

# Wormhole beyond Horndeski

1806.xxxx

S. Mironov, V. Volkova

INR RAS

Russia, Valday, May 31, 2018

Why do we study wormholes?

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We want to build a **teleport**!

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We want to build a **teleport**!  
And a **time machine**!  
And a **Universe** in the lab!

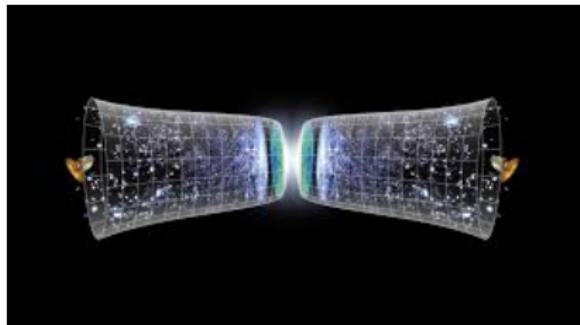
## Why Horndeski theories?

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It can break NEC in a healthy way.

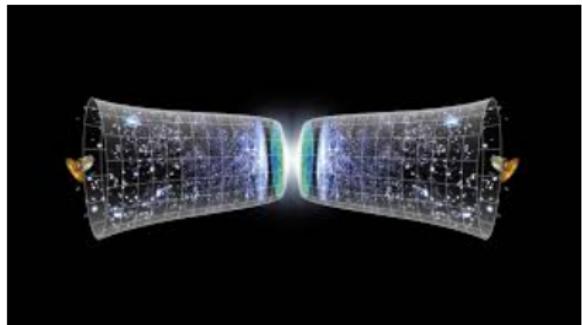
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*Radial axis* →

