Dense quark matter with chiral imbalance: NJL-model consideration

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Introduction

Introduction

Models with four-fermion interactions

Isospin asymmetry is the well-known property of dense quark matter, which exists in the compact stars and is produced in heavy ion collisions. On the other hand, the chiral imbalance between left- and right- handed quarks is another highly anticipated phenomenon that could occur in the dense quark matter.

To investigate dense quark under these conditions we use Nambu–Jona-Lasinio (NJL) model and take into account:

- Baryon – μ_B chemical potential to investigate non-zero density
- Isospin – μ_I chemical potential to investigate non-zero isotopic imbalance
- Chiral isospin – μ_{I5} chemical potential to investigate chiral isotopic imbalance
- Non-zero bare quark mass ($m_0 \neq 0$) to promote real threshold to pion condensation phase
- Non-zero temperature ($T \neq 0$) in order to make our investigation applicable to hot dense quark matter and compare our NJL-model analysis with the known lattice results

The model and its thermodynamical potential

Lagrangian of the model

$$\mathcal{L} = \bar{q}\Big[\gamma^\nu \mathrm{i}\partial_\nu - m_0 + \frac{\mu_B}{3}\gamma^0 + \frac{\mu_I}{2}\tau_3\gamma^0 + \frac{\mu_{I5}}{2}\tau_3\gamma^0\gamma^5\Big]q + \frac{G}{N_c}\Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5\vec{\tau}q)^2\Big]$$

Definitions

- q is the flavor doublet $q = (q_u, q_d)^T$
- q_u and q_d are four-component Dirac spinors as well as color N_c-plets[a]
- τ_k $(k = 1, 2, 3)$ are Pauli matrices
- m_0 is the diagonal matrix in flavor space with bare quark masses (from the following $m_u = m_d = m_0$)

[a] The summation over flavor, color, and spinor indices is implied

Lagrangian of the model

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Notations

- μ_B is a baryon number chemical potential
- μ_I is taken into account to promote non-zero imbalance between u and d quarks
- μ_{I5} is stands to promote chiral isospin imbalance between $u_{L(R)}$ and $d_{L(R)}$
- G is coupling constant

Lagrangian of the model

$$\mathcal{L} = \bar{q}\Big[\gamma^\nu \mathrm{i}\partial_\nu - m_0 + \frac{\mu_B}{3}\gamma^0 + \frac{\mu_I}{2}\tau_3\gamma^0 + \frac{\mu_{I5}}{2}\tau_3\gamma^0\gamma^5\Big]q + \frac{G}{N_c}\Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5\vec{\tau}q)^2\Big]$$

Symmetries in particular case ($m_0 = \mu_B = \mu_I = \mu_{I5} = 0$)

Lagrangian is invariant under transformations from the $SU(2)_L \times SU(2)_R$ group, which is also inherent in 2-flovor QCD in chiral limit

Symmetries in the chiral case ($m_0 = 0$; $\mu_B \neq 0, \mu_I \neq 0, \mu_{I5} \neq 0$)

Symmetry of the Lagrangian is reduced to $U_B(1) \times U_{I_3}(1) \times U_{AI_3}(1)$

Symmetries in the general case ($m_0 \neq 0$; $\mu_B \neq 0, \mu_I \neq 0, \mu_{I5} \neq 0$)

Symmetry remain intact in general case $U_B(1) \times U_{I_3}(1)$

Semi-bosonized version of the Lagrangian

Let us introduce the semi-bosonized version of the Lagrangian that contains only quadratic powers of fermionic fields as well as auxiliary bosonic fields $\sigma(x)$, $\pi_a(x)$, :

$$\widetilde{\mathcal{L}} = \bar{q}\Big[\gamma^\rho \mathrm{i}\partial_\rho - m_0 + \mu\gamma^0 + \nu\tau_3\gamma^0 + \nu_5\tau_3\gamma^0\gamma^5 - \sigma - \mathrm{i}\gamma^5\pi_a\tau_a\Big]q - \frac{N_c}{4G}\Big[\sigma\sigma + \pi_a\pi_a\Big]$$

Bosonic fields

$$\sigma(x) = -2\frac{G}{N_c}(\bar{q}q); \quad \pi_a(x) = -2\frac{G}{N_c}(\bar{q}\mathrm{i}\gamma^5\tau_a q)$$

The new notations of chemical potentials

$$\mu \equiv \frac{\mu_B}{3};\ \nu \equiv \frac{\mu_I}{2};\ \nu_5 \equiv \frac{\mu_{I5}}{2}$$

Note that the composite bosonic field $\pi_3(x)$ can be identified with the physical $\pi^0(x)$-meson field, whereas the physical $\pi^\pm(x)$-meson fields are the following combinations of the composite fields, $\pi^\pm(x) = (\pi_1(x) \pm i\pi_2(x))/\sqrt{2}$.

