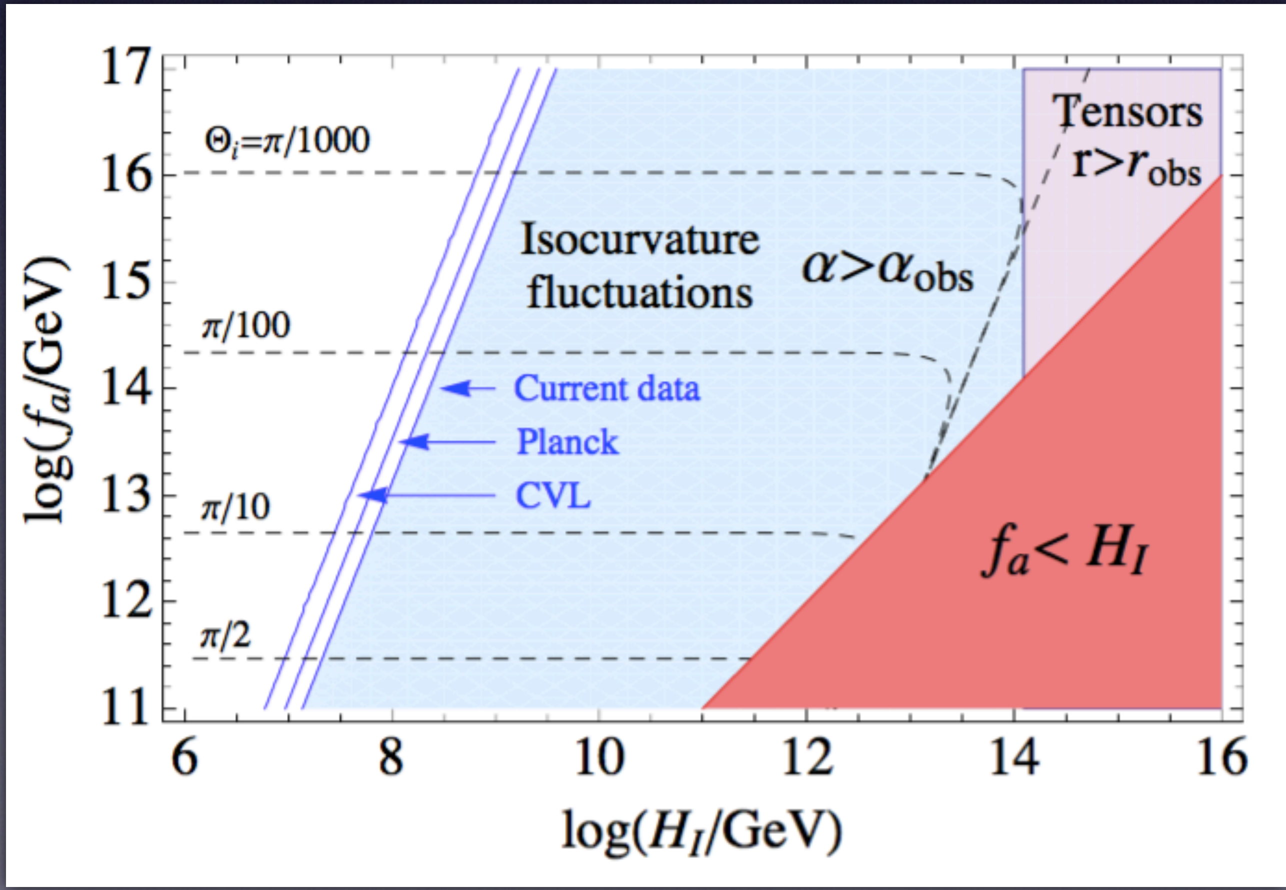
- Relic axion abundance depends on f_a and initial angle θ_i.



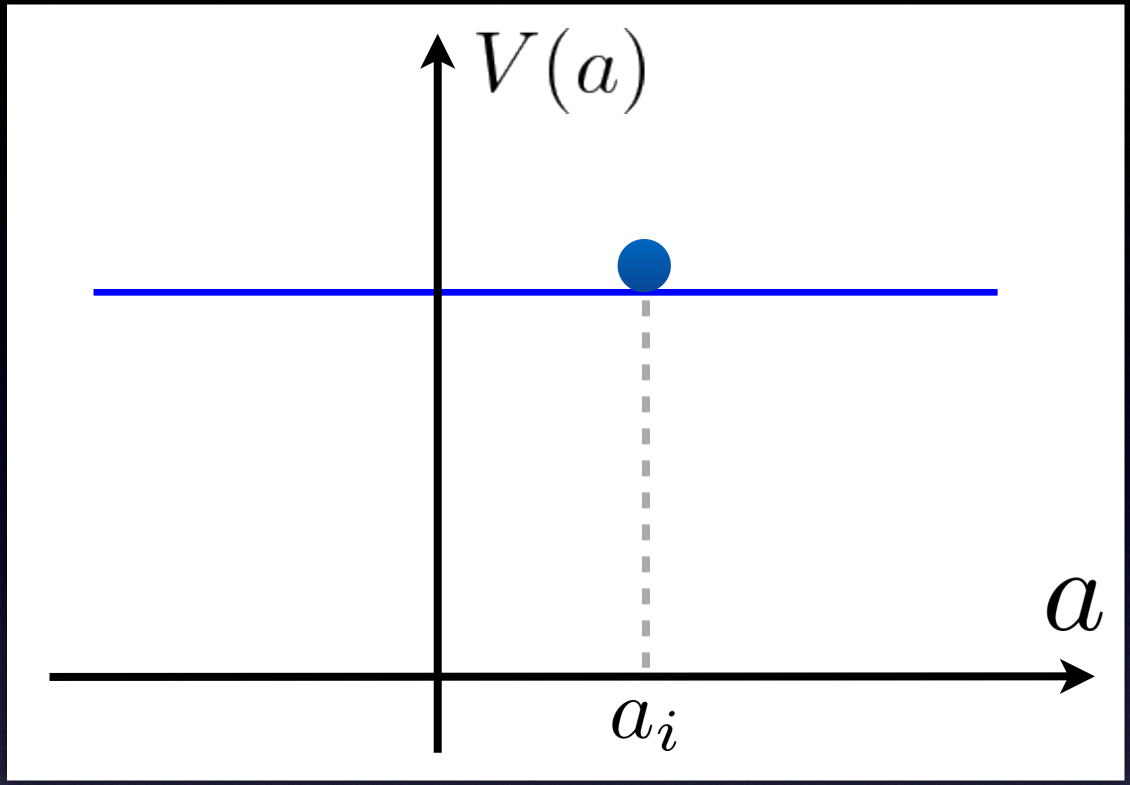
Pre-inflationary PQ symmetry breaking scenario

- Re-alignment mechanism:
  Axion field starts to oscillate at
  $$m_a(T_{\rm osc}) \approx 3H(T_{\rm osc})$$

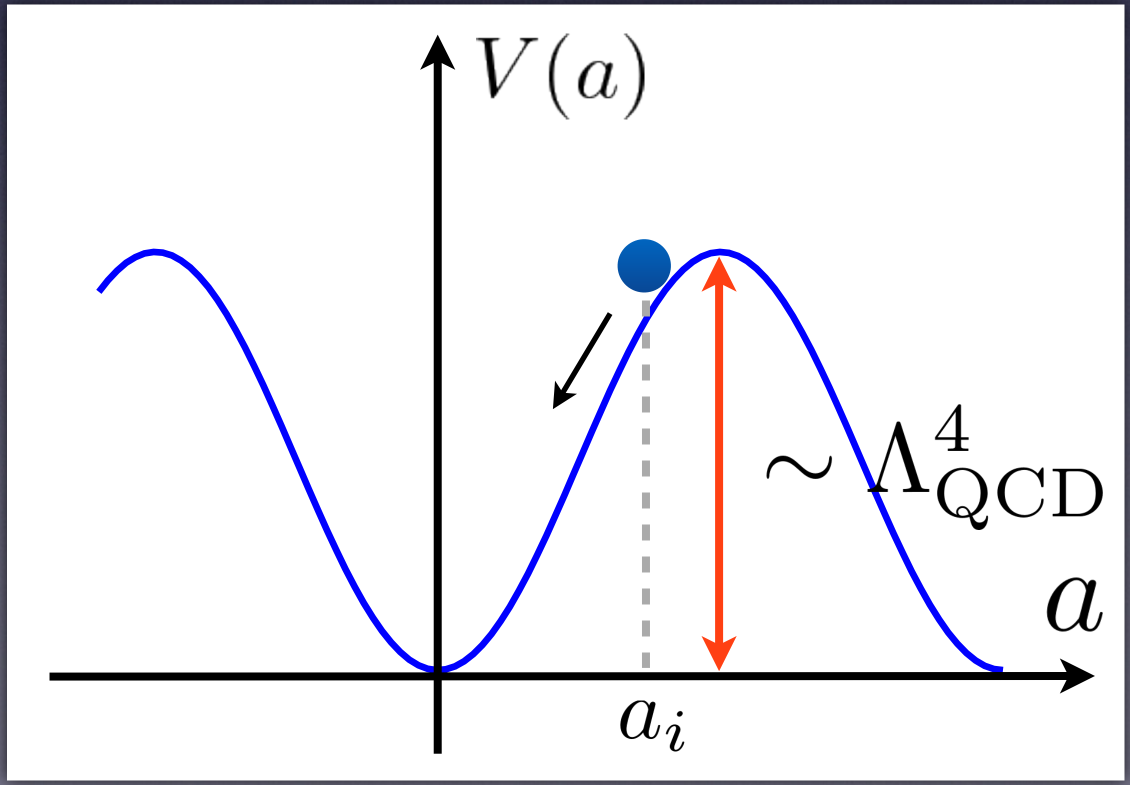
- Severe constraints from isocurvature fluctuations if inflationary scale is sufficiently high.



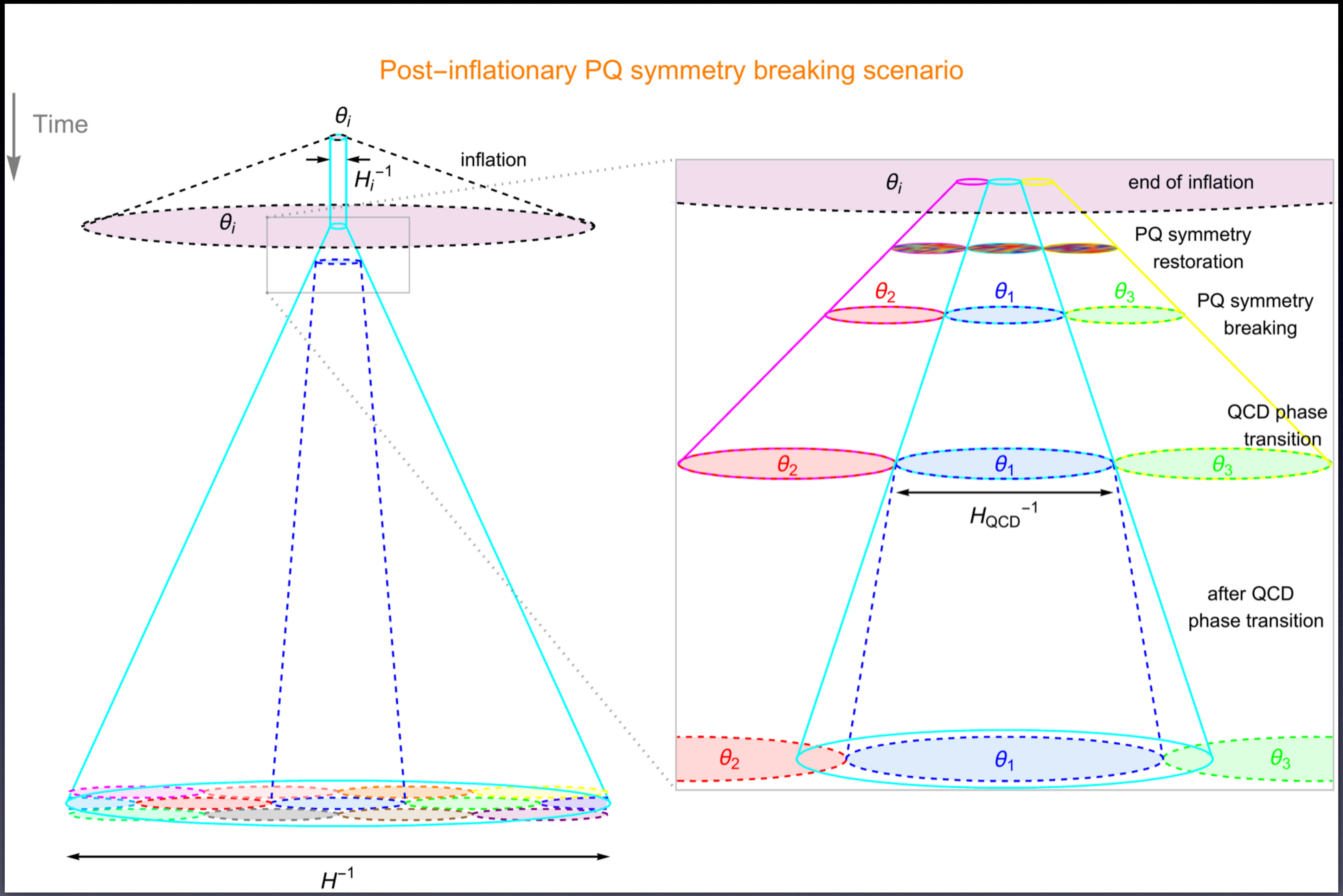
Hamann, Hannestad, Raffelt and Wong (2009)



QCD phase transition



Post-inflationary PQ symmetry breaking scenario



- Present observable universe contains many different patches with different values of θ_i .
- Topological defects (strings and domain walls) are formed.
- Relic axion density should be estimated by summing over all possible field configurations.

