

RELAXATION TO LOCAL EQUILIBRIUM IN RELATIVISTIC HEAVY-ION COLLISIONS

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UiO : Universitetet i Oslo



Phase diagram

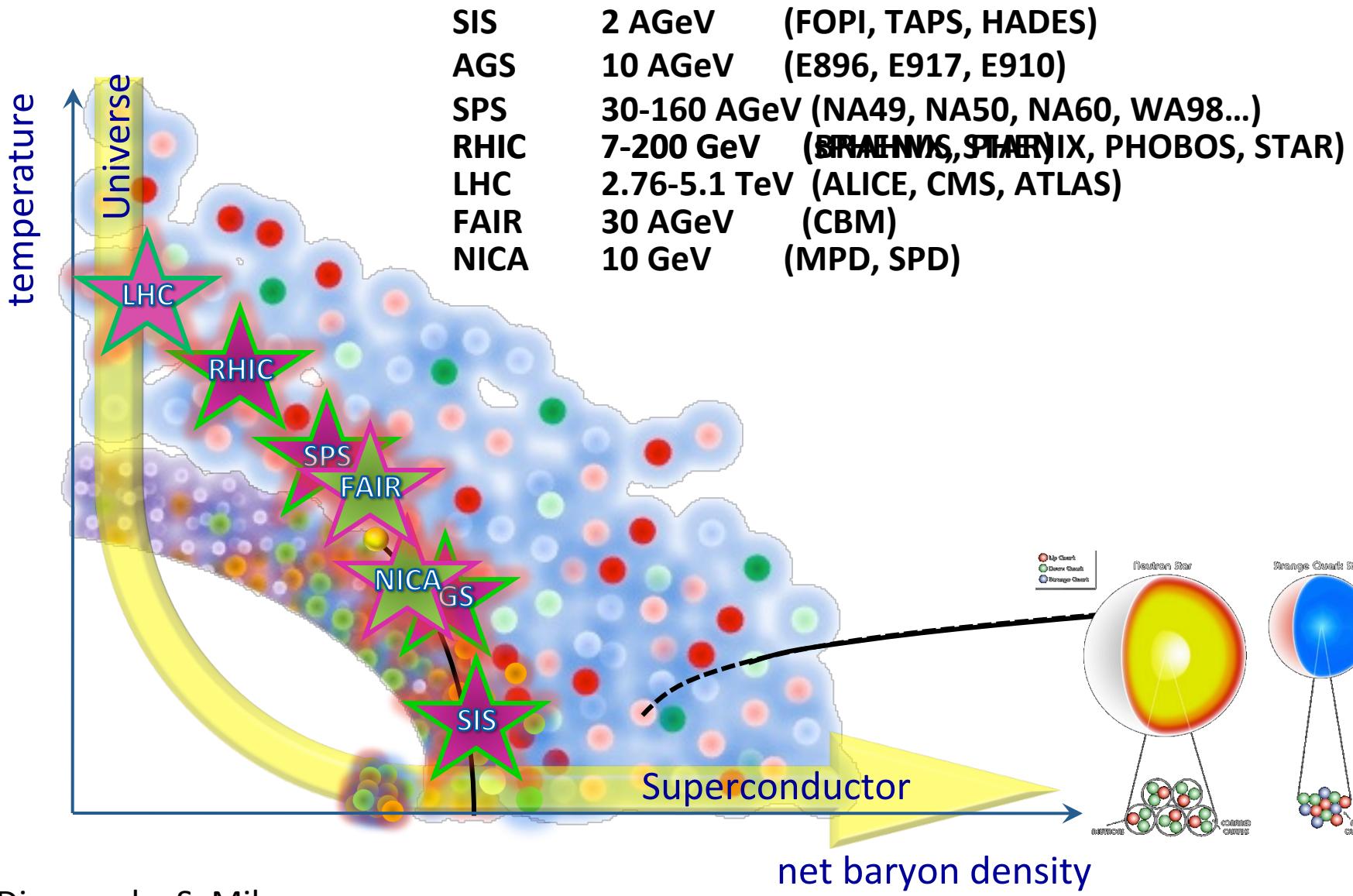
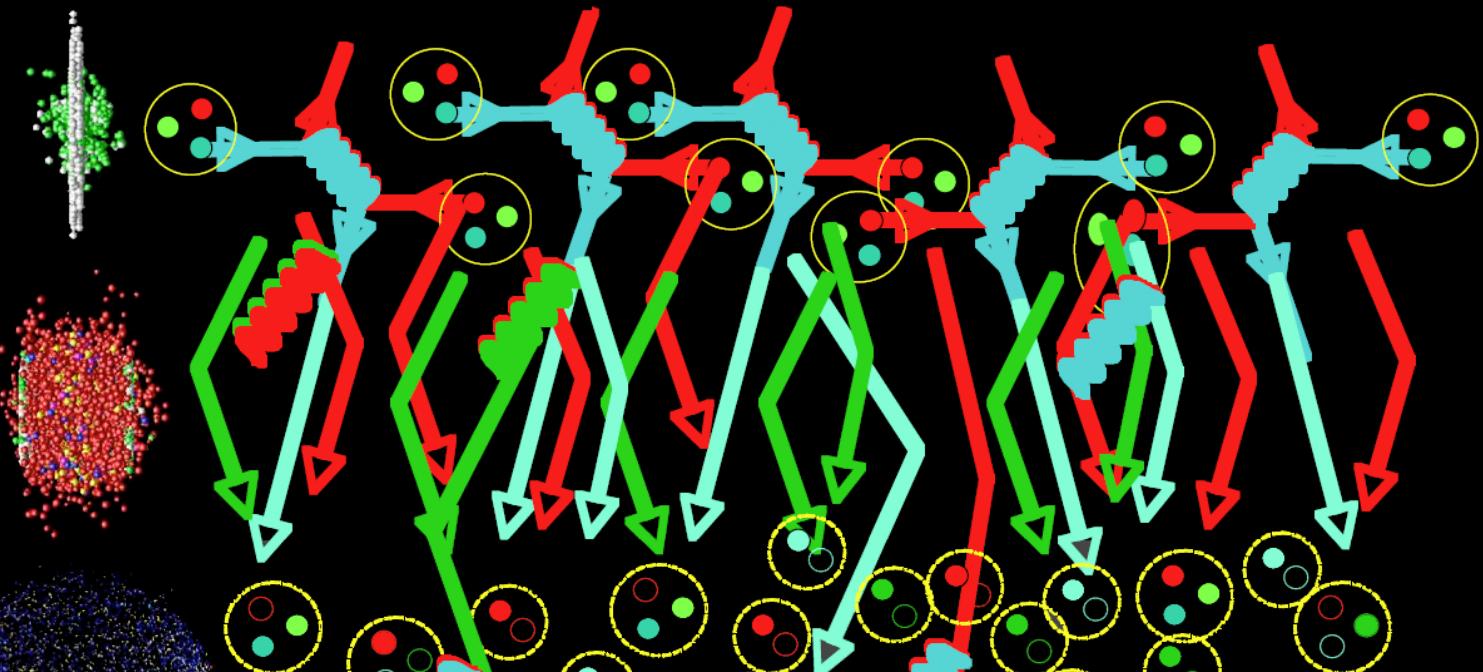


Diagram by S. Milov

Dynamic Regimes

Parton distribution,
Nuclear geometry
Nuclear shadowing



Jet fragmentation
functions

Hadron rescattering

Thermal freeze-out

Hadron decays

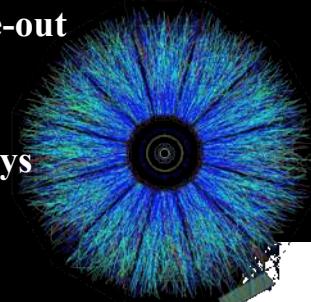
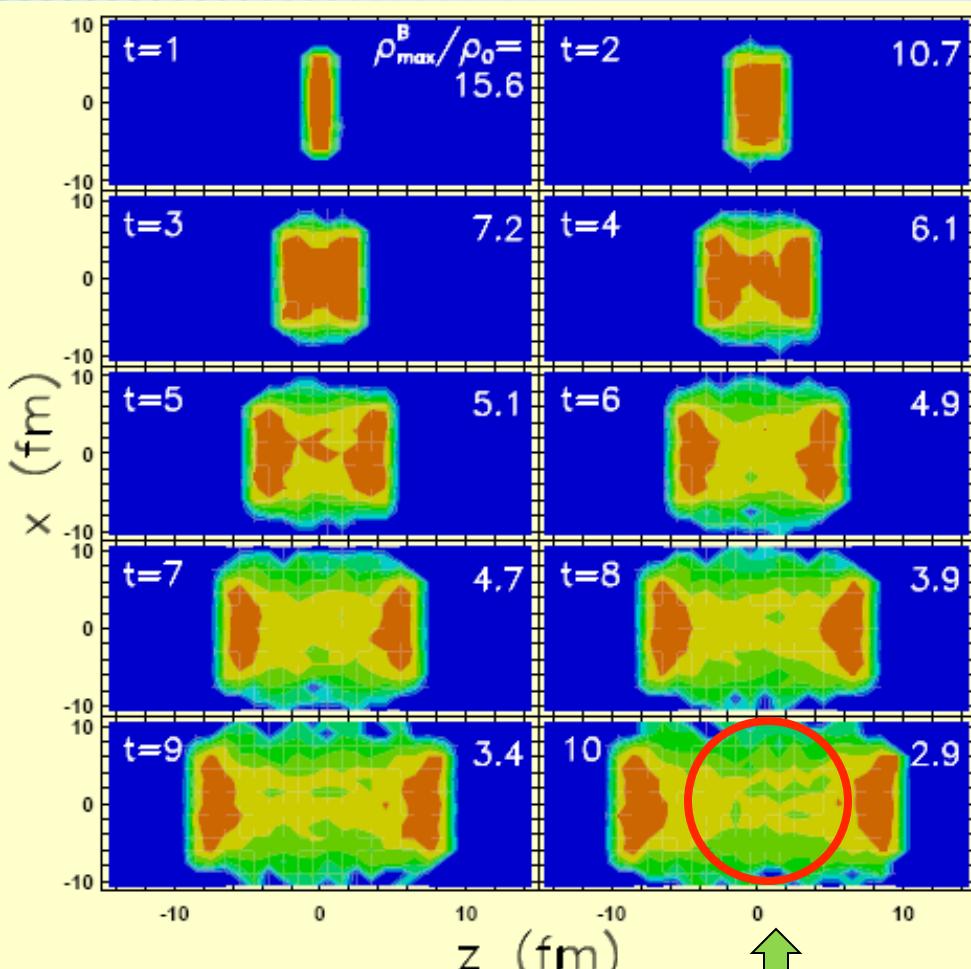


Diagram by Peter Steinberg

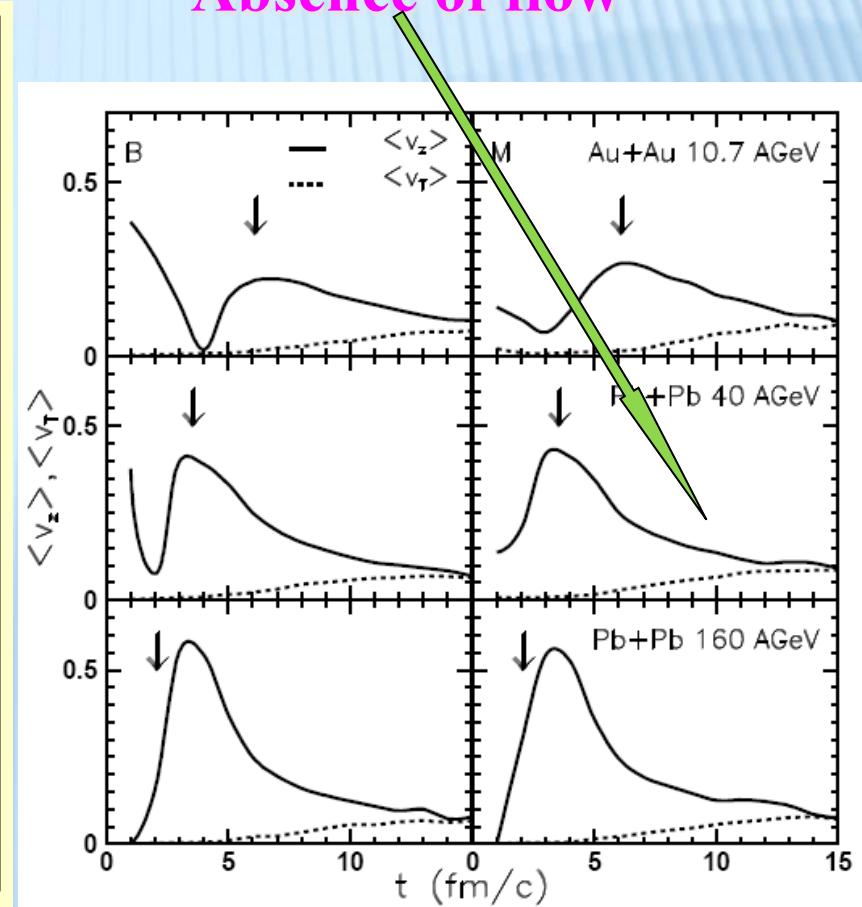
**Central cell:
Relaxation to
(local) equilibrium**

Pre-equilibrium Stage

Homogeneity of baryon matter



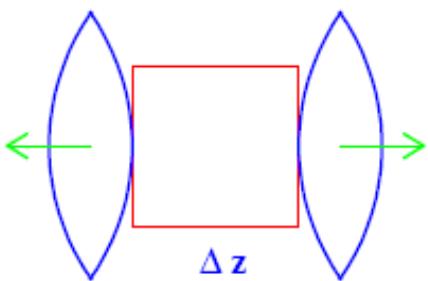
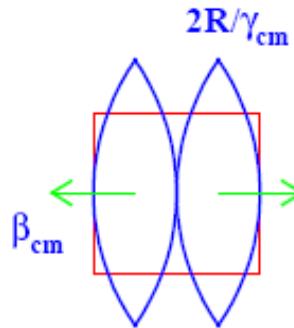
Absence of flow



L.Bravina et al., PRC 60 (1999) 024904

The local equilibrium in the central zone is quite possible

Equilibration in the Central Cell



$$t^{\text{cross}} = 2R/(\gamma_{\text{cm}} \beta_{\text{cm}})$$

$$t^{\text{eq}} \geq t^{\text{cross}} + \Delta z/(2\beta_{\text{cm}})$$



Kinetic equilibrium:
Isotropy of velocity distributions
Isotropy of pressure

Thermal equilibrium:
Energy spectra of particles are described by Boltzmann distribution

L.Bravina et al., PLB 434 (1998) 379;
JPG 25 (1999) 351

$$\frac{dN_i}{4\pi p E dE} = \frac{V g_i}{(2\pi\hbar)^3} \exp\left(\frac{\mu_i}{T}\right) \exp\left(-\frac{E_i}{T}\right)$$

Chemical equilibrium:

Particle yields are reproduced by SM with the same values of (T, μ_B, μ_S) :

$$N_i = \frac{V g_i}{2\pi^2 \hbar^3} \int_0^\infty p^2 dp \exp\left(\frac{\mu_i}{T}\right) \exp\left(-\frac{E_i}{T}\right)$$

Statistical model of ideal hadron gas

input values

output values

$$\varepsilon^{\text{mic}} = \frac{1}{V} \sum_i E_i^{\text{SM}}(T, \mu_B, \mu_S),$$

$$\rho_B^{\text{mic}} = \frac{1}{V} \sum_i B_i \cdot N_i^{\text{SM}}(T, \mu_B, \mu_S),$$

$$\rho_S^{\text{mic}} = \frac{1}{V} \sum_i S_i \cdot N_i^{\text{SM}}(T, \mu_B, \mu_S).$$

Multiplicity 

Energy 

Pressure 

Entropy density 

$$N_i^{\text{SM}} = \frac{V g_i}{2\pi^2 \hbar^3} \int_0^\infty p^2 f(p, m_i) dp,$$

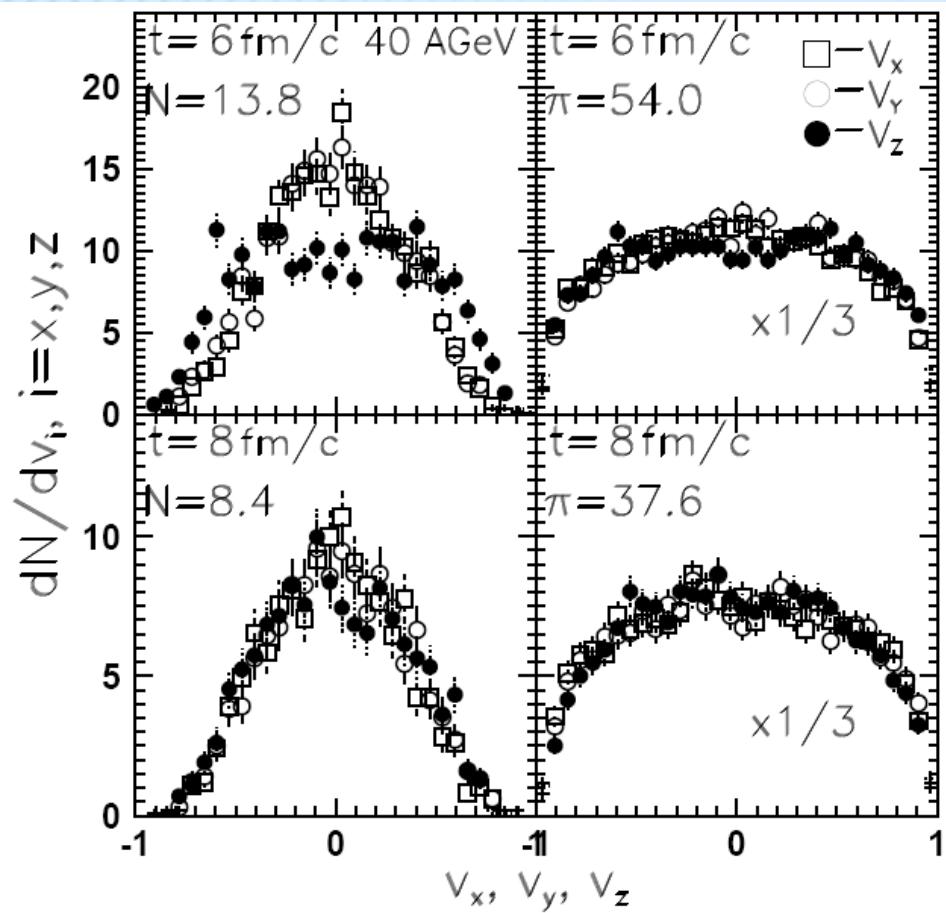
$$E_i^{\text{SM}} = \frac{V g_i}{2\pi^2 \hbar^3} \int_0^\infty p^2 \sqrt{p^2 + m_i^2} f(p, m_i) dp$$

$$P^{\text{SM}} = \sum_i \frac{g_i}{2\pi^2 \hbar^3} \int_0^\infty p^2 \frac{p^2}{3(p^2 + m_i^2)^{1/2}} f(p, m_i) dp$$

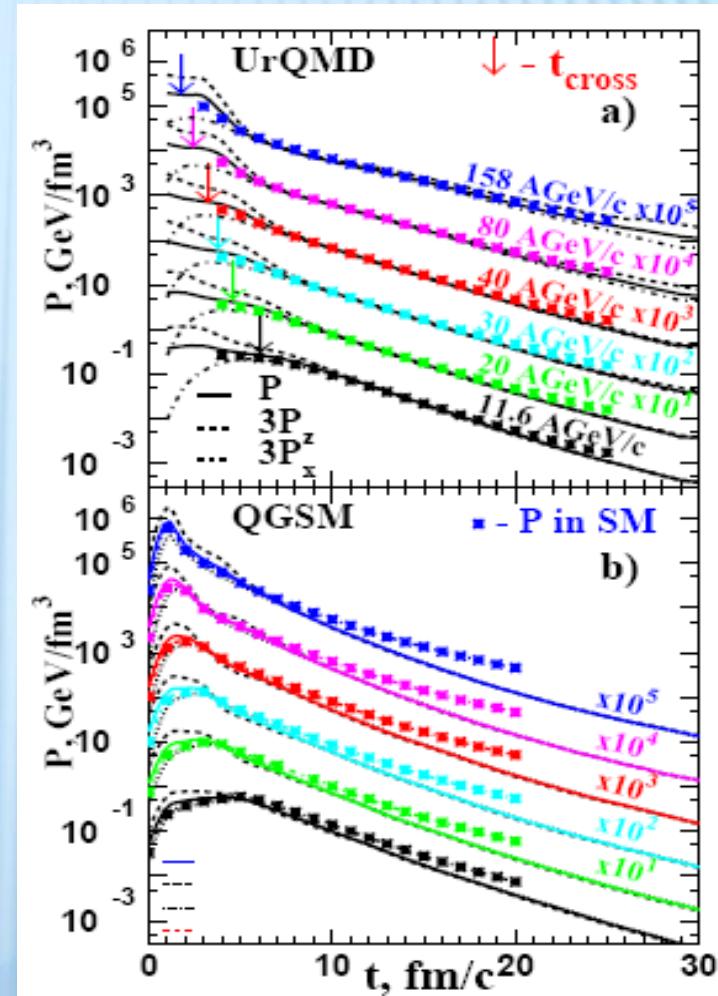
$$s^{\text{SM}} = - \sum_i \frac{g_i}{2\pi^2 \hbar^3} \int_0^\infty f(p, m_i) [\ln f(p, m_i) - 1] p^2 dp$$

Kinetic Equilibrium

Isotropy of velocity distributions

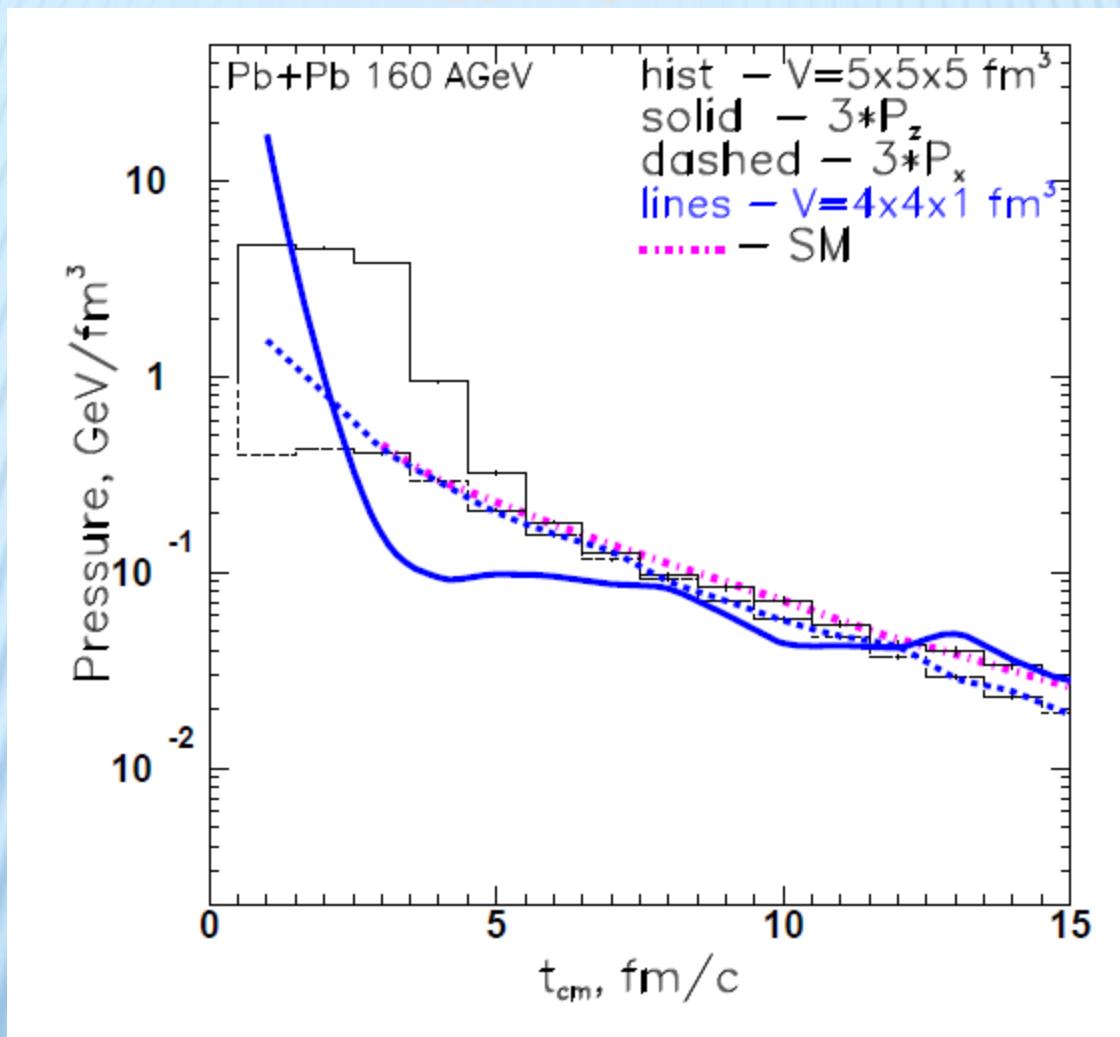


Isotropy of pressure



Kinetic Equilibrium in a Small Cell

Isotropy of pressure

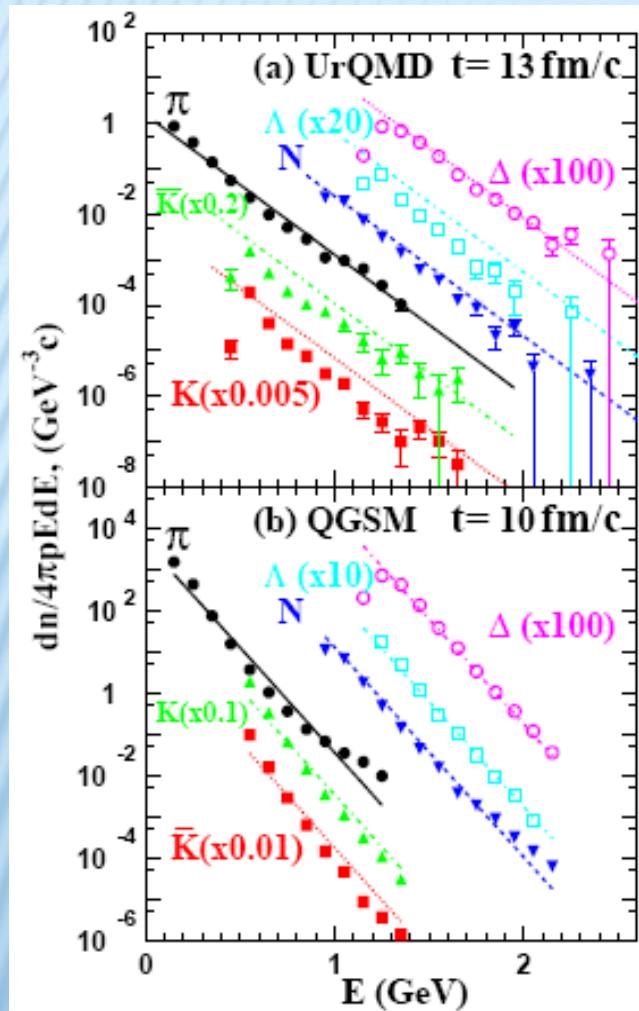


L. Bravina et al., JPG 25 (1999) 351

Longitudinal and transverse pressures in a small cell ($4 \times 4 \times 1 \text{ fm}^3$) converge at the same rate as those in a larger cell ($5 \times 5 \times 5 \text{ fm}^3$)

