QCD motivated meson models with a chiral imbalance

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Outline

Introduction

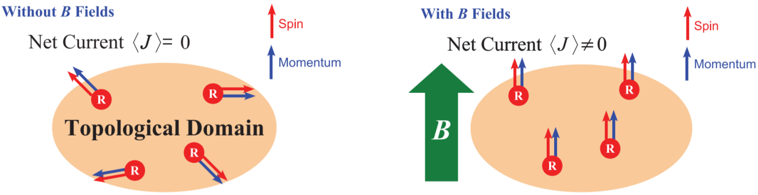
Parity Symmetry Breaking (CP-breaking)

- CP-odd condensates as regular phase at finite density $\Longleftrightarrow$ **baryonic chemical potential**
  - Migdal A B Zh. Eksp. Teor. Fiz. 61 (1971); Lee T D and Wick G C Phys. Rev. D 9 , 2291 (1974)
  - Andrianov A A, Andrianov V A and Espriu D Phys. Lett. B 678, 416 (2009)
- Topological fluctuation as regular phase at finite density or temperature $\Longleftrightarrow$ **chiral chemical potential**
  - Kharzeev D E, Pisarski R D and Tytgat M H G Phys. Rev. Lett. 81, 512 (1998)
  - Buckley K , Fugleberg T and Zhitnitsky A Phys. Rev. Lett. 84, 4814 (2000)
  - Andrianov A A, Andrianov V A, Espriu D and Planells X Phys. Lett. B 710 (2012)
  - Yamamoto A Phys. Rev. D 84 (2011) 114504; Phys. Rev. Lett. 107 (2011) 031601

Introduction

Detection of CP-breaking

- Chiral magnetic effect $j = \frac{e^2}{2\pi^2} \mu_5 B$
  - Wang G. Experimental Overview of the Search for Chiral Effects at RHIC. Journal of Physics: Conference Series, 779, 1 (2017)
  - Search for the Chiral Magnetic Effect in Relativistic Heavy-Ion Collisions. Jie Zhao, Int. J. Mod. Phys. A, 33, 1830010 (2018)



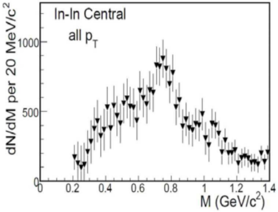
- Anomalous escape of dileptons
  - **NA60 Collaboration**. Eur. Phys. J. C, 2009, vol 61 (4); **STAR Collaboration**. Phys. Rev. Lett., 2014, vol 113 (2); **PHENIX Collaboration**. Phys. Rev. C, 2010, 40, vol 81 (3)
  - Andrianov A.A., Andrianov V.A., Espriu D., Planells X. Analysis of dilepton angular distributions in a parity breaking medium. Phys. Rev. D, 2014, vol. 90

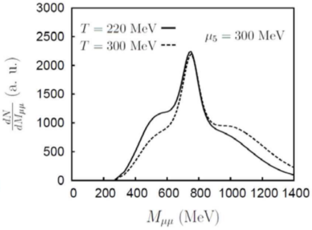
Massive vector mesons split into three polarizations with masses:

$m^2_{V,+} < m^2_{V,L} < m^2_{V,-}$

Anomalies in dilepton production in meson decays:

$\rho, \omega \to e^+e^-, \eta, \eta' \to \gamma\, e^+e^-, \omega \to \pi^0 e^+e^-$





Introduction
Detection of CP-breaking

- Processes which violate the party explicitly
  - Andrianov A.A., Espriu D., Planells X. An effective QCD Lagrangian in the presence of an axial chemical potential. Eur. Phys. J. C, 2013, vol. 73 (1)
    CP-symmetry: $\eta \to \pi^+\pi^-\pi^0$, $\eta \to 3\pi^0$, $\eta' \to \pi^+\pi^-\eta$
    CP-breaking: $\eta \to \pi^+\pi^-$, $\eta' \to \pi^+\pi^-$
- Asymmetry of photon polarization
  - Kawaguchi M., Harada M., Matsuzaki S., Ouyan R. Charged pions tagged with polarized photons probing strong CP violation in a chiral-imbalance medium. Phys. Rev. C, 2017, vol. 95 (6)
    $\pi^+\gamma \to \pi^+\gamma$

