

On photon splitting in Lorentz-violating QED

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- Lorentz Violation (LV) is mostly motivated by different approaches to construct quantum theory of gravity
- LV in matter sector is considered in the framework of effective field theory.
- Effects, appeared under hypothesis of LV, is the modification of cross-sections and particle decays.
- One of the effects is the changes of kinematics, allowing several reactions, forbidden in LI case, to occur.
- Well known example is **photon decay** into electron-positron pair.

The model

Extra LV term, quartic on space derivatives, for a photon, quartic on space derivative for photon and suppressed by the second power of LV mass scale M_{LV}

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \mp \frac{1}{2M_{LV}^2}F_{ij}\Delta^2F^{ij} + i\bar{\psi}\gamma^\mu D_\mu\psi - m\bar{\psi}\psi.$$

Come to following DR

$$E^2 = p^2 + \frac{p^4}{M_{LV}^2}.$$

The bound from the absence of photon decay

- $\gamma \rightarrow e^+e^-$ is forbidden in LI, but allowed in LV if the photon energy exceed a certain threshold
- in terms of effective photon mass

$$m_{\gamma,eff}^2 \equiv E^2 - p^2 = \frac{p^4}{M_{LV}^2}$$

If "photon mass" $m_{\gamma,eff} \geq 2m_e$ reaction is possible!

$$M_{LV} > \frac{E_\gamma^2}{2m_e}$$

Leads to following constraint on M_{LV} ¹

$$M_{LV,\gamma} > 2.8 \times 10^{12} \text{ GeV.}$$

¹H. Martínez-Huerta and A. Pérez-Lorenzana, arXiv:1610.00047 [astro-ph.HE]

The photon splitting

- In $m_{\gamma,\text{eff}} < 2m_e$ case, the photon decay $\gamma \rightarrow e^+e^-$ is kinematically forbidden
- Photon decay with several photons in the final state $\gamma \rightarrow n\gamma$, so-called **photon splitting**, is kinematically allowed whenever the photon dispersion relation is superluminal
- Splitting to two photons $\gamma \rightarrow 2\gamma$ do not occur due to the Furry theorem
- Thus the main splitting process is the photon decay to 3 photons $\gamma \rightarrow 3\gamma$

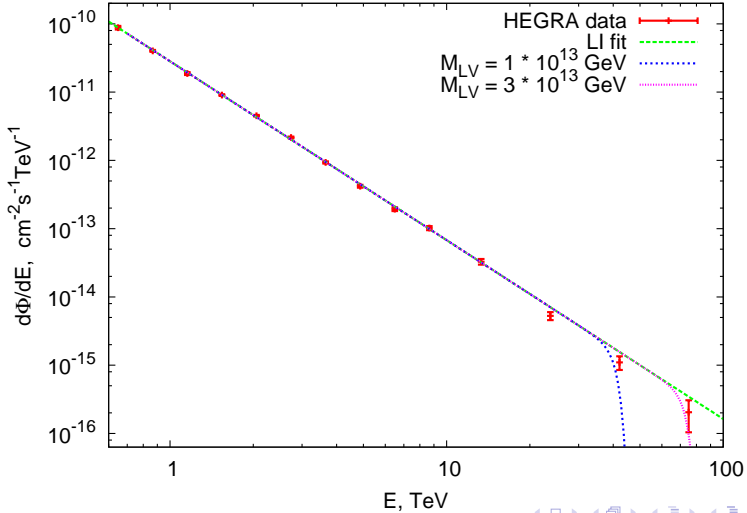
The width of photon splitting has been estimated by Gelmini Nussinov and Yaguna ². They used notation of effective photon mass with calculations in the artificial rest frame of massive photon, followed by subsequent boost back to laboratory frame

$$\mathcal{L}_{E-H} = \frac{2\alpha^2}{45m_e^4} \left[\left(\frac{1}{2} F_{\mu\nu} F^{\mu\nu} \right)^2 + 7 \left(\frac{1}{8} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \right)^2 \right].$$

$$\Gamma(\gamma \rightarrow 3\gamma) \simeq 0.3 \times 10^{-20} \frac{E_\gamma^{19}}{m_e^8 M_{LV}^{10}}.$$

²G. Gelmini, S. Nussinov and C. E. Yaguna, JCAP **0506** (2005) 012 [hep-ph/0503130].

- Using expression for $\Gamma(\gamma \rightarrow 3\gamma)$ photon free path distance can be obtained.
- Knowing distance to the source and max energy of incoming photons (the cut-off energy) constraint on M_{LV} can be found.
- 20 TeV from blazar Mrk 501 : $M_{LV} > 8 \cdot 10^{12}$ GeV
- 80 TeV from Crab nebula : $M_{LV} > 4 \cdot 10^{13}$ GeV



$$k_1^{\parallel} = p \frac{1+x+y}{3}, \quad k_2^{\parallel} = p \frac{1+x-y}{3}, \quad k_3^{\parallel} = p \frac{1-2x}{3}$$
$$k_1^{\parallel} + k_2^{\parallel} + k_3^{\parallel} = p, \quad \vec{k}_1^{\perp} + \vec{k}_2^{\perp} + \vec{k}_3^{\perp} = 0$$

Phase Volume:

$$\Phi = \frac{3}{p} \int \frac{dx dy d^2 k_1^{\perp} d^2 k_2^{\perp}}{((1+x)^2 - y^2)(1-2x)} \delta(E - E_1 - E_2 - E_3) |M|^2.$$

We introduce dimensionless variables β_1, β_2 :

$$k_1^{\perp} = \frac{2p^2}{3M} \cdot \beta_1, \quad k_2^{\perp} = \frac{2p^2}{3M} \cdot \beta_2.$$