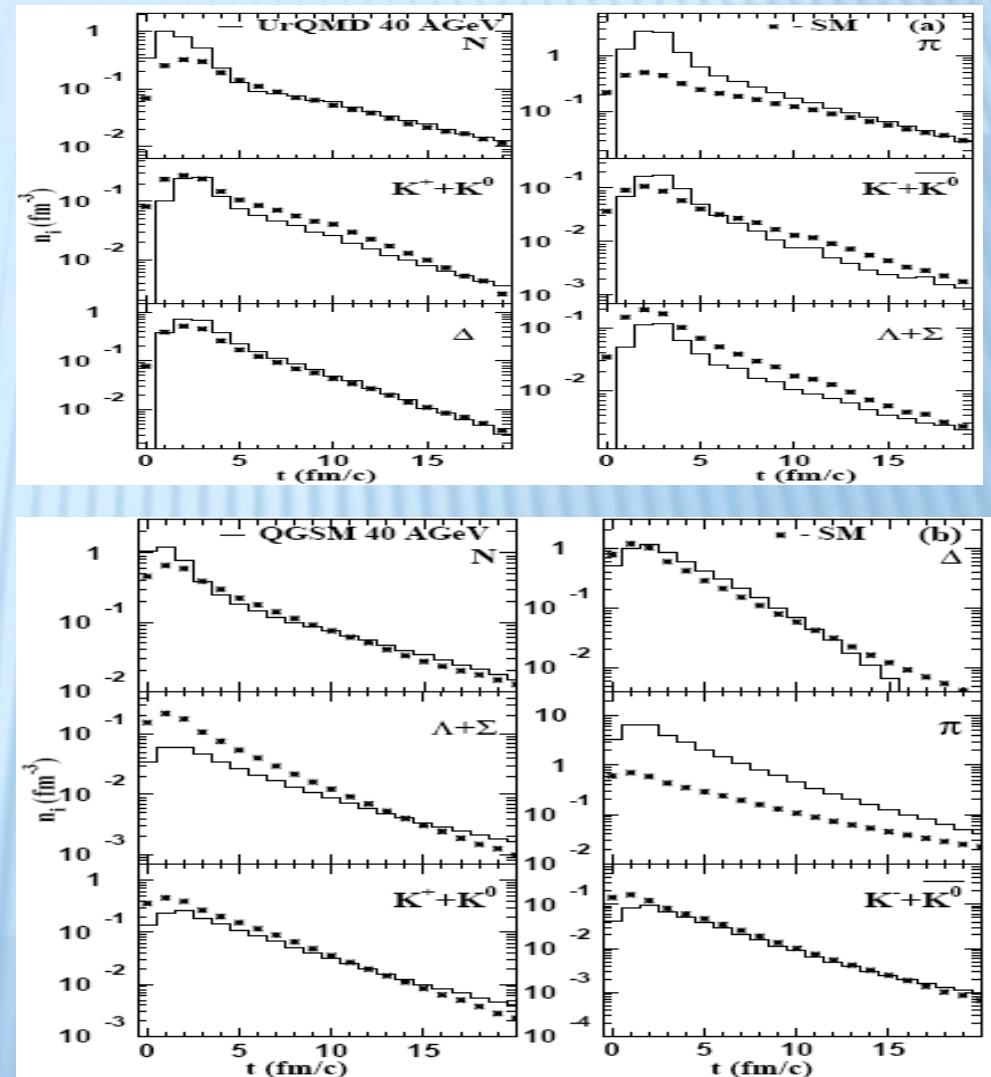
Thermal and Chemical Equilibrium

Boltzmann fit to the energy spectra



PRC 78 (2008) 014907

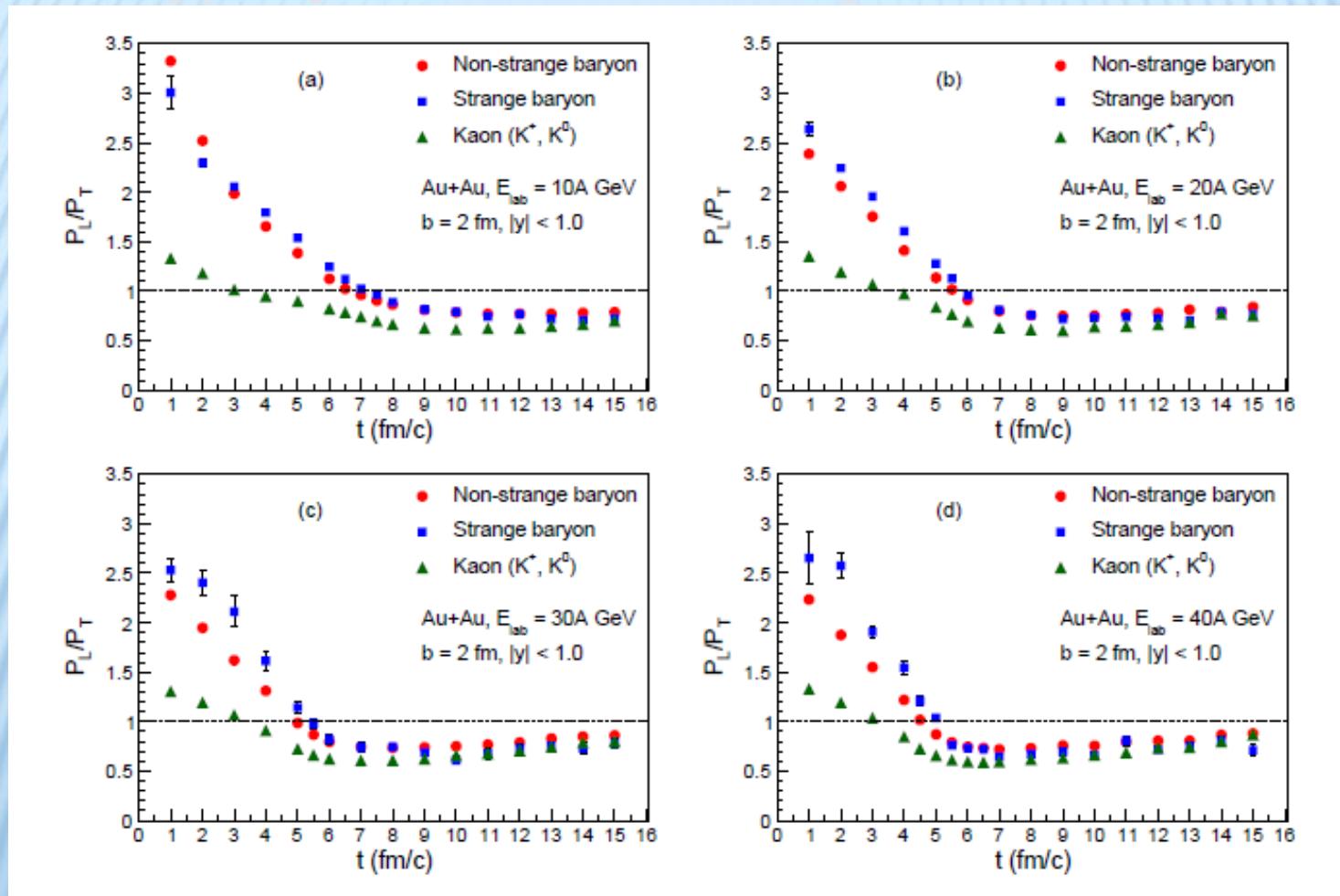
Particle yields



Thermal and chemical equilibrium seems to be reached

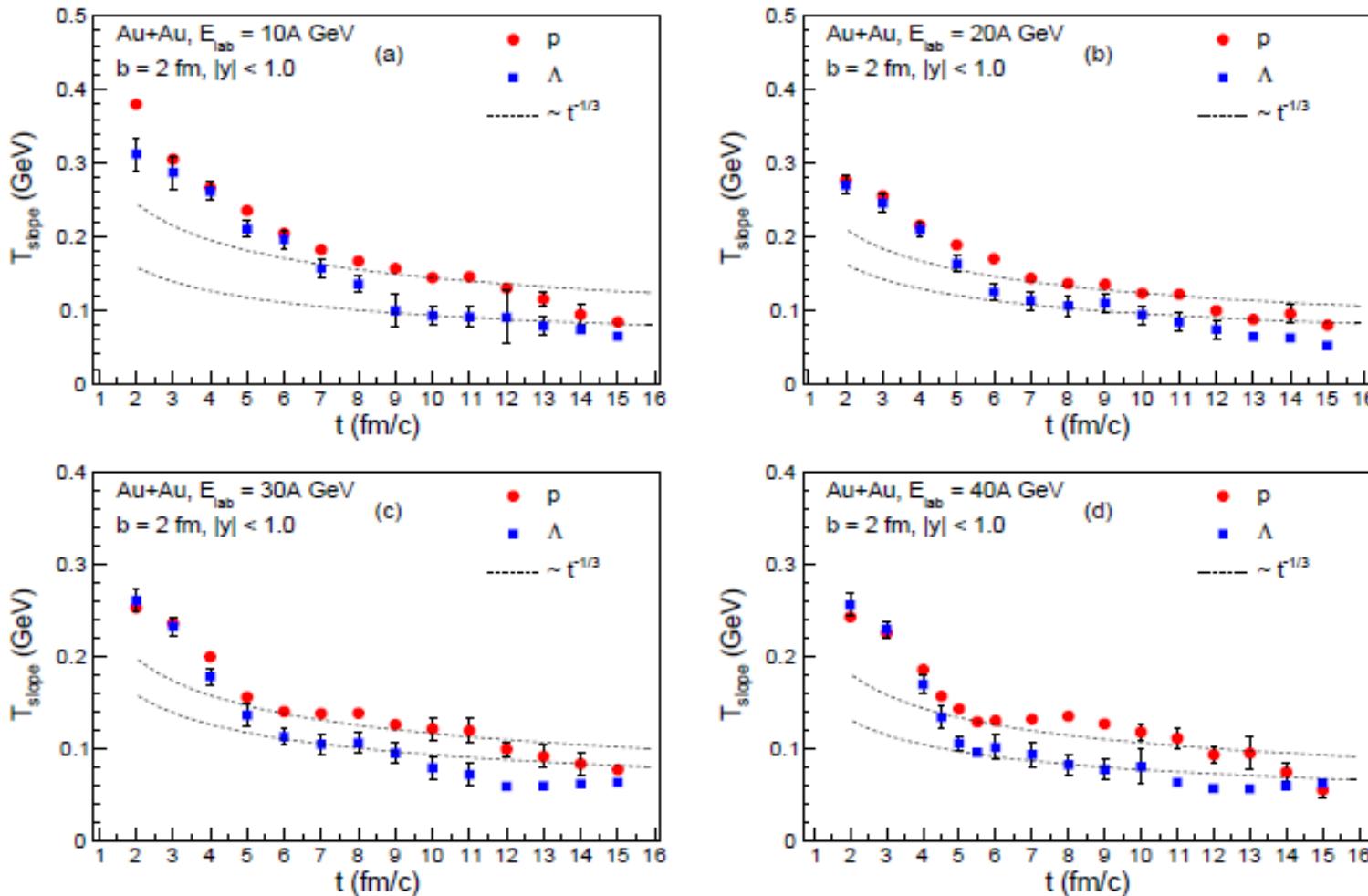
Other approaches

Isotropy of pressure for hadron species



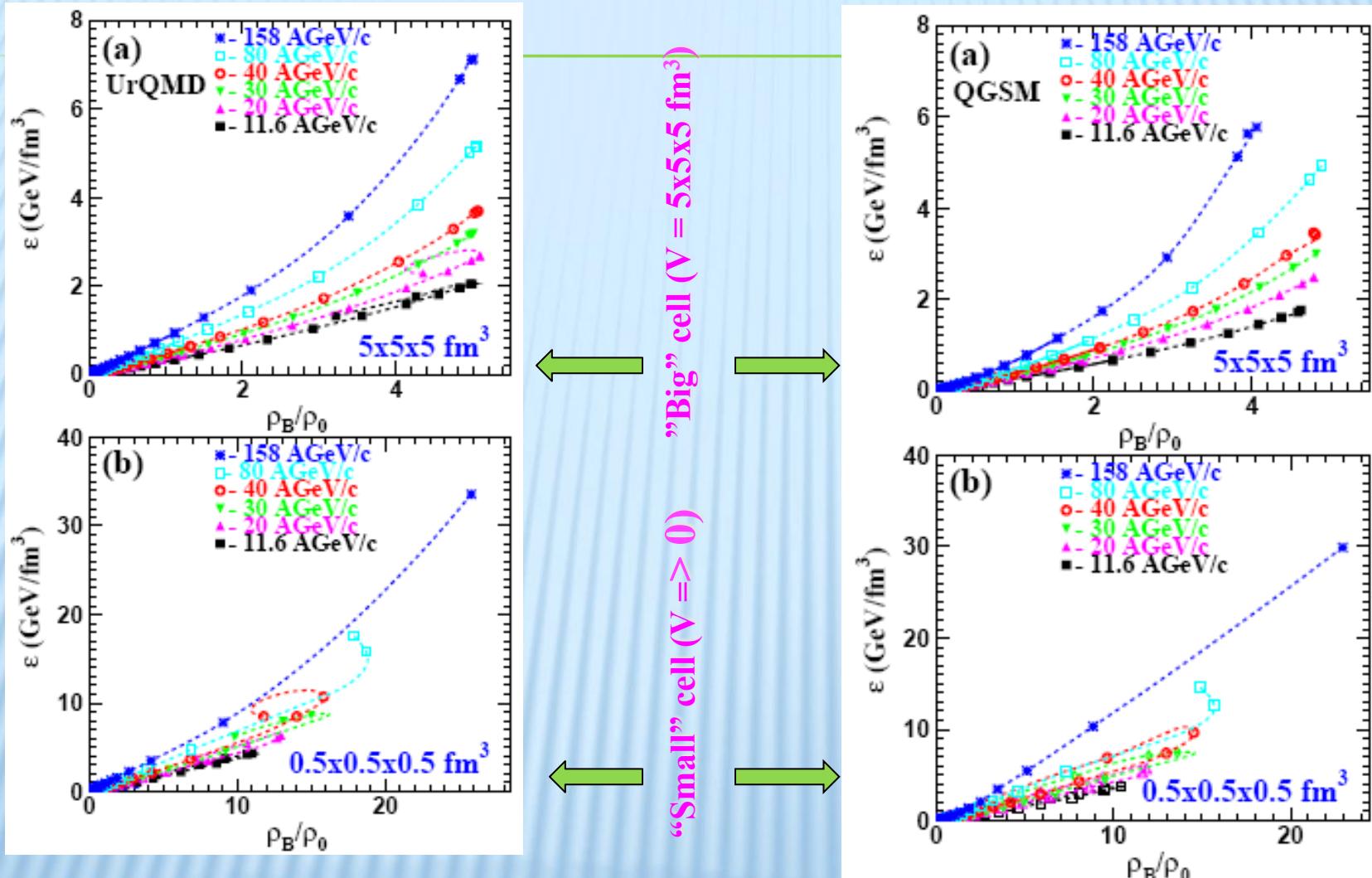
Other approaches

Fit of inverse slope parameter T_{slope} to Bjorken model



Fit to $t^{-1/3}$ (1D, dotted line) and t^{-1} (3D, dashed line)

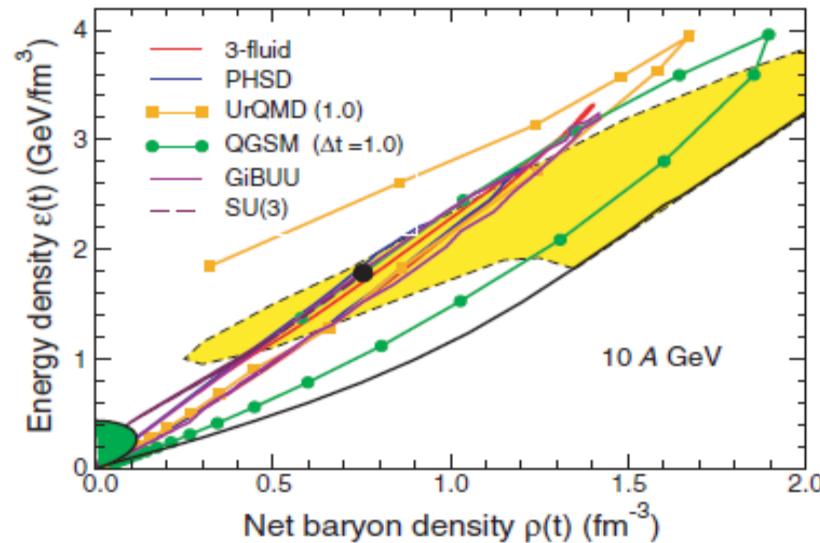
How dense can be the medium?