Introduction
Formulation of the problem

Aim of work:

study of the environment with the chiral chemical potential within the framework of Nambu-Jona-Lasinio Model and QCD motivated sigma model

The results of this work were published in:

- Andrianov A.A., Andrianov V.A., Espriu D., Putilova A.E., Iakubovich A.V. *Decays of light mesons triggered by chiral chemical potential.* Acta Physica Polonica B, 2017, Vol. 10, No. 4, P. 977-982.
- Andrianov A.A., Andrianov V.A., Espriu D., Putilova A.E., Iakubovich A.V. *Exotic meson decays in the environment with chiral imbalance.* European Physical Journal: Web of Conferences, 2017, Vol. 158, No. 03012, 10 p.
- Andrianov A.A., Andrianov V.A., Espriu D., Putilova A.E., Iakubovich A.V. *QCD with chiral chemical vector: models versus lattice.* Physics of Particles and Nuclei Letters, 2018, Vol. 15, No. 4, P. 357–361.

Introduction

Topological fluctuations in the restricted region

$$\partial^{\mu} J_{5,\mu} = 2\, i\, \bar{\psi}\, \hat{m}_q \gamma_5\, \psi + \frac{N_f\, g^2}{8\pi^2} \epsilon_{\mu\nu\alpha\beta}\, tr(G_{\mu\nu} G_{\alpha\beta}) \qquad (1)$$

$$\mathcal{L}_{\theta} = \theta \frac{g^2}{16\pi^2} \epsilon_{\mu\nu\alpha\beta}\, tr(G_{\mu\nu} G_{\alpha\beta}) \qquad (2)$$

Hypothesis: $\theta = \theta(x)$ - is pseudoscalar field

$$\mathcal{L}_{\theta} = \frac{-1}{2N_f} J_{5,\mu}\, \partial^{\mu}\theta(x) - \frac{\theta(x)}{N_f}\, i\, \bar{\psi}\, \hat{m}_q \gamma_5\, \psi \quad . \qquad (3)$$

$$\frac{1}{2N_f} \partial^0 \theta(t) = \mu_5 = const \qquad (4)$$

$$\partial^{\mu} J_{5,\mu} = 2\, i\, \bar{q}\, \hat{m}_q \gamma_5\, q + \frac{N_f\, g^2}{8\pi^2} \epsilon_{\mu\nu\alpha\beta}\, tr(G_{\mu\nu} G_{\alpha\beta}) \qquad (5)$$

$$Q_5(t_2) - Q_5(t_1) + \int_{\partial V} d^3\Sigma^i J_{5,i} =$$
$$= 2\, i \int_{t_1}^{t_2} dt \int_V d^3x\ \bar{q}\, \hat{m}_q \gamma_5\, q + \frac{1}{2N_f}(Q_t(t_2) - Q_t(t_1)) + \frac{1}{2N_f} \int_{\partial V} d^3\Sigma^i J_{t,i} \qquad (6)$$

$$\triangle Q_5 = \frac{1}{2N_f} \triangle Q_t \qquad (7)$$

Introduction

Left-right oscillations of quarks

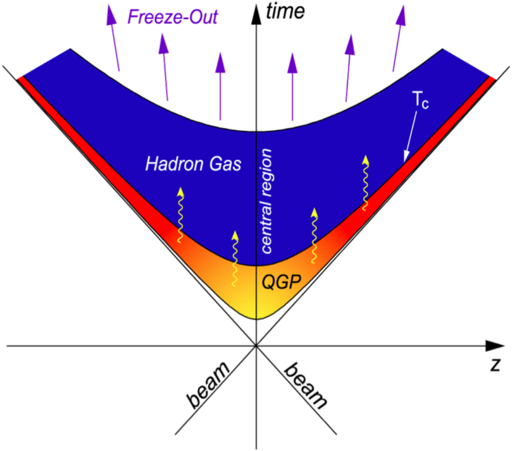
Fireball lifetime $\approx$ 10 fm

Oscillation time for u/d-quark $\approx \frac{1}{m_{u/d}} \approx \frac{1}{5\ \text{MeV}} \approx$ 40 fm

