Short- and long-range rapidity correlations in the model with a lattice in transverse plane

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Outline

- ♢ Short- and long-range rapidity correlations
- ♢ "Volume" fluctuations and the strongly intensive observables
- ♢ Strongly intensive observable $\Sigma(n_F, n_B)$
- ♢ $\Sigma(n_F, n_B)$ in the model with independent identical strings
- ♢ $\Sigma(n_F, n_B)$ for windows separated in azimuth and rapidity
- ♢ The model with string fusion on transverse lattice (grid)
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- ♢ $\Sigma(n_F, n_B)$ with charges
- ♢ Connection with Balance Function (BF)
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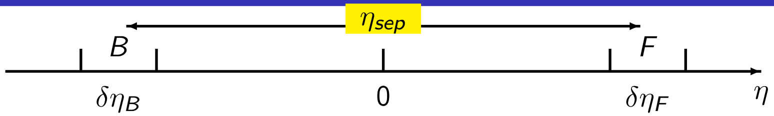
Short- and long-range rapidity correlations



Forward-Backward Rapidity Correlations: $(k_z, \mathbf{k}_\perp) \Rightarrow (\eta, \mathbf{k}_\perp)$

$$\eta \equiv \frac{1}{2} \ln \frac{k_0 + k_z}{k_0 - k_z} \,, \qquad \eta' \equiv \frac{1}{2} \ln \frac{|\mathbf{k}| + k_z}{|\mathbf{k}| - k_z} = -\ln \mathrm{tg} \left(\frac{\theta^*}{2} \right)$$

The correlation coefficient:

$$b_{BF} = \frac{\langle FB \rangle - \langle F \rangle \langle B \rangle}{\langle F^2 \rangle - \langle F \rangle^2} = \frac{cov(F, B)}{D_F} \,,$$

A. Capella and A. Krzywicki, Phys.Rev.D**18**, 4120 (1978)

The locality of strong interaction in rapidity $\Rightarrow$
Short-Range FB Correlations (SRC) (between particles from a same string)

Event-by-event variance in the number of cut pomerons (strings) $\Rightarrow$
Long-Range FB Correlations (LRC) at large η_{sep}

Traditional Observables

Traditional FB correlation:
B, F $\Rightarrow$ n_B, n_F - the extensive variables $\Rightarrow$ b_{nn}

$$b_{nn} = \frac{\langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle}{\langle {n_F}^2 \rangle - \langle n_F \rangle^2} = \frac{cov(n_F, n_B)}{D_{n_F}}$$

Strongly influenced by "volume" fluctuations.

The b_{nn} is connected with two-particle correlation function C_2, canonically defined as

$$C_2(\eta_1, \eta_2) \equiv \frac{\rho_2(\eta_1, \eta_2)}{\rho(\eta_1)\rho(\eta_2)} - 1 \ ,$$

where

$$\rho(\eta) \equiv \frac{dN}{d\eta} \ , \qquad \rho_2(\eta_1, \eta_2) \equiv \frac{d^2 N}{d\eta_1 \, d\eta_2} \ .$$

are the single and double inclusive particle distributions.

Connection between FBC and C_2

For small $\delta\eta_F$-$\delta\eta_B$ observation windows we have:

$$C_2(\eta_F,\eta_B) = \frac{\langle n_F n_B\rangle - \langle n_F\rangle\langle n_B\rangle}{\langle n_F\rangle\langle n_B\rangle} = \frac{cov(n_F,n_B)}{\langle n_F\rangle\langle n_B\rangle} = \frac{D_{n_F}}{\langle n_F\rangle\langle n_B\rangle} b_{nn} \approx \frac{b_{nn}}{\langle n_B\rangle} \,.$$

We have used that for small windows: $D_{n_F} \approx \langle n_F\rangle$.

Also influenced by "volume" fluctuations.

To suppress the influence of trivial "volume" fluctuations
we have to go from traditional extensive variables n_F and n_B
to new intensive variables, e.g. event-mean transverse momenta p_F and p_B
of all particles (n_F and n_B) in the intervals $\delta\eta_F$ and $\delta\eta_B$
(see e.g. [V.V., EPJ Web of Conf. 125, 04022 (2016)])

OR to study more sophisticated correlation observables,
e.g. the strongly intensive observable $\Sigma(n_F, n_B)$.

Strongly intensive observable $\Sigma(n_F, n_B)$

We define the strongly intensive observable $\Sigma(n_F, n_B)$ between multiplicities in forward (n_F) and backward (n_B) windows in accordance with [*M.I.Gorenstein, M.Gazdzicki, Phys.Rev.C84(2011)014904*] as

$$\Sigma(n_F, n_B) \equiv \frac{1}{\langle n_F\rangle + \langle n_B\rangle}[\langle n_F\rangle\, \omega_{n_B} + \langle n_B\rangle\, \omega_{n_F} - 2\,\mathrm{cov}(n_F\, n_B)]\ , \qquad (1)$$

where

$$\mathrm{cov}(n_F, n_B) \equiv \langle n_F n_B\rangle - \langle n_F\rangle\langle n_B\rangle\ , \qquad (2)$$

and ω_{n_F} and ω_{n_B} are the corresponding scaled variances of the multiplicities:

$$\omega_n \equiv \frac{D_n}{\langle n\rangle} = \frac{\langle n^2\rangle - \langle n\rangle^2}{\langle n\rangle}\ . \qquad (3)$$