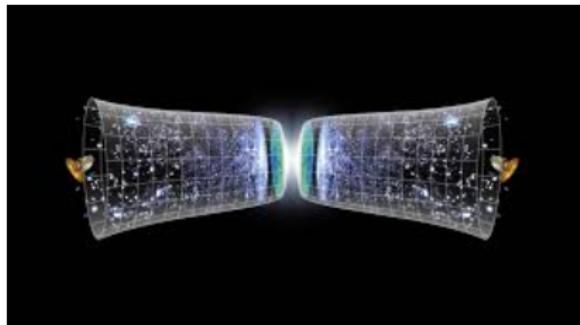


*Time axis* →

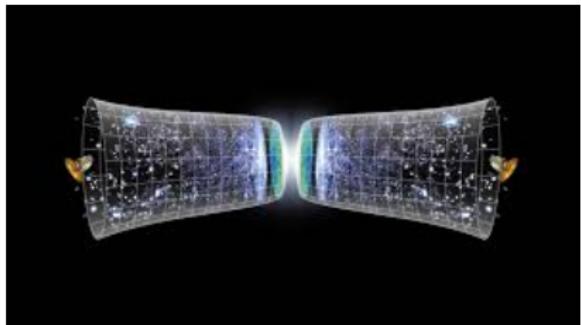


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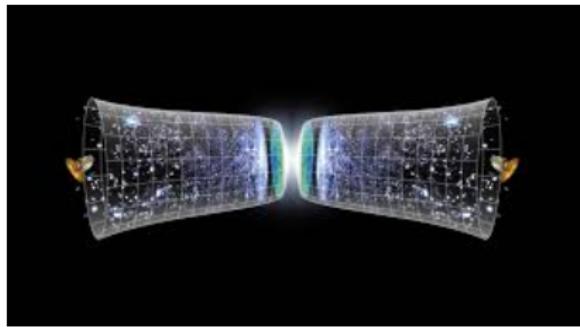
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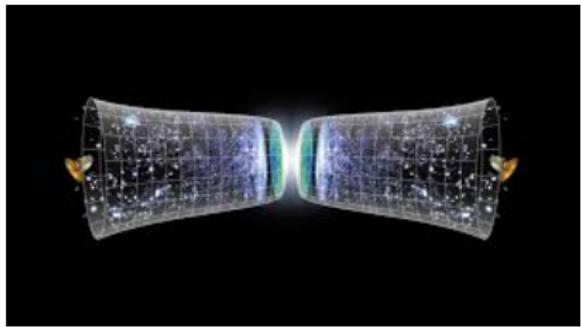
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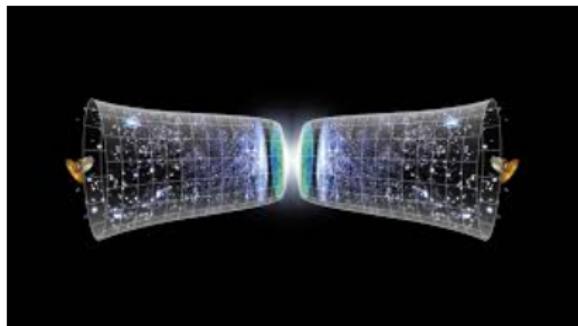
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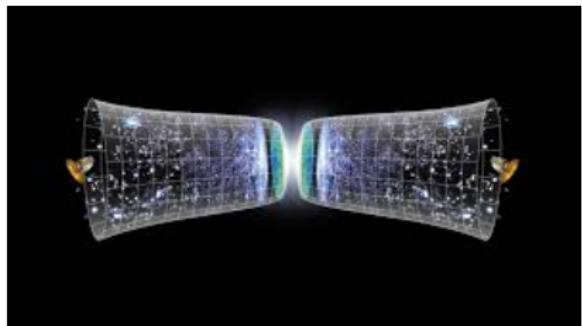
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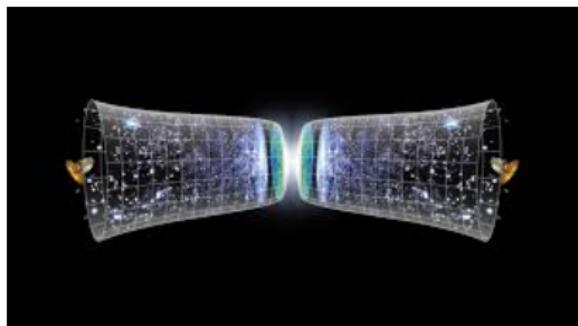
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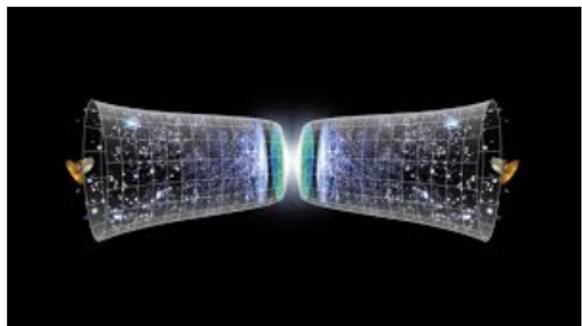
Healthy bounce in beyond Horndeski

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No-go for bounce in Horndeski

?