Effective action

The effective action $\mathcal{S}_{\text{eff}}(\sigma, \pi_a)$ of the considered model, after path inegration in the mean-field approximation has the following form:

$$\mathcal{S}_{\text{eff}}(\sigma(x), \pi_a(x)) = -N_c \int d^2x \left[\frac{\sigma^2(x) + \pi_a^2(x)}{4G} \right] - \mathrm{i} N_c \mathrm{Tr}_{sfx} \ln D,$$

where

$$D \equiv \gamma^\nu \mathrm{i}\partial_\nu - m_0 + \mu\gamma^0 + \nu\tau_3\gamma^0 + \nu_5\tau_3\gamma^0\gamma^5 - \sigma(x) - \mathrm{i}\gamma^5\pi_a(x)\tau_a.$$

Effective action

The effective action $\mathcal{S}_{\text{eff}}(\sigma, \pi_a)$ of the considered model, after path inegration in the mean-field approximation has the following form:

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The ground state expectation values $\langle\sigma\rangle$, $\langle\pi_a\rangle$, of the composite bosonic fields are determined by the saddle point equations:

$$\frac{\delta \mathcal{S}_{\text{eff}}}{\delta \sigma} = 0; \quad \frac{\delta \mathcal{S}_{\text{eff}}}{\delta \pi_a} = 0$$

Thermodynamic potential (TDP)

In the leading order of the large-N expansion TDP is defined by the following expression:

$$\int d^3x\Omega(\sigma,\pi_a) = -\frac{1}{N_c}\mathcal{S}_{\text{eff}}\{\sigma,\pi_a\}\Big|_{\sigma=\langle\sigma\rangle,\pi_a=\langle\pi_a\rangle}$$

Let us note that due to a $U_{I_3}(1)\times U_{AI_3}(1)$ invariance of the model in the chiral limit and $U_{I_3}(1)$ invariance at physical point, the TDP depends effectively only on the combinations, $\sigma^2+\pi_3^2$ and $\pi_1^2+\pi_2^2$ in the chiral limit, and $\pi_1^2+\pi_2^2$ at the physical point (where $\langle\sigma(x)\rangle\neq 0$ and $\langle\pi_3(x)\rangle=0$). So in both cases, without loss of generality, one can put $\pi_2=\pi_3=0$, and study the TDP as a function of only two variables:

$$\langle\sigma(x)\rangle = M-m_0,\quad \langle\pi_1(x)\rangle=\Delta,\quad \langle\pi_2(x)\rangle=0,\quad \langle\pi_3(x)\rangle=0.$$

It is also important that we investigate charged pion condensation (PC), that also violate charged $U_Q(1)$ symmetry. So charged PC phase is also superconduct or superfluid phase.

Calculation of the TDP

After all possible analytical calculations, we have the following form for the TDP:

$$\Omega(M,\Delta) = \frac{(M-m_0)^2+\Delta^2}{4G} - \sum_{i=1}^{4}\int_0^{\Lambda}\frac{p^2dp}{2\pi^2}|\eta_i| -$$
$$T\sum_{i=1}^{4}\int_0^{\Lambda}\frac{p^2dp}{2\pi^2}\Big\{\ln(1+e^{-\frac{1}{T}(|\eta_i|-\mu)})+\ln(1+e^{-\frac{1}{T}(|\eta_i|+\mu)})\Big\},$$

where η_i are the roots of the following polynomial:

$$\begin{aligned}
&\big(\eta^4-2a\eta^2-b\eta+c\big)\big(\eta^4-2a\eta^2+b\eta+c\big)=0,\\
&a=M^2+\Delta^2+|\vec{p}|^2+\nu^2+\nu_5^2;\\
&b=8|\vec{p}|\nu\nu_5;\\
&c=a^2-4|\vec{p}|^2(\nu^2+\nu_5^2)-4M^2\nu^2-4\Delta^2\nu_5^2-4\nu^2\nu_5^2.
\end{aligned}$$