$\Sigma(n_F, n_B)$ for symmetric reaction and symmetric windows

For symmetric reaction and symmetric observation windows $\delta\eta_F=\delta\eta_B=\delta\eta$:

$$\langle n_F \rangle = \langle n_B \rangle \equiv \langle n \rangle \ , \qquad \omega_{n_F} = \omega_{n_B} \equiv \omega_n \tag{4}$$

and

$$\Sigma(n_F, n_B) = \omega_n - \mathrm{cov}(n_F, n_B)/\langle n \rangle = \frac{\langle n^2 \rangle - \langle n_F n_B \rangle}{\langle n \rangle} =$$
$$= \frac{D_n - \mathrm{cov}(n_F, n_B)}{\langle n \rangle} = \frac{\mathrm{cov}(n_F, n_F) - \mathrm{cov}(n_F, n_B)}{\langle n \rangle} \ . \tag{5}$$

Connection with FBC coefficient b_{nn}:

$$\Sigma(n_F, n_B) = \omega_n \, (1 - b_{nn}) \tag{6}$$

$\Sigma(n_F, n_B)$ through two-particle correlation function C_2

$$\omega_n = D_n/\langle n\rangle = 1 + \langle n\rangle\, I_{FF}\ , \qquad \mathrm{cov}(n_F, n_B)/\langle n\rangle = \langle n\rangle\, I_{FB}\ , \tag{7}$$

$$\Sigma(n_F, n_B) = \omega_n - \mathrm{cov}(n_F, n_B)/\langle n\rangle = 1 + \langle n\rangle\,[I_{FF} - I_{FB}]\ , \tag{8}$$

where

$$I_{FF} = \frac{1}{\delta\eta_F^2}\int_{\delta\eta_F} d\eta_1 \int_{\delta\eta_F} d\eta_2\ C_2(\eta_1 - \eta_2) \quad \to C_2(0)$$

$$I_{FB} = \frac{1}{\delta\eta_F\,\delta\eta_B}\int_{\delta\eta_F} d\eta_1 \int_{\delta\eta_B} d\eta_2\ C_2(\eta_1 - \eta_2) \quad \to C_2(\eta_{sep})$$

The last limit is valid for the small windows: $\delta\eta_F = \delta\eta_B = \delta\eta \ll \eta_{corr}$, then

$$\boxed{\Sigma(n_F, n_B) = 1 + \langle n\rangle[C_2(0) - C_2(\eta_{sep})]}$$

$$\omega_n = 1 + \langle n\rangle\, C_2(0)\ , \tag{9}$$

For FBC coefficient b_{nn} we had in [V.V.,Nucl.Phys.A939(2015)21]:

$$b_{nn} = \frac{\langle n\rangle \mathrm{cov}(n_F, n_B)}{\omega_n} = \frac{\langle n\rangle\, I_{FB}}{1 + \langle n\rangle\, I_{FF}} \to \frac{\langle n\rangle\, C_2(\eta_{sep})}{1 + \langle n\rangle\, C_2(0)} \approx \langle n\rangle\, C_2(\eta_{sep})$$

The model with independent identical strings

[*M.A. Braun, C. Pajares, V.V.V., Phys. Lett. B* ***493****, 54 (2000)*]

1) The number of strings, N, fluctuates event by event around some mean value, $\langle N\rangle$, with some scaled variance, $\omega_N = D_N/\langle N\rangle$.

Intensive observable does not depends on $\langle N\rangle$.
Strongly intensive observable does not depends on $\langle N\rangle$ and ω_N.

2) The fragmentation of each string contributes event-by-event to the forward and backward observation rapidity windows, $\delta\eta_F$, and $\delta\eta_B$, the μ_F and μ_B charged particles correspondingly, which fluctuate around some mean values, $\langle\mu_F\rangle$ and $\langle\mu_B\rangle$, with some scaled variances, $\omega_{\mu_F} = D_{\mu_F}/\langle\mu_F\rangle$ and $\omega_{\mu_B} = D_{\mu_B}/\langle\mu_B\rangle$.

The observation rapidity windows are separated by some rapidity interval: $\eta_{sep} = \Delta\eta$ - the distance between the centers of the $\delta\eta_F$ and $\delta\eta_B$.

Clear that in this model (and the same for n_B):

$$\langle n_F\rangle = \langle\mu_F\rangle\langle N\rangle = \langle N\rangle\mu_0, \qquad \omega_{n_F} = \omega_{\mu_F} + \langle\mu_F\rangle\omega_N ,$$

Two-particle correlation function of a string

Along with the observed standard two-particle correlation function:

$$C_2(\eta_1,\eta_2) \equiv \frac{\rho_2(\eta_1,\eta_2)}{\rho(\eta_1)\rho(\eta_2)} - 1 \ , \tag{10}$$

where

$$\rho(\eta) = \frac{dN_{ch}}{d\eta} \ , \qquad \rho_2(\eta_1,\eta_2) = \frac{d^2 N_{ch}}{d\eta_1\, d\eta_2} \tag{11}$$

one can introduce the string two-particle correlation function, $\Lambda(\eta_1,\eta_2)$, characterizing correlation between particles, produced from the one string:

$$\Lambda(\eta_1,\eta_2) \equiv \frac{\lambda_2(\eta_1,\eta_2)}{\lambda(\eta_1)\lambda(\eta_2)} - 1 \ . \tag{12}$$

The $\Lambda(\eta_1,\eta_2)$ haracterizes the string decay properties

($z-\eta$ correspondence)

*[X.Artru,Phys.Rept.**97**(1983)147, V.V.,arXiv:0812.0604]*

Connection between the two-particle correlation functions

In this model we have the following connection:

$$C_2(\eta_1, \eta_2) = \frac{\omega_N + \Lambda(\eta_1, \eta_2)}{\langle N \rangle}$$

[*V.V., Nucl.Phys.A939(2015)21*]. (Note that one often looses the constant part $\omega_N / \langle N \rangle$ of C_2, using di-hadron correlation approach.)