No-go breaks in beyond Horndeski



Healthy bounce in beyond Horndeski

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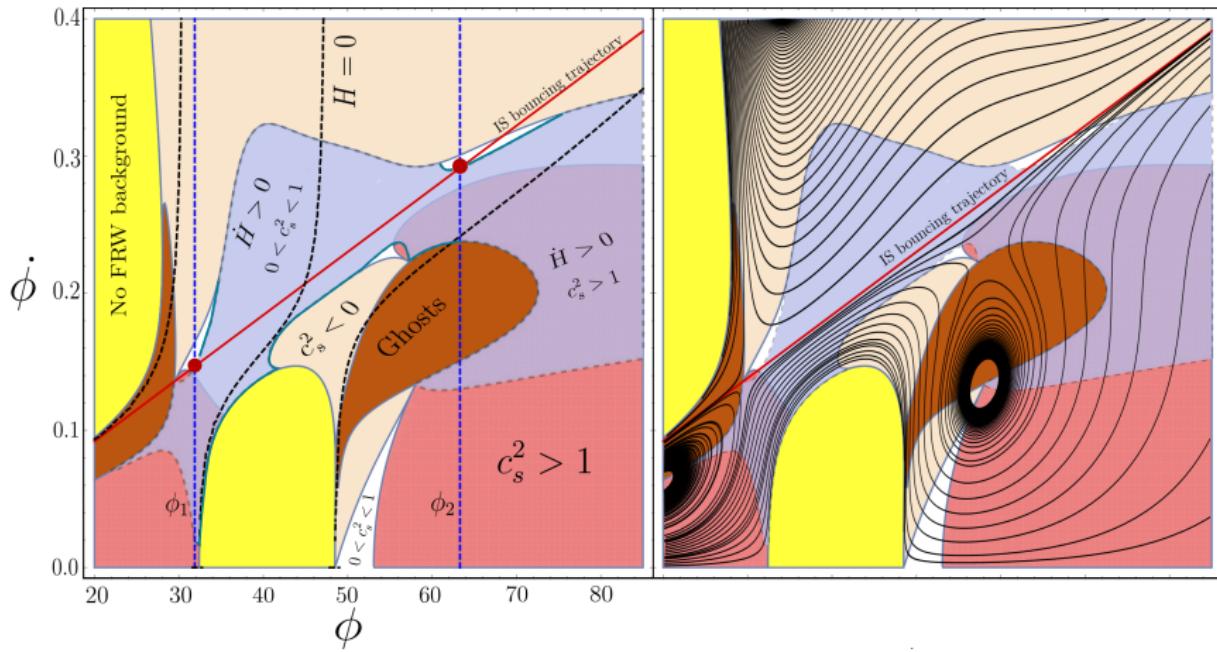
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These solutions require strong finetuning.



Finetuning for  $L^{(2)}$  probably means pathologies in  $L^{(3)}$ .

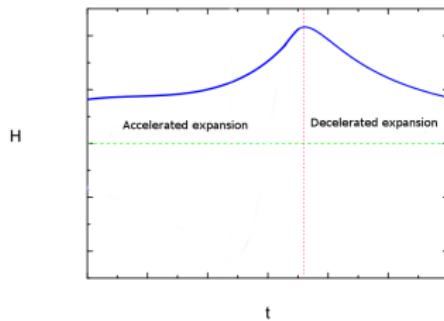
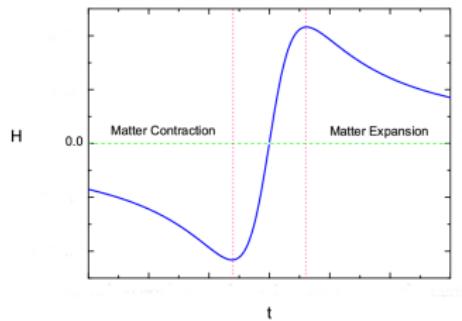