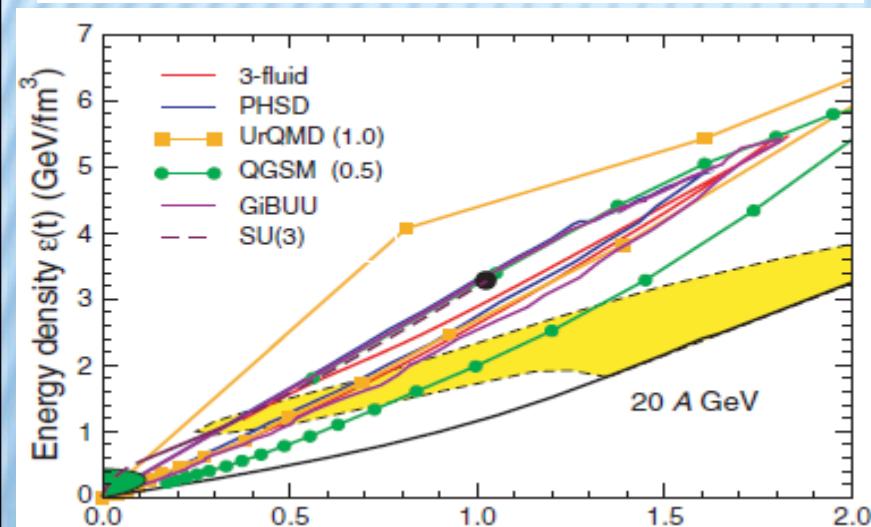
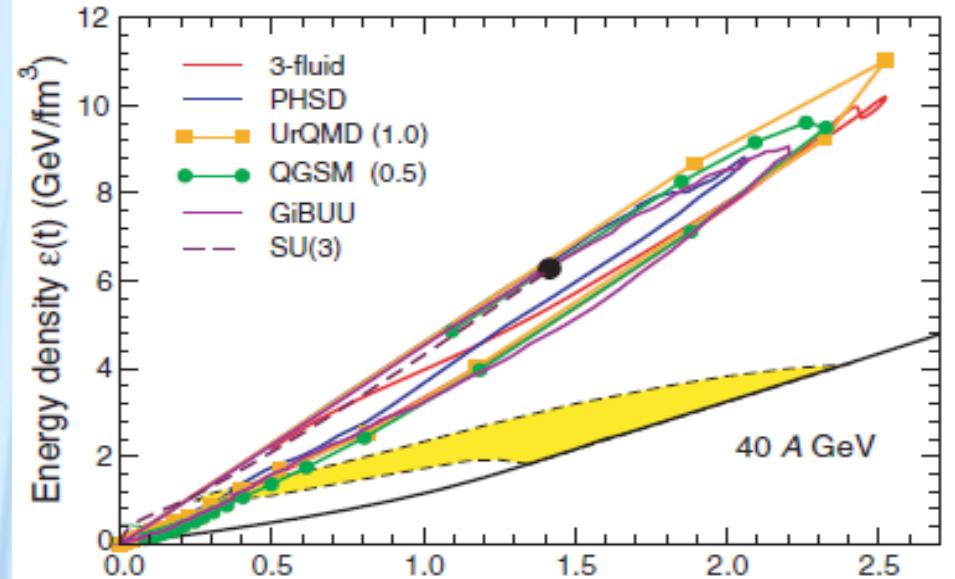
Dramatic differences at the non-equilibrium stage; after beginning of kinetic equilibrium the energy densities and the baryon densities are the same for "small" and "big" cell

Comparison between models

The phase trajectories at the center of a head-on Au+Au collisions



I. Arsene et al., PRC 75 (2007) 034902

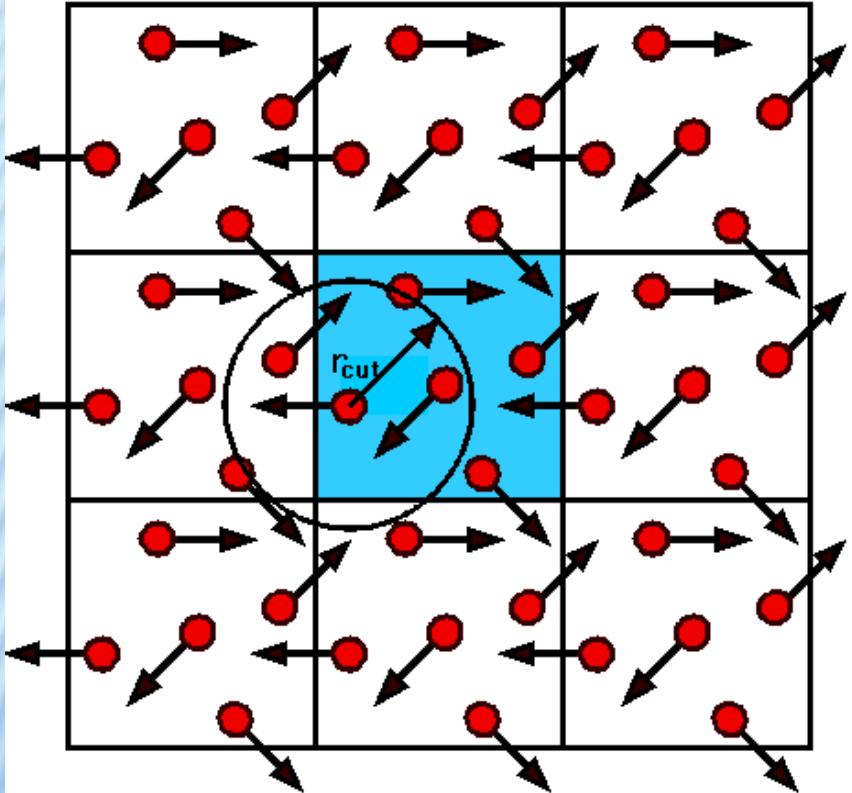


Green area : freeze-out region;
Yellow area : the phase coexistence
region from schematic EOS that has
a critical point at final density

Different models exhibit a large degree of mutual agreement

Infinite hadron gas: a box with periodic boundary conditions

BOX WITH PERIODIC BOUNDARY CONDITIONS



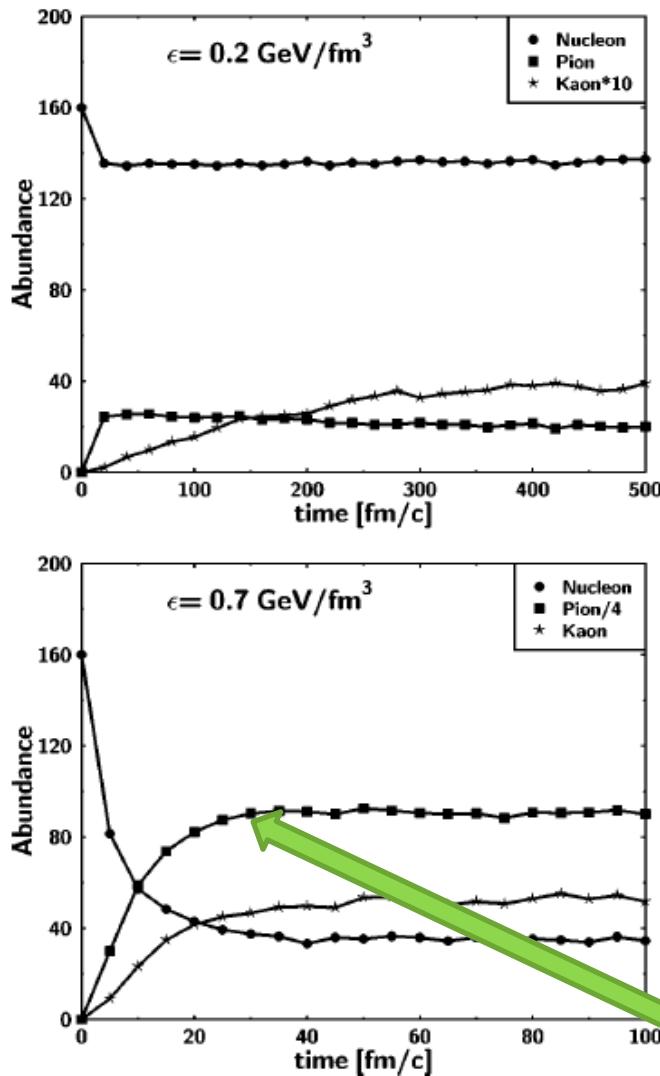
M.Belkacem et al., PRC 58, 1727 (1998)

Model employed: UrQMD
55 different baryon species
(N , Δ , hyperons and their
resonances with
 $m \leq 2.25 \text{ GeV}/c^2$),
32 different meson species
(including resonances with
 $m \leq 2 \text{ GeV}/c^2$) and their
respective antistates.
For higher mass excitations
a string mechanism is invoked.

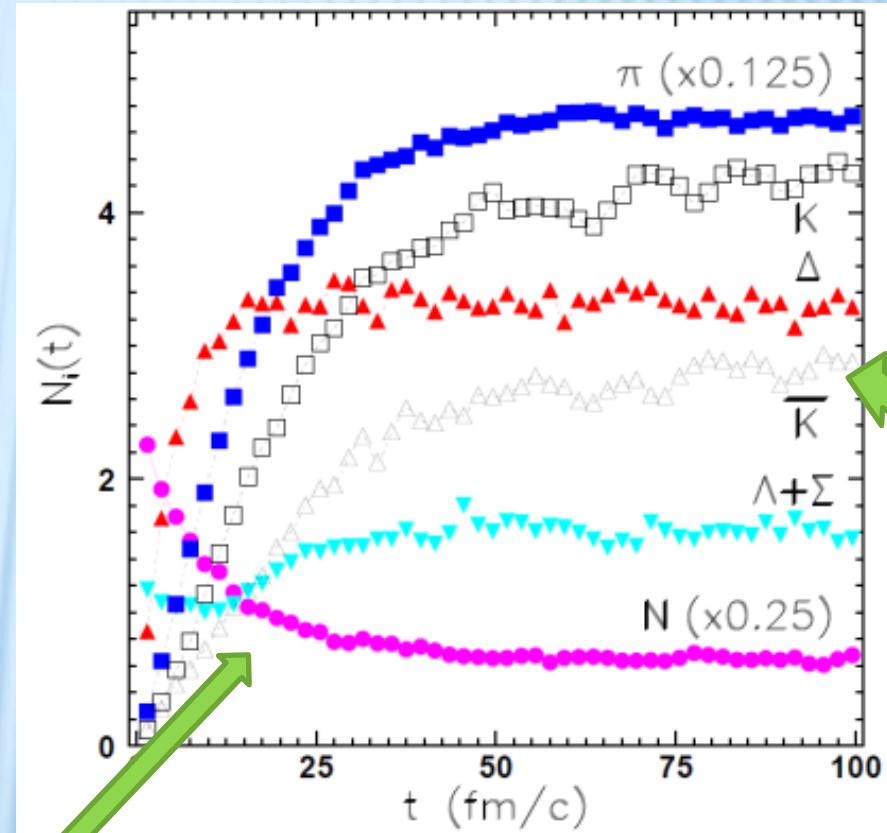
Initialization: (i) nucleons are uniformly distributed in a configuration space;
(ii) Their momenta are uniformly distributed in a sphere with random radius and then rescaled to the desired energy density.

Test for equilibrium: particle yields and energy spectra

BOX: PARTICLE ABUNDANCES



M.Belkacem et al., PRC 58, 1727 (1998)

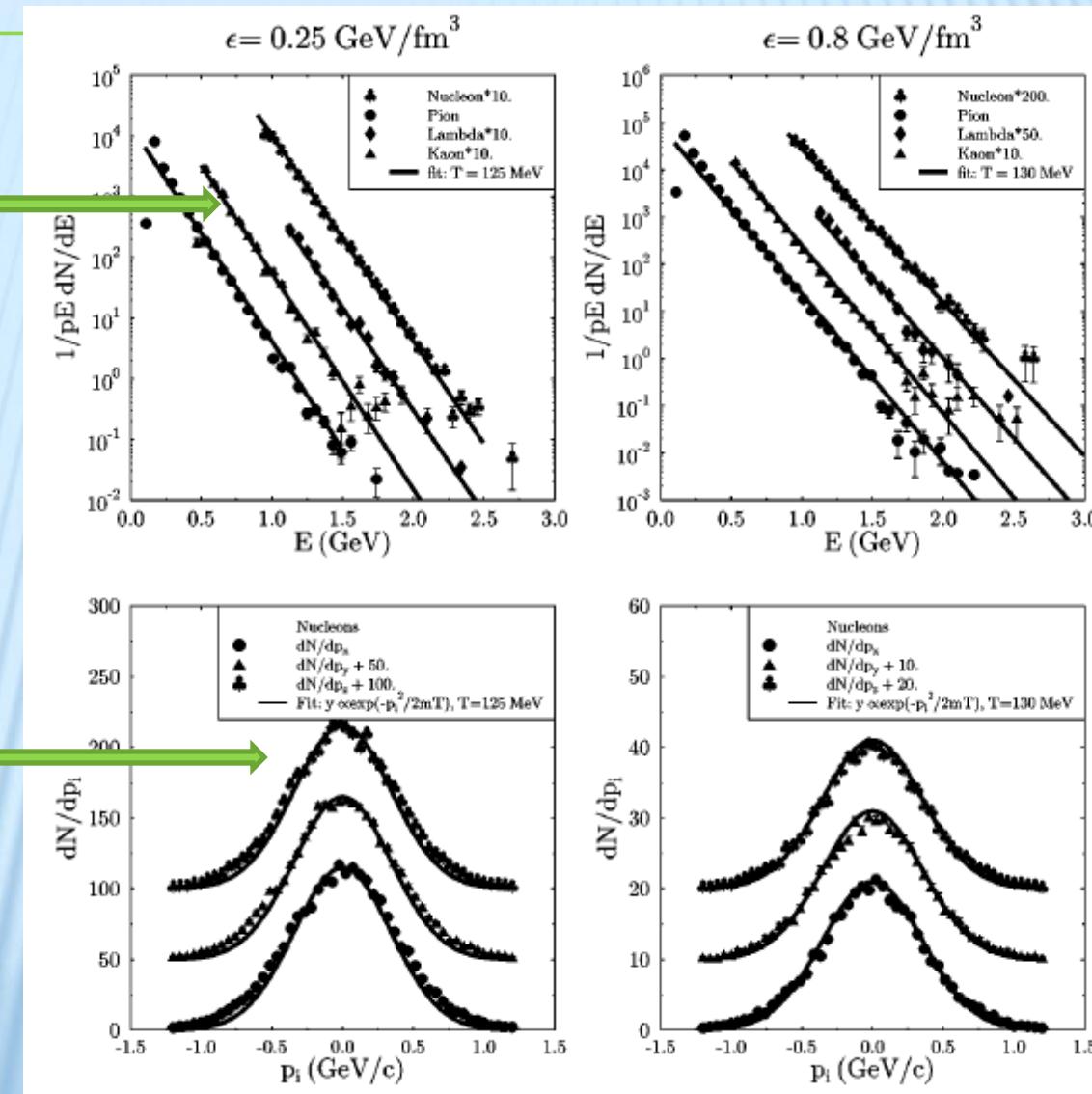


L.Bravina et al., PRC 62, 064906 (2000)

Saturation of yields after a certain time. Strange hadrons are saturated longer than others .