Fiiting parameters

Since the NJL model is a non-renormalizable theory we have to use fitting parameters for the quantitative investigation of the system. We use the following, widely used parameters:

$$m_0 = 5,5\,\mathrm{MeV}; \qquad G = 15.03\,\mathrm{GeV}^{-2}; \qquad \Lambda = 0.65\,\mathrm{GeV}.$$

In this case at $\mu = \nu = \nu_5 = 0$ one gets for constituent quark mass the value $M = 309\,\mathrm{MeV}$.

Phases

To define the ground state of the system one should find the coordinates (M_0, Δ_0) of the global minimum point (GMP) of the TDP. We also interested in the quark number density: $n_q = -\frac{\partial\Omega(M_0,\Delta_0)}{\partial\mu}$. We have found the following phases in the system:

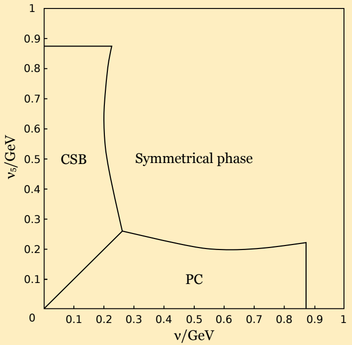
- $M = 0; \Delta = 0; n_q = 0$ – symmetrical phase (it could be realized only in chiral limit $m_0 = 0$)
- $M \neq 0; \Delta = 0; n_q = 0$ – chiral symmetry breaking phase (**CSB**)
- $M \neq 0; \Delta \neq 0; n_q = 0$ – pion condensation phase with zero quark density (**PC**) ($M = 0$ in chiral lim.)
- $M \neq 0; \Delta = 0; n_q \neq 0$ – chiral symmetry breaking phase with non-zero quark density ($\mathbf{CSB_d}$)
- $M \neq 0; \Delta \neq 0; n_q \neq 0$ – pion condensation phase with non-zero quark density ($\mathbf{PC_d}$)
- $M \approx m_0; \Delta = 0; n_q \neq 0$ – partially restored (**CSB**) phase with non-zero quark density ($\mathbf{CSB_{dr}}$)