## Null Energy Condition

$$T_{\mu\nu} k^\mu k^\nu \geq 0$$

## Friedmann equations

$$\dot{H} = -4\pi G(p + \rho) + \frac{\kappa}{a^2}$$



Bounce and genesis require NEC-violation

## Penrose theorem

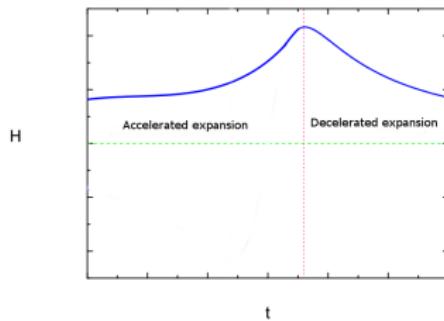
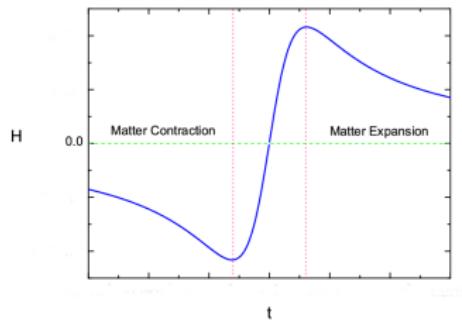
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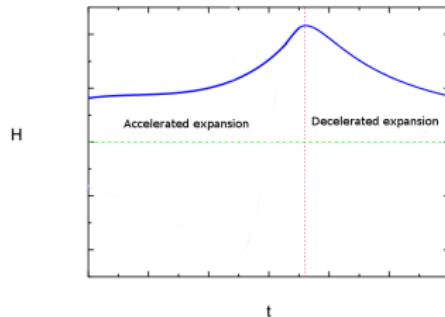
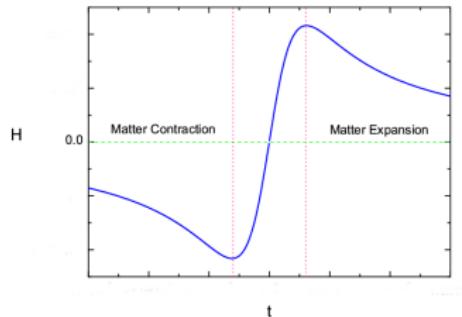
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Hence we need to consider Lagrangians with second derivatives:

- Deal with higher derivative equations
- Get 2 derivatives equations only

$$\mathcal{L} = F(\pi, X) + K(\pi, X)\square\pi$$

here  $X = \partial_\mu\pi\partial^\mu\pi$

$$\delta\mathcal{L} = F_\pi \delta\pi + F_X \delta X + K_\pi \square \pi \delta\pi + \underline{K_X \square \pi \delta X} + K \square \delta\pi =$$

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= ...only second derivatives

# Horndeski and Beyond Horndeski

$$S = \int d^4x \sqrt{-g} (\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_{\mathcal{BH}}),$$

$$\mathcal{L}_2 = F(\pi, X),$$

$$\mathcal{L}_3 = K(\pi, X) \square \pi,$$

$$\mathcal{L}_4 = -G_4(\pi, X)R + 2G_{4X}(\pi, X) \left[ (\square \pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu} \right],$$

$$\mathcal{L}_5 = G_5(\pi, X)G^{\mu\nu}\pi_{;\mu\nu} + \frac{1}{3}G_{5X} \left[ (\square \pi)^3 - 3\square \pi \pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu\nu}\pi^{;\mu\rho}\pi_{;\rho}^{\nu} \right],$$

$$\begin{aligned} \mathcal{L}_{\mathcal{BH}} = & F_4(\pi, X)\epsilon^{\mu\nu\rho}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'} + \\ & + F_5(\pi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'}\pi_{;\sigma\sigma'} \end{aligned}$$

where  $\pi$  is the Galileon field,  $X = g^{\mu\nu}\pi_{,\mu}\pi_{,\nu}$ ,  $\pi_{,\mu} = \partial_\mu\pi$ ,  $\pi_{;\mu\nu} = \nabla_\nu\nabla_\mu\pi$ ,  
 $\square\pi = g^{\mu\nu}\nabla_\nu\nabla_\mu\pi$ ,  $G_{4X} = \partial G_4/\partial X$