Oscillation time for s-quark $\approx \frac{1}{m_s} \approx \frac{1}{150\ \text{MeV}} \approx$ 1 fm

$$\partial^\mu J_{5,\mu} = 2i\bar{\psi}\hat{m}_s\gamma_5\psi + \frac{N_f g^2}{8\pi^2}\epsilon_{\mu\nu\alpha\beta} tr(G_{\mu\nu}G_{\alpha\beta}) = 0 \qquad (8)$$

Model
Relativistic heavy-ion collisions



Quark-gluon plasma $\Longleftrightarrow$ **Nambu-Jona-Lasinio model**

Hadron gas $\Longleftrightarrow$ **Sigma model**

Model
Chiral Nambu-Jona-Lasinio model

$$SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_V \otimes U(1)_A \otimes \mathcal{T} :$$

$$\mathcal{L} = \bar{\psi}(\not{\partial} + m - \mu_5 \gamma_0 \gamma_5)\psi - \frac{G_1}{N_c}\left((\bar{\psi}\,\psi)^2 + (\bar{\psi}\,i\,\gamma_5\,\vec{\tau}\,\psi)^2\right) - -\frac{G_2}{N_c}\,(\bar{\psi}\,\vec{\tau}\,\psi)^2$$

$$\bar{\psi}\,\psi \Longleftrightarrow \sigma \Longleftrightarrow (J^P, I) = (0^+, 0)$$
$$\bar{\psi}\,\gamma_5\,\vec{\tau}\,\psi \Longleftrightarrow \pi \Longleftrightarrow (J^P, I) = (0^-, 1)$$
$$\bar{\psi}\,\vec{\tau}\,\psi \Longleftrightarrow a_0 \Longleftrightarrow (J^P, I) = (0^+, 1)$$

Model
Chiral sigma model

$$\mathcal{L}_m = \frac{1}{4} Tr\,(D_\mu H\,(D^\mu H)^\dagger) + \frac{b}{2} Tr\,[\,\hat{m}(H + H^\dagger)] + \frac{M^2}{2} Tr\,(H H^\dagger)$$
$$- \frac{\lambda_1}{2} Tr\,\left[(H H^\dagger)^2\right] - \frac{\lambda_2}{4}\left[Tr\,(H H^\dagger)\right]^2 + \frac{c}{2}(\det H + \det H^\dagger) \quad (9)$$

$$H = \xi\,\Sigma\,\xi \quad (10)$$

$$\Sigma = \begin{bmatrix} v + \sigma + a_0 & \sqrt{2}\,a_+ \\ \sqrt{2}\,a_- & v + \sigma - a_0 \end{bmatrix} \qquad \xi = \exp\left(i\,\frac{\vec{\pi}\vec{\tau}}{2\,f_\pi}\right) \quad (11)$$

Model

Chiral sigma model

$$\mathcal{L}_m = \frac{1}{4} Tr\,(D_\mu H\,(D^\mu H)^\dagger) + \frac{b}{2} Tr\,[\hat{m}(H + H^\dagger)] + \frac{M^2}{2} Tr\,(HH^\dagger)$$

$$- \frac{\lambda_1}{2} Tr\,\left[(HH^\dagger)^2\right] - \frac{\lambda_2}{4} [Tr\,(HH^\dagger)]^2 + \frac{c}{2}(\det H + \det H^\dagger) \quad (9)$$

$$D_\mu H = \partial_\mu H - i\mathcal{L}_\mu H + iH\mathcal{R}_\mu \quad (12)$$

$$\mathcal{L}_\mu = e\,Q_{em}\,A_\mu - \mu_5\,\delta_{\mu 0}\,I \qquad \mathcal{R}_\mu = e\,Q_{em}\,A_\mu + \mu_5\,\delta_{\mu 0}\,I \quad (13)$$