Phase portraits of the model

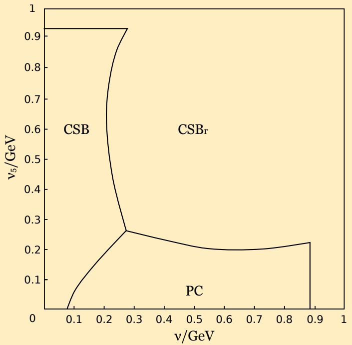
(ν, ν_5)-phase portraits

$\mu = 0$ MeV

Chiral limit $m_0 = 0$



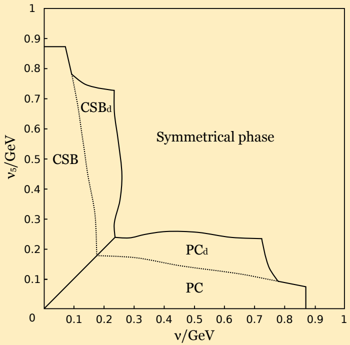
Physical point $m_0 \neq 0$



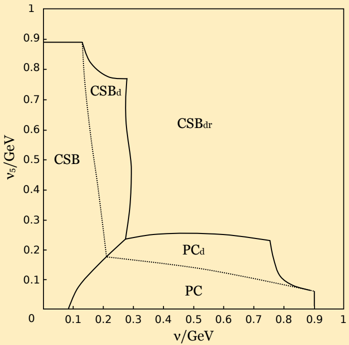
(ν, ν_5)-phase portraits

$\mu = 150$ MeV

Chiral limit $m_0 = 0$



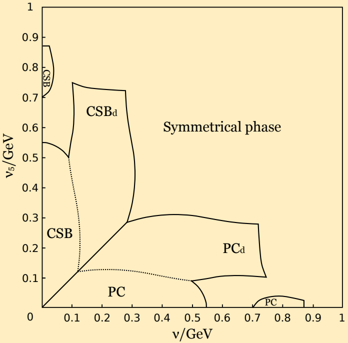
Physical point $m_0 \neq 0$



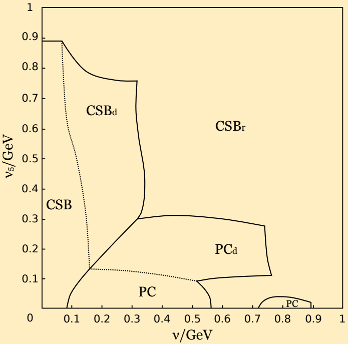
(ν, ν_5)-phase portraits

$\mu = 200$ MeV

Chiral limit $m_0 = 0$



Physical point $m_0 \neq 0$

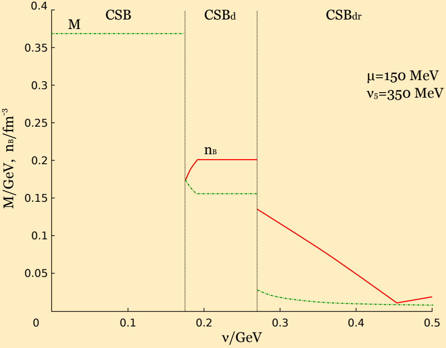


Certain dual symmetry, that we have observed in the chiral limit, is broken explicitly. Nevertheless duality is still relatively instructive even at the physical point. On the other hand, the results become more physically adequate due to the threshold $\nu^c = m_\pi/2 \approx 70MeV$ to the PC phase.

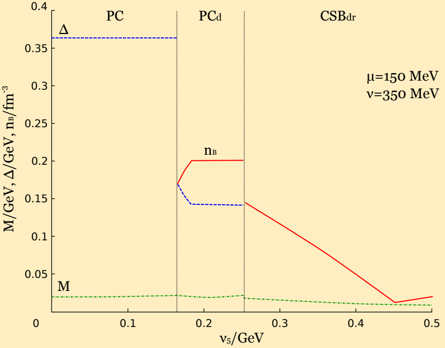
Gaps and Density

Slices of the (ν, ν_5) phase portrait at $\mu = 150$ MeV

$\mu = 150$ MeV; $\nu_5 = 350$ MeV



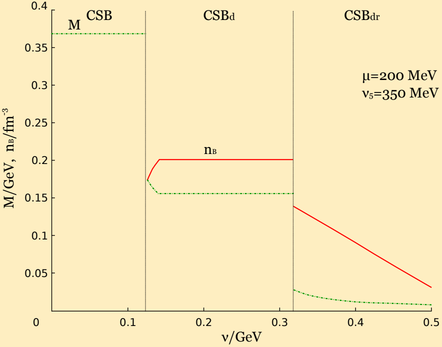
$\mu = 150$ MeV; $\nu = 350$ MeV



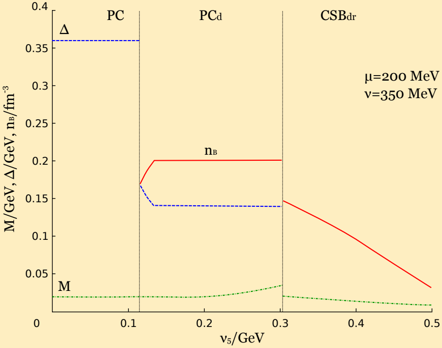
One can easily see that the quark matter in $\mathrm{PC_d}$-phase has baryon density approximately equal to the density of the ordinary nuclear matter. Duality is still relatively instructive feature to investigate phase portrait.

Slices of the (ν, ν_5) phase portrait at $\mu = 200$ MeV

$\mu = 200$ MeV; $\nu_5 = 350$ MeV



$\mu = 200$ MeV; $\nu = 350$ MeV



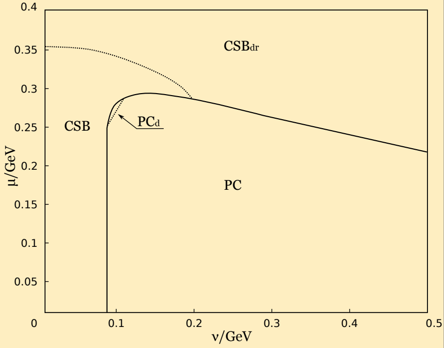
One can easily see that the quark matter in $\mathrm{PC_d}$-phase has baryon density approximately equal to the density of the ordinary nuclear matter. Duality is still relatively instructive feature to investigate phase portrait.