$$ds^2 = dt^2 - a(t)^2 d\vec{x}^2 \quad (2)$$

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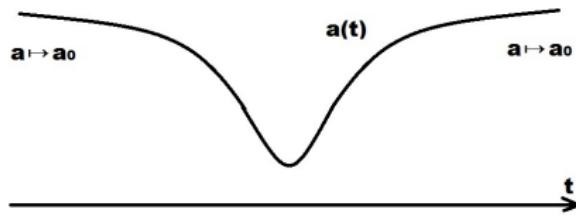
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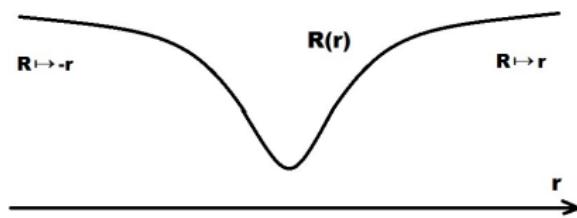
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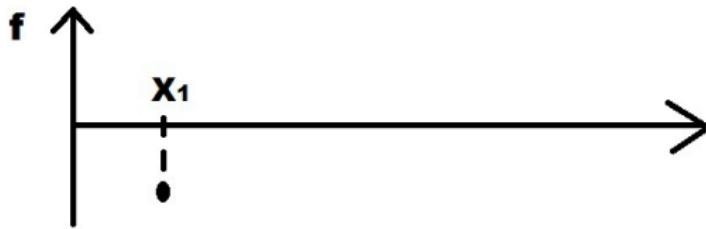
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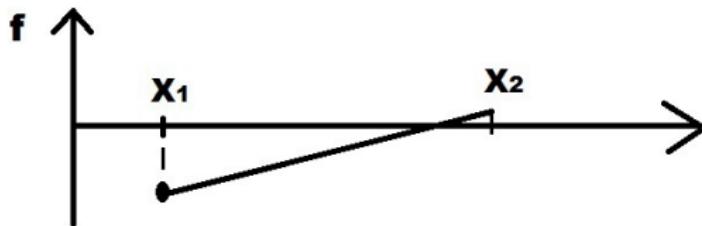
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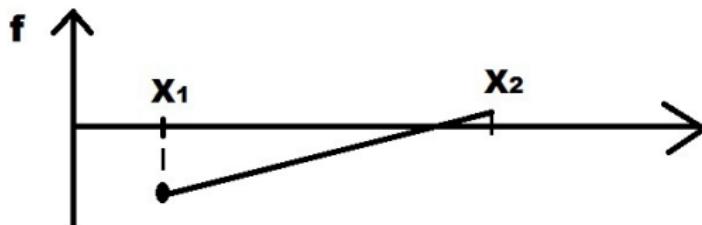
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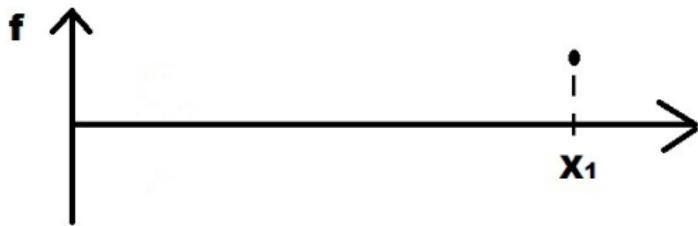
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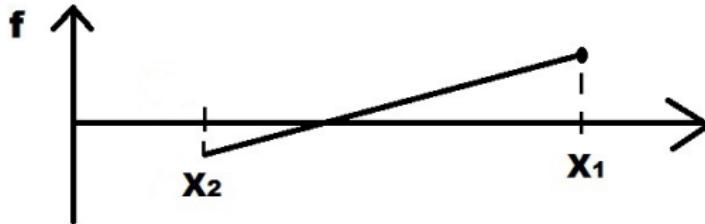
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If  $f(x_1) > 0$       Then  $f(x_2) \leq 0$  ( $x_2 = x_1 - \frac{f(x_1)}{\epsilon}$ )

$$S = \int dt d^3x a^3 \left[ \frac{\mathcal{G}_T}{8} \left( \dot{h}_{ik}^T \right)^2 - \frac{\mathcal{F}_T}{8a^2} \left( \partial_i h_{kl}^T \right)^2 + \mathcal{G}_S \dot{\zeta}^2 - \mathcal{F}_S \frac{(\nabla \zeta)^2}{a^2} \right]$$

The speeds of sound for tensor and scalar perturbations are, respectively,

$$c_T^2 = \frac{\mathcal{F}_T}{\mathcal{G}_T}, \quad c_S^2 = \frac{\mathcal{F}_S}{\mathcal{G}_S}$$