$$Q_{em} = \frac{1}{2}\tau_3 + \frac{1}{6} I \quad (14)$$

Model

Chiral Wess–Zumino–Witten model

$$W_- = -\frac{iN_c}{96\pi^2}\int_0^1 dx_5 \int d^4x \epsilon^{\mu\nu\sigma\lambda\rho}\, Tr\Big[-j^-_\mu F^L_{\nu\sigma}F^L_{\lambda\rho} - j^+_\mu F^R_{\nu\sigma}F^R_{\lambda\rho}$$
$$-\frac{1}{2}\, j^+_\mu F^L_{\nu\sigma}\, U(x_5) F^R_{\lambda\rho}\, U^\dagger(x_5) - \frac{1}{2}\, j^+_\mu F^R_{\nu\sigma}\, U^\dagger(x_5) F^L_{\lambda\rho}\, U(x_5)$$
$$+\, iF^L_{\mu\nu}\, j^-_\sigma j^-_\lambda j^-_\rho + iF^R_{\mu\nu}\, j^+_\sigma j^+_\lambda j^+_\rho + \frac{2}{5} j^-_\mu j^-_\nu j^-_\sigma j^-_\lambda j^-_\rho\Big] \qquad (15)$$

$$U = \exp\left(i\,\vec{\pi}\vec{\tau}/f_\pi\right), \qquad (16)$$

$$j^-_\mu = (D_\mu U)U^\dagger,\ j^+_\mu = U^\dagger(D_\mu U),\ \mathcal{L}_5 = \mathcal{R}_5 = 0 \qquad (17)$$

$$F^L_{\mu\nu} = \partial_\mu \mathcal{L}_\nu - \partial_\nu \mathcal{L}_\mu + [\mathcal{L}_\mu, \mathcal{L}_\nu] \qquad (18)$$
$$F^R_{\mu\nu} = \partial_\mu \mathcal{R}_\nu - \partial_\nu \mathcal{R}_\mu + [\mathcal{R}_\mu, \mathcal{R}_\nu]$$

Results

Explicit form of Lagrangian up to second order on a inverse f_π

Sigma meson:

$$\frac{1}{2}\,\partial_\mu\sigma\,\partial^\mu\sigma - \frac{1}{2}\,m_\sigma^2\sigma^2 \tag{19}$$

Uncharged mesonic sector:

$$\frac{1}{2}\,\partial_\mu a_0\,\partial^\mu a_0 + \frac{1}{2}\,\partial_\mu\pi_0\partial^\mu\pi_0 - \frac{1}{2}m_a^2\,(a_0)^2 - \frac{1}{2}m_\pi^2\,(\pi_0)^2 - 4\mu_5\,\dot{\pi}_0 a_0 \tag{20}$$

Charged mesonic sector:

$$\partial_\mu a_-\,\partial^\mu a_+ + \partial_\mu\pi_-\partial^\mu\pi_+ - m_a^2 a_- a_+ - m_\pi^2\pi_-\pi_+ \boxed{-4\mu_5\,\dot{\pi}_+ a_- - 4\mu_5\,\dot{\pi}_- a_+} \tag{21}$$

Vertices for uncharged mesonic sector:

$$\frac{1}{v}\,\sigma\,\partial_\mu\pi_0\partial^\mu\pi_0 + -\frac{4\mu_5}{v}\,\sigma\,\dot{\pi}_0 a_0 - \frac{b\,m}{v^2}\,\sigma\,(\pi_0)^2 \tag{22}$$

Vertices for charged mesonic sector with field of σ meson:

$$\frac{2}{v}\,\sigma\,\partial_\mu\pi_-\partial^\mu\pi_+ - \frac{4\mu_5}{v}\,\sigma\,\dot{\pi}_+ a_- - \frac{4\mu_5}{v}\,\sigma\,\dot{\pi}_- a_+ - \frac{2b\,m}{v^2}\,\sigma\,\pi_-\pi_+ - 24\lambda_1 v\,\sigma a_- a_+ - 8\lambda_2\,v\,\sigma\,a_- a_+ \tag{23}$$