ν_5 does promote $\mathrm{PC_d}$-phase

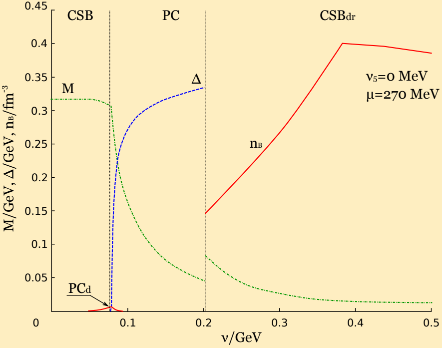
(μ, ν)-phase portraits

$\nu_5 = 0$ MeV

(μ, ν)-phase portrait



Slice at $\mu = 270$ MeV

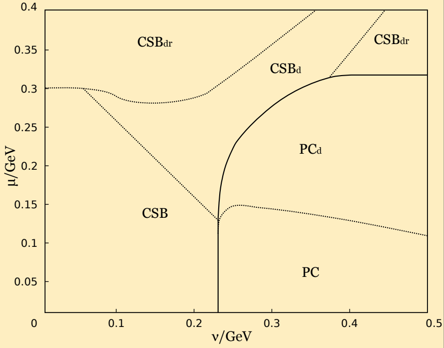


It is evident from the figures that PC$_d$ phase exist in the very small region of the phase portrait.

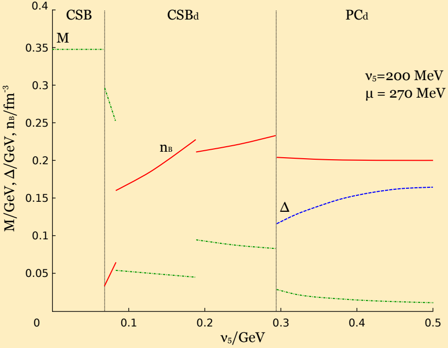
(μ, ν)-phase portraits

$\nu_5 = 200$ MeV

(μ, ν)-phase portrait



Slice at $\mu = 270$ MeV



One can see from that non-zero isospin chiral potential ν_5 does promote the PC_d phase in a wide range of the parameters.

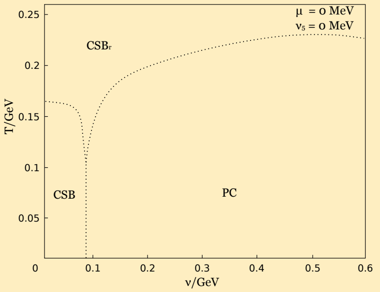
Non-zero temperature $T \neq 0$

comparison with the lattice simulations

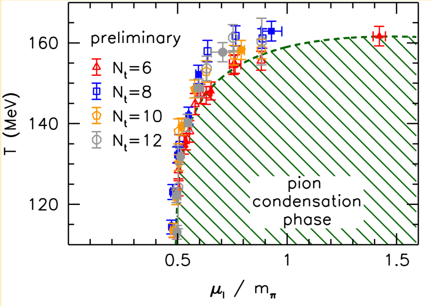
(ν, T)-phase portraits

$\nu_5 = 0$ MeV

(ν, T)-phase portrait



Lattice [1611.06758] Brandt et al.

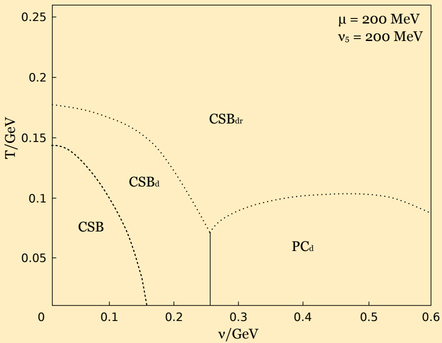


Qualitatively comparable with the first principle lattice simulation.