BOX: ENERGY SPECTRA AND MOMENTUM DISTRIBUTIONS

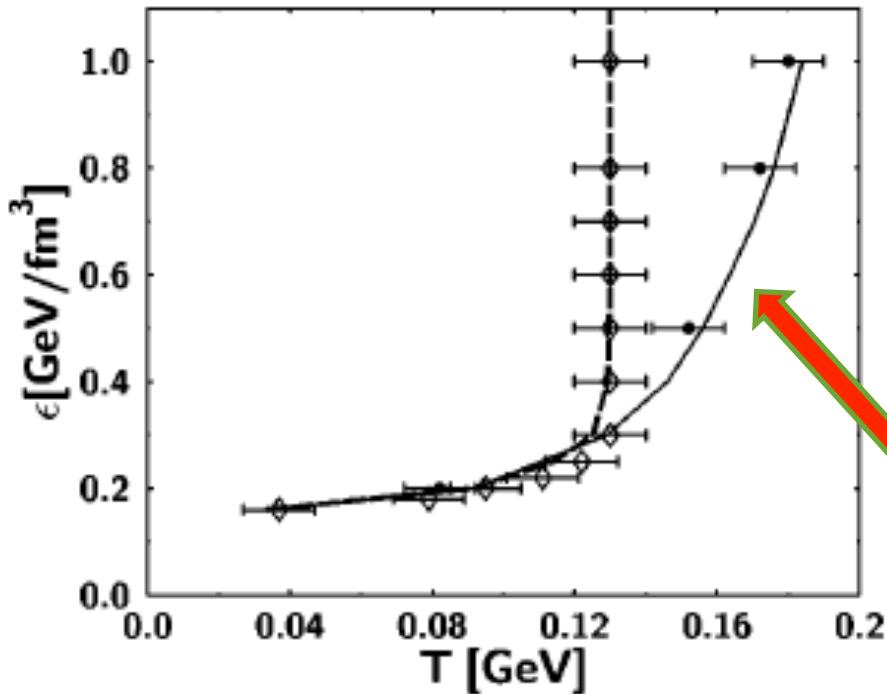
Fit to Boltzmann distributions $\sim \exp(-E/T)$



Fit to Gaussian distributions $\sim \exp(-p^2/2mT)$

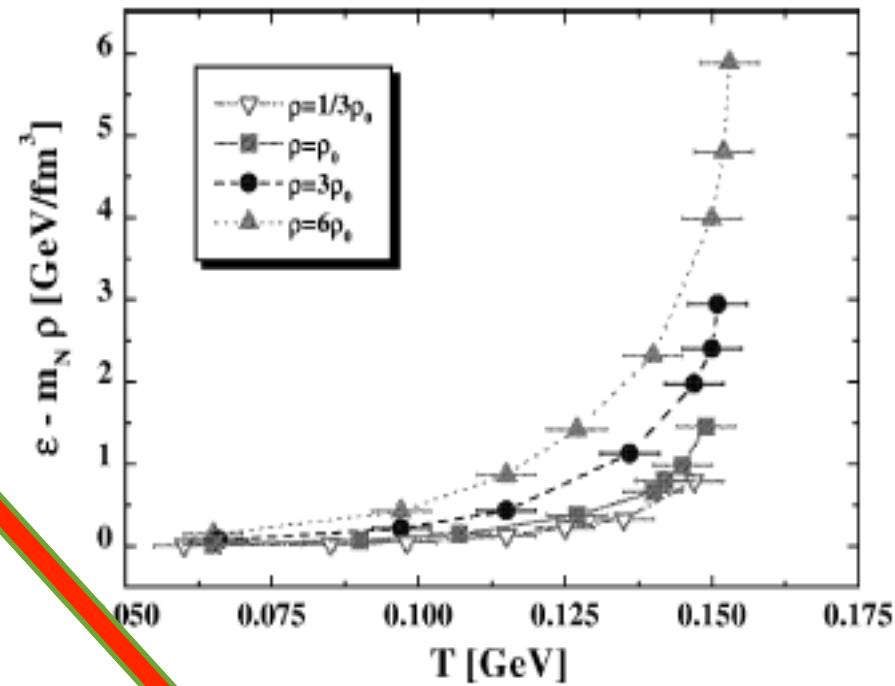
BOX: HAGEDORN-LIKE LIMITING TEMPERATURE

M.Belkacem et al., PRC 58, 1727 (1998)



UrQMD

HSD



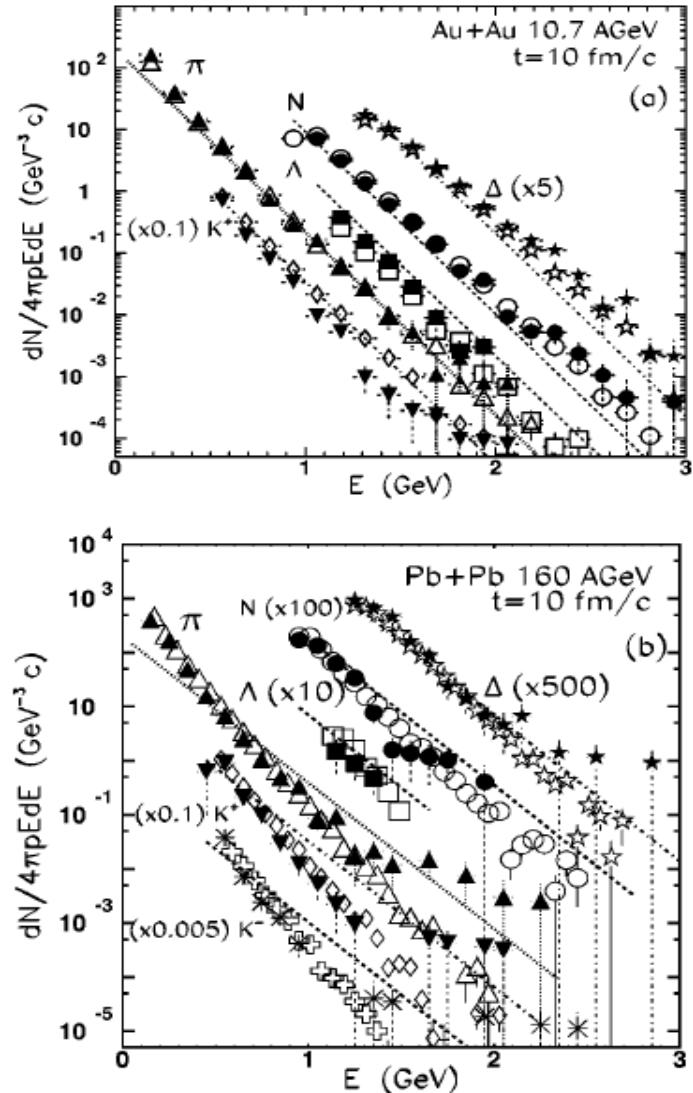
E.Bratkovskaya et al., NPA 675, 661 (2000)

A rapid rise of T at low ϵ and saturation at high energy densities.
Saturation temperature depends on number of resonances in the
model. W/o strings and many-N decays – no limiting T is observed.

Comparison of cell and box calculations

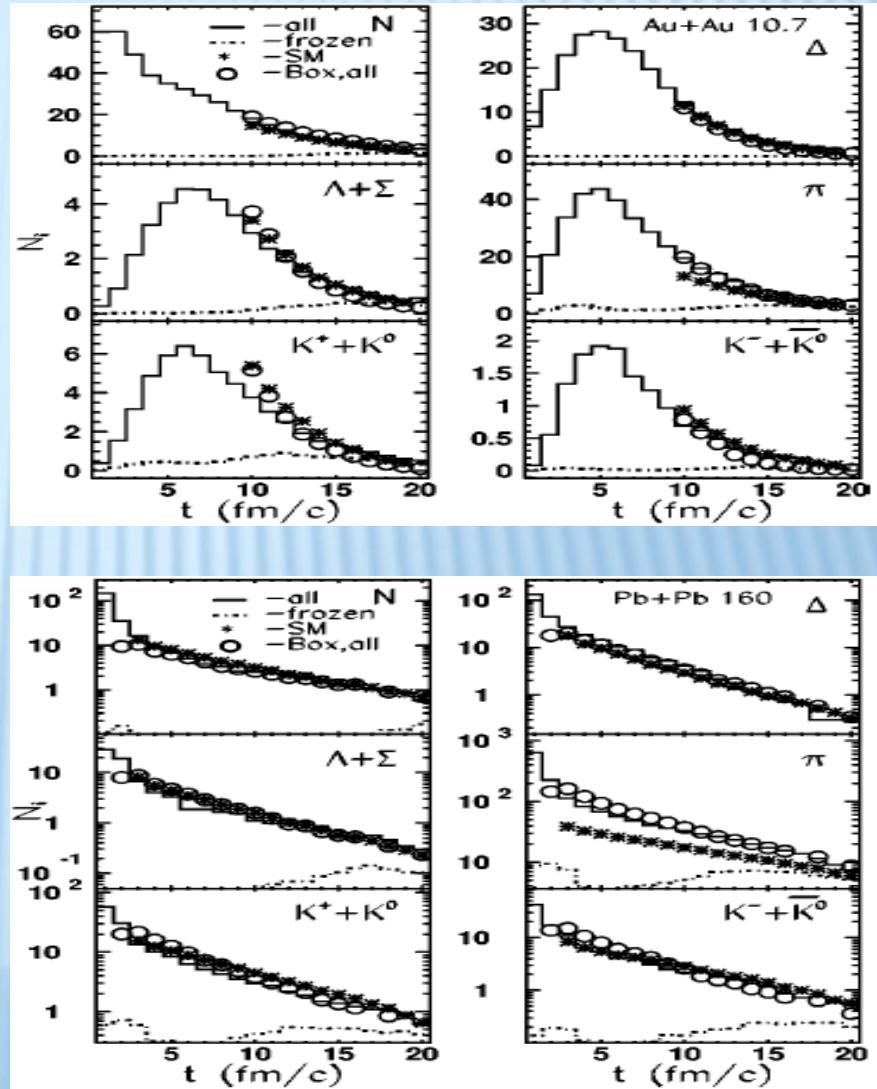
THERMAL AND CHEMICAL EQUILIBRIUM

Boltzmann fit to the energy spectra



L.Bравина et al., PRC 62, 064906 (2000)

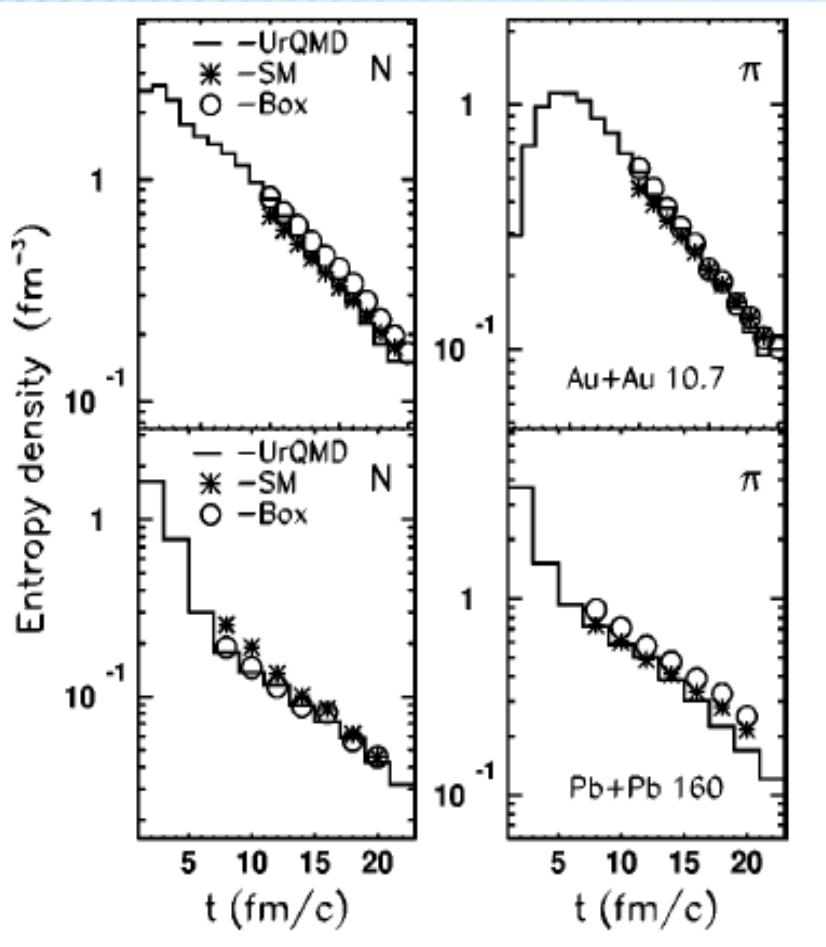
Hadron yields



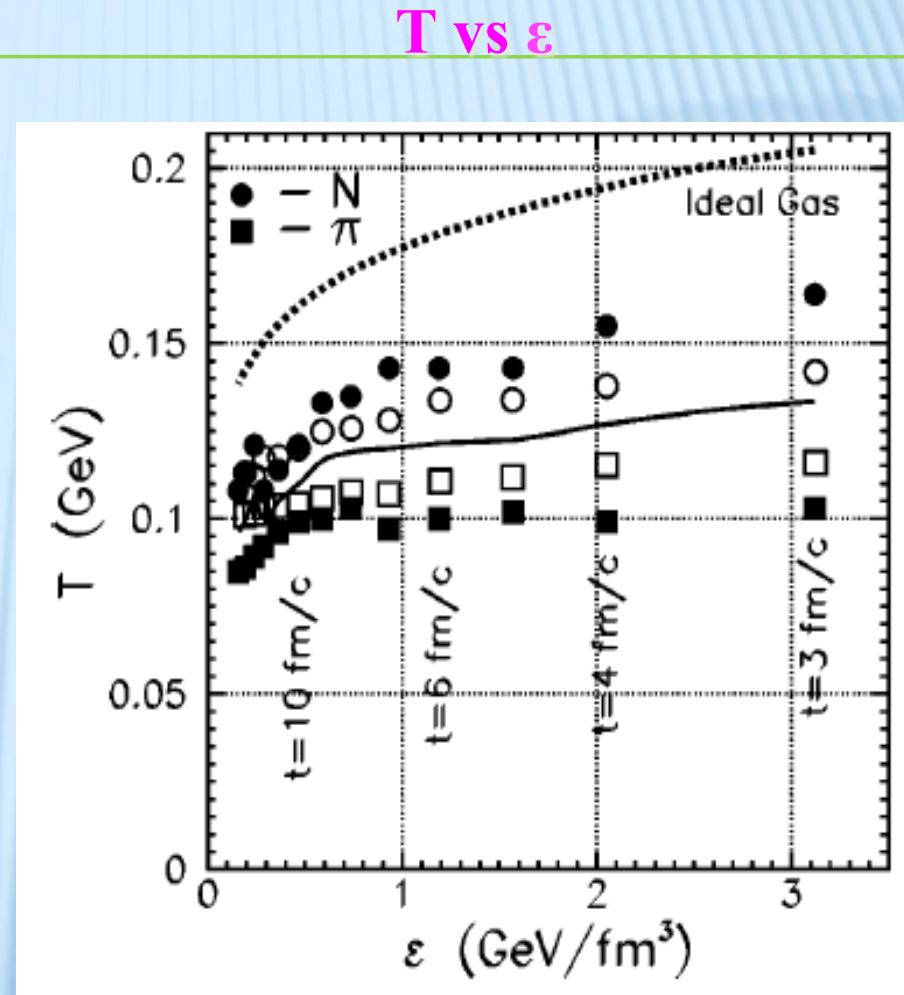
Box calculations are on the top of the cell results

