Holographic anisotropic background with confinement-deconfinement phase transition

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based on I.Ya. Aref'eva and K. Rannu,
JHEP (2018), arXiv:1802.05652 [hep-th]

Motivation

Holography needs:

- Natural theory with metric as solution of EOM.
- Non-zero chemical potential $\mu \neq 0$.

I. Ya. Aref'eva **"Holography for HIC at LHC and NICA"**
arXiv:1612.08928

$$ds^2 = L^2 \, \frac{b(z)}{z^2} \left[- \, g(z) dt^2 + dx^2 + z^{2-\frac{2}{\nu}} \left(dy_1^2 + dy_2^2 \right) + \frac{dz^2}{g(z)} \right]$$

I.A., K.R. arXiv:1802.05652

AGG solution

$$ds^2 = \frac{L^2}{z^2}\left[- g(z)dt^2 + dx^2 + z^{2-\frac{2}{\nu}}\left(dy_1^2 + dy_2^2\right) + \frac{dz^2}{g(z)}\right]$$

$\mu = 0,\ b(z) = 1,\ g(z) = 1$ *Aref'eva, Golubtsova* JHEP **1504** 011

$\mu = 0,\ b(z) = 1,\ g(z) \neq 1$ *Aref'eva, Golubtsova, Gourgoulhon* JHEP **1609** 142

Since phenomenology requires $\mu \neq 0,\ b \neq 1,\ \nu \neq 1$ *Aref'eva 1612.08928*

OUR GOAL

$\Downarrow$

Generalizations:

$\downarrow$ $\downarrow$

$\mu \neq 0$ $b(z) \neq 1$

Action in Einstein frame, metric ansatz

$$S = \frac{1}{16\pi G_5} \int d^5x \, \sqrt{-g} \, \times$$
$$\times \left[R - \frac{f_1(\phi)}{4} \, F_{(1)}^2 - \frac{f_2(\phi)}{4} \, F_{(2)}^2 - \frac{1}{2} \, \partial_\mu \phi \partial^\mu \phi - V(\phi) \right]$$

$$A_\mu^{(1)} = A_t(z) \delta_\mu^0 \qquad F^{(2)} = q \; dy^1 \wedge dy^2 \qquad \phi = \phi(z)$$

$$ds^2 = L^2 \, \frac{b(z)}{z^2} \left[- \, g(z) dt^2 + dx^2 + z^{2-\frac{2}{\nu}} \left(dy_1^2 + dy_2^2 \right) + \frac{dz^2}{g(z)} \right]$$

Boundary conditions:

$$g(0) = 1, \;\; g(z_h) = 0 \qquad A_t(0) = \mu, \;\; A_t(z_h) = 0 \qquad \phi(z_h) = 0$$

$\nu = 1$: *M.-W. Li, Y. Yang arXiv:1703.09184*

Equations of motion

$$\phi'' + \phi' \left(\frac{g'}{g} + \frac{3b'}{2b} - \frac{\nu+2}{\nu z} \right) + \frac{z^2 A_t'^{\,2}}{2bg} \frac{\partial f_1}{\partial \phi} - \frac{q^2 z^{-2+\frac{4}{\nu}}}{2bg} \frac{\partial f_2}{\partial \phi} - \frac{b}{z^2 g} \frac{\partial V}{\partial \phi} = 0$$

$$A_t'' + A_t' \left(\frac{b'}{2b} + \frac{f_1'}{f_1} - \frac{2-\nu}{\nu z} \right) = 0$$

1-st: $$g'' + g' \left(\frac{3b'}{2b} - \frac{1}{z} - \frac{2}{\nu z} \right) - \frac{z^2}{b} f_1 A_t'^{\,2} = 0$$

2-nd: $$b'' - \frac{3b'^{\,2}}{2b} + \frac{2b'}{z} - \frac{4b}{3\nu z^2} \left(1 - \frac{1}{\nu} \right) + \frac{b}{3} \phi'^{\,2} = 0$$

3-rd: $$2g' \left(1 - \frac{1}{\nu} \right) + g \left(1 - \frac{1}{\nu} \right) \left(\frac{3b'}{b} - \frac{4}{z} - \frac{4}{\nu z} \right) + \frac{q^2 z^{-1+\frac{4}{\nu}}}{b} f_2 = 0$$