Axionic string

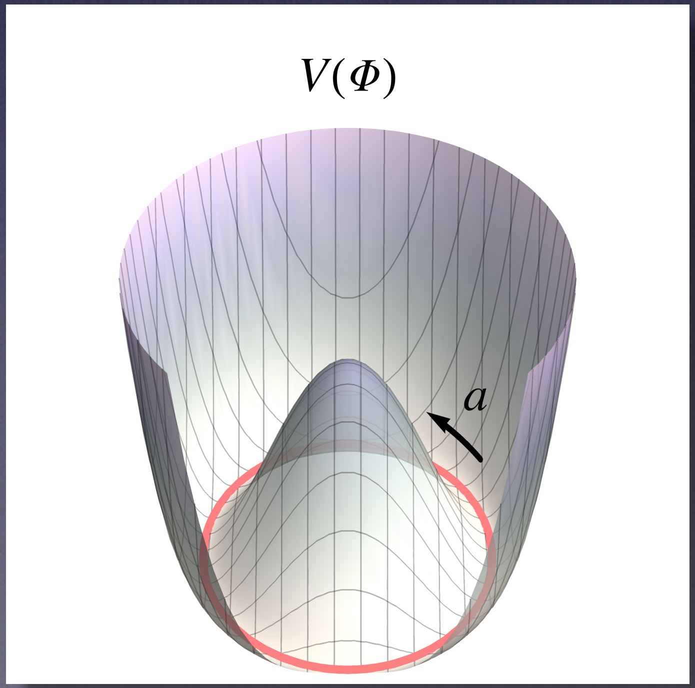
- Peccei-Quinn field (complex scalar field)

$$\Phi = |\Phi| e^{ia(x)/v_{\mathrm{PQ}}}$$

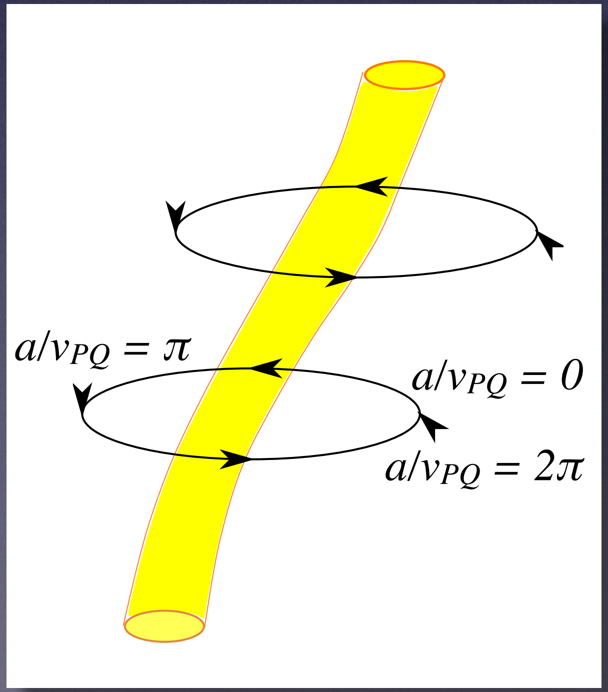
$a(x)$: axion field

- Spontaneous breaking of global U(1)$_{\mathrm{PQ}}$ symmetry

$$V(\Phi) = \lambda \left(|\Phi|^2 - \frac{v_{\mathrm{PQ}}^2}{2} \right)^2$$



field space

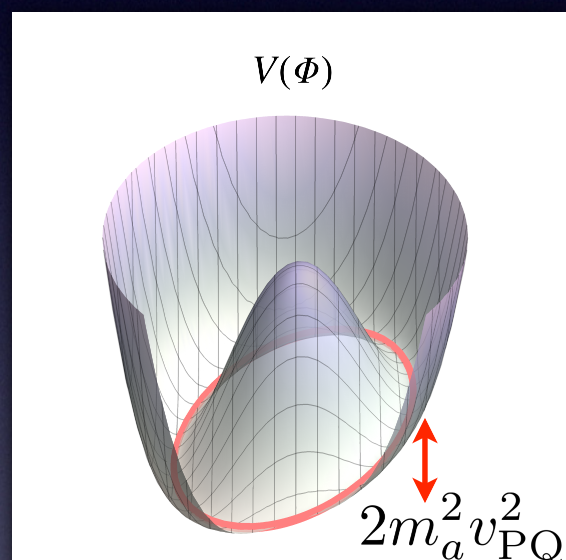


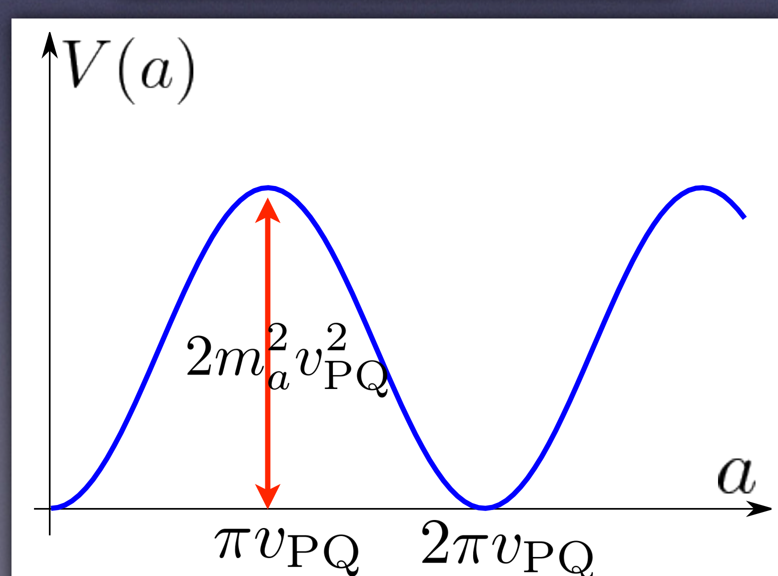
coordinate space

Axionic domain wall

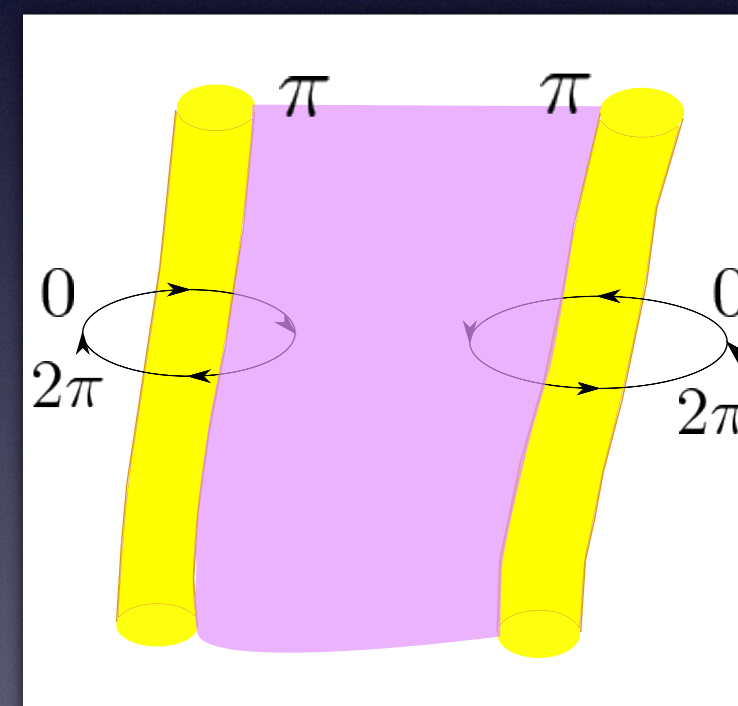
- Mass of the axion (QCD effect ; $T \lesssim 1\mathrm{GeV}$)

$$V(\Phi) = \lambda \left(|\Phi|^2 - \frac{v_{\mathrm{PQ}}^2}{2} \right)^2 + \boxed{m_a^2 v_{\mathrm{PQ}}^2 (1 - \cos(a/v_{\mathrm{PQ}}))}$$





field space



coordinate space

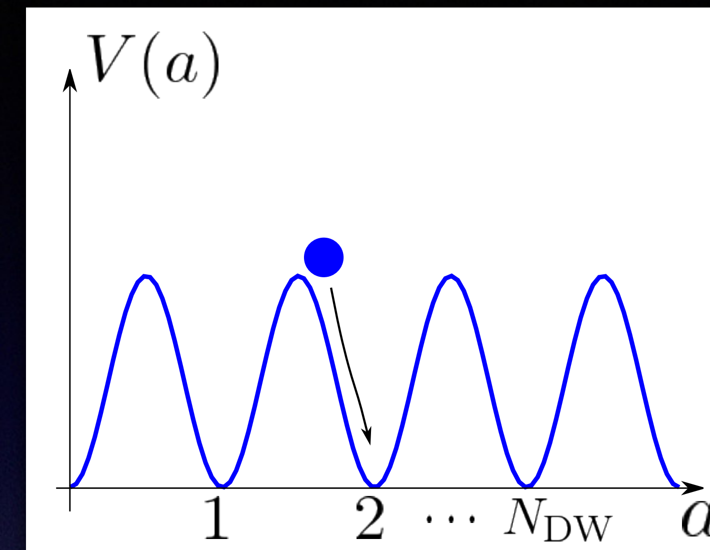
Strings attached by domain walls.

Domain wall problem

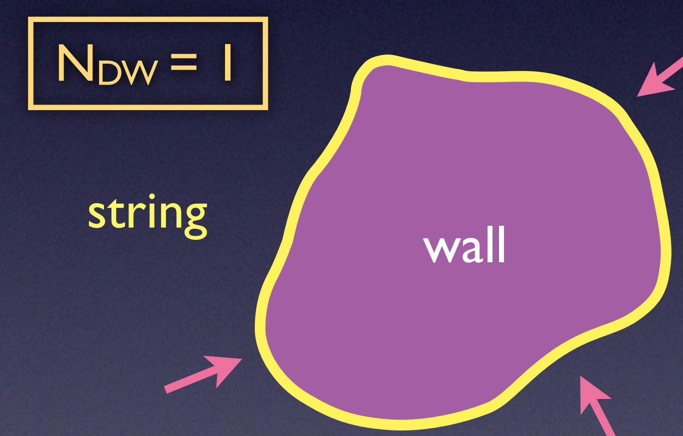
- Domain wall number $N_{\rm DW}$
  - $N_{\rm DW}$ degenerate vacua

$$\boxed{V(a) = \frac{m_a^2 v_{\rm PQ}^2}{N_{\rm DW}^2}\left(1-\cos\left(N_{\rm DW}\frac{a}{v_{\rm PQ}}\right)\right)}$$

$N_{\rm DW}$: integer determined by QCD anomaly



- If $N_{\rm DW} = 1$, string-wall systems are unstable.
  - They collapse soon after the formation.



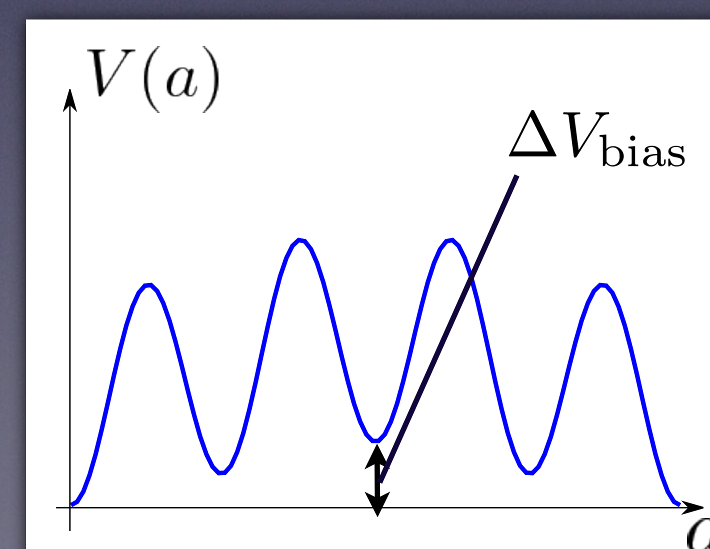
- If $N_{\rm DW} > 1$, string-wall systems are stable.
  - coming to overclose the universe.

Zel'dovich, Kobzarev and Okun (1975)

  - We may avoid this problem by introducing an energy bias (walls become unstable). Sikivie (1982)

$$V(a) = \frac{m_a^2 v_{\rm PQ}^2}{N_{\rm DW}^2}\left(1-\cos\left(\frac{N_{\rm DW}a}{v_{\rm PQ}}\right)\right) + \underline{\Delta V_{\rm bias}}$$

lifts degenerate vacua

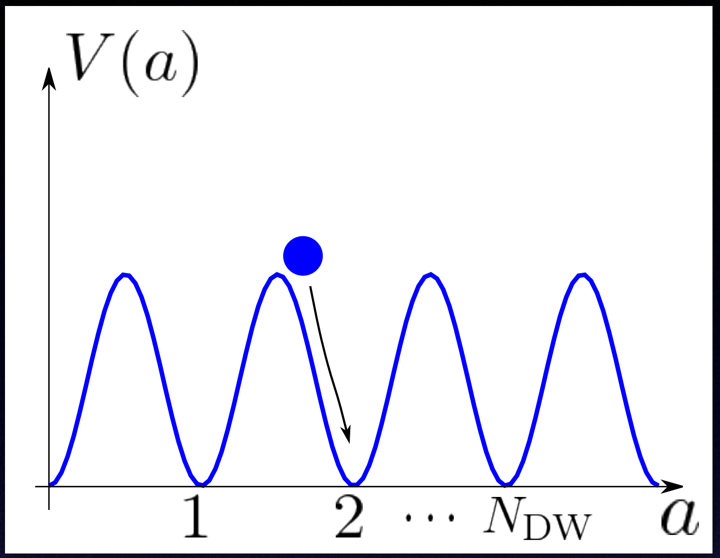


Domain wall problem

- Domain wall number N_{DW}
  - N_{DW} degenerate vacua

$$\boxed{V(a) = \frac{m_a^2 v_{PQ}^2}{N_{DW}^2}\left(1-\cos\left(N_{DW}\frac{a}{v_{PQ}}\right)\right)}$$

$N_{\rm DW}$: integer determined by QCD anomaly



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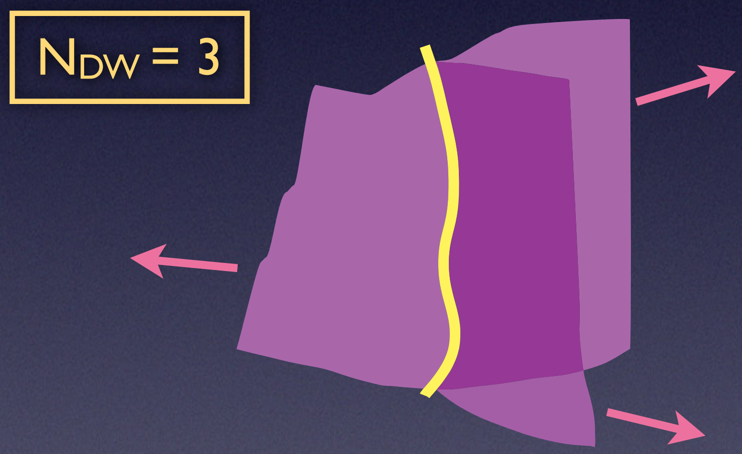
  - They collapse soon after the formation.