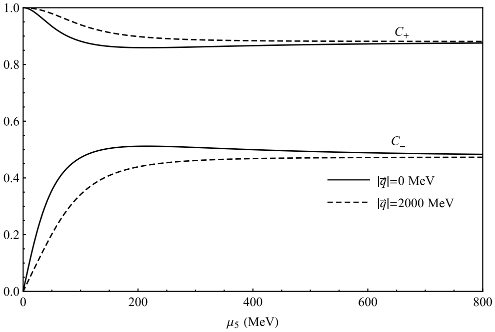
Vertices for charged mesonic sector with electromagnetic field:

$$ie\,\partial_\mu\pi_+\pi_- A^\mu - ie\,\pi_+\partial_\mu\pi_- A^\mu + ie\,\partial_\mu a_+ a_- A^\mu - ie\,a_+\partial_\mu a_- A^\mu \tag{24}$$

Results
Coefficients of diagonalization

$$C_\pm = \frac{1}{\sqrt{2}}\sqrt{1 \pm \frac{m_a^2 - m_\pi^2}{\sqrt{(m_a^2 - m_\pi^2)^2 + (8\mu_5 q_0)^2}}} \quad (25)$$

$$\begin{pmatrix} \tilde{a}_0 \\ \tilde{\pi}_0 \end{pmatrix} = \begin{pmatrix} C_+ & -C_- \\ -i\,C_- & -i\,C_+ \end{pmatrix}^{-1} \begin{pmatrix} a_0 \\ \pi_0 \end{pmatrix} \quad (26)$$

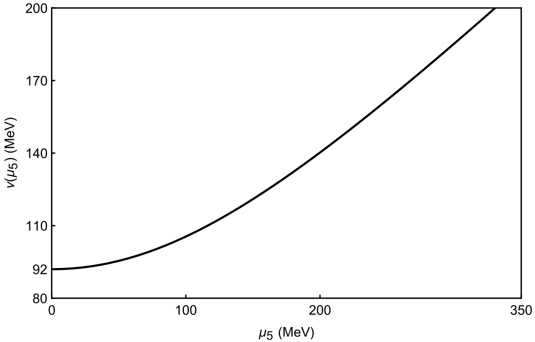


Results

The equation for the mass gap: quark condensate

$$\langle \bar{\psi}\psi \rangle = -b \cdot v(\mu_5)$$

$$v(\mu_5) \approx v_0 + v_1 m = \sqrt{\frac{M^2 + 2\mu_5^2 + c}{2(\lambda_1 + \lambda_2)}} + \frac{b}{2(M^2 + 2\mu_5^2 + c)}\, m$$



From equation:

$$\begin{cases} m_\sigma^2 = -2\left(M^2 - 6\,(\lambda_1+\lambda_2)v^2 + c + 2\mu_5^2\right) \\ m_a^2 = -2\left(M^2 - 2\,(3\lambda_1+\lambda_2)v^2 - c + 2\mu_5^2\right) \\ m_\pi^2 = \dfrac{2\,b\,m}{v} \\ v(\mu_5) = \sqrt{\dfrac{M^2 + 2\mu_5^2 + c}{2(\lambda_1+\lambda_2)}} + \dfrac{b}{2(M^2+2\mu_5^2+c)}\,m \end{cases},$$

we can find λ_1, λ_2, c и b. We get two sets of parameters:

1) $\lambda_1 = 1.64850 \times 10$, $\lambda_2 = -1.31313 \times 10$, $c = -4.46874 \times 10^4$ MeB^2, $b = 1.61594 \times 10^5$ MeB^2

2) $\lambda_1 = 1.79298 \times 10$, $\lambda_2 = -1.52985 \times 10$, $c = -8.13732 \times 10^4$ MeB^2, $b = 1.61594 \times 10^5$ MeB^2

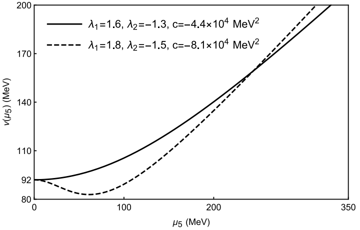


Figure 1: $v(\mu_5)$ from sigma model.