At midrapidities, implying uniform rapidity distribution:

$$\lambda(\eta) = \mu_0 = \frac{\langle \mu_F \rangle}{\delta y_F} = \frac{\langle \mu_B \rangle}{\delta y_B}, \qquad \rho(\eta) = \frac{dN_{ch}}{d\eta} = \rho_0 = \frac{\langle n_F \rangle}{\delta y_F} = \frac{\langle n_B \rangle}{\delta y_B} = \langle N \rangle \mu_0$$

and the correlation functions depends only on a difference of rapidities:

$$\eta_{sep} = \eta_1 - \eta_2 = \Delta\eta$$

We suppose that the string correlation function

$$\Lambda(\Delta\eta) \to 0, \text{ when } \Delta\eta \gg \eta_{corr} \; ,$$

where the η_{corr} is the correlation length.

$\Sigma(n_F, n_B)$ for small observation windows

For small observation windows, of a width $\delta\eta \ll \eta_{corr}$, we find [V.V.,Nucl.Phys.A939(2015)21]:

$$\omega_n = D_n/\langle n\rangle = 1 + \mu_0\delta\eta\,[\Lambda(0) + \omega_N]\ , \qquad (13)$$

$$\mathrm{cov}(n_F, n_B)/\langle n\rangle = \mu_0\delta\eta\,[\Lambda(\Delta\eta) + \omega_N]\ , \qquad (14)$$

$$\Sigma(n_F, n_B) = \omega_n - \mathrm{cov}(n_F, n_B)/\langle n\rangle = 1 + \mu_0\delta\eta\,[\Lambda(0) - \Lambda(\Delta\eta)]\ , \qquad (15)$$

where $\Delta\eta = \eta_F - \eta_B = \eta_{sep}$ is a distance between the centers of the forward an backward observation windows. For a single string we have

$$\omega_\mu = D_\mu/\langle\mu\rangle = 1 + \mu_0\delta\eta\,\Lambda(0)\ , \qquad (16)$$

$$\mathrm{cov}(\mu_F, \mu_B)/\langle\mu\rangle = \mu_0\delta\eta\,\Lambda(\Delta\eta)\ , \qquad (17)$$

$$\Sigma(\mu_F, \mu_B) = \omega_\mu - \mathrm{cov}(\mu_F, \mu_B)/\langle\mu\rangle = 1 + \mu_0\delta\eta\,[\Lambda(0) - \Lambda(\Delta\eta)]\ , \qquad (18)$$

So in $\Sigma(n_F, n_B)$ we have the cancelation of the contributions from the fluctuation of the number of strings, ω_N, and it becames strongly intensive:

$$\Sigma(n_F, n_B) = \Sigma(\mu_F, \mu_B)$$

Strongly intensive observable $\Sigma(\mu_F, \mu_B)$

In general case the strongly intensive variable for a single string is defined similarly to $\Sigma(n_F, n_B)$ by

$$\Sigma(\mu_F, \mu_B) \equiv \frac{1}{\langle \mu_F \rangle + \langle \mu_B \rangle} [\langle \mu_F \rangle \, \omega_{\mu_B} + \langle \mu_B \rangle \, \omega_{\mu_F} - 2 \operatorname{cov}(\mu_F, \mu_B)] \; . \quad (19)$$

It depends only on properties of a single string.
So in the model with independent identical strings for symmetric reaction and small symmetric observation windows we found for $\Sigma(n_F, n_B)$:

$$\boxed{\Sigma(\eta_{sep}) = 1 + \mu_0 \delta\eta [\Lambda(0) - \Lambda(\eta_{sep})]}$$

We see that really the $\Sigma(\eta_{sep})$ is strongly intensive quantity.
It does not depend on $\langle N \rangle$ and ω_N.

Properties of Σ in model with independent identical strings

$$\Sigma(\eta_{sep}) = 1 + \mu_0 \delta\eta [\Lambda(0) - \Lambda(\eta_{sep})]$$

The $\Sigma(0) = 1$ and increases with the gap between windows, η_{sep},
because the $\Lambda(\eta_{sep})$ decrease with η_{sep},
as the correlations in string go off with increase of η_{sep}.

The rate of the $\Sigma(\eta_{sep})$ growth with η_{sep} is proportional
to the width of the observation window $\delta\eta$ and μ_0 -
- the multiplicity produced from one string.

The model predicts saturation of the $\Sigma(\eta_{sep})$ on the level

$$\Sigma(\eta_{sep}) = 1 + \mu_0 \delta\eta \, \Lambda(0) = \omega_\mu$$

at large η_{sep}, as $\Lambda(\eta_{sep}) \to 0$ at the $\eta_{sep} \gg \eta_{corr}$,
where the η_{corr} is a string correlation length.