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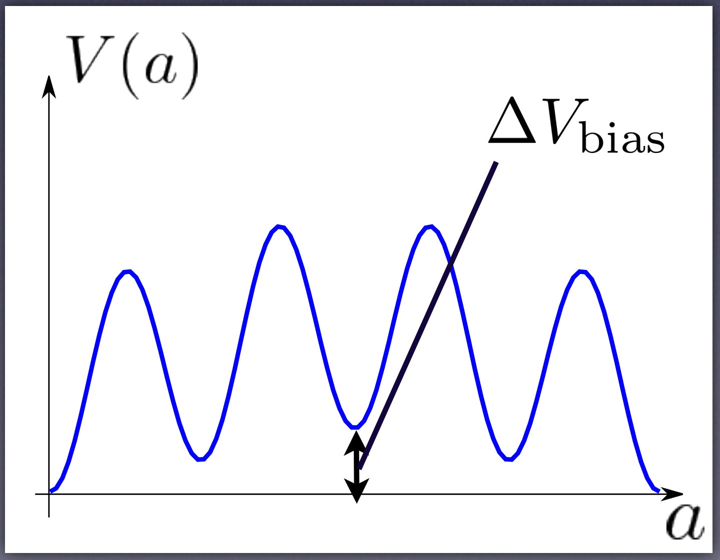
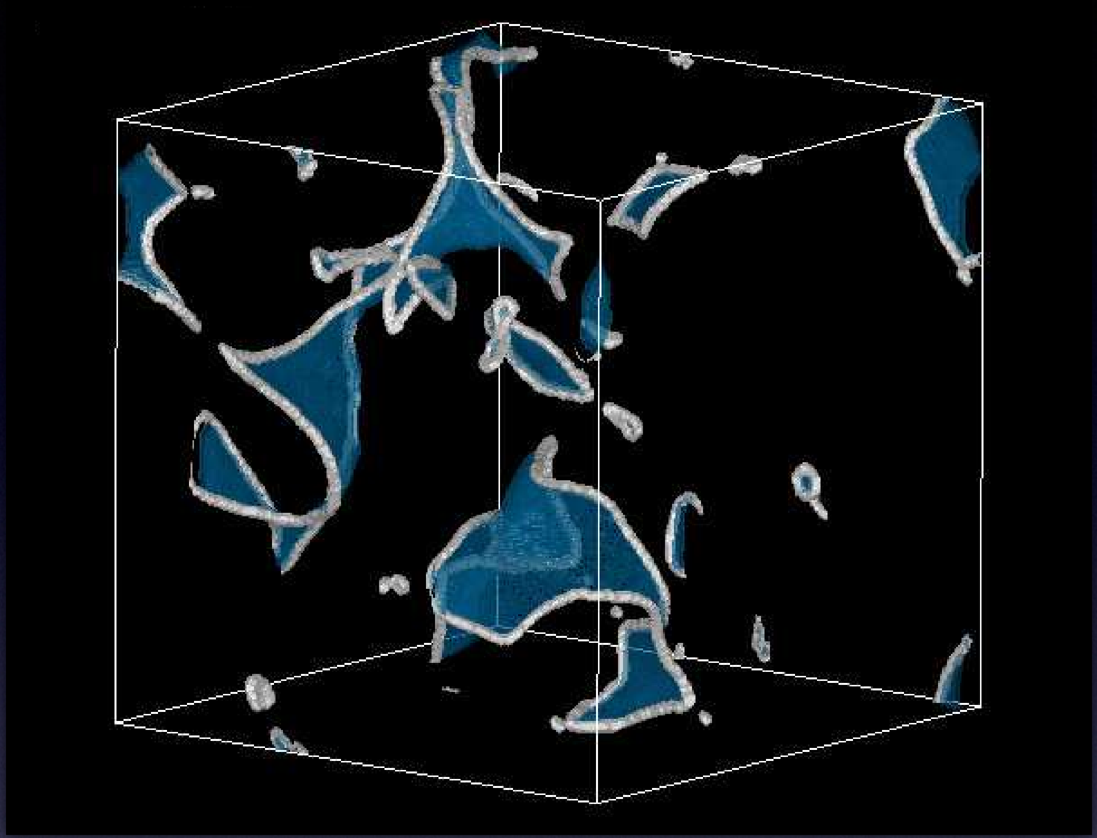
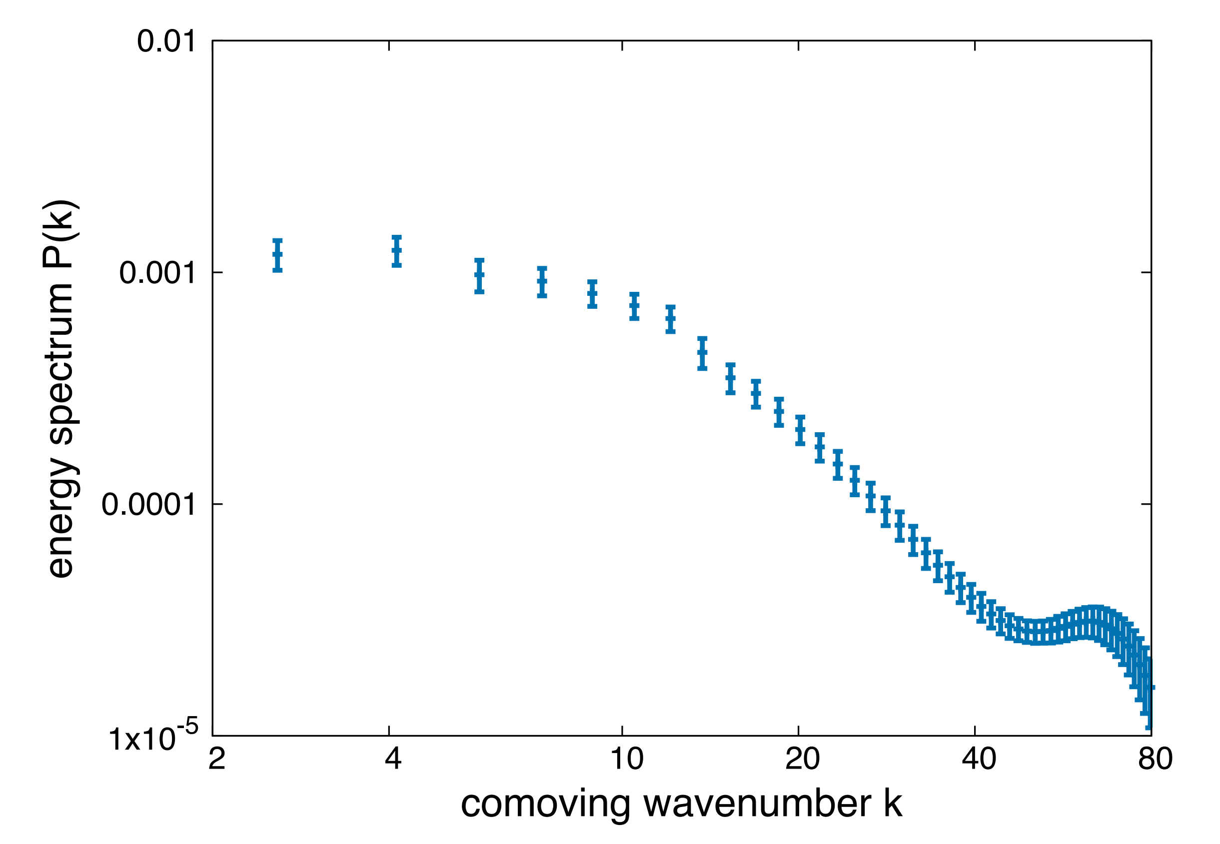


Numerical simulation : N_{DW} = 1

Hiramatsu, Kawasaki, KS, and Sekiguchi (2012), Kawasaki, KS, and Sekiguchi (2015)





- Spectrum of radiated axions is estimated based on the field theoretic lattice simulations.
- Total axion dark matter abundance including the contribution from string-wall systems:

$$\Omega_a h^2 \approx 1.6^{+1.0}_{-0.7} \times 10^{-2} \left(\frac{f_a}{10^{10}\,\mathrm{GeV}}\right)^{1.165}$$

- Constraint on the axion mass:

$$\Omega_a \leq \Omega_{\mathrm{CDM}}$$

$$\Omega_{\mathrm{CDM}} h^2 \simeq 0.12$$

$$\boxed{\begin{aligned} f_a &\lesssim (3.8\text{–}9.9) \times 10^{10}\,\mathrm{GeV} \\ m_a &\gtrsim (0.6\text{–}1.5) \times 10^{-4}\,\mathrm{eV} \end{aligned}}$$

Effect of high string tension ?


The dark-matter axion mass

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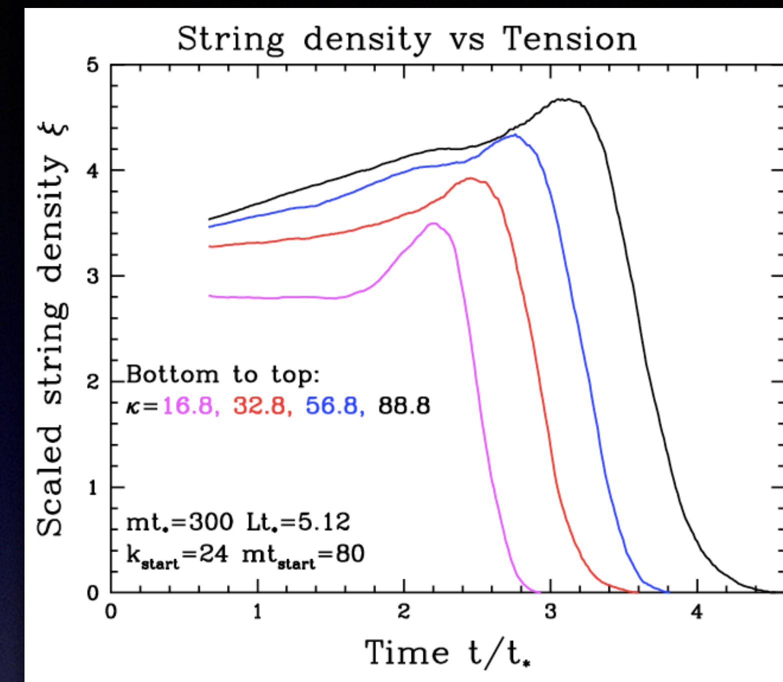


- Alternative simulation method Klaer and Moore, 1707.05566, 1708.07521

  Realizing high string tension $\propto \ln(v_{\rm PQ}/H) \sim 50\text{-}70$

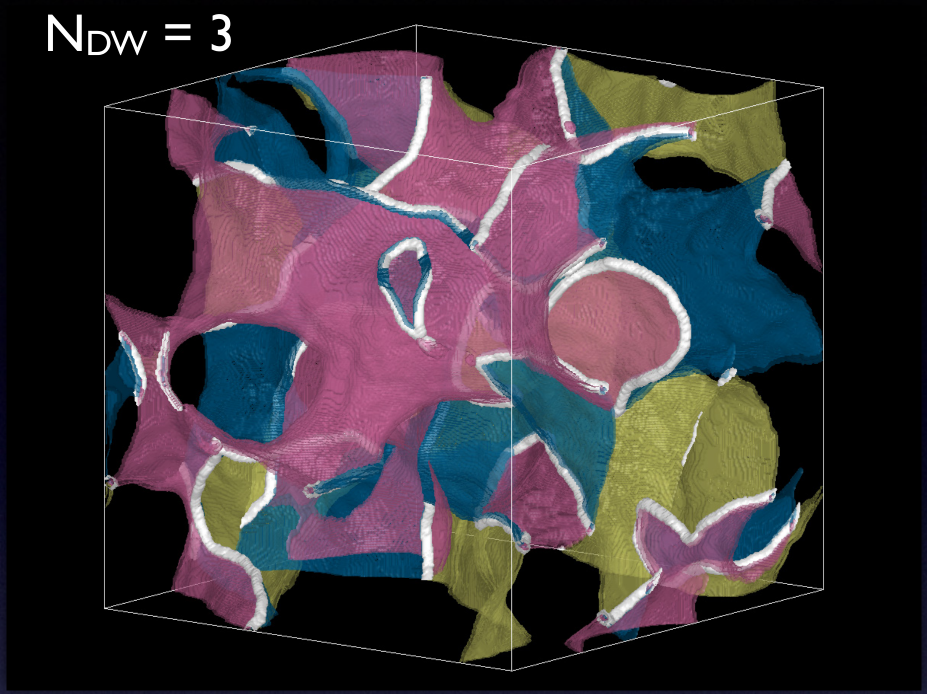
  that cannot be simulated in the conventional method ($v_{\rm PQ}/H \lesssim \mathcal{O}(100)$).

- String density increases,
  but the axion production becomes less efficient.

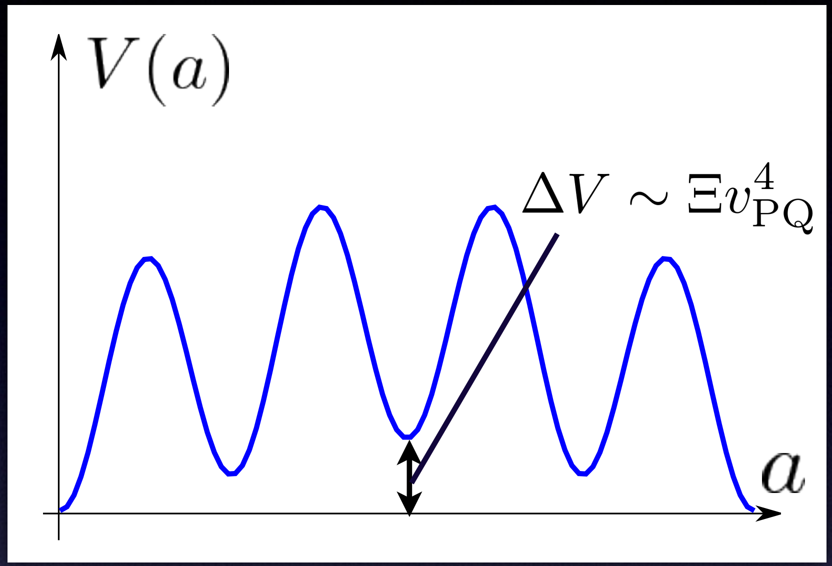
- Smaller axion DM mass:

$$\boxed{\begin{aligned} f_a &= (2.21 \pm 0.29) \times 10^{11}\ \mathrm{GeV} \\ m_a &= (2.62 \pm 0.34) \times 10^{-5}\ \mathrm{eV} \end{aligned}}$$

Models with N_{DW} > 1



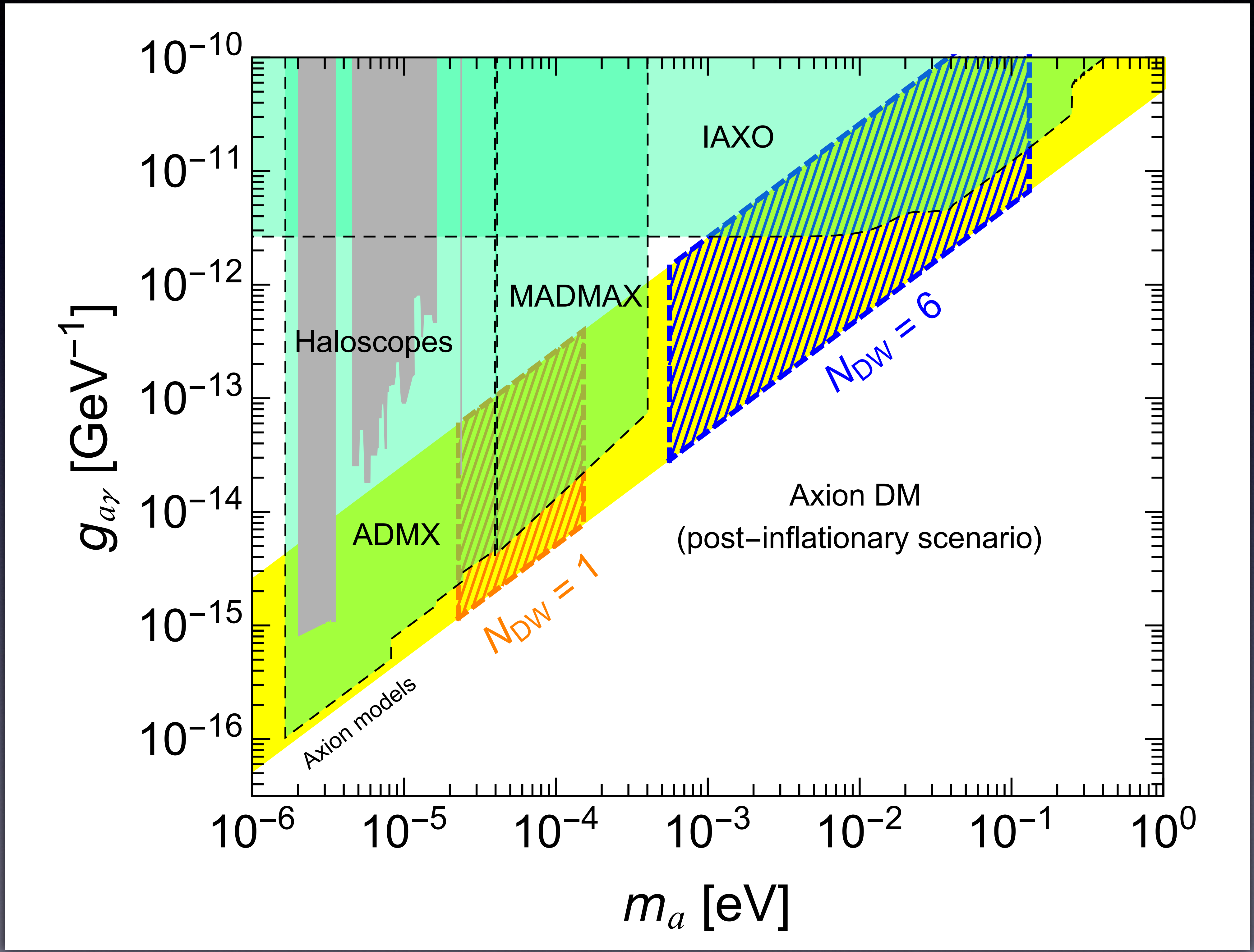
Hiramatsu, Kawasaki, KS and Sekiguchi (2013), Kawasaki, KS and Sekiguchi (2015), Ringwald and KS (2016)



- Domain walls are long-lived and decay due to the explicit symmetry breaking term: $$\Delta V = -\Xi v_{\rm PQ}^3 \left(\Phi e^{-i\Delta} + {\rm h.c.}\right)$$
- The contribution from long-lived domain walls leads to the possibility that axions explain CDM at lower F_a or larger m_a.
$$\Omega_a h^2 \simeq (3.4\text{–}6.2) \times N_{\rm DW}^{-2} \left(\frac{\Xi}{10^{-52}}\right)^{-1/2} \left(\frac{f_a}{10^9\ {\rm GeV}}\right)^{-1/2}$$
- Several constraints on the explicit symmetry breaking term.
(Some (mild) tuning of parameters is required.)