THERMAL AND CHEMICAL EQUILIBRIUM

Partial entropy densities



L.B. et al., PRC 62, 064906 (2000)



Note the difference between T_N and T_π

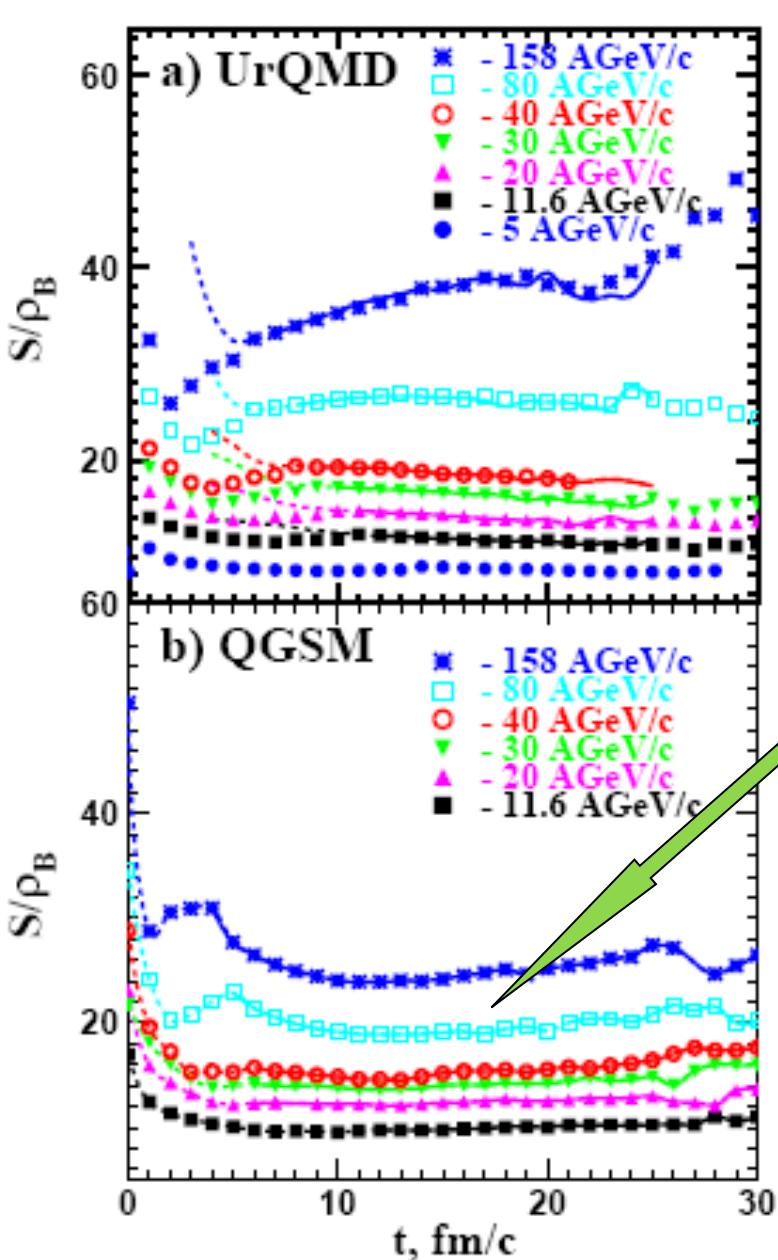
Box calculations are on the top of the cell results

Equation of State

T vs. energy,

etc

Isentropic expansion

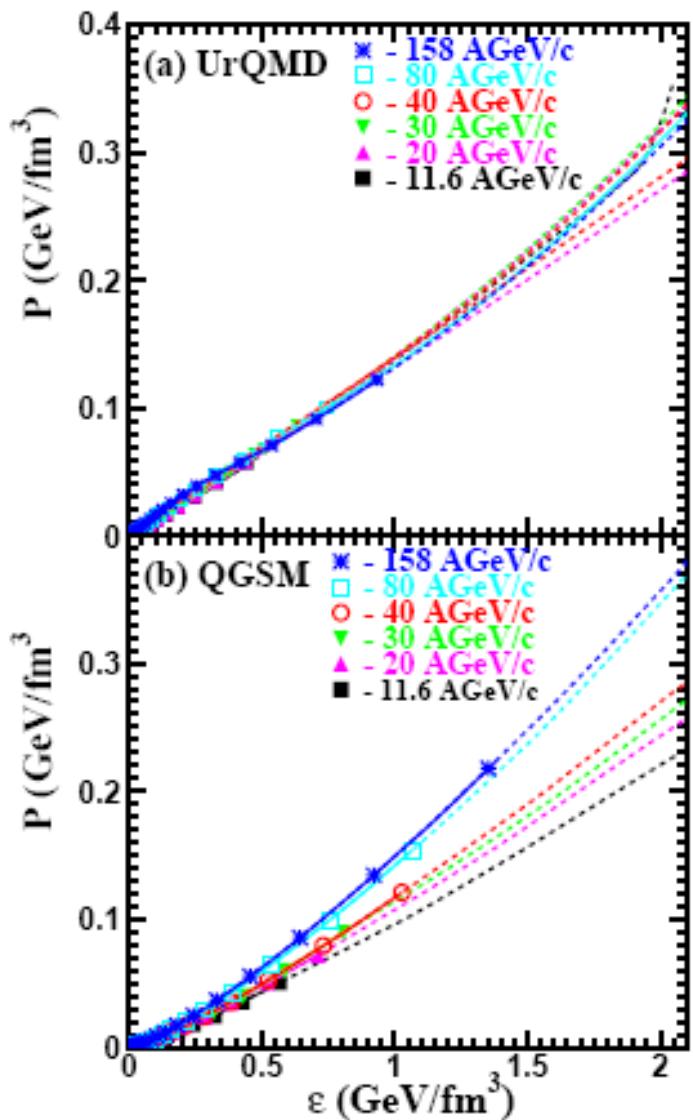


Expansion proceeds isentropically (with constant entropy per baryon). This result supports application of hydrodynamics

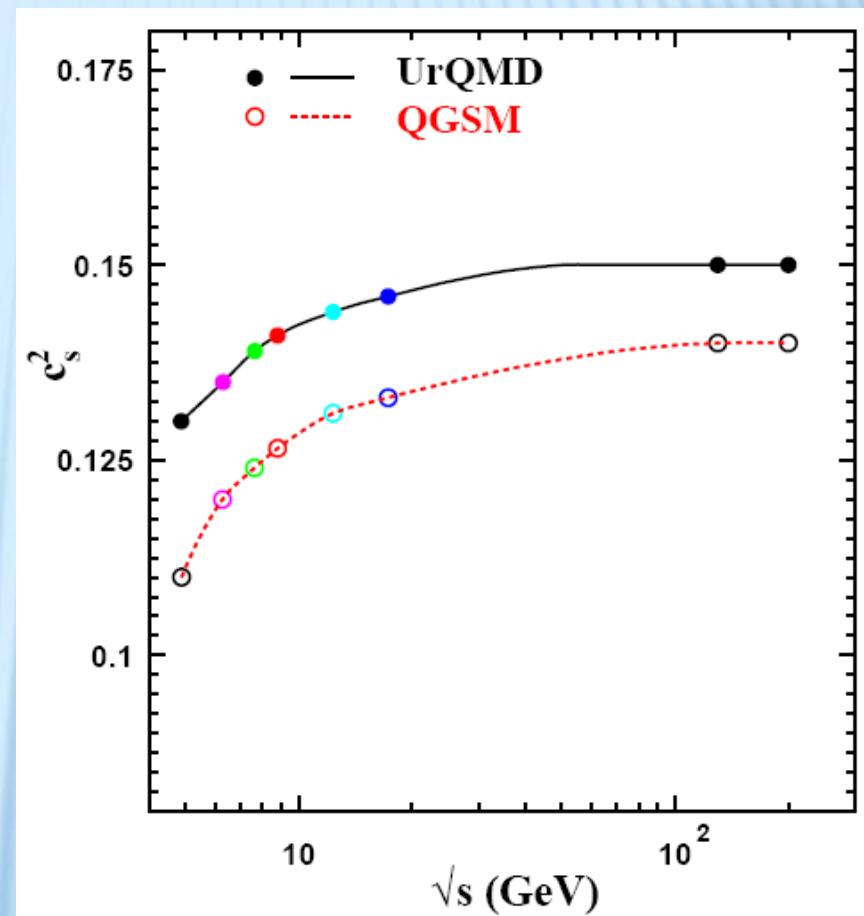
$s/\rho_B = \text{const} = 12(\text{AGS}), 20(40), 38(\text{SPS})$

Equation of State in the cell

pressure vs. energy

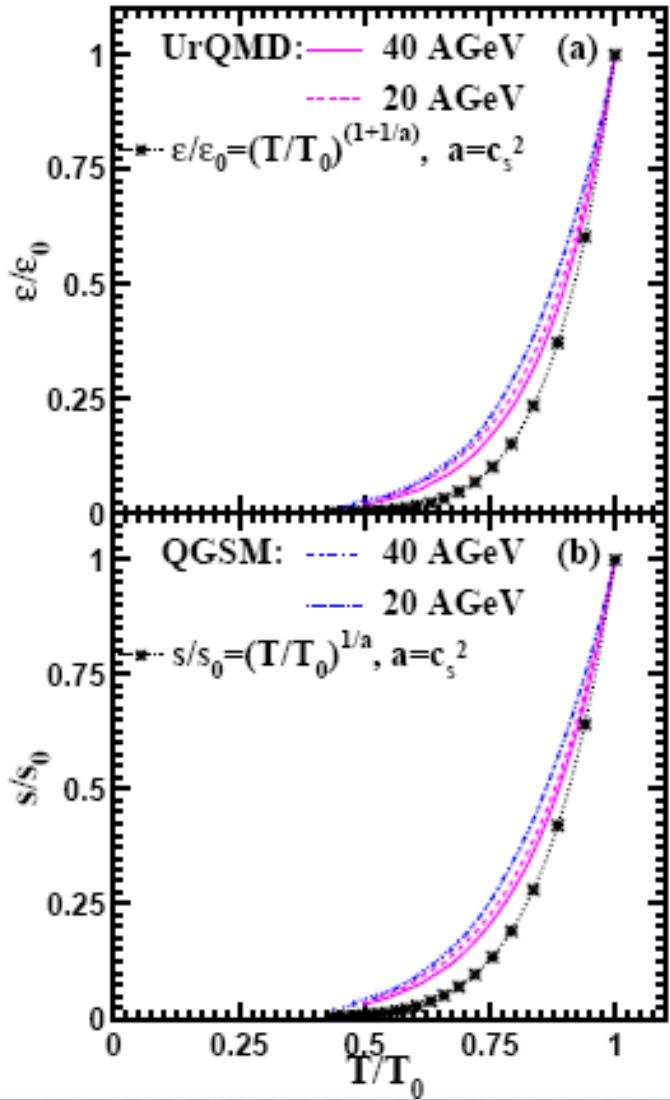


sound velocity



$P/\varepsilon = 0.13(\text{AGS}), \textcolor{red}{0.14(40)}, \textcolor{blue}{0.146(\text{SPS})}, \textcolor{green}{0.15(\text{RHIC})}$

Equation of State: energy and entropy densities vs. T



$$dP = a d\epsilon$$

$$d\epsilon = T ds$$

$$dP = s dT$$

Zero chem. potential

$$\frac{\epsilon}{\epsilon_0} = \left(\frac{T}{T_0} \right)^{\frac{1+a}{a}} \quad (1)$$

$$\frac{s}{s_0} = \left(\frac{T}{T_0} \right)^{\frac{1}{a}}$$

$$dP = a d\epsilon$$

$$d\epsilon = T ds + \mu d\rho$$

$$dP = s dT + \rho d\mu$$

$$ds = b d\rho$$

Non-zero chem. potential

$$\frac{\epsilon}{\epsilon_0} = \left(\frac{bT + \mu}{bT_0 + \mu_0} \right)^{\frac{a+1}{a}} \quad (2)$$

$$\frac{s}{s_0} = \left(\frac{bT + \mu}{bT_0 + \mu_0} \right)^{\frac{1}{a}}$$

If $\mu = cT$ then (2) is transformed to (1)