A healthy and stable solution requires correct signs for kinetic and gradient terms as well as subluminal propagation:

$$\mathcal{G}_T > \mathcal{F}_T > 0, \quad \mathcal{G}_S > \mathcal{F}_S > 0$$

These coefficients are combinations of Lagrangian functions and have non-trivial relations

$$\begin{aligned} \mathcal{G}_S &= \frac{\Sigma \mathcal{G}_T^2}{\Theta^2} + 3\mathcal{G}_T, & \mathcal{G}_S &= \frac{\Sigma \mathcal{G}_T^2}{\Theta^2} + 3\mathcal{G}_T, \\ \mathcal{F}_S &= \frac{1}{a} \frac{d\xi}{dt} - \mathcal{F}_T, & \Rightarrow & \mathcal{F}_S &= \frac{1}{a} \frac{d\xi}{dt} - \mathcal{F}_T, \\ \xi &= \frac{a \mathcal{G}_T^2}{\Theta}. & \xi &= \frac{a (\mathcal{G}_T - \mathcal{D}\dot{\pi}) \mathcal{G}_T}{\Theta}. \end{aligned} \tag{4}$$

## No-go theorem for bounce in Horndeski theory

M. Libanov, S. M and V. Rubakov, 1605.05992

R. Kolevatov and S. M., 1607.04099

T. Kobayashi, 1606.05831

S. Akama and T. Kobayashi, 1701.02926

## No-go theorem for bounce breaks in beyond Horndeski

Y. Cai, Y. Wan, H. Li, T. Qiu and Y. Piao, 1610.03400

P. Creminelli, D. Pirtskhalava, L. Santoni and E. Trincherini, 1610.04207

Y. Cai and Y. S. Piao, 1705.03401

R. Kolevatov, SM, N. Sukhov, V. Volkova, 1705.06626

## No-go theorem for Wormholes in Horndeski theory

V. Rubakov, 1601.06566

O. Evseev, O. Melichev, 1711.04152

## No-go theorem for Wormholes breaks in beyond Horndeski

arXiv:!? Vika, hurry!

$$\mathcal{L} = \frac{1}{2}\mathcal{K}_{ij}\dot{\nu}^i\dot{\nu}^j - \frac{1}{2}\mathcal{G}_{ij}\nu^{i'}\nu^{j'} - Q_{ij}\nu^i\nu^{j'} - \frac{1}{2}\mathcal{M}_{ij}\nu^i\nu^j, \quad (5)$$

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$$\mathcal{K}_{11} > 0, \quad \det(\mathcal{K}) > 0, \quad \mathcal{G}_{11} > 0, \quad \det(\mathcal{G}) > 0. \quad (6)$$

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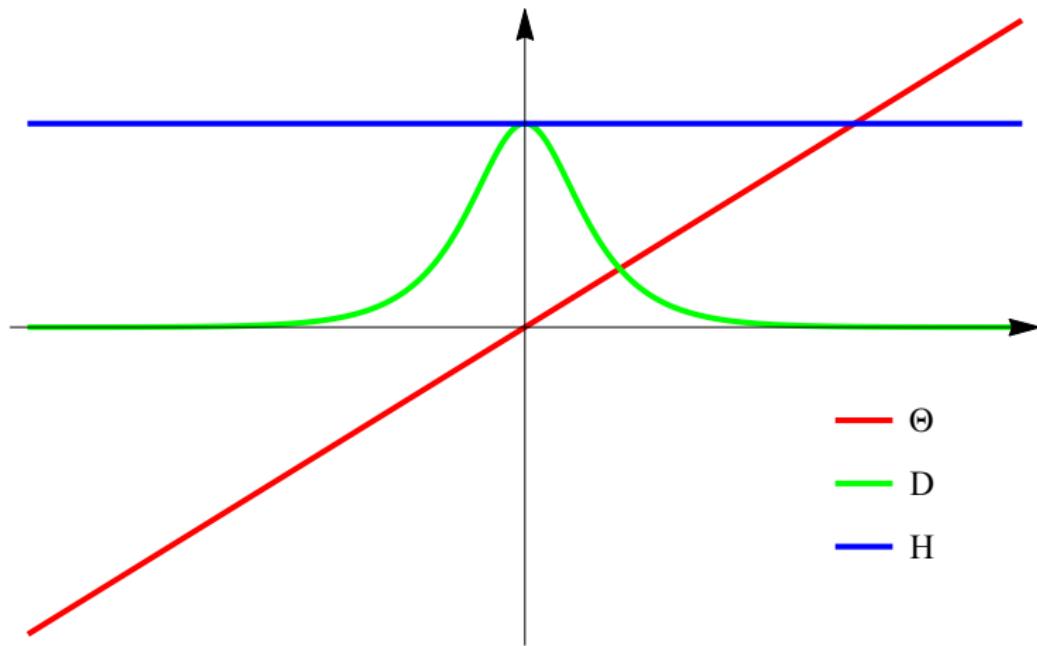
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$$\begin{aligned} \mathcal{P} &= \left[ \frac{(R\mathcal{H})^2}{\Theta} \right]' & \mathcal{P} &= \left[ \frac{R^2\mathcal{H}(\mathcal{H}-\mathcal{D})}{\Theta} \right]' \\ &\Rightarrow && \\ \det K \sim \mathcal{F}(2\mathcal{P} - \mathcal{F}) &> 0 & \det K \sim (\mathcal{F} - \mathcal{Q})(2\mathcal{P} - \mathcal{F}) - \mathcal{Q}^2 &> 0 \\ & & \mathcal{Q} &\sim \frac{\mathcal{D}'}{R'} \end{aligned} \quad (7)$$