Search for axion dark matter

Search space in photon coupling $g_{a\gamma} \sim \alpha/(2\pi F_a)$ vs. mass m_a



Axion and ALP dark matter

Axion and axion-like particles (ALPs)

- There might exist several axion-like field in low energy effective theory. We generalize the previous considerations:

$$a \;\to\; \{a_i\} = \{a, \varphi_{i'}\} \qquad i = 1, 2, \ldots \text{ number of axion-like fields}$$

- Couplings to gluons and photons now become

$$\mathcal{L} \supset -\frac{\alpha_s}{8\pi} \boxed{\sum_i C_{ig} \frac{a_i}{f_{a_i}}} G^b_{\mu\nu} \tilde{G}^{b\mu\nu} - \frac{\alpha}{8\pi} \sum_i C_{i\gamma} \frac{a_i}{f_{a_i}} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$$\to -\frac{\alpha_s}{8\pi} \boxed{\frac{a}{f_a}} G^b_{\mu\nu} \tilde{G}^{b\mu\nu} - \frac{\alpha}{8\pi} \sum_i C_{i\gamma} \frac{a_i}{f_{a_i}} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

- We define the field a that couples to gluons as QCD axion.

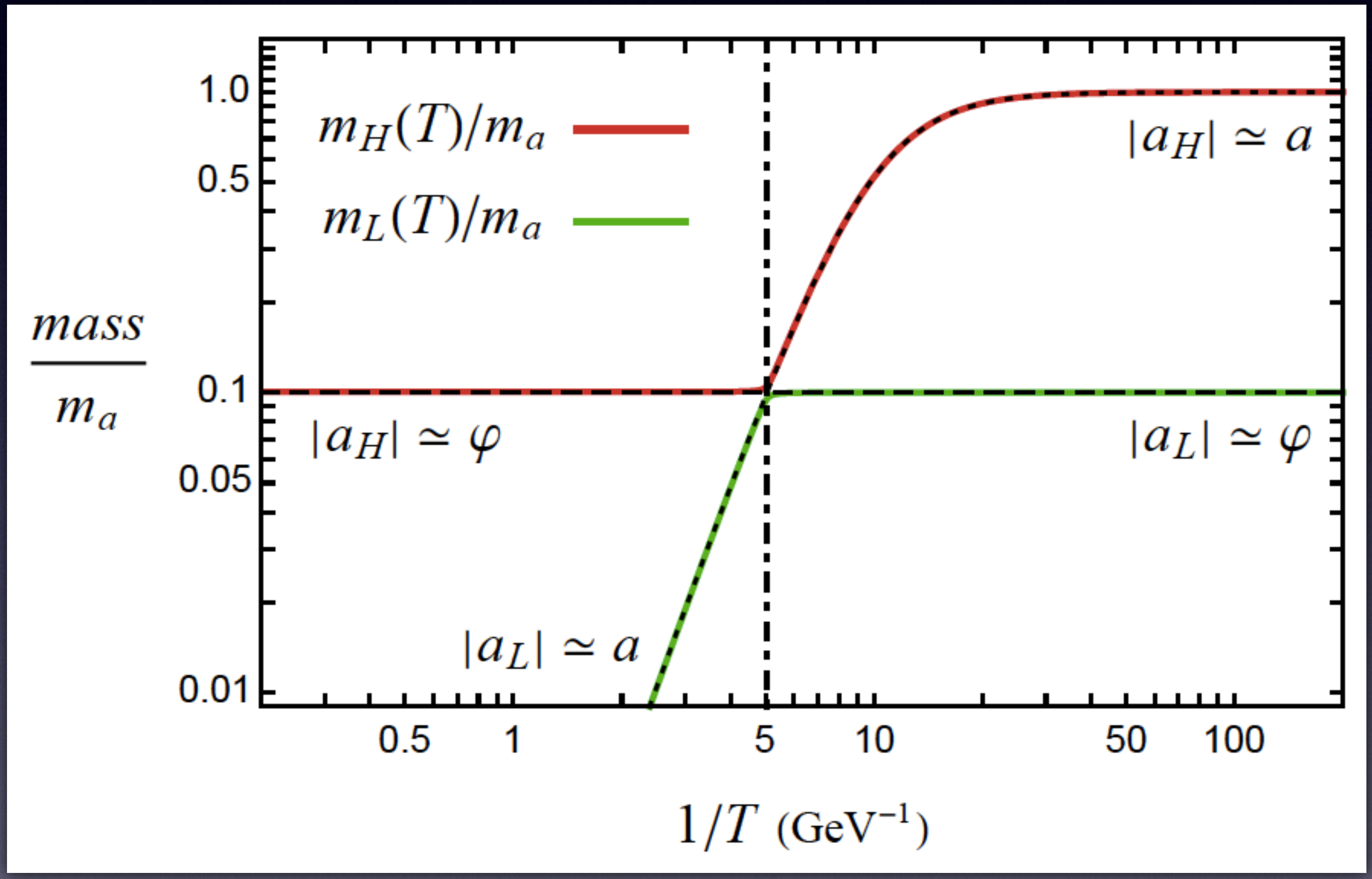
$$\frac{a}{f_a} \equiv \sum_i C_{ig} \frac{a_i}{f_{a_i}}$$

- Fields $\varphi_{i'}$ that do not couple to QCD (but may still couple to photons) are referred to as ALPs.

Mass mixing between axion and ALP

$$V_{\rm mix}(a,\varphi) = m_\varphi^2 f_\varphi^2 \left[1 - \cos\left(\frac{a}{f_a} + \frac{\varphi}{f_\varphi}\right)\right]$$

Level crossing behavior of mass eigenvalues when $f_\varphi \ll f_a$ and $m_\varphi \ll m_a$.

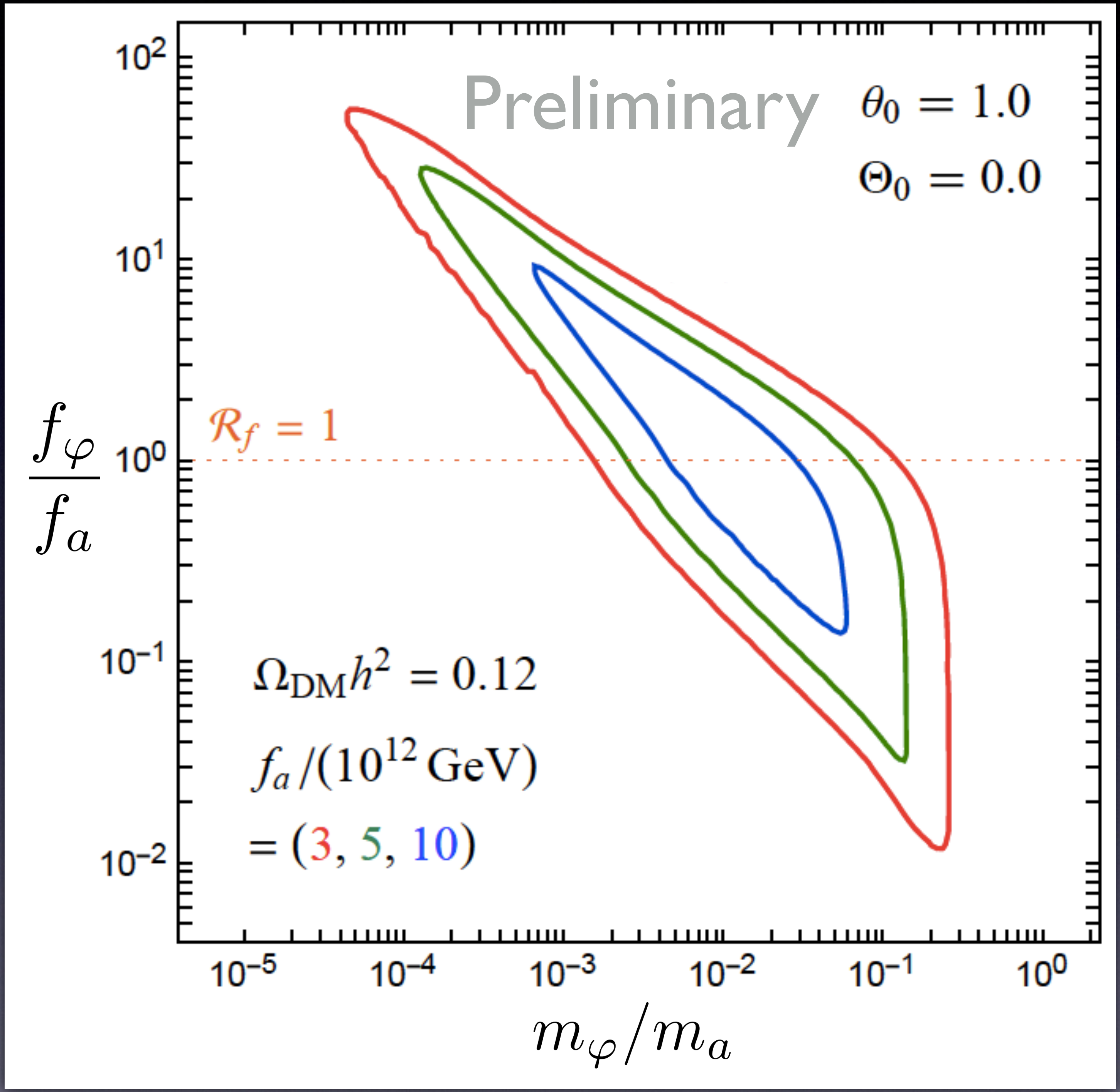


Ho, KS and Takahashi, work in progress

Adiabatic conversion:
Dark matter (mostly the lighter eigenmode) is produced as QCD axion, but behaves like ALP at the present time.

Prediction for ALP dark matter

Ho, KS and Takahashi, work in progress



The ALP can become the main constituent of dark matter at $f_\varphi \ll f_a$:
The coupling to photons can be enhanced.

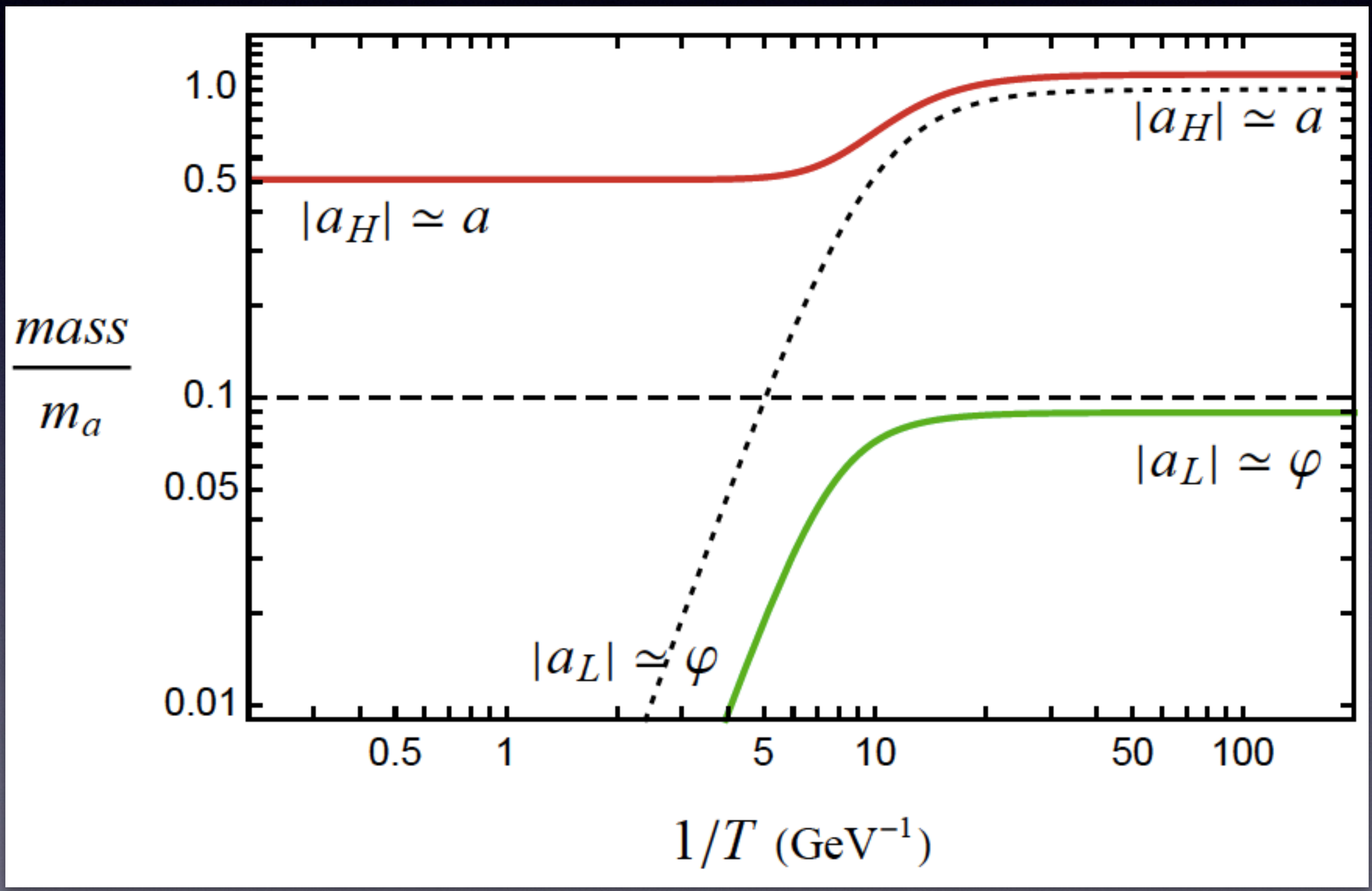
Summary

- Axion is a well motivated hypothetical particle and a good candidate of dark matter.
- Predictions for dark matter strongly depend on the early history of the universe.
  - A variety of contributions from topological defects if the PQ symmetry was broken after inflation.
  - Enhancement of couplings due to the adiabatic conversion between the axion and ALP.
- Mass ranges can be probed in the future experiments.