4-th: $$-V - \frac{z^4}{2b^2} A_t'^{\,2} f_1 - \frac{3z^2 b' g'}{2b^2} - \frac{3z^2 g b'^{\,2}}{b^3} + \frac{9zgb'}{2\nu b^2} + \frac{15zgb'}{2b^2} + \\ + \frac{zg'}{\nu b} + \frac{2zg'}{b} + \frac{z^2 g \phi'^{\,2}}{2b} - \frac{8g}{\nu b} - \frac{4g}{b} = 0$$

General anisotropic solution $b(z) = \exp P(z)$, $\nu \neq 1$

$$f_1 = z^{-2+\frac{2}{\nu}}$$

$$A_t = \tilde{\mu} \int_z^{z_h} e^{-\frac{P(\xi)}{2}} \xi d\xi, \quad \tilde{\mu} = \frac{\mu}{\int_0^{z_h} e^{-\frac{P(\xi)}{2}} \xi d\xi}$$

$$g = 1 + \tilde{\mu}^2 \int_0^z e^{-\frac{3P(\xi)}{2}} \left(\int_0^\xi e^{-\frac{P(\chi)}{2}} \chi d\chi \right) \xi^{1+\frac{2}{\nu}} d\xi - \\ - \frac{1 + \tilde{\mu}^2 \int_0^{z_h} e^{-\frac{3P(\xi)}{2}} \left(\int_0^\xi e^{-\frac{P(\chi)}{2}} \chi d\chi \right) \xi^{1+\frac{2}{\nu}} d\xi}{\int_0^{z_h} e^{-\frac{3P(\xi)}{2}} \xi^{1+\frac{2}{\nu}} d\xi} \int_0^z e^{-\frac{3P(\xi)}{2}} \xi^{1+\frac{2}{\nu}} d\xi$$

$$\phi(z) = C_5 + \int_0^z \sqrt{- 3P''(\xi) + \frac{3}{2} P'^2(\xi) - \frac{6}{\xi} P'(\xi) + 4 \frac{\nu - 1}{\xi^2 \nu^2}} d\xi$$

Solution for anisotropic metric ansatz $b(z) = \exp(cz^2/2)$, $\nu \neq 1$

$$f_1 = z^{-2+\frac{2}{\nu}} \qquad A_t(z) = \mu \, \frac{e^{-\frac{cz^2}{4}} - e^{-\frac{cz_h^2}{4}}}{1 - e^{-\frac{cz_h^2}{4}}}$$

$$g(z) = 1 - \frac{z^{2+\frac{2}{\nu}}}{z_h^{2+\frac{2}{\nu}}} \, \frac{\mathfrak{G}(\frac{3}{4}cz^2)}{\mathfrak{G}(\frac{3}{4}cz_h^2)} - \frac{\mu^2 c z^{2+\frac{2}{\nu}} e^{\frac{cz_h^2}{2}}}{4\left(1 - e^{\frac{cz_h^2}{4}}\right)^2} \, \mathfrak{G}(cz^2) \, +$$

$$+ \frac{\mu^2 c z^{2+\frac{2}{\nu}} e^{\frac{cz_h^2}{2}}}{4\left(1 - e^{\frac{cz_h^2}{4}}\right)^2} \, \frac{\mathfrak{G}(\frac{3}{4}cz^2)}{\mathfrak{G}(\frac{3}{4}cz_h^2)} \, \mathfrak{G}(cz_h^2),$$

$$\mathfrak{G}(x) = x^{-1-\frac{1}{\nu}} \, \gamma\left(1 + \frac{1}{\nu}, x\right) = \sum_{n=0}^{\infty} \frac{(-1)^n \, x^n}{n!(1 + n + \frac{1}{\nu})}$$

$b(z)$ similar to phenom. model of O. Andreev, V.I. Zakharov arXiv:0604204

Scalar field $\phi(z)$ for anisotropic metric $b(z) = \exp(cz^2/2)$, $\nu \neq 1$

$$\phi = \frac{1}{2\sqrt{2}\,\nu}\Bigg\{\sqrt{3c^2\nu^2z^4 - 18c\,\nu^2z^2 + 8\,(\nu-1)} \, -$$

$$- \sqrt{3c^2\nu^2z_h^4 - 18c\,\nu^2z_h^2 + 8\,(\nu-1)} + 2\sqrt{2\,(\nu-1)}\ln\left(\frac{z^2}{z_h^2}\right)$$