—  $\Theta$   
—  $D$   
—  $H$

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but, at the same time infinitesimal deviation from this solution destroys  $L^{(2)}$

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## Conclusion

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## **Healthy Wormhole?**

**THANK YOU FOR YOUR ATTENTION!**

$$\mathcal{G}_T = 2G_4 - 4G_{4X}X + G_{5\pi}X - 2HG_{5X}X\dot{\pi},$$

$$\mathcal{F}_T = 2G_4 - 2G_{5X}X\ddot{\pi} - G_{5\pi}X,$$

$$\mathcal{D} = 2F_4X\dot{\pi} + 6HF_5X^2,$$

$$\hat{\mathcal{G}}_T = \mathcal{G}_T + \mathcal{D}\dot{\pi},$$

$$\begin{aligned} \Theta = & -K_X X\dot{\pi} + 2G_4H - 8HG_{4X}X - 8HG_{4XX}X^2 + G_{4\pi}\dot{\pi} + 2G_{4\pi X}X\dot{\pi} - \\ & - 5H^2G_{5X}X\dot{\pi} - 2H^2G_{5XX}X^2\dot{\pi} + 3HG_{5\pi}X + 2HG_{5\pi X}X^2 + \\ & + 10HF_4X^2 + 4HF_{4X}X^3 + 21H^2F_5X^2\dot{\pi} + 6H^2F_{5X}X^3\dot{\pi}, \end{aligned}$$

$$\begin{aligned} \Sigma = & F_X X + 2F_{XX}X^2 + 12HK_X X\dot{\pi} + 6HK_{XX}X^2\dot{\pi} - K_\pi X - K_{\pi X}X^2 - \\ & - 6H^2G_4 + 42H^2G_{4X}X + 96H^2G_{4XX}X^2 + 24H^2G_{4XXX}X^3 - \\ & - 6HG_{4\pi}\dot{\pi} - 30HG_{4\pi X}X\dot{\pi} - 12HG_{4\pi XX}X^2\dot{\pi} + 30H^3G_{5X}X\dot{\pi} + \\ & + 26H^3G_{5XX}X^2\dot{\pi} + 4H^3G_{5XXX}X^3\dot{\pi} - 18H^2G_{5\pi}X - 27H^2G_{5\pi X}X^2 - \\ & - 6H^2G_{5\pi XX}X^3 - 90H^2F_4X^2 - 78H^2F_{4X}X^3 - 12H^2F_{4XX}X^4 - \\ & - 168H^3F_5X^2\dot{\pi} - 102H^3F_{5X}X^3\dot{\pi} - 12H^3F_{5XX}X^4\dot{\pi}. \end{aligned}$$