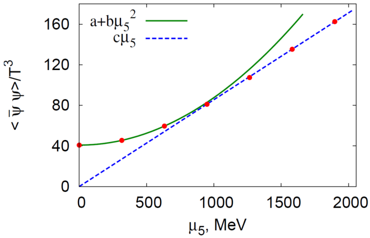
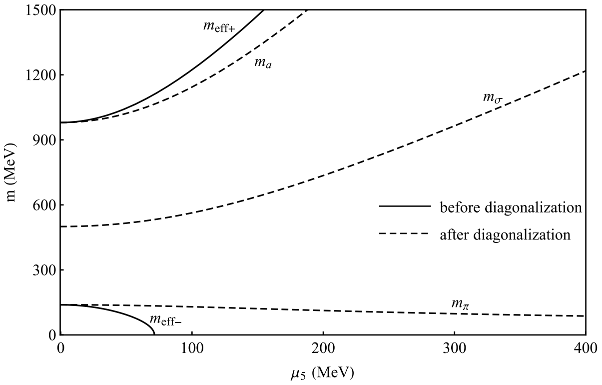


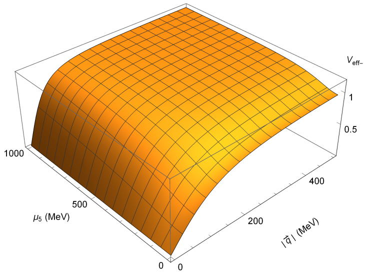
Figure 2: $v(\mu_5)$ from lattice model for nonzero T. (Braguta V.V., Kotov A.Yu. Catalysis of dynamical chiral symmetry breaking by chiral chemical potential. Phys. Rev. D, 2016, vol. 93 (10), pp.)

$$m^2_{eff\mp} = \mp\frac{1}{2}\sqrt{(16\mu_5^2 + m_a^2 + m_\pi^2)^2 - 4m_a^2\,m_\pi^2 + 64\mu_5^2\,|\vec{q}|^2} + \\ + \frac{1}{2}(16\,\mu_5^2 + m_a^2 + m_\pi^2\,)$$

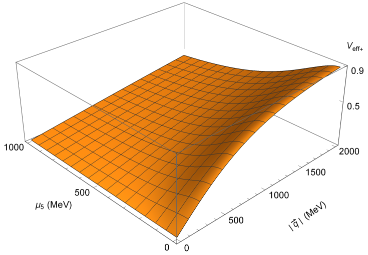


Results
Group velocity

$$V_{eff-} = \frac{|\vec{q}| + \frac{1}{2}\frac{\partial m^2_{eff-}}{\partial |\vec{q}|}}{\sqrt{|\vec{q}|^2 + m_{eff-}(|\vec{q}|)}}$$



$$V_{eff+} = \frac{|\vec{q}| + \frac{1}{2}\frac{\partial m^2_{eff+}}{\partial |\vec{q}|}}{\sqrt{|\vec{q}|^2 + m_{eff+}(|\vec{q}|)}}$$



Summary

Topological fluctuations $\Rightarrow$ Axial chemical potential $\Rightarrow$ Hadronic physics

- Effective mass of the a meson is **more** than the mass of a meson in the vacuum
- Effective mass of the π meson is **less** than the mass of π meson in the vacuum
- The effective mass of the π meson may become imaginary quantity
- $\uparrow$ chiral chemical potential $\Rightarrow$ growth of the violation of chiral symmetry
- $\uparrow$ baryonic chemical potential $\Rightarrow$ chiral symmetry partially restores

Thank you for attention

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Part I

Appendix

In more detail about quantization of fields

$$\tilde{a}_0(k) = \delta(k^2 - m^2_{eff+})F_{a_0}(k) \qquad \tilde{\pi}_0(k) = \delta(k^2 - m^2_{eff-})F_{\pi_0}(k)$$
$$\tilde{a}_-(k) = \delta(k^2 - m^2_{eff+})F_{a_-}(k) \qquad \tilde{a}_+(k) = \delta(k^2 - m^2_{eff+})F_{a_+}(k)$$
$$\tilde{\pi}_-(k) = \delta(k^2 - m^2_{eff-})F_{\pi_-}(k) \qquad \tilde{\pi}_+(k) = \delta(k^2 - m^2_{eff-})F_{\pi_+}(k)$$