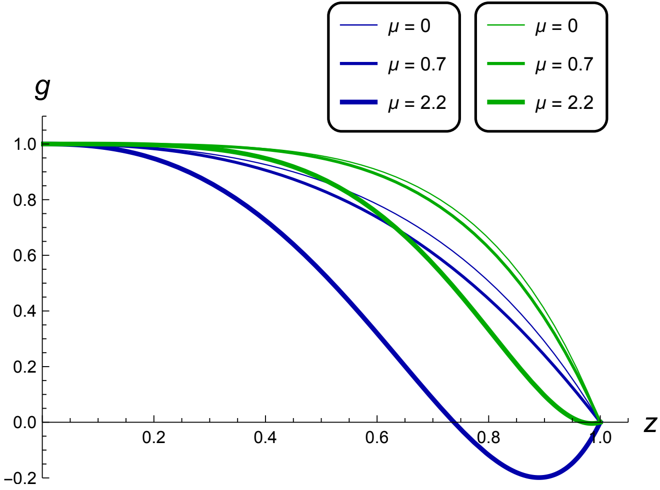
$$- 3\sqrt{3}\,\nu\ln\left(\frac{\sqrt{3c^2\nu^2z^4 - 18c\,\nu^2z^2 + 8\,(\nu-1)} - \sqrt{3}\,\nu\,(3-cz^2)}{\sqrt{3c^2\nu^2z_h^4 - 18c\,\nu^2z_h^2 + 8\,(\nu-1)} - \sqrt{3}\,\nu\,(3-cz_h^2)}\right) \, -$$

$$- 2\sqrt{2\,(\nu-1)} \, \times$$

$$\times \ln\left(\frac{9c\,\nu^2z^2 - 8\,(\nu-1) - \sqrt{2\,(\nu-1)}\,\sqrt{3c^2\nu^2z^4 - 18c\,\nu^2z^2 + 8\,(\nu-1)}}{9c\,\nu^2z_h^2 - 8\,(\nu-1) - \sqrt{2\,(\nu-1)}\,\sqrt{3c^2\nu^2z_h^4 - 18c\,\nu^2z_h^2 + 8\,(\nu-1)}}\right)\Bigg\}$$

Stable solution $\Rightarrow c < 0$

Blackening function $g(z)$ for various μ
$z_h = 1,\ c = -1,\ \nu = 1,\ 4.5$



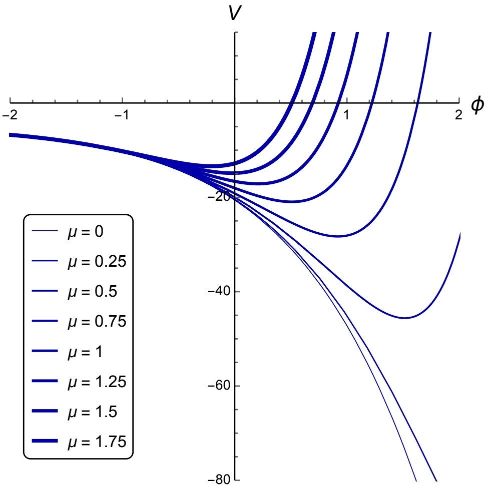
Scalar potential $V(\phi)$ for anisotropic metric $b(z) = \exp(cz^2/2)$, $\nu \neq 1$

$$V(\phi, \mu, \nu) \approx V_0(\nu) - C_7(\mu, \nu) e^{K_1(\nu)\phi} + C_8(\mu, \nu) e^{K_2(\nu)\phi}$$

$$V_0(4.5) = -\,0.5778$$
$$K_1(4.5) = 0.7897, \quad K_2(4.5) = 2.0995$$
$$C_7(\mu, 4.5) = 23.0779 + 2.4236\mu^2$$
$$C_8(\mu, 4.5) = 0.0575 + 4.9919\mu^2$$