The pair correlation function of a single string

The parametrization for the pair correlation function $\Lambda(\eta,\phi)$ of a single string (reflecting the Schwinger mechanism of a string decay, was suggested in [V.V.,Nucl.Phys.A939(2015)21]:

$$\Lambda(\eta,\phi) = \Lambda_1 e^{-\frac{|\eta|}{\eta_1}} e^{-\frac{\varphi^2}{\varphi_1^2}} + \Lambda_2 \left(e^{-\frac{|\eta-\eta_0|}{\eta_2}} + e^{-\frac{|\eta+\eta_0|}{\eta_2}} \right) e^{-\frac{(|\varphi|-\pi)^2}{\varphi_2^2}} . \qquad (20)$$

This formula has the nearside peak, characterizing by parameters Λ_1, η_1 and φ_1, and the awayside ridge-like structure, characterizing by parameters Λ_2, η_2, η_0 and φ_2 (two wide overlapping hills shifted by $\pm\eta_0$ in rapidity, η_0 - the mean length of a string decay segment). We imply that in formula (20)

$$|\varphi| \leq \pi . \qquad (21)$$

If $|\varphi| > \pi$, then we use the replacement $\varphi \to \varphi + 2\pi k$, so that (21) was fulfilled. With such completions the $\Lambda(\eta,\phi)$ meets the following properties

$$\Lambda(-\eta,\phi) = \Lambda(\eta,\phi) , \quad \Lambda(\eta;-\phi) = \Lambda(\eta,\phi) , \quad \Lambda(\eta,\phi+2\pi k) = \Lambda(\eta,\phi) \qquad (22)$$

Fitting the model parameters by FBC in small windows

$\Lambda(\eta_{sep}, \phi_{sep})$ was fitted by the ALICE b_{nn} pp data with FB windows of small acceptance, $\delta\eta = 0.2, \delta\phi = \pi/4$, separated in azimuth and rapidity *[ALICE collab.,JHEP 05(2015)097]*. It gives for the parameters:

$\sqrt{s}$, TeV		0.9	2.76	7.0
LRC	$\mu_0\omega_N$	0.7	1.4	2.1
SRC	$\mu_0\Lambda_1$	1.5	1.9	2.3
	η_1	0.75	0.75	0.75
	ϕ_1	1.2	1.15	1.1
	$\mu_0\Lambda_2$	0.4	0.4	0.4
	η_2	2.0	2.0	2.0
	ϕ_2	1.7	1.7	1.7
	η_0	0.9	0.9	0.9

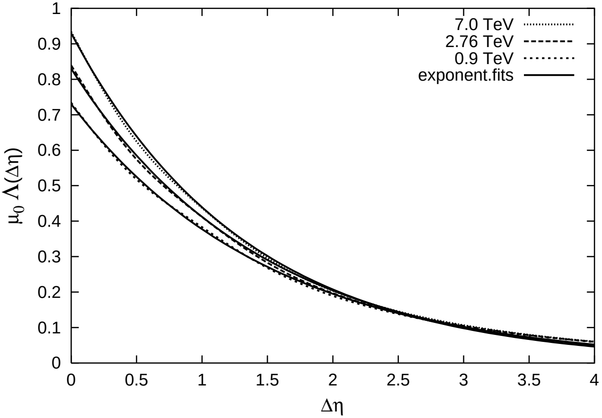
$\omega_N = \frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle}$ is the e-by-e scaled variance of the number of strings,
μ_0 is the average rapidity density of the charged particles from one string,
$i = 1$ corresponds to the nearside and $i = 2$ to the awayside contributions,
η_0 is the mean length of a string decay segment.

[V.V.,Nucl.Phys.A939(2015)21]

The stringtg correlation function $\Lambda(\Delta\eta)$

Then we find $\Lambda(\Delta\eta)$ integrating over azimuth:

$$\Lambda(\eta_{sep}) = \frac{1}{\pi}\int_0^{\pi} \Lambda(\eta_{sep}, \phi_{sep})\, d\phi_{sep} \ .$$



The stringt correlation function $\Lambda(\Delta\eta)$

The obtained dependencies in this fugure for three initial energies are well approximated by the exponent:

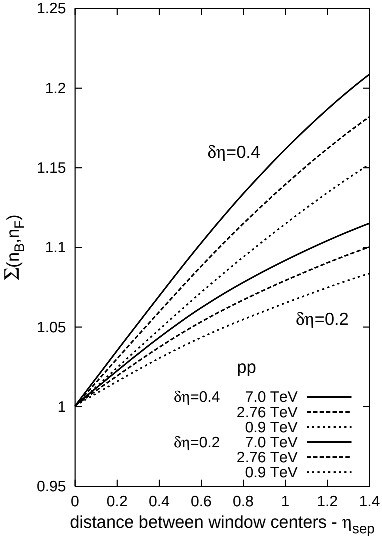
$$\Lambda(\Delta\eta) = \Lambda_0 e^{-\frac{|\Delta\eta|}{\eta_{corr}}} , \tag{23}$$

with the parameters presented in the table:

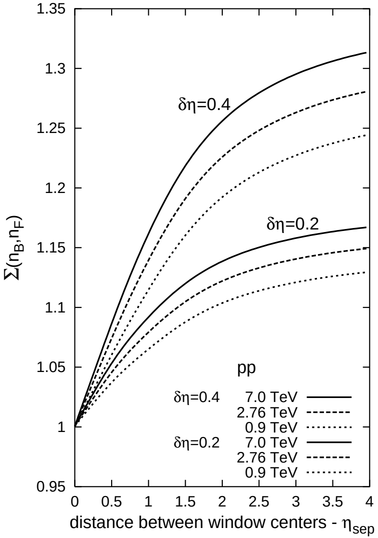
$\sqrt{s}$, TeV	0.9	2.76	7.0
$\mu_0\Lambda_0$	0.73	0.83	0.93
η_{corr}	1.52	1.43	1.33

We see that the correlation length, η_{corr}, decreases with the increase of collision energy.
This can be interpreted as a signal of an increase with energy of the admixture of strings of a new type - the fused strings in pp collisions (see below).