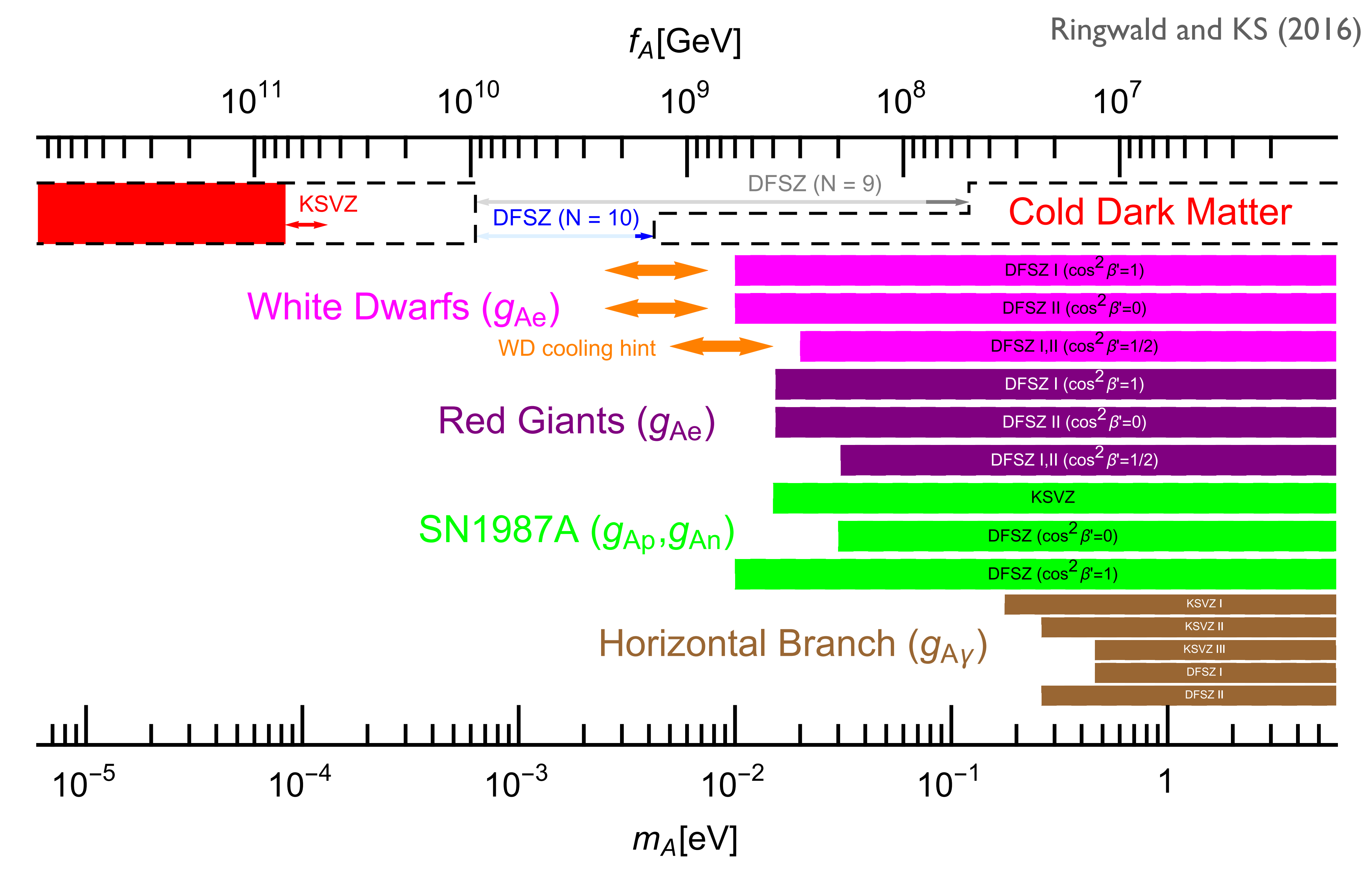
Backup slides

Behavior of mass eigenvalues (other cases)

For $1 \ll f_\varphi / f_a \ll m_a / m_\varphi$

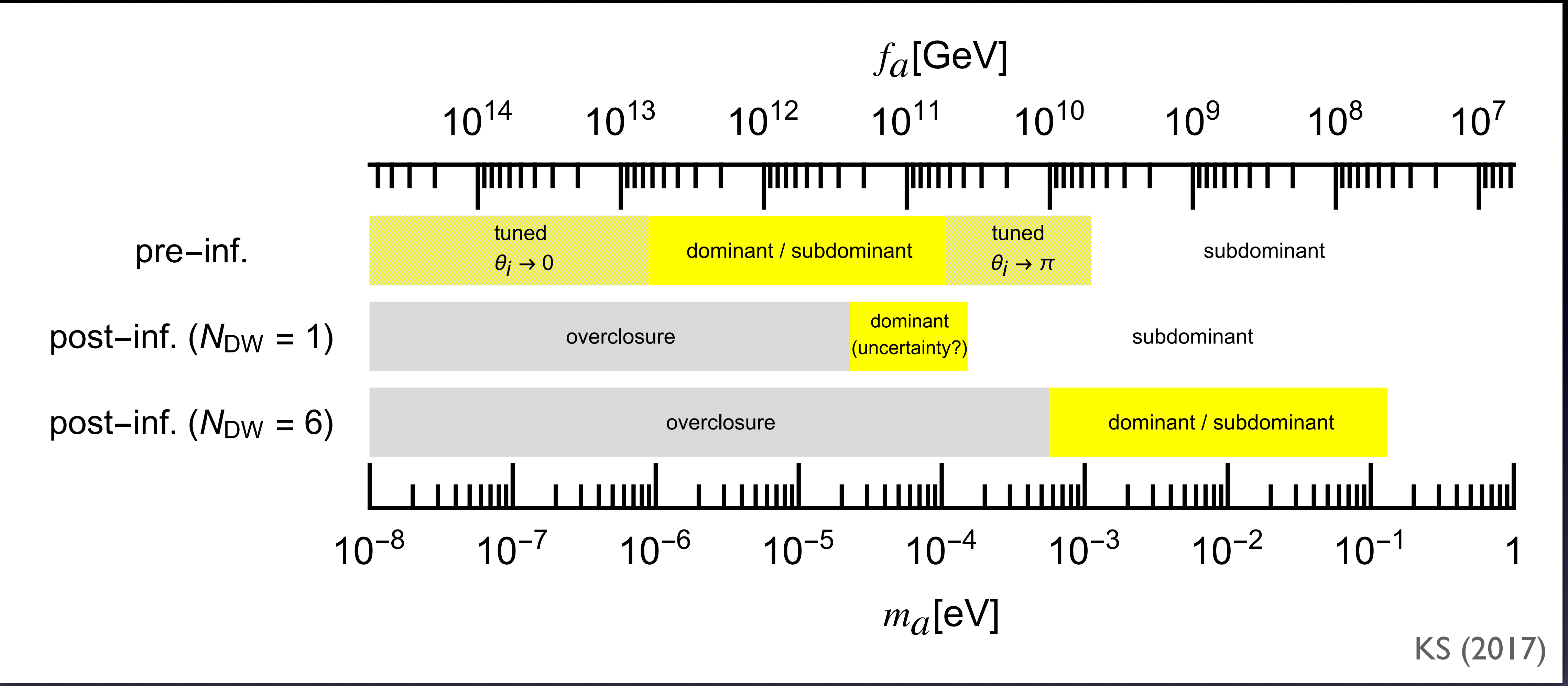


Astrophysical and cosmological constraints



- Astrophysical observations give lower (upper) bounds on $F_a\,(m_a)$
- Dark matter abundance gives upper (lower) bounds on $F_a\,(m_a)$ [and also a lower (upper) bound for DFSZ models]

Axion dark matter mass: summary



- Pre-inflationary PQ symmetry breaking scenario :

  Lower mass ranges, depending on initial misalignment angle

- Post-inflationary PQ symmetry breaking scenario (N_{DW} = 1) :

  Potentially large systematic uncertainty in numerical simulations

- Post-inflationary PQ symmetry breaking scenario (N_{DW} > 1) :

  Higher mass ranges due to the production from long-lived domain walls

Axion production from topological defects

Davis (1986); Harari and Sikivie (1987); Davis and Shellard (1989); Hagmann and Sikivie (1991); Battye and Shellard (1994); Yamaguchi, Kawasaki, and Yokoyama (1999); Hagmann, Chang, and Sikivie (2001)

- $N_{DW} = 1$:
  String-wall systems collapse soon after the formation of domain walls. This happens around the time of QCD phase transition.

- The relic axion density is given by

$$\rho_a(t_{\mathrm{today}}) = m_a n_a(t_{\mathrm{decay}}) \left(\frac{R(t_{\mathrm{decay}})}{R(t_{\mathrm{today}})} \right)^3$$

$R(t)$: scale factor

where $$n_a(\underline{t_{\mathrm{decay}}}) \sim \frac{\rho_a(t_{\mathrm{decay}})}{\langle E_a(t_{\mathrm{decay}}) \rangle} \sim \frac{\rho_{\mathrm{defects}}(t_{\mathrm{decay}})}{\langle E_a(t_{\mathrm{decay}}) \rangle}$$

Time at the decay of defects

- ρ_{defects} is given by the scaling solution

$$\boxed{\rho_{\mathrm{string}} = \xi \frac{\mu}{t^2}}$$

"$\mathcal{O}(\xi)$ strings in a horizon volume"

μ : string energy per length

- The mean energy $\langle E_a(t_{\mathrm{decay}}) \rangle$ depends on the energy spectrum of radiated axions.

Annihilation mechanism of domain walls

The energy bias acts as a pressure force p_V on the wall

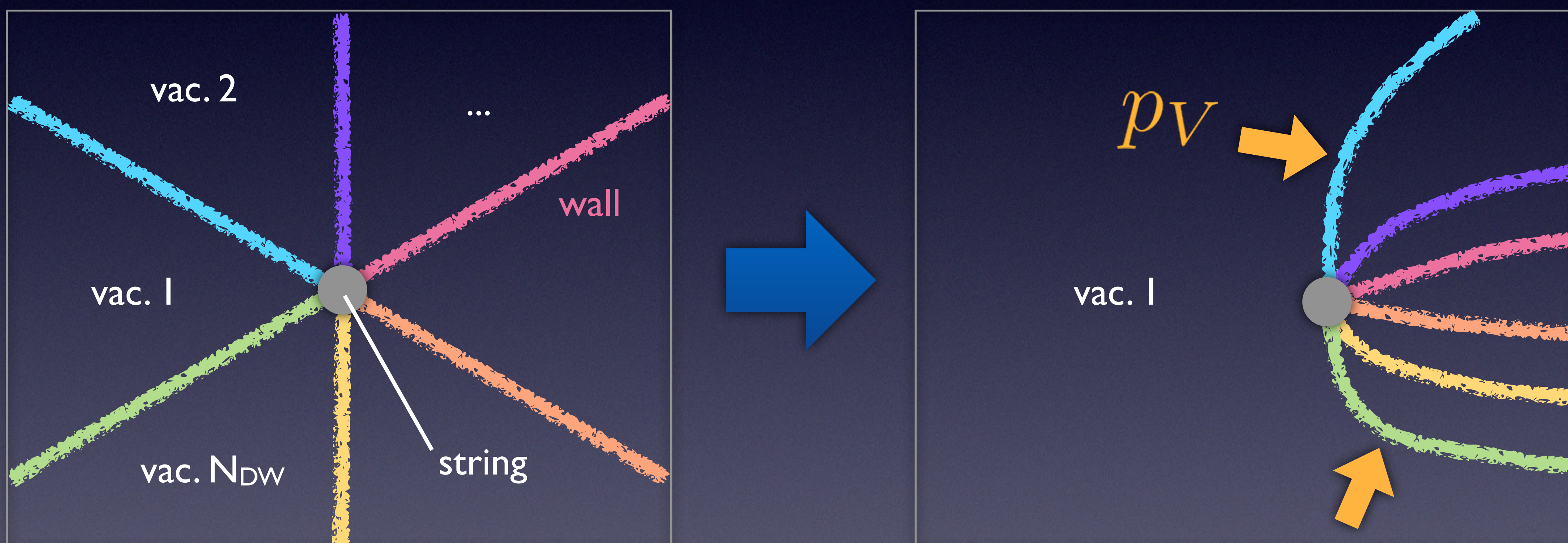
$$p_V \sim \Delta V_{\rm bias}$$

Annihilation occurs when the tension p_T becomes comparable with the pressure p_V

$$p_T \sim \sigma_{\rm wall}/R \sim m_a v_{\rm PQ}^2/N_{\rm DW}^2 R$$

R : curvature radius of walls

$\sigma_{\rm wall}$: surface mass density of walls



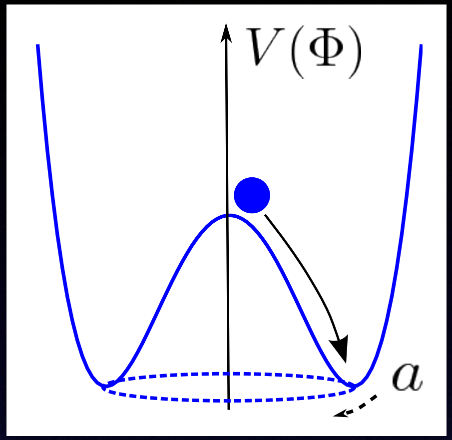
Annihilation time

$$t_{\rm ann} \sim R|_{p_V=p_T} \sim \frac{m_a v_{\rm PQ}^2}{N_{\rm DW}^2 \Delta V_{\rm bias}}$$

$$\sim \mathcal{O}(10^{-4})\,{\rm sec}\left(\frac{6}{N_{\rm DW}}\right)^4\left(\frac{10^{-51}}{\Delta V_{\rm bias}/v_{\rm PQ}^4}\right)\left(\frac{10^9\,{\rm GeV}}{F_a}\right)^3$$

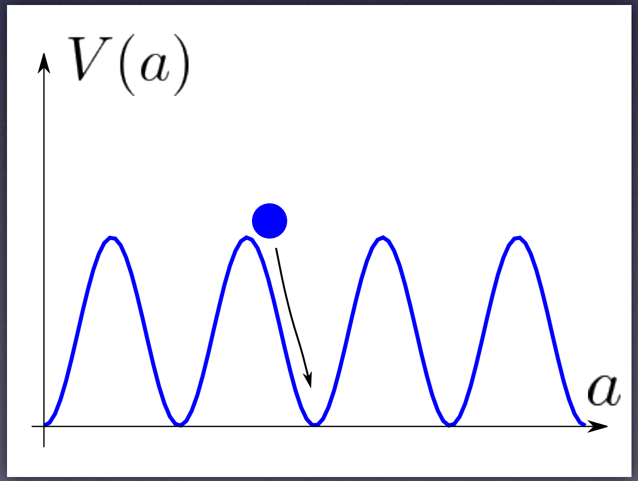
Production of axions in the early universe