Scalar potential $V(\phi)$ for various μ

$z_h = 1,\ c = -1,\ \nu = 4.5$



Anisotropic solution
$b(z) = 1,\ \nu \neq 1,\ A_t(z) = 0$

$$g(z) = 1 - \left(\frac{z}{z_h}\right)^{2+\frac{2}{\nu}}$$

$$f_2(z) = \frac{4z^{-4/\nu}}{q^2}\,\frac{(\nu-1)(1+3\nu+2\nu^2)}{\nu^2\ (1+2\nu)}$$

$$\phi(z) = C_5 \pm 2\,\frac{\sqrt{\nu-1}}{\nu}\ \log(z)$$

$$V(z) = -\ 2\,\frac{(1+\nu)(1+2\nu)}{\nu^2}$$

Corresponds to AGG solution

(I.Ya. Aref'eva, A.A. Golubtsova and E. Gourgoulhon arXiv:1601.06046)

RG flow

$$E = E_0 L B \text{ — energy scale}$$
$$\lambda = e^{\phi} \text{ — string coupling function}$$

$z \to w$:

$$ds^2 = -\, B(w)^2 g(w) dt^2 + B(w)^2 dx^2 + R(w)\left(dy_1^2 + dy_2^2\right) + \frac{dw^2}{g(w)}$$

$$E_o L = 1$$

$$\Downarrow$$

$$X(\phi) = \frac{B}{B'}\frac{\phi'}{3}, \quad Y(\phi) = \frac{1}{4}\frac{g'}{g}\frac{B}{B'}, \quad Z(\phi) = \frac{1}{4}\frac{R'}{R}\frac{B}{B'}$$

$$H_1(\phi) = \frac{A_t'}{B}, \quad H_2(\phi) = \frac{q}{R}$$

Generalization of: U. Gursoy, E. Kiritsis, L. Mazzanti and F. Nitti arXiv:0812.0792

RG flow equations

$$\mathcal{Z} = 1 - \frac{9}{4} X^2 + 2Y + 8Z + 8YZ + 4Z^2$$

$$\frac{dX}{d\phi} = -\frac{2}{9}\,\mathcal{Z}\left(1 + \frac{1}{X}\,\frac{2\partial_\phi V - H_1^2\,\partial_\phi f_1 + H_2^2\,\partial_\phi f_2 - 2XH_2^2 f_2}{2V + H_1^2 f_1 + H_2^2 f_2}\right)$$

$$\frac{dY}{d\phi} = -\frac{2Y}{9X}\,\mathcal{Z}\left(1 + \frac{1}{2Y}\,\frac{3H_1^2 f_1 - 4YH_2^2 f_2}{2V + H_1^2 f_1 + H_2^2 f_2}\right)$$

$$\frac{dZ}{d\phi} = \frac{1-2Z}{9X}\,\mathcal{Z}\left(1 + \frac{1+4Z}{1-2Z}\,\frac{H_2^2 f_2}{2V + H_1^2 f_1 + H_2^2 f_2}\right)$$

$$\frac{dH_1}{d\phi} = -\left(\frac{1+4Z}{3X} + \frac{\partial_\phi f_1}{f_1}\right) H_1$$

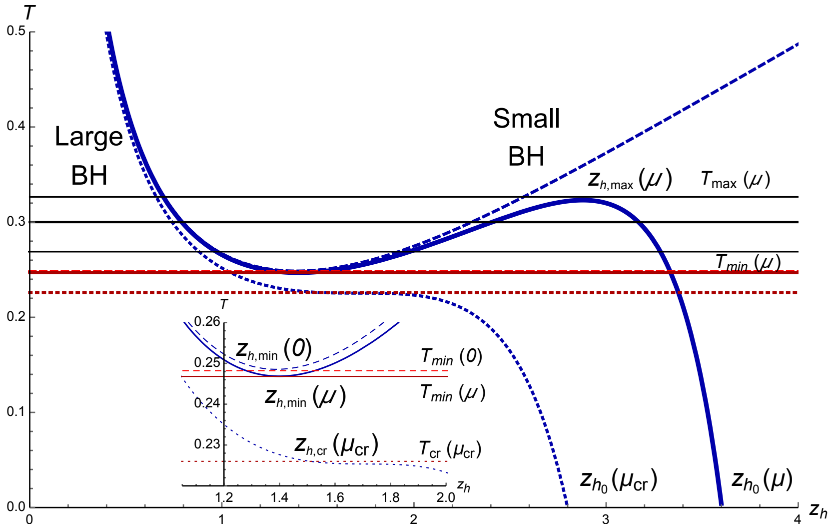
$$\frac{dH_2}{d\phi} = -\frac{4Z}{3X}\,H_2$$