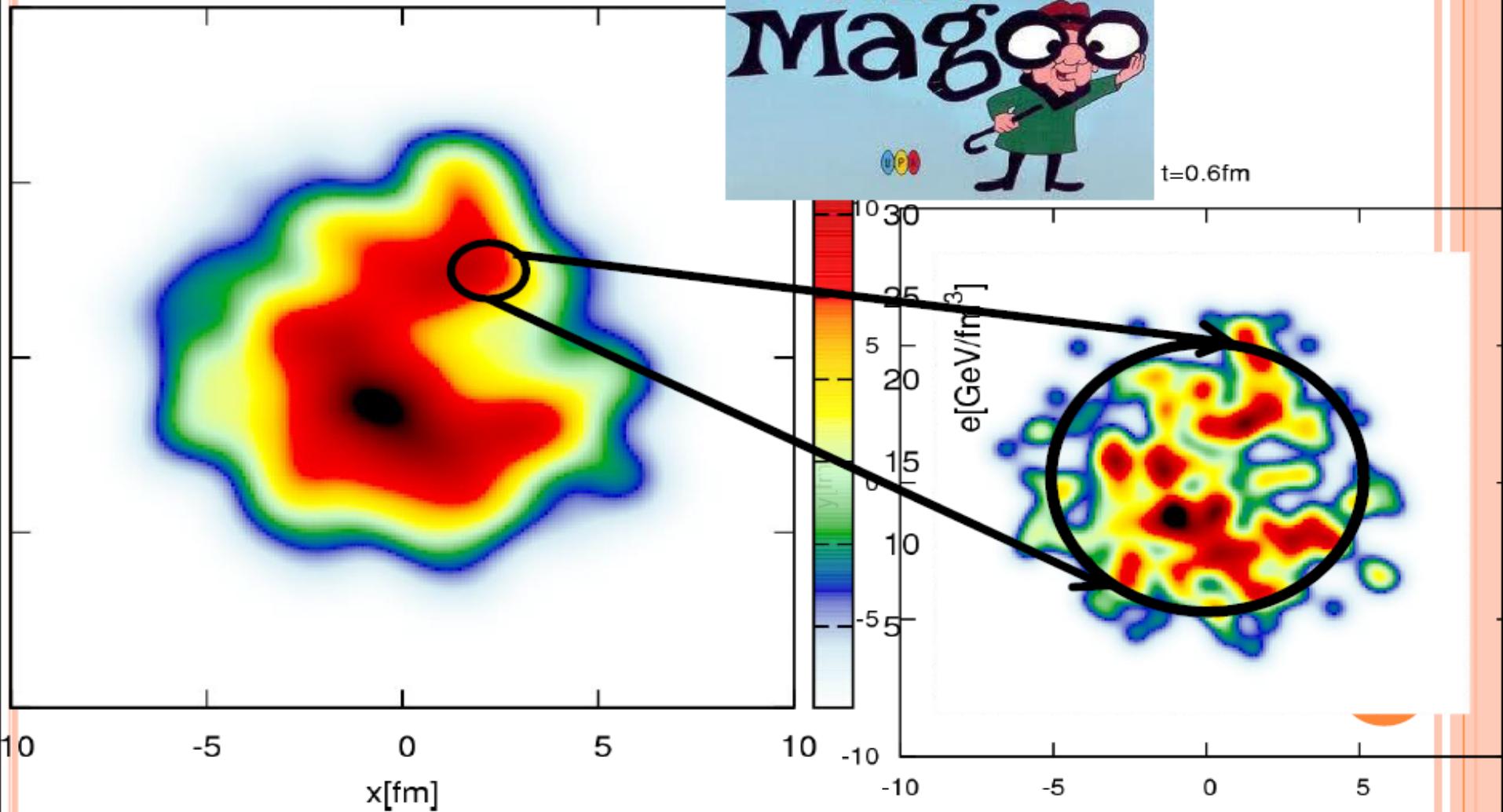
No difference between the models

**Modification of
the analysis
(small cells
or coarse-graining)**

Example of coarse-graining

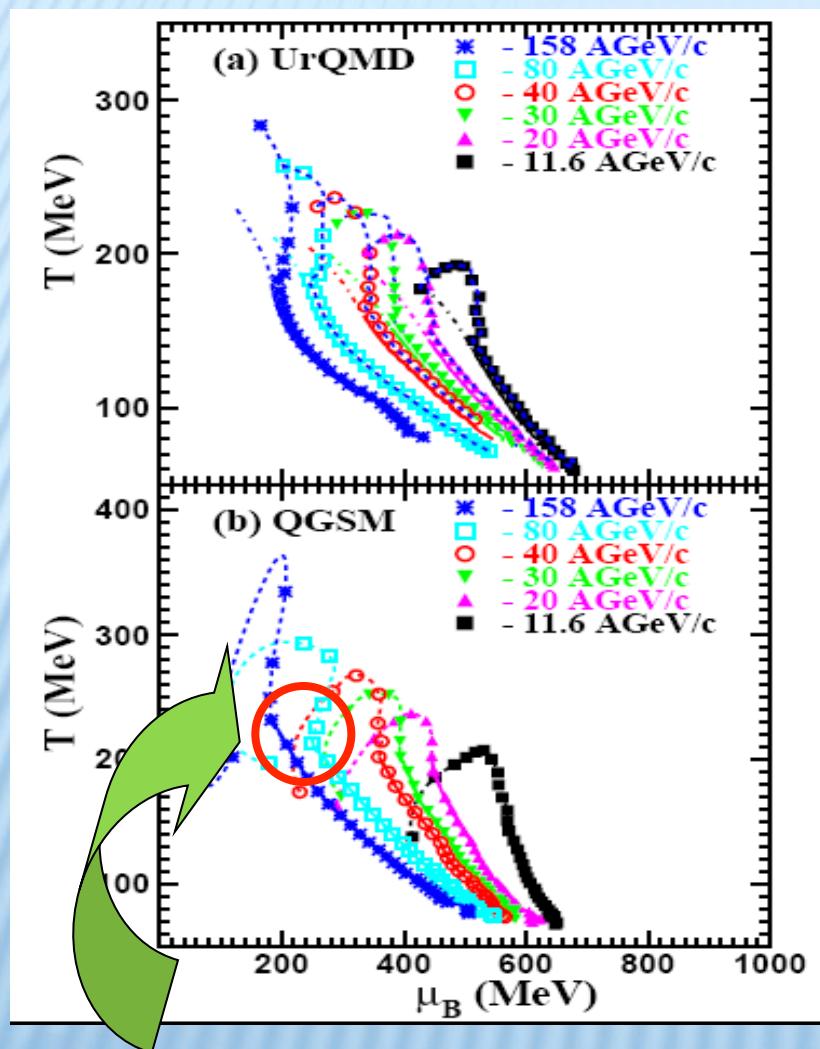
T. Kodama, conf. NeD/TURIC'2012

COARSE GRAINING AND RESOLUTION $t=0.6\text{fm}$

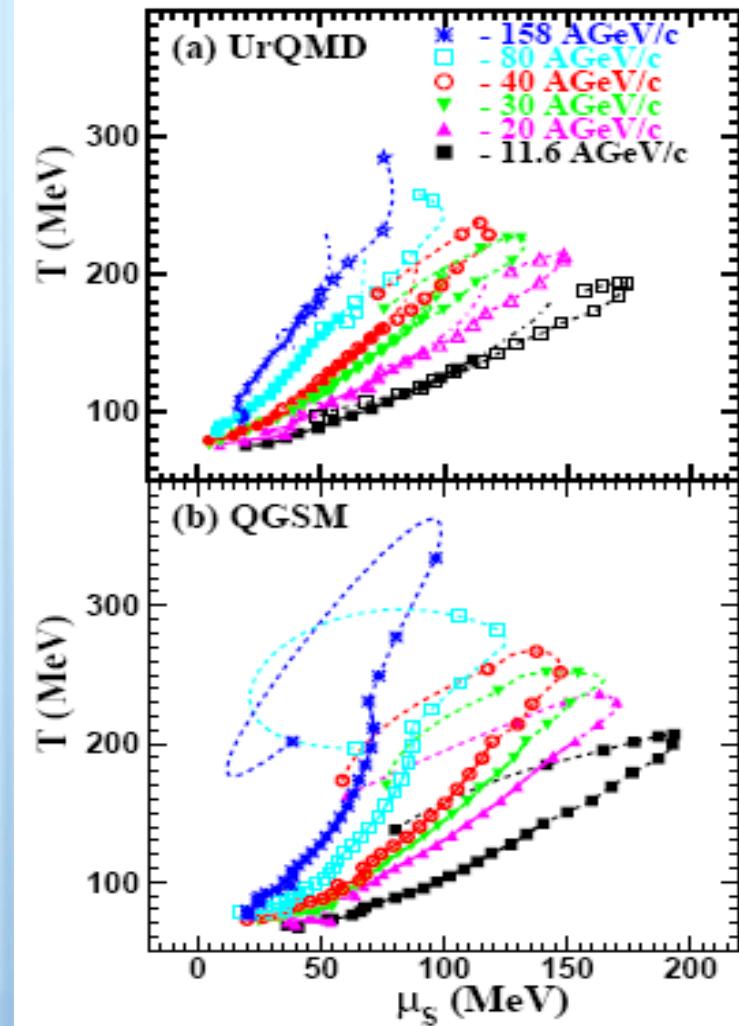


$t=0.6\text{fm}$

EOS in the cell: observation of knee temperature vs. chemical potentials



L.B. et al., PRC 78 (2008) 014907

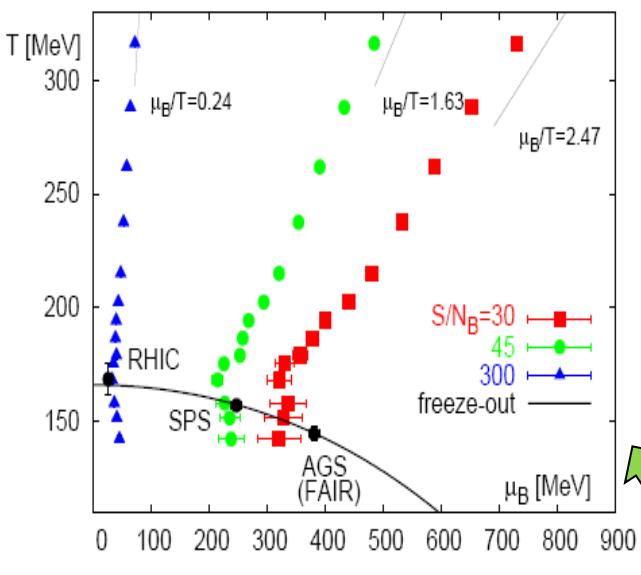


The “knee” in MC simulations appears at chemical FO

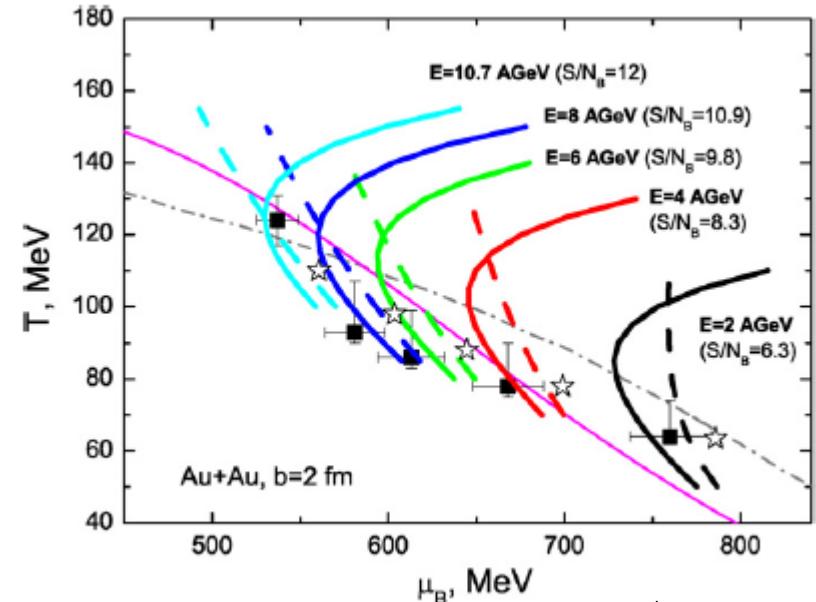
Observation of the ‘knee’ in other models

temperature vs. chemical potentials

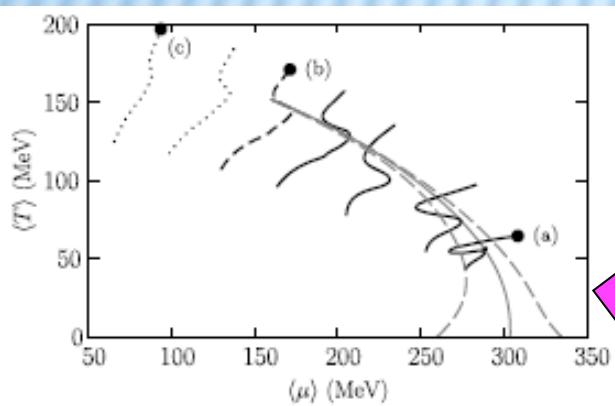
S. Ejiri et al., PRD 73 (2006) 054506



A. Khvororsukhin et al., NPA 791 (2007) 180



C. Herold et al., NPA 925 (2014) 14



The «knee» is observed in
 (i) 2-flavor lattice QCD;
 (ii) σ - ω - ρ model with scaled hadron masses;
 (iii) chiral fluid dynamics model with Polyakov loop

Further box application

COLD NUCLEON MATTER:

nucleon gas at low temperatures (20–50 MeV) at densities of order of a normal nuclear density $n_0=0.16 \text{ fm}^3$ is studied.

A. Motornenko et al., JPG 45 (2018) 035101

Green-Kubo formalism is used to extract shear viscosity η :

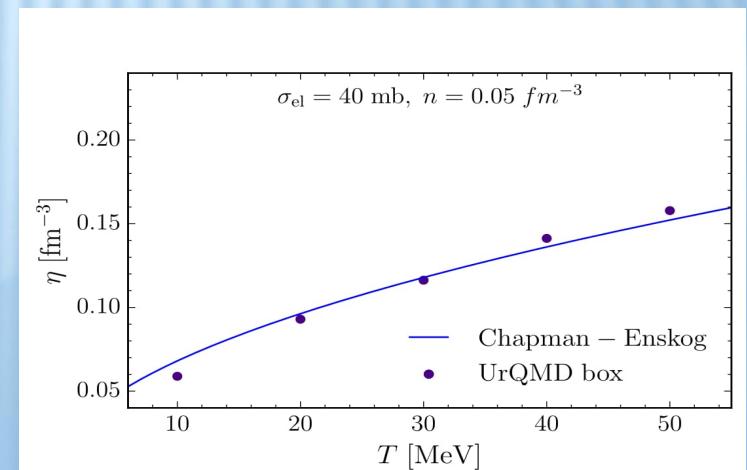
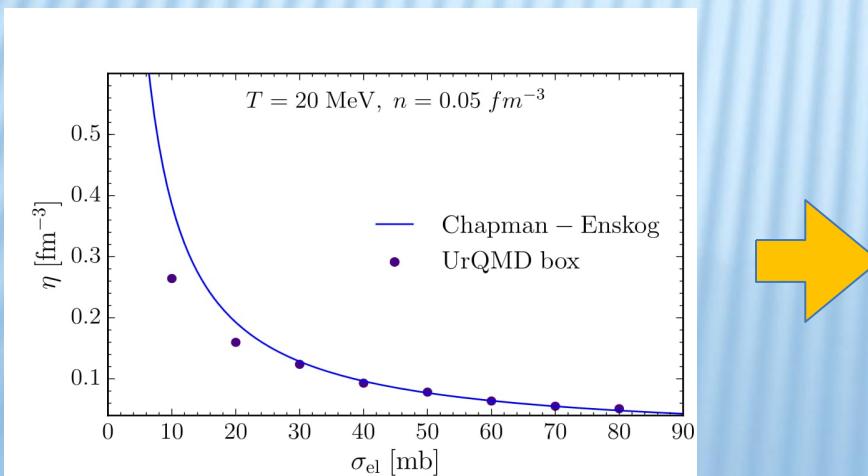
$$\eta = \frac{V}{T} \int_0^\infty dt \langle \pi^{xy}(t) \pi^{xy}(0) \rangle, \quad \pi^{ij} = T^{ij} - P\delta^{ij}$$

Chapman-Enskog approximation:

$$\eta_{CE} = \frac{5}{64\sqrt{\pi}} \frac{\sqrt{mT}}{d^2}$$

Pressure is calculated from the stress-energy tensor:

$$P = \frac{1}{3} \sum_{i=1}^3 T^{ii}, \quad T^{\alpha\beta} = \int_V \frac{p^\alpha(x)p^\beta(x)}{p^0(x)}$$