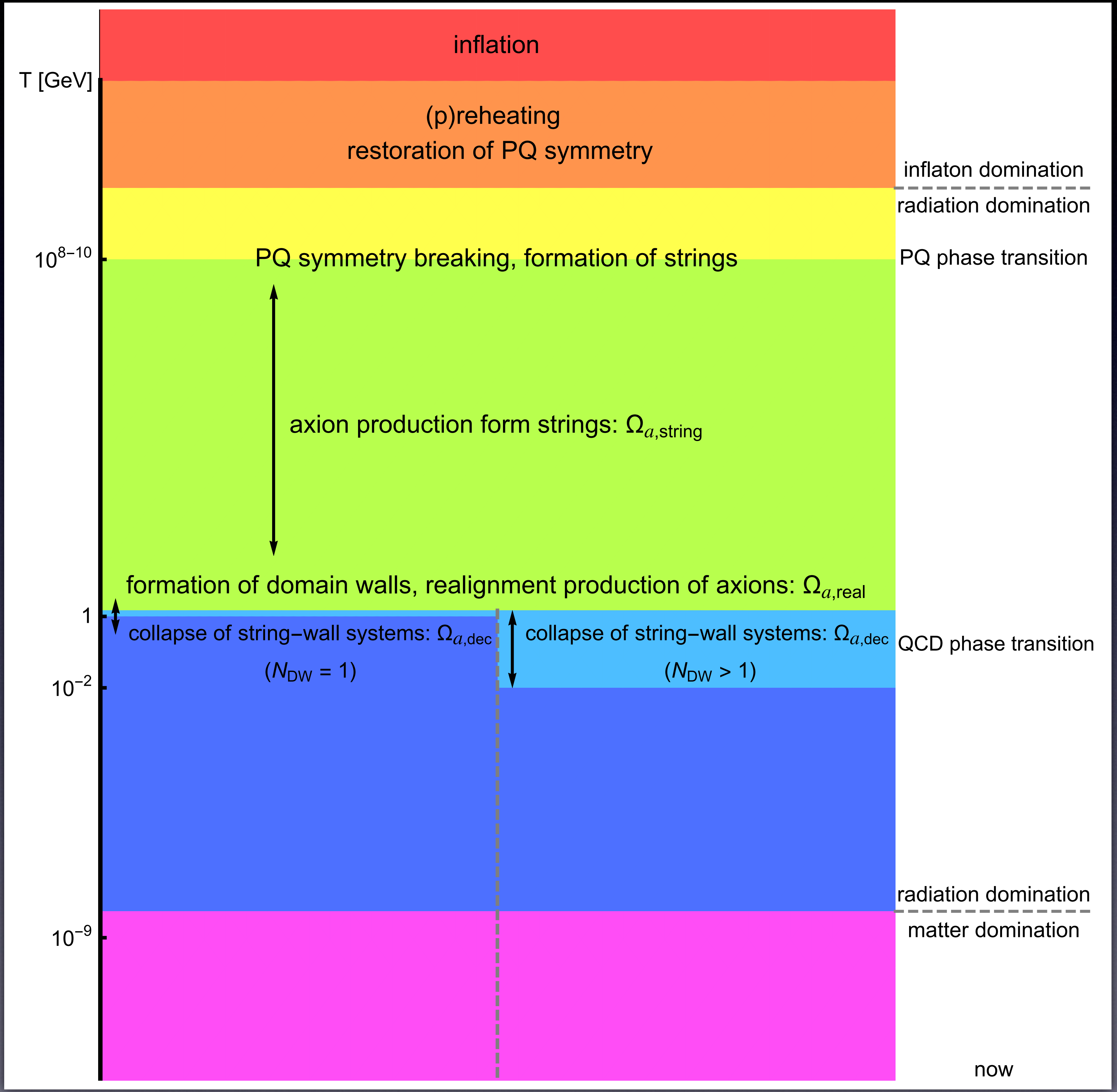
(post-inflationary PQ symmetry breaking scenario)



$$T \lesssim F_a \simeq 10^{8-11}\,\mathrm{GeV}$$

$$T \lesssim 1\,\mathrm{GeV}$$

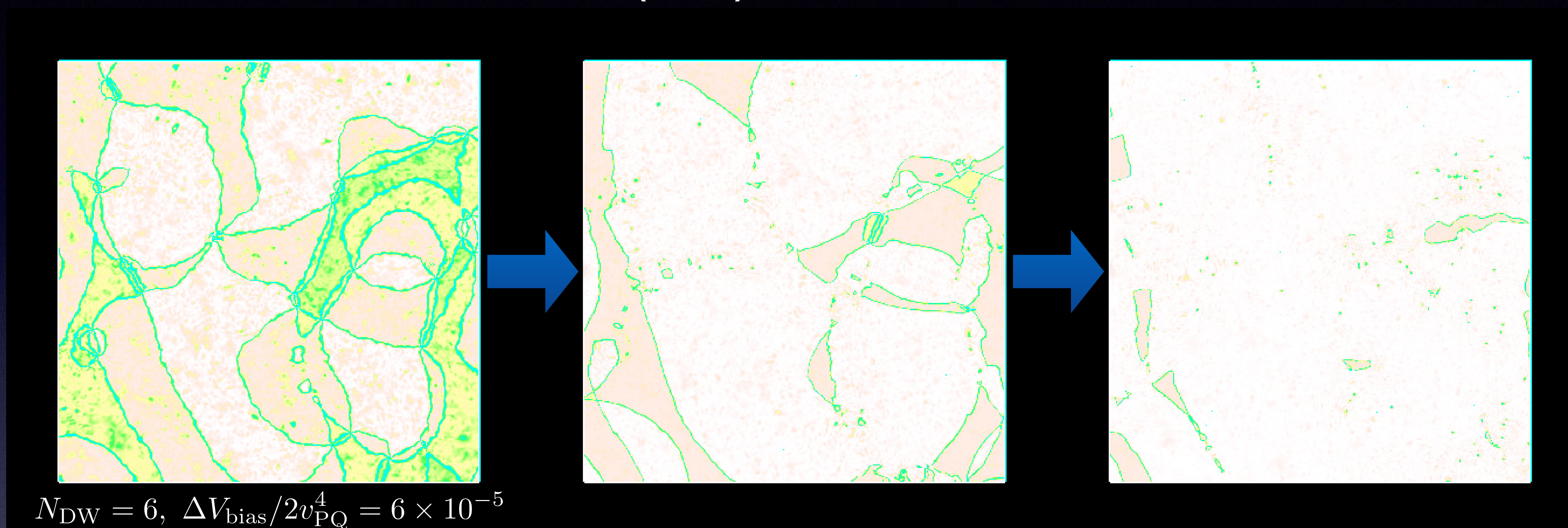




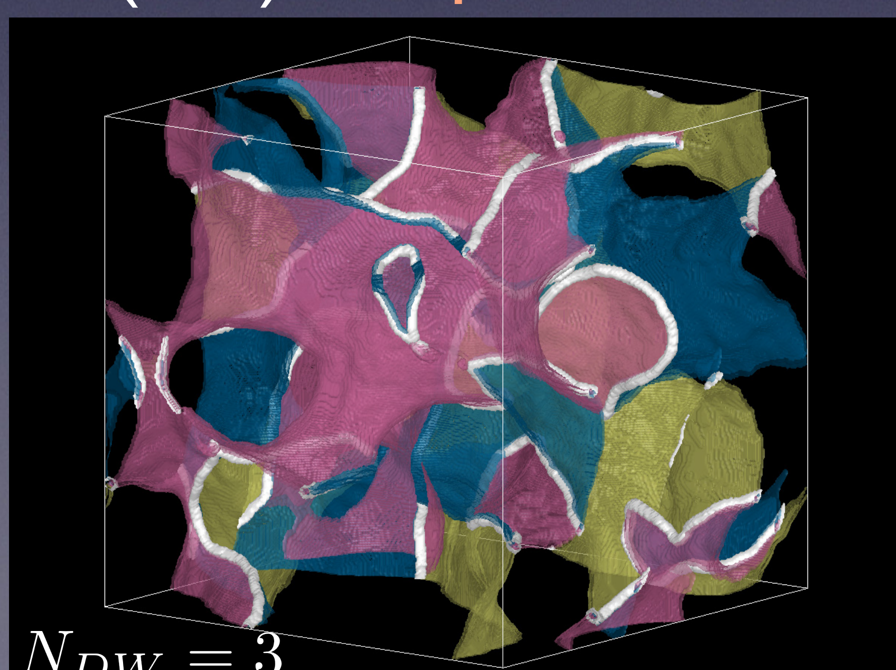
Numerical simulation : $N_{DW} > 1$

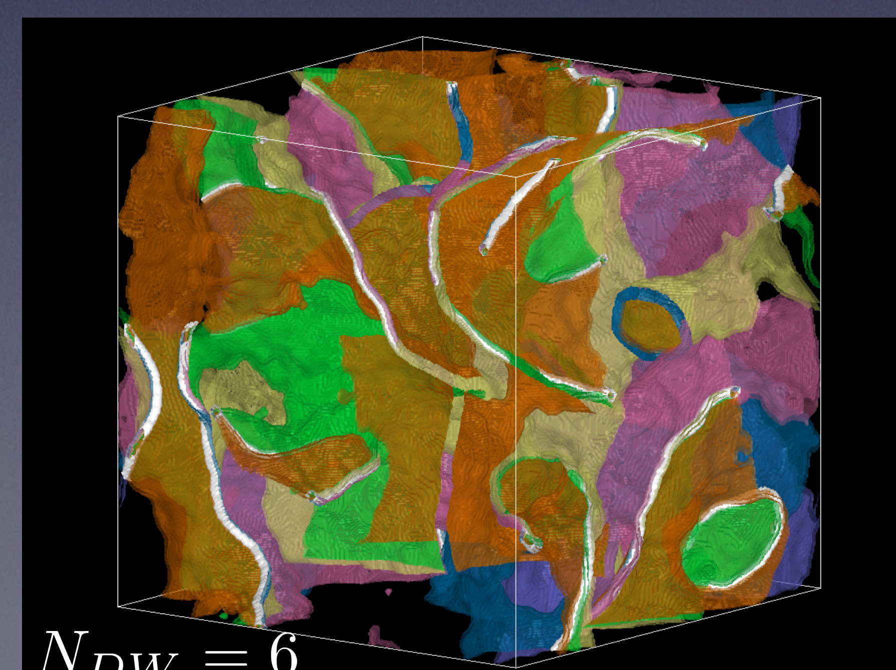
Hiramatsu, Kawasaki, KS and Sekiguchi (2013)
Kawasaki, KS and Sekiguchi (2015)

- 8192^2, 16384^2, 32768^2 (2D) → decay time of domain walls



- 512^3 (3D) → spectrum of radiated axions





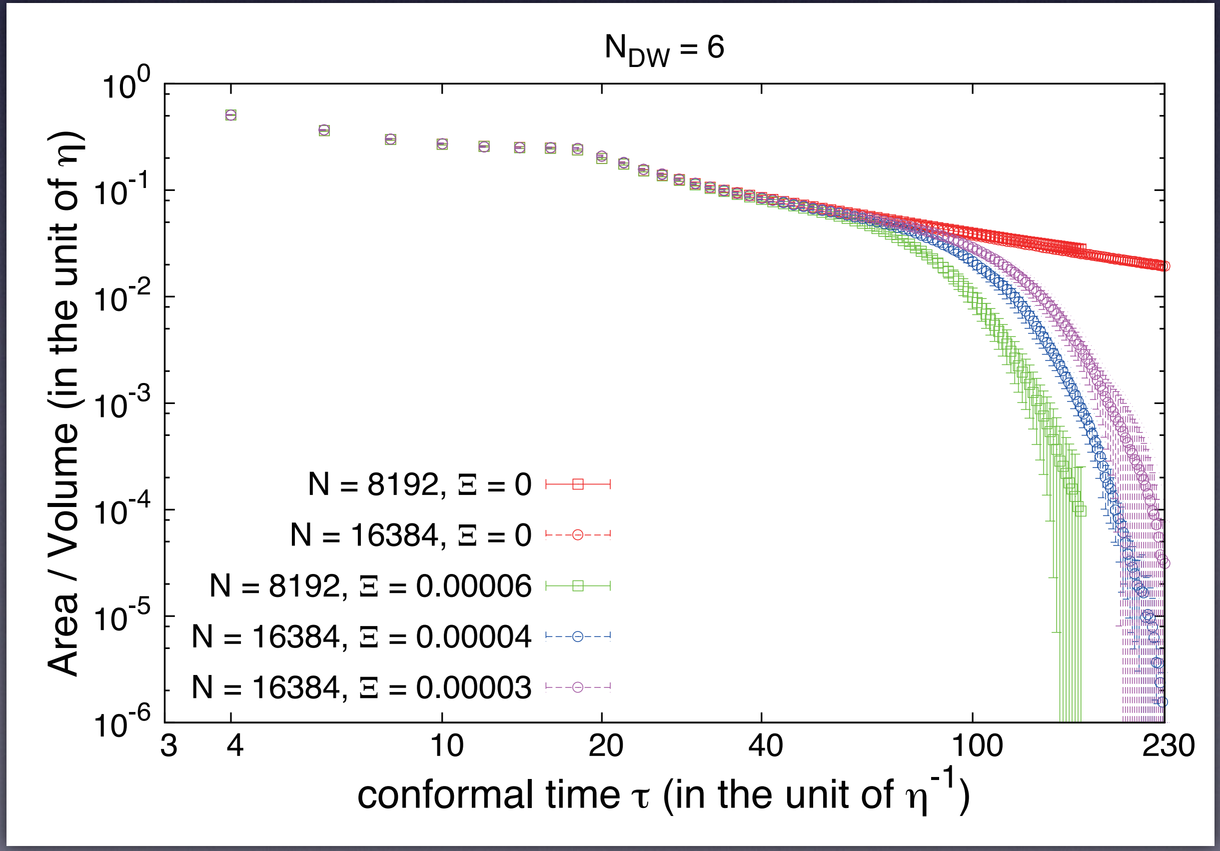
Evolution of long-lived domain walls

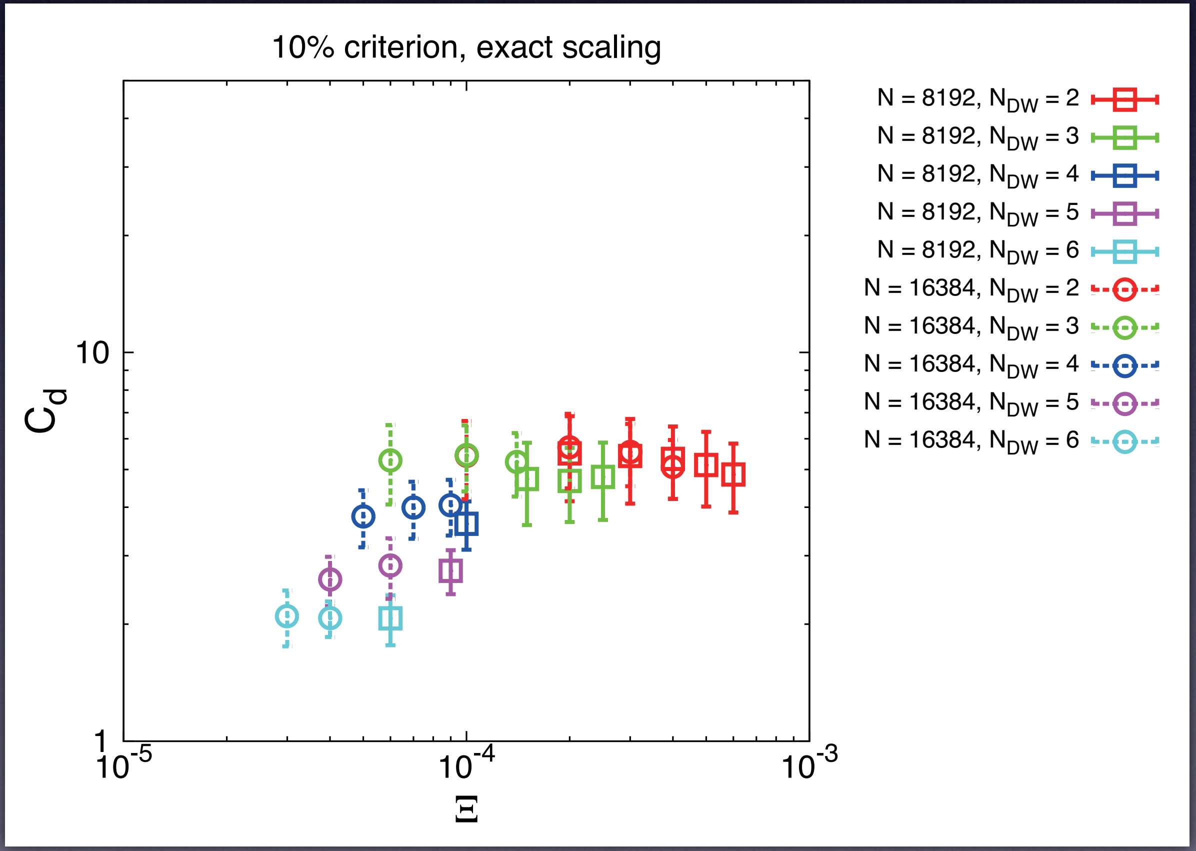
- Walls obey scaling solution if $\Xi = 0$ $\quad \rho_{\rm wall} = \mathcal{A}\dfrac{\sigma}{t}$
- Decay time (estimated from the condition $\underset{\text{pressure}}{\Xi v_{\rm PQ}^4} \gtrsim \underset{\text{tension}}{\mathcal{A}\sigma/t}$)

$$t_{\rm dec} = C_d \frac{\mathcal{A}\sigma}{\Xi v_{\rm PQ}^4 [1 - \cos(2\pi N/N_{\rm DW})]}$$

- C_d is determined from numerical simulation

$$\left.\frac{A/V(\Xi)}{A/V(\Xi=0)}\right|_{t_{\rm dec}} = 0.1$$





$\Rightarrow$ $C_d \simeq 2$–5

Constraints

- CP violation

The higher dimensional operator shifts the minimum of the potential and spoils the original Peccei-Quinn solution to the strong CP problem.

$$\frac{\langle a \rangle}{F_a} \simeq \frac{\frac{N|g|N_{\rm DW}^{N-1}}{(\sqrt{2})^{N-2}} \left(\frac{F_a}{M_{\rm Pl}}\right)^{N-2} M_{\rm Pl}^2 \sin\Delta_D}{m_a^2 + \frac{N^2|g|N_{\rm DW}^{N-2}}{(\sqrt{2})^{N-2}} \left(\frac{F_a}{M_{\rm Pl}}\right)^{N-2} M_{\rm Pl}\cos\Delta_D} < 7\times 10^{-12}$$

where $\Delta_D \propto \mathrm{Arg}(g)$

→ Large N is required

- Dark matter abundance

Long-lived domain walls produce too much cold axions.