Thermodynamics for anisotropic solution $b(z) = \exp(cz^2/2)$, $\nu \neq 1$

$$s = \frac{e^{\frac{3cz_h^2}{4}}}{4z_h^{1+\frac{2}{\nu}}}$$

$$T = \frac{e^{-\frac{3cz_h^2}{4}}}{2\pi z_h}\left|\frac{1}{\mathfrak{G}\left(\frac{3}{4}cz_h^2\right)} + \frac{\mu^2 c z_h^{2+\frac{2}{\nu}} e^{\frac{cz_h^2}{4}}}{4\left(1 - e^{\frac{cz_h^2}{4}}\right)^2}\left(1 - e^{\frac{cz_h^2}{4}}\frac{\mathfrak{G}\left(cz_h^2\right)}{\mathfrak{G}\left(\frac{3}{4}cz_h^2\right)}\right)\right|$$

Temperature $T(z_h, \mu)$ for $c = -1$, $\nu = 4.5$

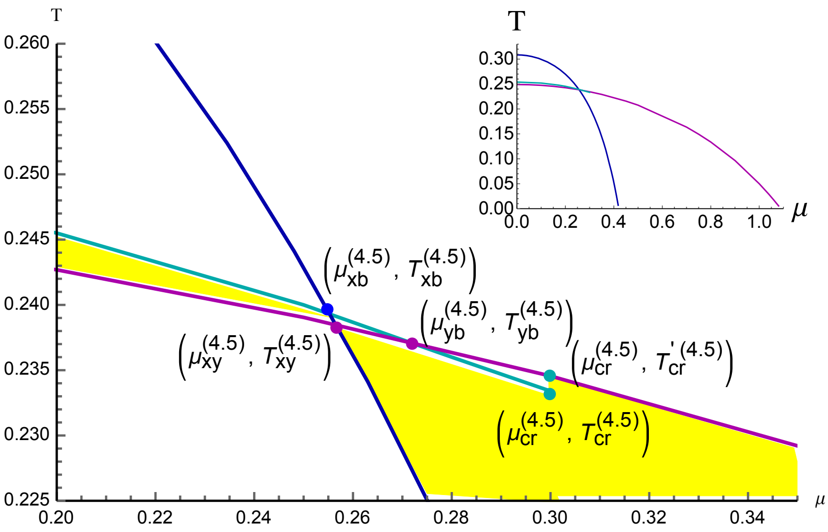


Confinement-deconfinement for $b(z) = \exp(cz^2/2)$, $\nu \neq 1$

$$\mathcal{DW}_x \equiv cz + \frac{1}{\nu z}\sqrt{\frac{2}{3}}\sqrt{3c\,\nu^2 z^2\left(\frac{cz^2}{2} - 3\right) + 4\nu - 4} + \frac{g'}{2g} - \frac{2}{z}\Big|_{z=z_{DWx}} = 0$$

$$\mathcal{DW}_y \equiv cz + \frac{1}{\nu z}\sqrt{\frac{2}{3}}\sqrt{3c\,\nu^2 z^2\left(\frac{cz^2}{2} - 3\right) + 4\nu - 4} + \frac{g'}{2g} - \frac{\nu+1}{\nu z}\Big|_{z=z_{DWy}} = 0$$

Confinement-deconfinement phase transition for $c = -1$, $\nu = 4.5$



Conclusions

- The AdS anisotropic black hole solution for 5-dimensional Maxwell-dilaton gravity is found.
- The AGG solution is generalized for the non-zero chemical potential case.
- Scalar field restricts the BH raduis and the warp-factor values.
- RG flow equations for anisotropic case are found.
- Confinement-deconfinement phase transition diagram is obtained [next talk]

To do:

- Investigate $P(z) \sim z^4$ to reconstruct Cornell potential.
- Shock-wave consideration
  (Aref'eva, Mamedov, KR, Arifulov) – in progress.
- Drag-force consideration (Aref'eva, KR, Slepov) – in progress.

Thank you
for your attention