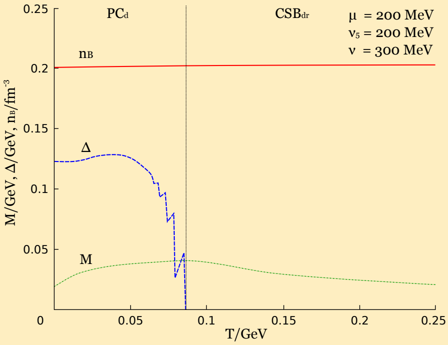
(ν, T)-phase portraits

$\nu_5 = 200$ MeV

(ν, T)-phase portrait



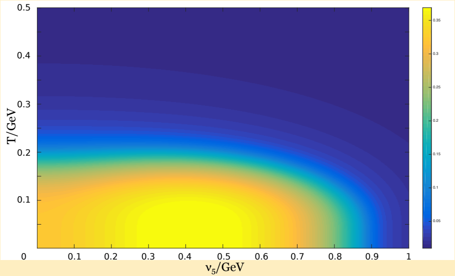
Slice at $\nu = 300$ MeV



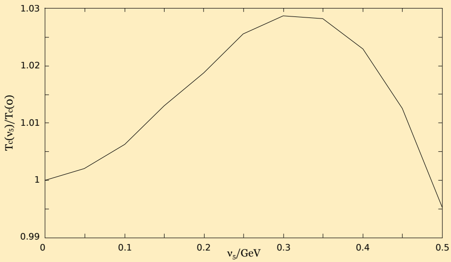
As one could expect, the system restores broken symmetries under non-zero temperature. Nevertheless, it is easy to see that $\mathrm{PC_d}$ phase still occupies wide range of parameters in the phase portrait

(ν_5, T)-phase portraits at $\nu = 0$ MeV

(ν_5, T)-phase portrait



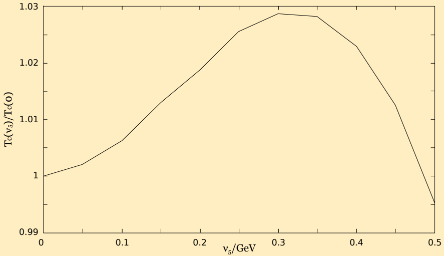
Critical temperature T_c



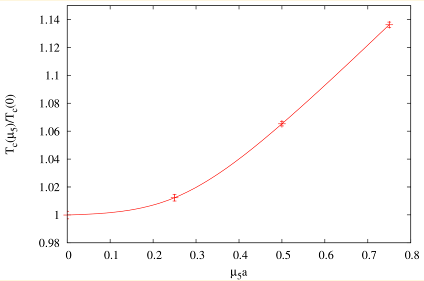
We have recently shown that introduction of the chiral chemical potential μ_5 into consideration (with the following term in the Lagrangian: $\frac{\mu_5}{2}\bar{q}\gamma^0\gamma^5 q$) leads to an additional dual-symmetry between $\mu_{I5} \longleftrightarrow \mu_5$ in the region where $\Delta = 0$. In other words, in the NJL model (1) we can certainly consider μ_{I5} as a μ_5 (only in the pure CSB phase). So we can compare our results with the known lattice calculations with μ_5 [1512.05873] (Braguta et al.).

(ν_5, T)-phase portraits at $\nu = 0$ MeV

Critical temperature T_c within NJL



Critical temperature T_c within Lattice



We have recently shown that introduction of the chiral chemical potential μ_5 into consideration (with the following term in the Lagrangian: $\frac{\mu_5}{2}\bar{q}\gamma^0\gamma^5 q$) leads to an additional dual-symmetry between $\mu_{I5} \longleftrightarrow \mu_5$ in the region where $\Delta = 0$. In other words, in the NJL model (1) we can certainly consider μ_{I5} as a μ_5 (only in the pure CSB phase). So we can compare our results with the known lattice calculations with μ_5 [1512.05873] (Braguta et al.).

Conclusions

- We have shown that there is a huge $\mathrm{PC_d}$ region in the phase portrait of the model promoted by ν_5
- The certain duality which exist in chiral limit is still instructive at the physical point
- Obtained phase portraits are in qualitative accordance with the recent lattice simulations