Σ for 2π azimuth windows



in ALICE TPC acceptance



in wider pseudorapidity range

$\Sigma(n_F, n_B)$ in windows separated in azimuth and rapidity

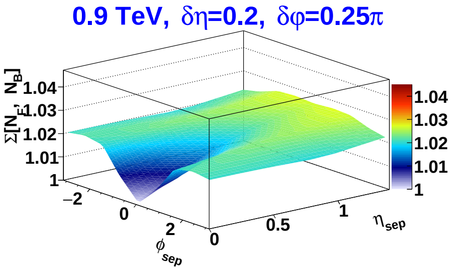
For small windows:

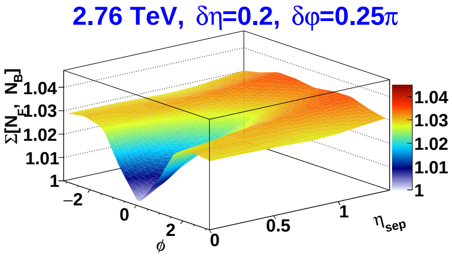
$$\boxed{\Sigma(\eta_{sep}, \phi_{sep}) = 1 + \frac{\delta\eta\,\delta\phi}{2\pi}\mu_0\,\left[\Lambda(0,0) - \Lambda(\eta_{sep}, \phi_{sep})\right]}$$

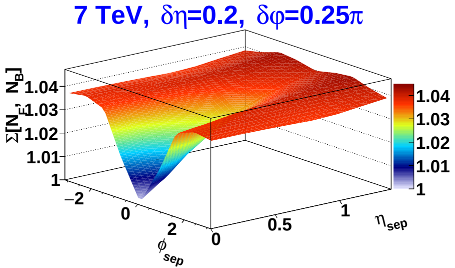
$$\Sigma(n_F, n_B) = \frac{D_n - \mathrm{cov}(n_F, n_B)}{\langle n\rangle} = \frac{\mathrm{cov}(n_F, n_F) - \mathrm{cov}(n_F, n_B)}{\langle n\rangle}\;.$$

This explains the general nature of the compensation for neighbor windows.

Σ for $\delta\eta\ \delta\phi$ windows separated in azimuth and rapidity - 1

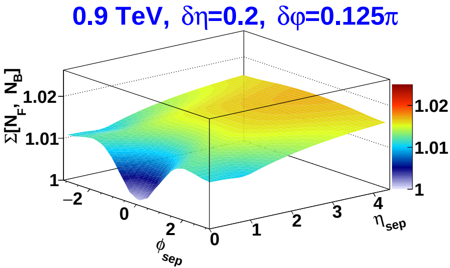


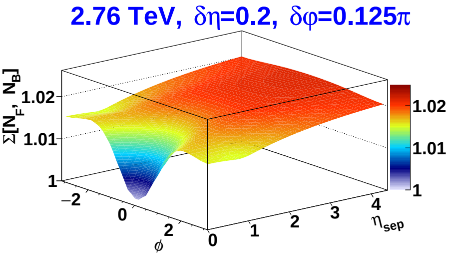


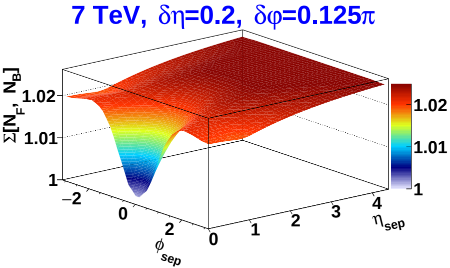


in ALICE TPC acceptance (ALICE experimental pp data analysis is in progress)

Σ for $\delta\eta\ \delta\phi$ windows separated in azimuth and rapidity - 2







with $\pi/8$ azimuth windows and for wider pseudorapidity range

String fusion effects

pp $\rightarrow$ pA $\rightarrow$ AA - the increase of the string density in transverse plain leads to the string fusion

M.A. Braun,C. Pajares, Phys.Lett. **B287**, 154 (1992);
Nucl. Phys. **B390**, 542 (1993).

$\Rightarrow$ Reduction of multiplicity, increase of transverse momenta.

N.S. Amelin, N. Armesto, M.A. Braun, E.G. Ferreiro, C. Pajares, Phys.Rev.Lett. **73**, 2813 (1994).

$\Rightarrow$ The influence on the Long-Range FB Correlations (LRC).

Various versions of string fusion

local fusion (overlaps)

M.A. Braun, C. Pajares Eur.Phys.J. **C16**, 349, (2000)

$$\langle n \rangle_k = \mu_0 \sqrt{k}\ S_k/\sigma_0\ , \qquad \langle p_t^2 \rangle_k = p_0^2 \sqrt{k}\ , \qquad k = 1, 2, 3, ... \qquad (24)$$

global fusion (clusters)

M.A. Braun, F. del Moral, C. Pajares, Phys.Rev. **C65**, 024907, (2002)

$$\langle p_t^2 \rangle_{cl} = p_0^2 \sqrt{k_{cl}}\ , \qquad \langle n \rangle_{cl} = \mu_0 \sqrt{k_{cl}} S_{cl}/\sigma_0\ , \qquad k_{cl} = k\sigma_0/S_{cl} \quad (25)$$

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the version of SFM with the finite lattice (grid) in transverse plane

V.V., Kolevatov R.S., hep-ph/0304295; hep-ph/0305136

Braun M.A., Kolevatov R.S., Pajares C., V.V., Eur.Phys.J. C32 (2004) 535

Domains in transverse area

The approach with string fusion on a transverse lattice (grid) was exploited later for a description of various phenomena (correlations, anisotropic azimuthal flows, the ridge) in high energy hadronic collisions in

ALICE collaboration et al., J. Phys. G **32** 1295 (2006), [Sect. 6.5.15]
V.V., Kolevatov R.S. Phys.of Atom.Nucl. **70** (2007) 1797; 1858
M.A. Braun, C. Pajares, Eur. Phys. J. C **71**, 1558 (2011)
M.A. Braun, C. Pajares, V.V., Nucl. Phys. A **906**, 14 (2013)
V.N. Kovalenko, Phys. Atom. Nucl. **76**, 1189 (2013)
M.A. Braun, C. Pajares, V.V., Eur. Phys. J. A **51**, 44 (2015)
V.V. V., Theor. Math. Phys. 184 (2015) 1271
V.V. V., Theor. Math. Phys. 190 (2017) 251

It leads to the splitting of the transverse area into domains with different, fluctuating values of color field within them.