Phase volume:

$$\Phi = \frac{8\pi}{9} \frac{p^4}{M^2} \int \frac{dx dy d\beta_1 d\beta_2}{((1+x)^2 - y^2) \sin \varphi_2} \frac{1}{\text{podstanovka}} |M|^2.$$

$$\cos \varphi_2 = \frac{2/3 f(x, y)(1 - 2x) - \frac{2-x+y}{1+x+y} \beta_1^2 - \frac{2-x-y}{1+x-y} \beta_2^2}{2\beta_1 \beta_2}, \quad |\cos \varphi_2| < 1$$

$$(p, k_{1,2}) = \frac{E_p^4}{9M^2} \left[(1+x \pm y) + \frac{6\beta_{1,2}^2}{1+x \pm y} + \frac{1}{6} (1+x \pm y)^3 \right]$$

$$(p, k_3) = \frac{E_p^4}{M^2} - (p, k_1) - (p, k_2)$$

$$(k_1, k_2) = \frac{2E_p^4}{9M^2} \left[\frac{\beta_1^2}{1+x+y} + \frac{\beta_2^2}{1+x-y} + \frac{(1+x)^4 - y^4}{18} - \frac{2}{3} f(x, y)(1-2x) \right]$$

$$(k_{1,2}, k_3) = \frac{1}{2M^2} (E_p^4 \pm E_2^4 \mp E_1^4 - E_3^4) - (p, k_{1,2})$$

where

$$f(x, y) = 1 - \frac{3x^2}{4} + \frac{x^3}{4} - (1+x^2)y^2$$

Preliminary!

$$|M|^2 = |M|_{LI}^2 + \delta|M|^2$$

$$\begin{aligned}
 |M|_{LI}^2 = & 24k_1^2 k_2^2 k_3^2 p^2 + 888(k_1, k_2)^2 (k_3, p)^2 + \\
 & 136k_3^2 p^2 (k_1, k_2)^2 + 720k_3^2 (k_1, k_2) (k_1, p) (k_2, p) + \\
 & + 2672(k_1, k_2) (k_1, p) (k_2, k_3) (k_3, p) + 2672(k_1, k_2) (k_1, k_3) (k_2, p) (k_3, p) + \\
 & 720p^2 (k_1, k_2) (k_1, k_3) (k_2, k_3) + \\
 & + 888(k_1, k_3)^2 (k_2, p)^2 + 136k_1^2 k_2^2 (k_3, p)^2 + \\
 & 2672(k_1, k_3) (k_1, p) (k_2, k_3) (k_2, p) + 136k_1^2 k_3^2 (k_2, p)^2 + 136k_2^2 k_3^2 (k_1, p)^2 + \\
 & + 720k_2^2 (k_1, k_3) (k_1, p) (k_3, p) + \\
 & 720k_1^2 (k_2, k_3) (k_2, p) (k_3, p) + 136k_1^2 p^2 (k_2, k_3)^2 + 136k_2^2 p^2 (k_1, k_3)^2
 \end{aligned}$$

$$\begin{aligned}
\delta|M|^2 = & \frac{222}{M^2} (|\vec{k}_1|^2 + |\vec{k}_2|^2 + |\vec{k}_3|^2 + |\vec{p}|^2) (k_1, k_2)^2 (k_3, p)^2 + \\
& \frac{720}{M^2} (|\vec{k}_3|^2 k_{30}^2 (k_1, k_2) (k_1, p) (k_2, p)) + \\
& \frac{2672}{M^2} (|\vec{k}_1|^2 + |\vec{k}_2|^2 + |\vec{k}_3|^2 + |\vec{p}|^2) (k_1, k_2) (k_1, p) (k_2, k_3) (k_3, p) + \\
& \frac{2672}{M^2} (|\vec{k}_1|^2 + |\vec{k}_2|^2 + |\vec{k}_3|^2 + |\vec{p}|^2) (k_1, k_2) (k_1, k_3) (k_2, p) (k_3, p) + \\
& \frac{720}{M^2} (|\vec{p}|^2 p_0^2) (k_1, k_2) (k_1, k_3) (k_2, k_3) + \\
& \frac{222}{M^2} (|\vec{k}_1|^2 + |\vec{k}_2|^2 + |\vec{k}_3|^2 + |\vec{p}|^2) (k_1, k_3)^2 (k_2, p)^2 + \\
& \frac{2672}{M^2} (|\vec{k}_1|^2 + |\vec{k}_2|^2 + |\vec{k}_3|^2 + |\vec{p}|^2) (k_1, k_3) (k_1, p) (k_2, k_3) (k_2, p) + \\
& \frac{720}{M^2} (|\vec{k}_2|^2 k_{20}^2) (k_1, k_3) (k_1, p) (k_3, p) + \\
& \frac{720}{M^2} (|\vec{k}_1|^2 k_{10}^2) (k_2, k_3) (k_2, p) (k_3, p)
\end{aligned}$$

- $\Phi \sim \frac{E_p^4}{M^2} \frac{1}{m_e^8} \left(\frac{E_p^4}{M^2}\right)^4 = \frac{E_p^{20}}{m_e^8 M^{10}}$, $\Gamma \sim E_p^{-1} \Phi$ - the same order as in Gelmini estimation
- $x=0$ $y=0$ (three equal momenta photons) case is dominating
- **Preliminary** We determined the coefficient in expression for phase volume

$$20 \times \left(\frac{8\pi}{9}\right) \left(\frac{\alpha^2}{90}\right)^4$$