Hiramatsu, Kawasaki, KS, and Sekiguchi (2013); Kawasaki, KS, and Sekiguchi (2015)

$$\Omega_a h^2 \simeq (3.4\text{–}6.2)\times N_{\rm DW}^{-2}\left(\frac{\Xi}{10^{-52}}\right)^{-1/2}\left(\frac{F_a}{10^9\ \mathrm{GeV}}\right)^{-1/2} \quad \text{with} \quad \Xi = \frac{|g|N_{\rm DW}^{N-4}}{(\sqrt{2})^N}\left(\frac{F_a}{M_{\rm Pl}}\right)^{N-4}$$

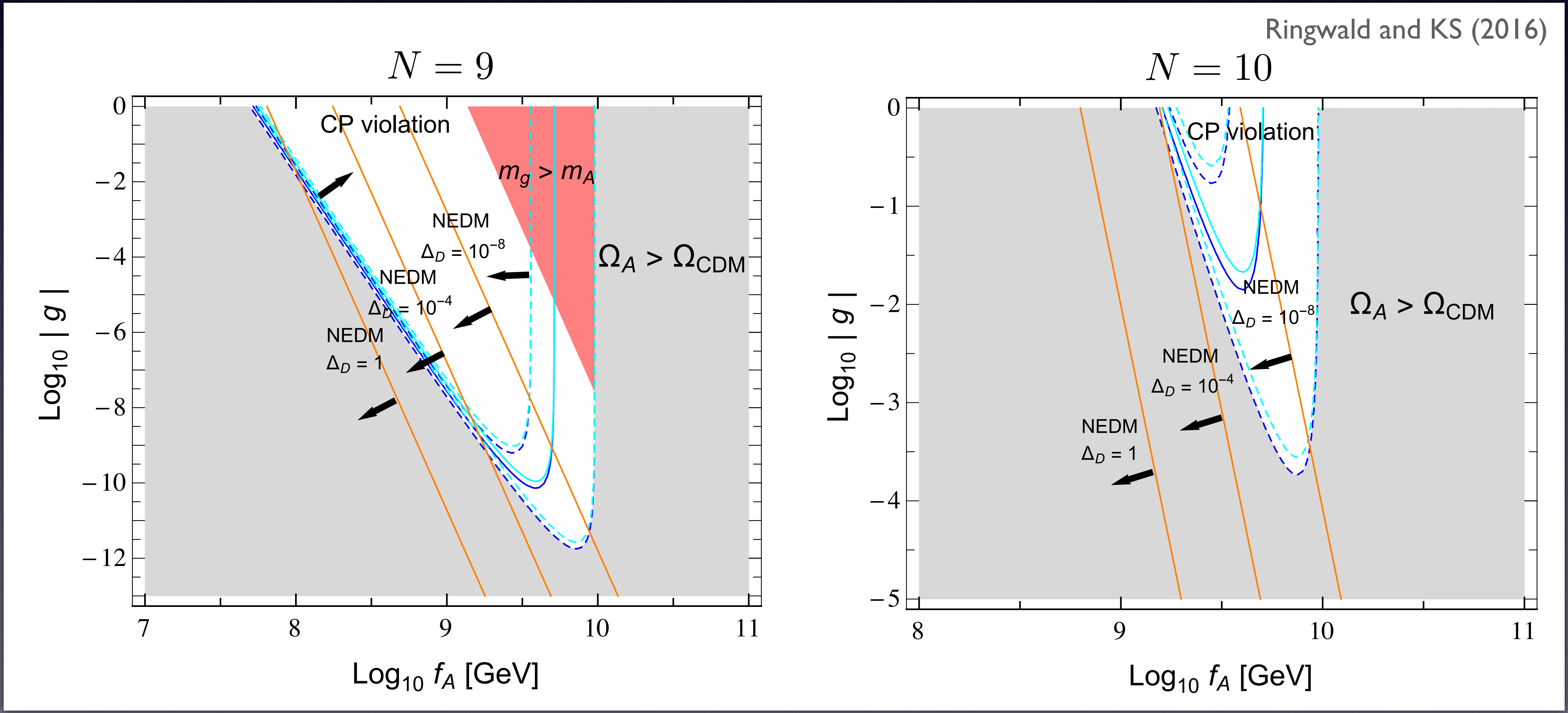
→ Small N is required

- Constraints on the energy bias (= on the coefficient g)

$$\Delta V_{\rm bias} = -\frac{|g| N_{\rm DW}^{N-4}}{(\sqrt{2})^{N-2}} \left(\frac{F_a}{M_{\rm Pl}}\right)^{N-4} v_{\rm PQ}^4 \cos\left(N\frac{a}{v_{\rm PQ}} + \Delta_D\right) \Leftarrow \mathcal{L} \supset \frac{g}{M_{\rm Pl}^{N-4}}\Phi^N + {\rm h.c.}$$

- Loopholes appear if the order of the operator is N = 9 or 10, but some tuning of the phase parameter Δ_D is required.

- With a mild tuning, axions can explain total dark matter abundance in the small F_a range.



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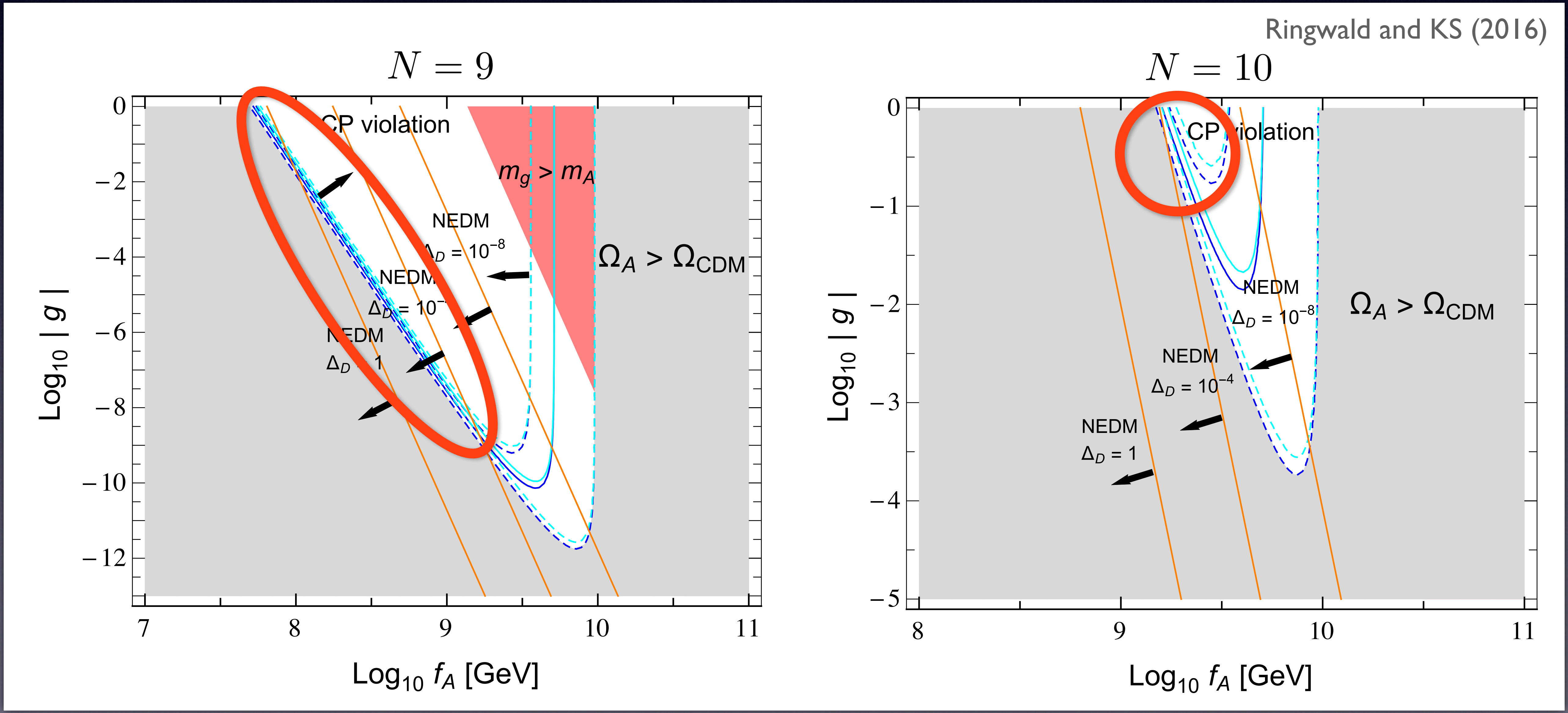


Radiation of axions

- Compute power spectrum by using data of scalar field $\Phi(t, \mathbf{x})$ obtained by simulations

$$\frac{1}{2}\langle \dot{a}(t, \mathbf{k})^* \dot{a}(t, \mathbf{k}')\rangle = \frac{(2\pi)^3}{k^2}\delta^{(3)}(\mathbf{k} - \mathbf{k}')P(k, t)$$

$$\dot{a}(t, \mathbf{k}) = \int d^3\mathbf{x} e^{i\mathbf{k}\cdot\mathbf{x}} \dot{a}(t, \mathbf{x}) \qquad \dot{a}(t, \mathbf{x}) = \mathrm{Im}\left[\frac{\dot{\Phi}}{\Phi}(t, \mathbf{x})\right]$$

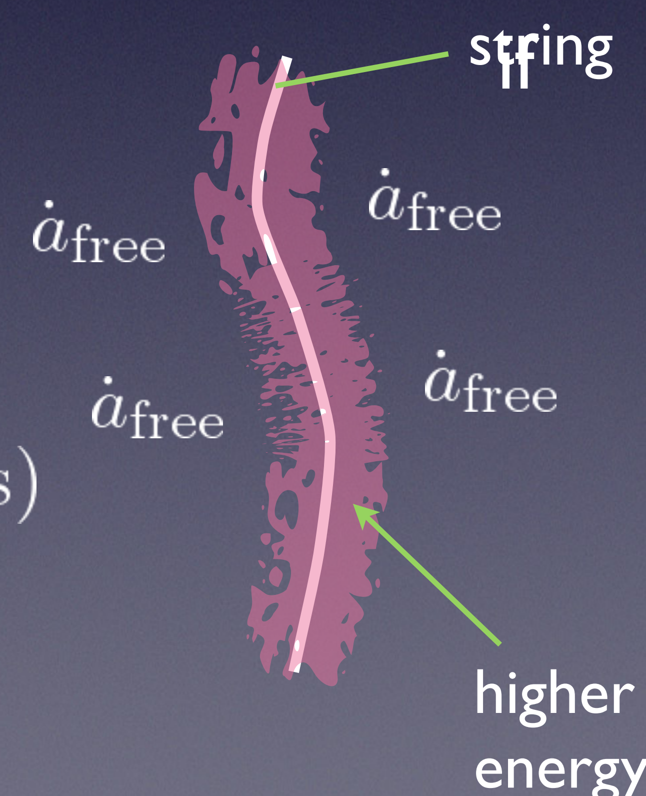
- We overestimate the energy of axions we include data on the defects

$$\dot{a}(t, \mathbf{x})$$

$$= \dot{a}_{\mathrm{free}}(t, \mathbf{x}) + (\text{contamination from defects})$$

radiated axions

higher energy

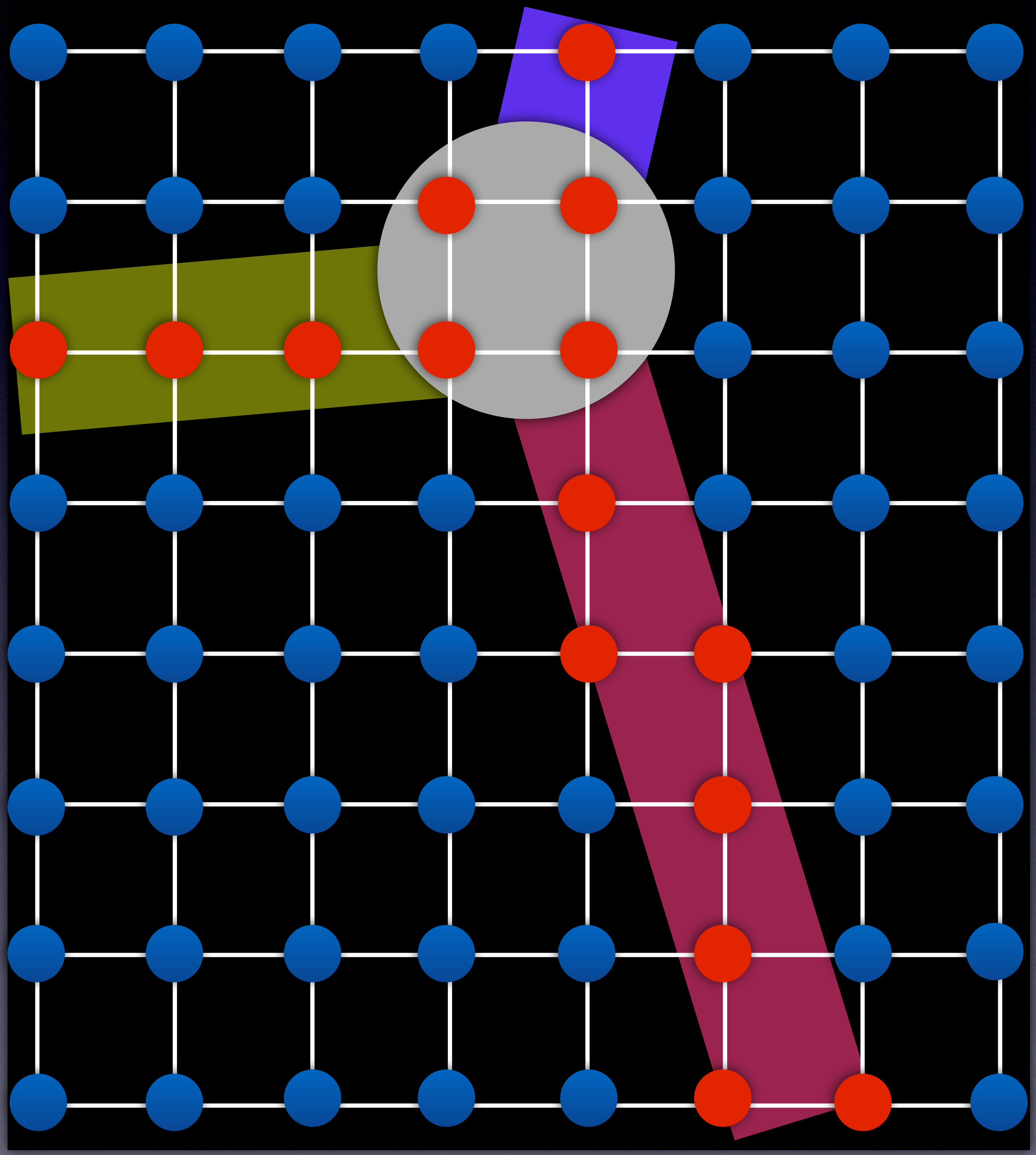


Masking analysis

Hiramatsu, Kawasaki, Sekiguchi, Yamaguchi and Yokoyama (2011)