What is similar to the models with the color field density variation in transverse plane based on the BFKL evolution

E. Levin, A.H. Rezaeian, Phys.Rev. D **84**, 034031 (2011)

or on the CGC approach *A.Kovner., M. Lublinsky*, Phys.Rev. D **83**, 034017 (2011)

$\Sigma(n_F, n_B)$ in the model with string fusion on transverse grid

In this model we found that

$$\Sigma(n_F, n_B) = \frac{\sum_{i=1}^{M} \sum_{N_i=1}^{\infty} P_i(N_i)\, \langle n_i^F \rangle_{N_i} \Sigma_{N_i}(n_i^F, n_i^B)}{\sum_{i=1}^{M} \sum_{N_i=1}^{\infty} P_i(N_i)\, \langle n_i^F \rangle_{N_i}} = \qquad (26)$$

$$= \frac{1}{\langle n_F \rangle} \sum_{i=1}^{M} \sum_{N_i=1}^{\infty} P_i(N_i)\, \langle n_i^F \rangle_{N_i} \Sigma_{N_i}(n_i^F, n_i^B) \ ,$$

where we have introduce the $\Sigma_{N_i}(n_i^F, n_i^B)$ for i-th cell with N_i strings by anology with (1) :

$$\Sigma_{N_i}(n_i^F, n_i^B) \equiv \frac{d_{N_i}(n_i^F) - \mathrm{cov}_{N_i}(n_i^F\, n_i^B)}{\langle n_i^F \rangle_{N_i}} \ , \qquad (27)$$

$\Sigma(n_F, n_B)$ in the model with string fusion on transverse grid

If else all M cells are equivalent, $P_i(N_i) = P(N_i)$, then

$$\Sigma(n_F, n_B) = \frac{M \sum_{N_1=1}^{\infty} P(N_1) \langle n_1^F \rangle_{N_1} \Sigma_{N_1}(n_1^F, n_1^B))}{M \sum_{N_1=1}^{\infty} P(N_1) \langle n_1^F \rangle_{N_1}} = \tag{28}$$

$$= \frac{1}{\langle n_1 \rangle} \sum_{k=1}^{\infty} P(k) \langle n_1 \rangle_k \Sigma_{N_1}(n_1^F, n_1^B) \ .$$

Using $\langle n \rangle = M \langle n_1 \rangle$ and $\langle n^{(k)} \rangle = M\, P(k) \langle n_1 \rangle_k$ it can be presented also as

$$\Sigma(n_F, n_B) = \sum_{k=1}^{\infty} \alpha_k\, \Sigma_k(\mu_F, \mu_B) \ , \qquad \alpha_k = \frac{\langle n^{(k)} \rangle}{\langle n \rangle} \ , \tag{29}$$

where k is a number of strings fused in a given sell and $\langle n^{(k)} \rangle$ is a mean number of particles produced from all sells with k fused strings. $\sum \alpha_k = 1$.
The same result was obtained in the model with two types of string in [*E.V.Andronov, Theor.Math.Phys.185(2015)1383*] for the long-range part of $\Sigma(n_F, n_B)$, when at $\Delta\eta \gg \eta_{corr}$ we have $\Sigma_k(\mu_F, \mu_B) = \omega_\mu^{(k)}$ with k =1,2.

String fusion effects

The same value of $\Sigma(n_F, n_B)$ in AA collisions, as in pp,
if we suppose the formation of the same strings in AA and pp collisions.
Because the $\Sigma(n_F, n_B)$ does not depends on the mean value, $\langle N \rangle$,
and the event-by-event fluctuations, ω_N, in the number of strings.
It depends only on string properties.

If we suppose the formation of new strings in AA collisions
(and may be in central pp collisions at high energy)
with some new characteristics, compared to pp collisions,
due to e.g. string fusion processes, then for a source with k fused strings

$$\Sigma_k(\eta_{sep}) = 1 + \mu_0^{(k)} \delta\eta [\Lambda_k(0) - \Lambda_k(\eta_{sep})]$$

For these fused strings we expect, basing on the string decay picture
[*V.V.*, Baldin ISHEPP XIX v.1(2008)276; arXiv:0812.0604]:

1) larger multiplicity from one string, $\mu_0^{(k)} > \mu_0$,

2) smaller correlation length, $\eta_{corr}^{(k)} < \eta_{corr}$.

String fusion effects

This corresponds to the analysis of the net-charge fluctuations in the framework of the string model for pp and AA collisions *[A.Titov, V.V., PoS(Baldin ISHEPP XXI)047(2012)]*.