In our case, masses of field depend on momentum, and normalization of wave packets is changed:

$$\int dk_0\, e^{ikx}\, \tilde{a}_0(k) = \frac{2\pi}{\sqrt{f_+}} (e^{ikx}\, a_0^+(\vec{k}) + e^{-ikx}\, a_0^-(\vec{k}))\Big|_{k_0 = k_0(\vec{k})} \tag{27}$$

$$\int dk_0\, e^{ikx}\, \tilde{\pi}_0(k) = \frac{2\pi}{\sqrt{f_-}} (e^{ikx}\, \pi_0^+(\vec{k}) + e^{-ikx}\, \pi_0^-(\vec{k}))\Big|_{k_0 = k_0(\vec{k})} \tag{28}$$

$$\int dk_0\, e^{ikx}\, \tilde{a}_-(k) = \frac{2\pi}{\sqrt{f_+}} (e^{ikx}\, \check{a}_-^+(\vec{k}) + e^{-ikx}\, a_-^-(\vec{k}))\Big|_{k_0 = k_0(\vec{k})} \tag{29}$$

$$\int dk_0\, e^{ikx}\, \tilde{\pi}_-(k) = \frac{2\pi}{\sqrt{f_-}} (e^{ikx}\, \check{\pi}_-^+(\vec{k}) + e^{-ikx}\, \pi_-^-(\vec{k}))\Big|_{k_0 = k_0(\vec{k})} \tag{30}$$

$$\int dk_0\, e^{ikx}\, \tilde{a}_+(k) = \frac{2\pi}{\sqrt{f_+}} (e^{ikx}\, a_+^+(\vec{k}) + e^{-ikx}\, \check{a}_+^-(\vec{k}))\Big|_{k_0 = k_0(\vec{k})} \tag{31}$$

$$\int dk_0\, e^{ikx}\, \tilde{\pi}_+(k) = \frac{2\pi}{\sqrt{f_-}} (e^{ikx}\, \pi_+^+(\vec{k}) + e^{-ikx}\, \check{\pi}_+^-(\vec{k}))\Big|_{k_0 = k_0(\vec{k})} \tag{32}$$

Quark condensate

Nambu and Jona-Lasinio model

$$\langle\bar{\psi}\psi\rangle = -(250\ \text{MeV})^3 = -153 \cdot 10^5\ \text{MeV}^3$$

(Vogl U., Weise W. The Nambu and Jona-Lasinio model: Its implications for Hadrons and Nuclei. Progress in Particle and Nuclear Physics, 1991, vol. 27, pp. 195-272.)

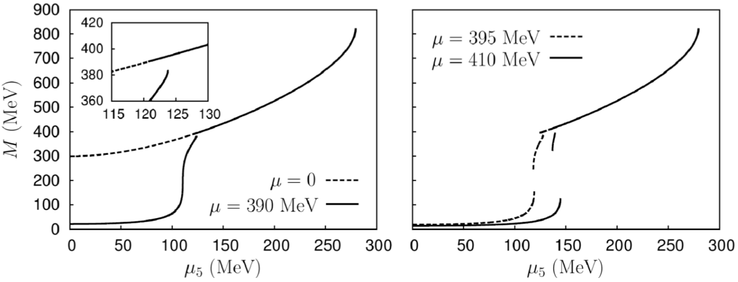
Investigative model
Let's use:

$$\begin{cases} GOR : m_\pi^2 = -\dfrac{2m}{v^2}\langle\bar{\psi}\psi\rangle \\ m_\pi^2 = \dfrac{2\,b\,m}{v} \\ b = 1.61594 \times 10^5\ \text{MeV}^2 \end{cases}$$

If $\mu_5 = 0$ then:

$$\langle\bar{\psi}\psi\rangle = -\,b \cdot v(\mu_5) = -147 \cdot 10^5\ \text{MeV}^3$$

Results for NJL model



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