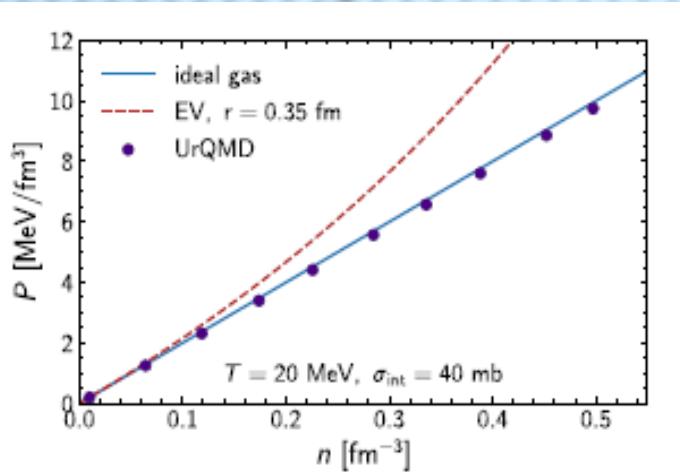


COLD NUCLEON MATTER: search for excluded volume effects

$$b = \frac{4}{3}\pi d^3$$

Average distance between colliding nucleons $\langle d \rangle = 0.7 \text{ fm}$

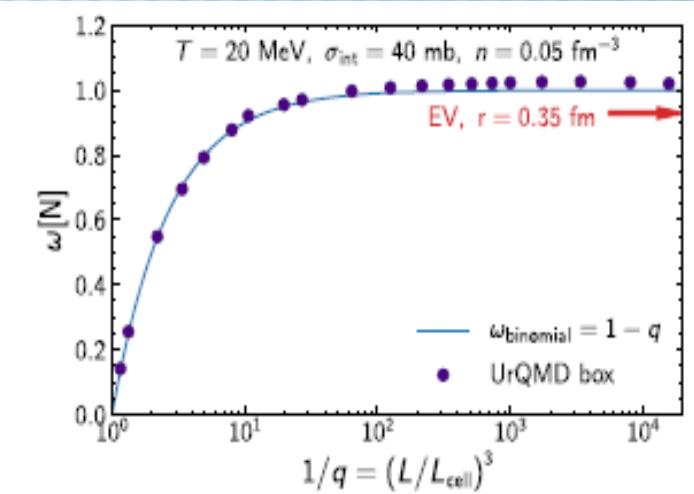
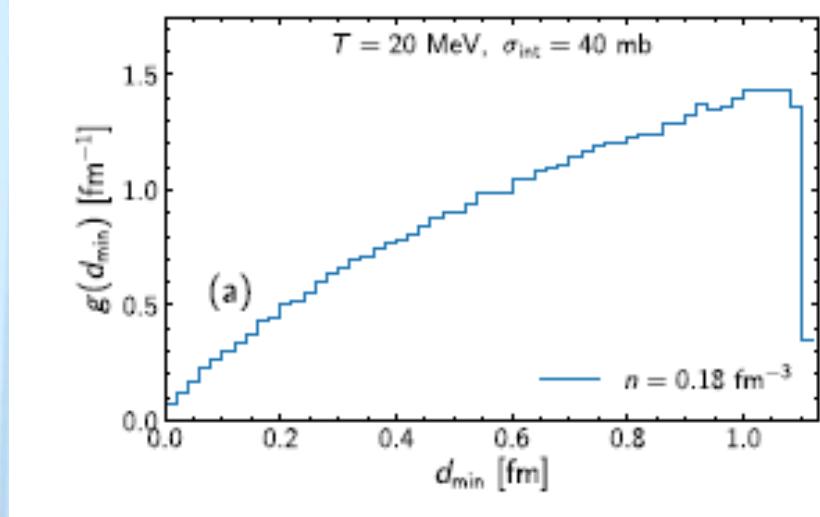
However, no traces of the excluded volume in EoS



$$p = \frac{nT}{1 - bn}$$

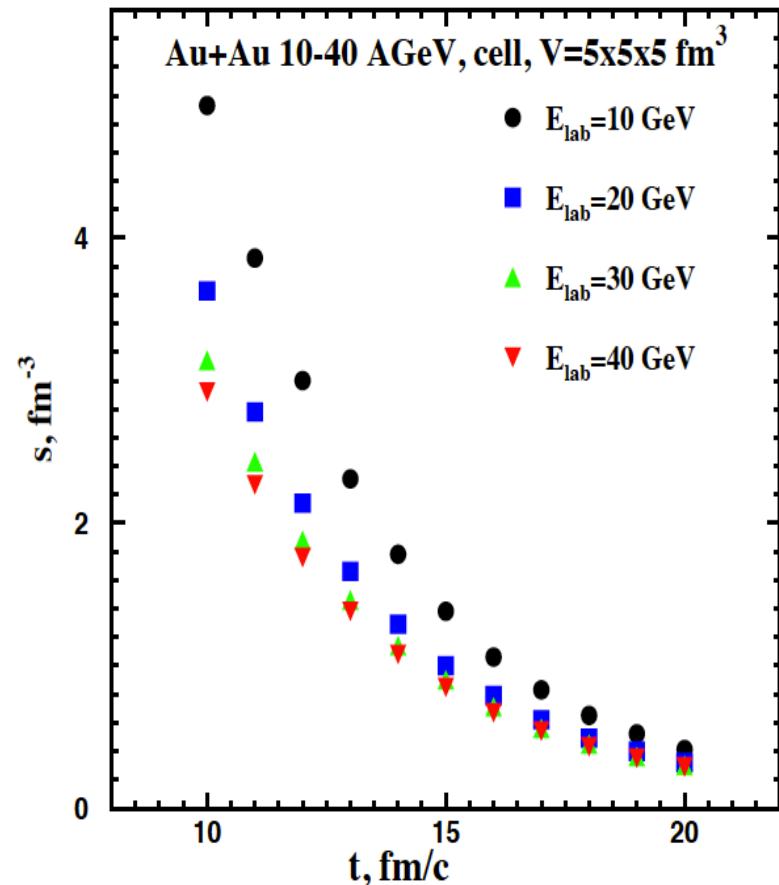
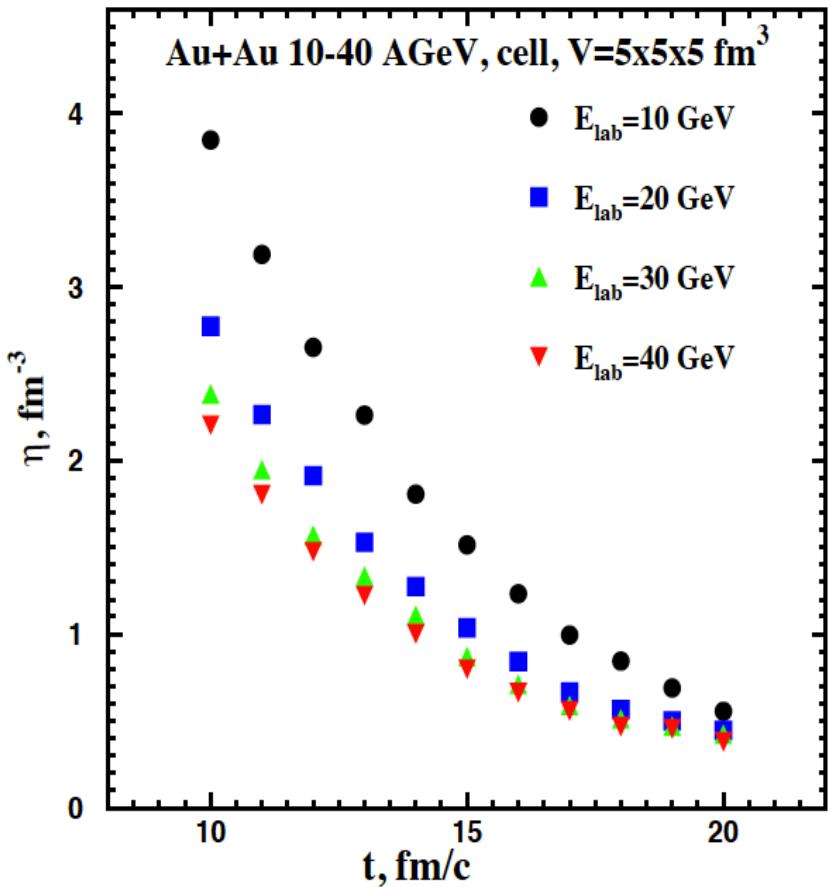
or in
particle
number
fluctuations

$$\omega = (1 - bn)^2$$



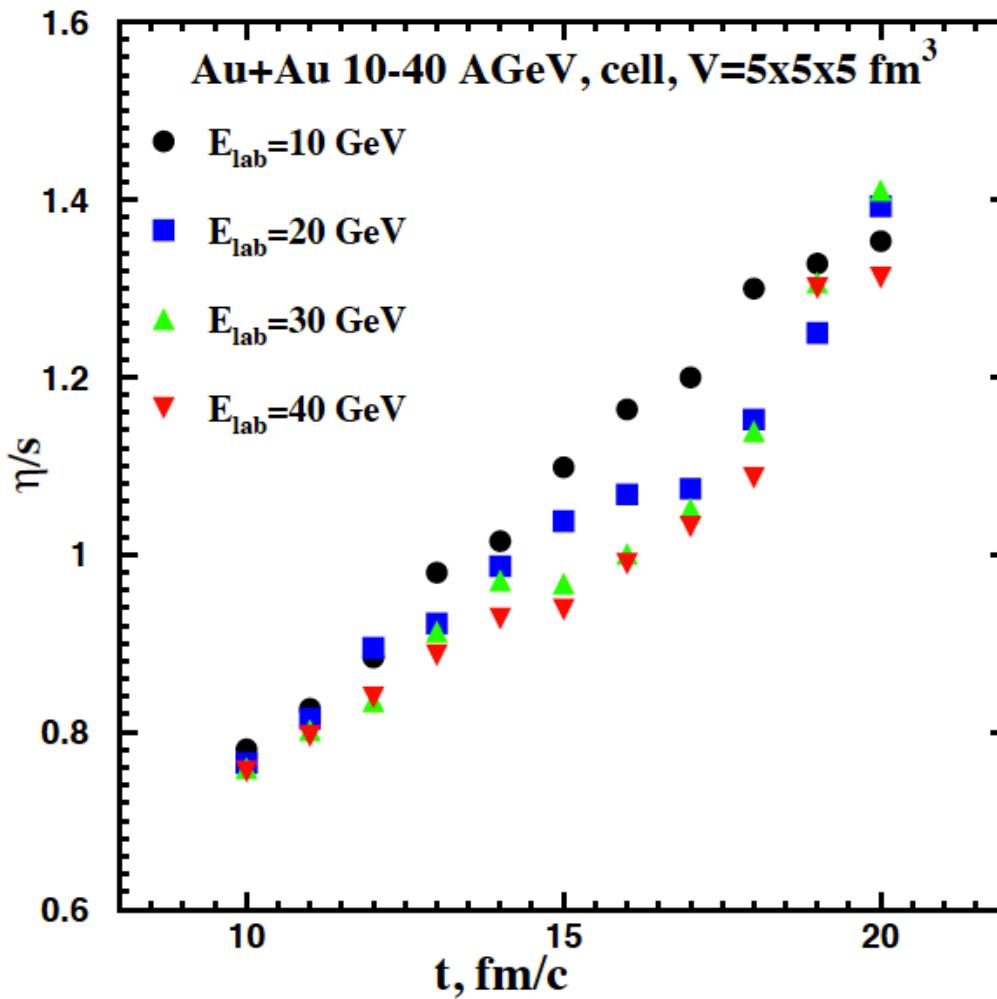
SHEAR VISCOSITY AND ENTROPY AT NICA

in collaboration with M. Teslyk



SHEAR VISCOSITY AND ENTROPY AT NICA

in collaboration with M. Teslyk



Conclusions

- *MC models favor formation of equilibrated matter for a period of 10-15 fm/c*
- *During this period the expansion of matter in the central cell proceeds isentropically with constant S/B (hydro!)*
- *The EOS has a simple form: $P/\varepsilon = \text{const}$, where the speed of sound squared varies from 0.12 (AGS) to 0.14 (40 AGeV), and to 0.15 (SPS & RHIC) \Rightarrow onset of saturation*
- *Agreement between the cell and box results; not always good between the cell/box and the SM*
- *T vs. μ : the knee structure which appears at the onset of equilibrium is related to chemical freeze-out*

Back-up Slides

Bibliography

Local thermodynamical equilibration in central Au + Au collisions at AGS

L.V. Bravina et al., *Phys. Lett. B* 434 (1998) 379-387; DOI: 10.1016/S0370-2693(98)00624-8

Equation of state, spectra and composition of hot and dense infinite hadronic matter in a microscopic transport model

M. Belkacem et al., *Phys. Rev. C* 58 (1998) 1727-1733; DOI: 10.1103/PhysRevC.58.1727

Local thermal and chemical equilibration and the equation of state in relativistic heavy ion collisions

L.V. Bravina et al., *J. Phys. G* 25 (1999) 351-361; DOI: 10.1088/0954-3899/25/2/024

Equilibrium and nonequilibrium effects in relativistic heavy ion collisions

L.V. Bravina et al., *Nucl. Phys. A* 661 (1999) 600-603; DOI: 10.1016/S0375-9474(99)85097-0

Local equilibrium in heavy ion collisions: Microscopic model versus statistical model analysis

L.V. Bravina et al., *Phys. Rev. C* 60 (1999) 024904; DOI: 10.1103/PhysRevC.60.024904

Equation of state of resonance rich matter in the central cell in heavy ion collisions at S^(1/2)=200-A/GeV

L.V. Bravina et al., *Phys. Rev. C* 63 (2001) 064902; DOI: 10.1103/PhysRevC.63.064902

Local equilibrium in heavy ion collisions: Microscopic analysis of a central cell versus infinite matter

L.V. Bravina et al., *Phys. Rev. C* 62 (2000) 064906; DOI: 10.1103/PhysRevC.62.064906

Chemical freezeout parameters at RHIC from microscopic model calculations

L.V. Bravina et al., *Nucl. Phys. A* 698 (2002) 383-386; DOI: 10.1016/S0375-9474(01)01385-9

Equilibration of matter near the QCD critical point

L.V. Bravina et al., *J. Phys. G* 32 (2006) S213-S221; DOI: 10.1088/0954-3899/32/12/S27

Equilibration of matter in relativistic heavy-ion collisions

L. Bravina et al., *Int. J. Mod. Phys. E* 16 (2007) 777-786; DOI: 10.1142/S0218301307006277

Bibliography (cont.)

Microscopic models and effective equation of state in nuclear collisions at FAIR energies

L.V. Bravina et al.. Phys. Rev. C78 (2008) 014907 ; DOI: 10.1103/PhysRevC.78.014907

Equation of state at FAIR energies and the role of resonances

E E Zabrodin et al.. J. Phys. G36 (2009) 064065; DOI: 10.1088/0954-3899/36/6/064065

Effective equation of state of hot and dense matter in nuclear collisions around FAIR energy

L. Bravina , E. Zabrodin , EPJ Web Conf. 95 (2015) 01003; DOI: 10.1051/epjconf/20159501003