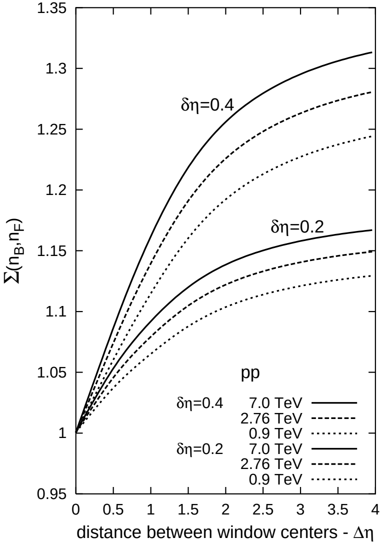
$$\Sigma_k(\eta_{sep}) = 1 + \mu_0^{(k)} \delta\eta [\Lambda_k(0) - \Lambda_k(\eta_{sep})]$$

Both factors lead to the steeper increase of $\Sigma_k(\eta_{sep})$ with η_{sep} in the case of AA collisions, compared to pp.

In reality - a mixture of fused and single strings:

$$\Sigma(n_F, n_B) = \sum_{k=1}^{\infty} \alpha_k \, \Sigma_k(\mu_F, \mu_B) \, , \qquad \alpha_k = \frac{\langle n^{(k)} \rangle}{\langle n \rangle} \, ,$$

Unfortunately in this case through the weighting factors $\alpha_k = \langle n^{(k)} \rangle / \langle n \rangle$ the observable $\Sigma(n_F, n_B)$ becomes dependent on collision conditions and, strictly speaking, can not be considered any more as strongly intensive.



Increase of the fused strings contribution to $\Sigma(n_F, n_B)$ with collision energy in pp collisions

$$\Lambda(\Delta\eta) = \Lambda_0 e^{-\frac{|\Delta\eta|}{\eta_{corr}}} ,$$

$\sqrt{s}$, TeV	0.9	2.76	7.0
$\mu_0\Lambda_0$	0.73	0.83	0.93
η_{corr}	1.52	1.43	1.33

$$\Sigma(n_F, n_B) = 1+$$

$$+\delta\eta \sum_{k=1}^{\infty} \alpha_k \, \mu_0^{(k)} \Lambda_0^{(k)} [1 - \exp\left(-|\Delta\eta|/\eta_{corr}^{(k)}\right)]$$

$\Sigma(\Delta\eta)$ with charges

(with E.Andronov)
In case of zero net-charges in both windows, which is a very good approximation for mid-rapidity region at LHC collision energies we have:

$$\Sigma(n_F, n_B) = \Sigma(n_F^+, n_B^+) + \Sigma(n_F^-, n_B^+) - \Sigma(n_F^+, n_F^-) \; . \tag{30}$$

For symmetric reaction and small windows:

$$\Sigma(n_F^+, n_B^+) = 1 + \frac{1}{2}\mu_0 \delta\eta(\Lambda^{++}(0) - \Lambda^{++}(\Delta\eta)) \; , \tag{31}$$

$$\Sigma(n_F^+, n_B^-) = 1 + \frac{1}{2}\mu_0 \delta\eta(\Lambda^{++}(0) - \Lambda^{+-}(\Delta\eta)) \quad [\star] \; , \tag{32}$$

$$\Sigma(n_F^+, n_F^-) = 1 + \frac{1}{2}\mu_0 \delta\eta(\Lambda^{++}(0) - \Lambda^{+-}(0)) \; . \tag{33}$$

$$\lambda^+(\eta) = \lambda^-(\eta) = \frac{1}{2}\lambda(\eta) \; , \tag{34}$$

$$\Lambda^{++}(\eta_1, \eta_2) = \Lambda^{--}(\eta_1, \eta_2) \; , \qquad \Lambda^{+-}(\eta_1, \eta_2) = \Lambda^{-+}(\eta_1, \eta_2) \; , \tag{35}$$

$\Lambda(\Delta\eta)$ with charges

$$\Lambda(\eta_1,\eta_2) \equiv \frac{\lambda(\eta_1,\eta_2)}{\lambda(\eta_1)\lambda(\eta_2)} - 1 =$$

$$= \frac{\lambda^{++}(\eta_1,\eta_2) + \lambda^{--}(\eta_1,\eta_2) + \lambda^{+-}(\eta_1,\eta_2) + \lambda^{-+}(\eta_1,\eta_2)}{[\lambda^{+}(\eta_1) + \lambda^{-}(\eta_1)][\lambda^{+}\eta_2) + \lambda^{-}(\eta_2)]} - 1 =$$

$$= \frac{2[\lambda^{++}(\eta_1,\eta_2) + \lambda^{+-}(\eta_1,\eta_2)]}{4\lambda^{+}(\eta_1)\lambda^{+}\eta_2)} - 1$$

$$\Rightarrow$$

$$\boxed{\Lambda(\Delta\eta) = \tfrac{1}{2}[\Lambda^{++}(\Delta\eta) + \Lambda^{+-}(\Delta\eta)]}$$

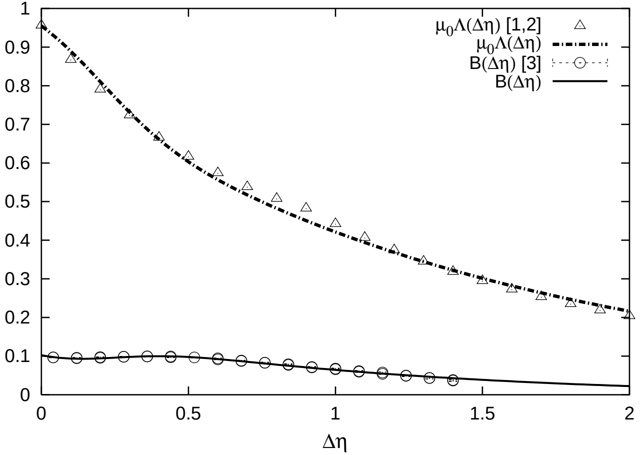
Connection with Balance Function (BF)