$a(x)$: contains contamination from defects ●

$a_{\rm free}(x)$: use masked data ● only

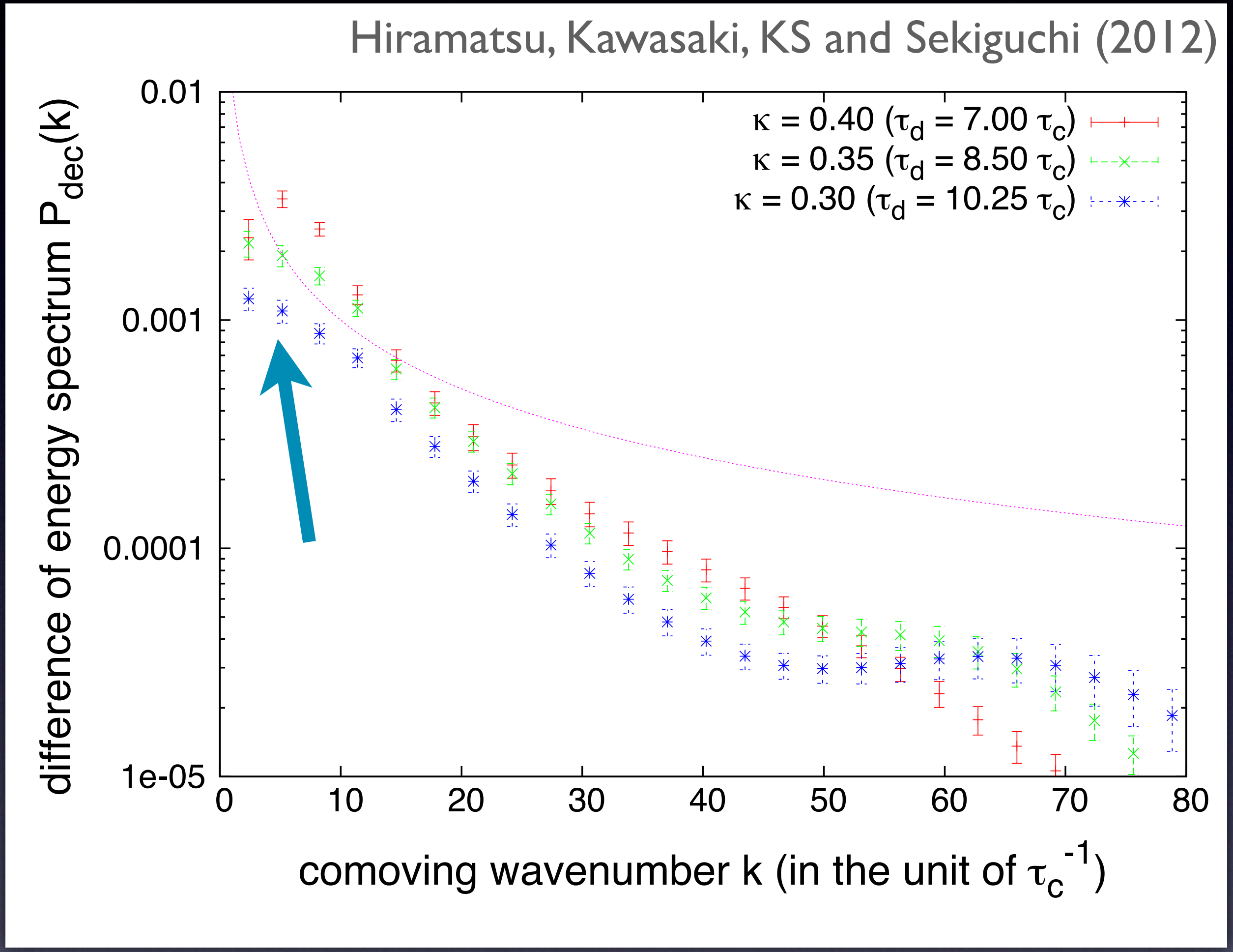


compute

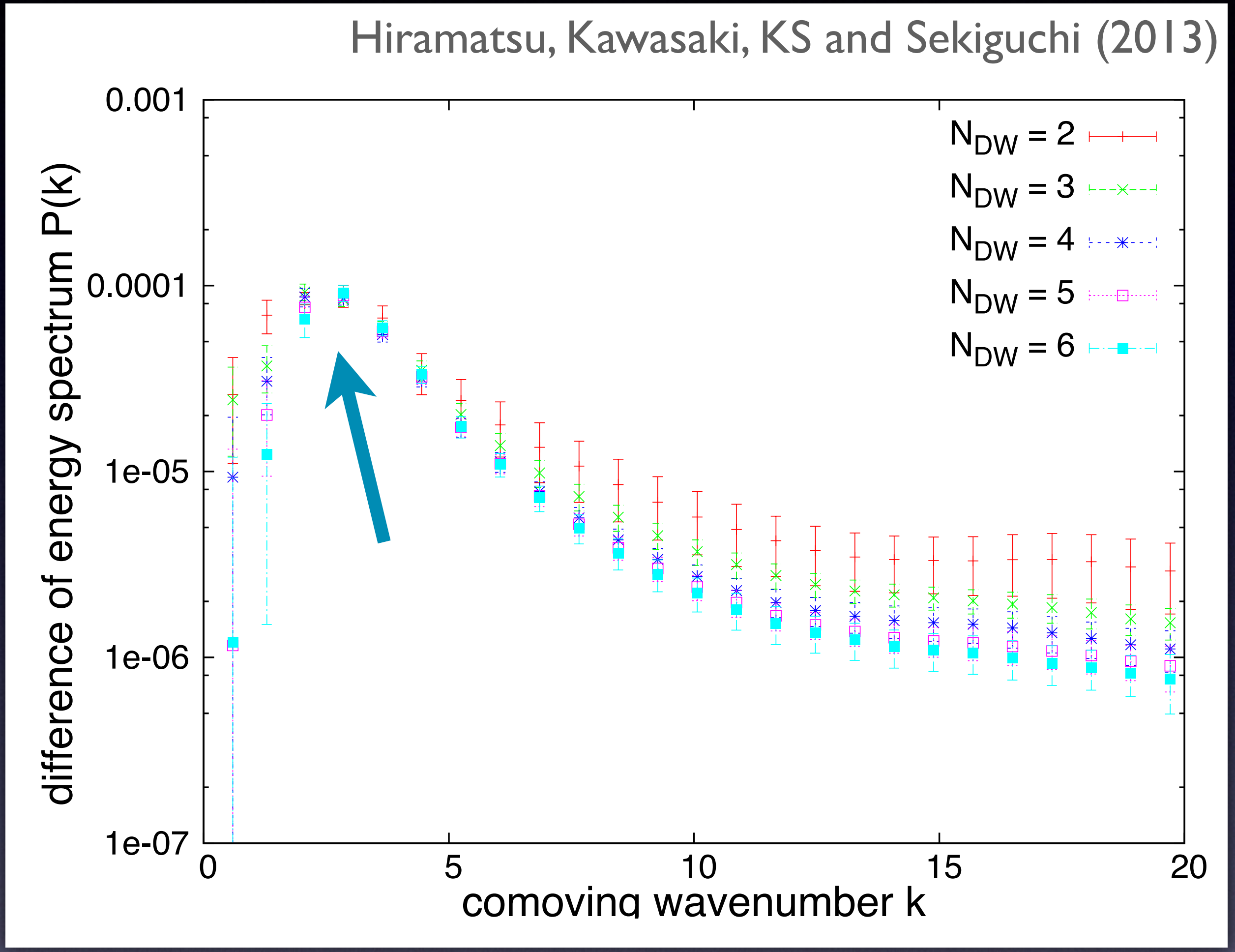
$$\frac{1}{2}\langle \dot{a}_{\rm free}(\mathbf{k})^* \dot{a}_{\rm free}(\mathbf{k}')\rangle = \frac{(2\pi)^3}{k^2}\delta^{(3)}(\mathbf{k}-\mathbf{k}')P(k)$$

Spectrum of axions

$N_{DW} = 1$



$N_{DW} > 1$



Peaked at

$$\langle E_a \rangle \simeq \mathcal{O}(1) \times m_a$$

(axions are mildly relativistic)

Contribution for relic CDM abundance

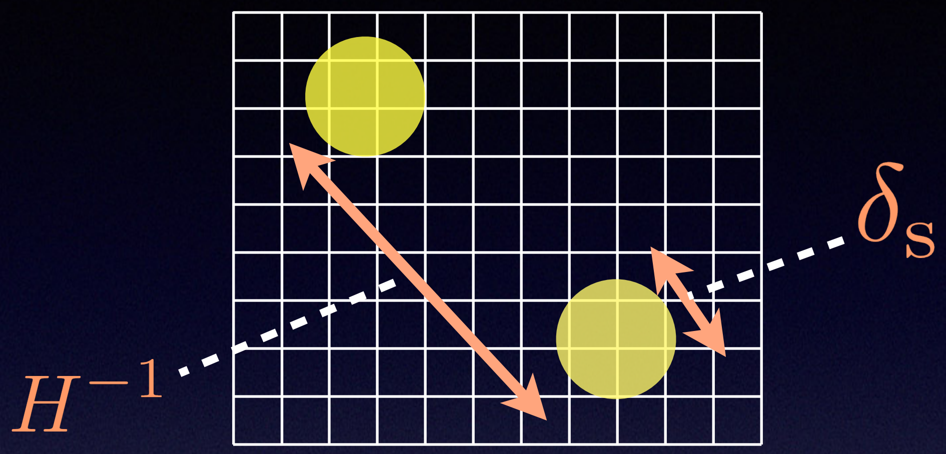
$$\rho_a(t_{\mathrm{today}}) = m_a \frac{\rho_a(t_{\mathrm{decay}})}{\langle E_a \rangle} \left(\frac{R(t_{\mathrm{decay}})}{R(t_{\mathrm{today}})} \right)^3$$

Technical limitations of lattice simulations

- We must consider two extremely different length scales.
  - Width of string core

    $$\delta_s \sim (\sqrt{\lambda} v_{PQ})^{-1} = \text{const.}$$

  - Hubble radius $H^{-1} \sim t$



- In order to follow the time evolution correctly, we must maintain

$$\delta_s > \text{lattice spacing} \propto R(t)$$

$$H^{-1} < \text{simulation box size}$$

- These conditions put a constraint on the simulation time:

$H^{-1}/\delta_s \lesssim 300$ for 512^3 lattice,

while $H^{-1}/\delta_s \sim \sqrt{\lambda} v_{PQ}/m_a(T_{QCD}) \sim 10^{30}$ at the realistic situation.

- To what extent can we believe the simulation results ?

Global nature of strings

- String tension acquires a large logarithmic correction due to the gradient energy:

$$\mu = \frac{\text{energy}}{\text{length}} = \int r dr \int_0^{2\pi} d\varphi \left[\left| \frac{\partial \Phi}{\partial r} \right|^2 + \left| \frac{1}{r} \frac{\partial \Phi}{\partial \varphi} \right|^2 + V(\Phi) \right]$$

$$\approx 2\pi \int r dr \left| \frac{1}{r} \frac{\partial \Phi}{\partial \varphi} \right|^2 \simeq \pi v_{\mathrm{PQ}}^2 \ln \left(H^{-1}/\delta_{\mathrm{s}} \right)$$

- When $H^{-1} \gg \delta_{\mathrm{s}}$, this is larger than the string radiation power: $P \sim v_{\mathrm{PQ}}^2$ Vilenkin and Vachaspati (1987)
- We expect that the radiation damping becomes less important in the limit of $H^{-1}/\delta_{\mathrm{s}} \gg 1$ Dabholkar and Quashnock (1990)

(cf. $\ln(H^{-1}/\delta_{\mathrm{s}}) \simeq 69 \quad \text{for} \quad H^{-1}/\delta_{\mathrm{s}} = 10^{30}$)

Strings might be denser.

Simulations with auxiliary fields

Klaer and Moore, JCAP10(2017)043 [arXiv:1707.05566]
Klaer and Moore, JCAP11(2017)049 [arXiv:1708.07521]

- Introduce two complex scalars and one U(1) gauge field:

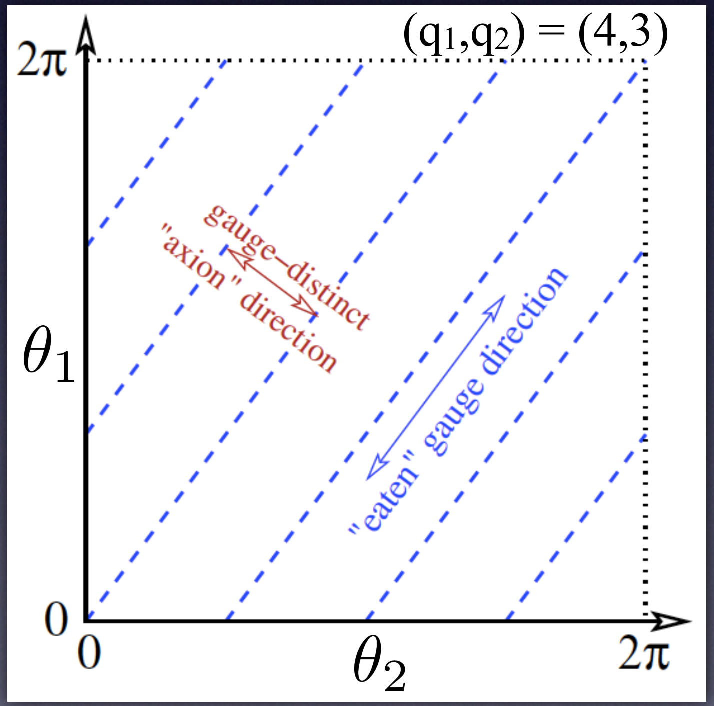
$$-\mathcal{L} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |(\partial_\mu - iq_1 e A_\mu)\,\Phi_1|^2 + |(\partial_\mu - iq_2 e A_\mu)\,\Phi_2|^2$$

$$+\lambda\left[\left(|\Phi_1|^2 - \frac{v^2}{2}\right)^2 + \left(|\Phi_2|^2 - \frac{v^2}{2}\right)^2\right] \qquad \text{with} \qquad q_1 \neq q_2$$

- Among two phases $\theta_1 = \mathrm{Arg}(\Phi_1)$ and $\theta_2 = \mathrm{Arg}(\Phi_2)$, one combination is eaten by A_μ, and the other is identified as massless axion with a decay constant

$$F_a = \frac{v}{\sqrt{q_1^2 + q_2^2}}$$

$$\theta_{\mathrm{axion}} = q_2\theta_1 - q_1\theta_2$$



- String tension is given by that of gauge string:

$$T \simeq 2\pi v^2$$

- Tension becomes relatively high compared with the coupling of strings to axions ($\propto F_a^2$):

$$\kappa \equiv \frac{T}{\pi F_a^2} \simeq 2(q_1^2 + q_2^2)$$