[ALICE collab., Eur.Phys.J.C 76(2016)86]

$$B(\Delta\eta, \Delta\phi) = \frac{1}{2}[C_{+-} + C_{-+} - C_{++} - C_{--}]$$

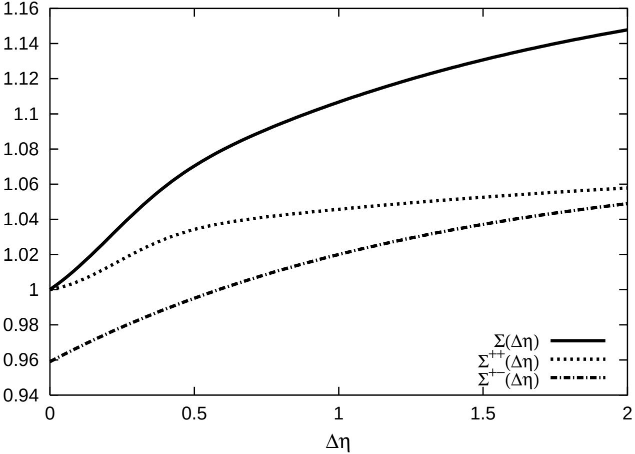
$$\boxed{B(\Delta\eta) = \tfrac{1}{4}[\Lambda^{+-}(\Delta\eta) - \Lambda^{++}(\Delta\eta)]}$$

$\Lambda(\Delta\eta)$ and $B(\Delta\eta)$



[1] ALICE collab.,JHEP 05(2015)097
[2] V.V., Nucl.Phys.A939(2015)21
[3] ALICE collab., Eur.Phys.J.C 76(2016)86 (70-80% centrality)

$\Sigma(\Delta\eta)$ with charges



$$\Sigma(n_F, n_B) = \Sigma(n_F^+, n_B^+) + \Sigma(n_F^-, n_B^+) - \Sigma(n_F^+, n_F^-) \ .$$

Conclusions

- The string model enables to understand the main features of the behavior of the strongly intensive observable $\Sigma(n_F, n_B)$. In particular the dependencies of this variable on the width of observation windows and the rapidity gap between them were found and its connection with the string two-particle correlation function was established.
- In the case with independent identical strings the model calculation confirms the strongly intensive character of this observable: it is independent of both the mean number of string and its fluctuation.
- In the case when the string fusion processes are taken into account and a formation of strings of a few different types takes place, it is shown, using a lattice in transverse plane, that this observable is equal to a weighted average of its values for different string types. Unfortunately in this case through the weighting factors the observable $\Sigma(n_F, n_B)$ starts to depend on collision conditions.

Backup slides

Backup slides

C_2 through multiplicities in two small windows

For two small windows $\delta\eta_1$ and $\delta\eta_2$ around η_1 and η_2 we have

$$\rho(\eta) = \frac{\langle n \rangle}{\delta\eta} \ , \qquad \rho_2(\eta_1, \eta_2) = \frac{\langle n_1 n_2 \rangle}{\delta\eta_1 \, \delta\eta_2} \ , \tag{36}$$

$$C_2(\eta_1, \eta_2) = \frac{\langle n_1 n_2 \rangle}{\langle n_1 \rangle \langle n_2 \rangle} - 1 \ , \tag{37}$$

where n_1 and n_2 are the event multiplisities in these windows $\delta\eta_1$ and $\delta\eta_2$. Note that when $\eta_1 = \eta_2 = \eta$, $\eta_{sep} = 0$, we have to use

$$\rho_2(\eta, \eta) = \frac{\langle n(n-1) \rangle}{\delta\eta^2} \ , \qquad C_2(0) = \frac{\langle n(n-1) \rangle}{\langle n \rangle^2} - 1 = \frac{\omega_n - 1}{\langle n \rangle} \ , \tag{38}$$

where n is the number of particles in small window $\delta\eta$ around the point η. (see e.g. [C.Pruneau,S.Gavin,S.Voloshin,Phys.Rev.C66(2002)044904] or [V.V.,Nucl.Phys.A939(2015)21]).

Connection between the two-particle correlation functions

In this model we have the following connection:

$$C_2(\eta_1, \eta_2) = \frac{\omega_N + \Lambda(\eta_1, \eta_2)}{\langle N \rangle}$$

[V.V.,Nucl.Phys.A939(2015)21]. (Note that one often looses the constant part $\omega_N/\langle N \rangle$ of C_2, obtaining C_2 by di-hadron correlation approach.)

At midrapidities, implying uniform rapidity distribution:

$$\rho(\eta) = \frac{dN_{ch}}{d\eta} = \rho_0 = \frac{\langle n_F \rangle}{\delta y_F} = \frac{\langle n_B \rangle}{\delta y_B} = \langle N \rangle \mu_0 \ , \qquad \mu_0 = \frac{\langle \mu_F \rangle}{\delta y_F} = \frac{\langle \mu_B \rangle}{\delta y_B}$$

the correlation functions depends only on a difference of rapidities:

$$\eta_{sep} = \eta_1 - \eta_2$$

Note that we use the two-particle correlation functions integrated over azimuth:

$$C_2(\eta_{sep}) = \frac{1}{\pi} \int_0^{\pi} C_2(\eta_{sep}, \phi_{sep}) \, d\phi_{sep} \ , \qquad \Lambda(\eta_{sep}) = \frac{1}{\pi} \int_0^{\pi} \Lambda(\eta_{sep}, \phi_{sep}) \, d\phi_{sep} \ .$$

Various versions of string fusion