One-loop correction to the photon velocity in Lorentz-violating QED.

Petr Satunin

INR
Institute for Nuclear Research
of the Russian Academy of Sciences

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Intro..

Dispersion relation as the pole of the propagator

Physical dispersion relation for a particle — pole of the propagator.

$iG^{\mu\nu} =$ 

Sum over 1-particle reduced diagrams for photon propagator.
No loop corrections to dispersion relation — in Lorentz-invariant (LI) theories.

LI may be violated by external classical field — magnetic or gravitational.
Non-trivial photon dispersion!

Shabad 1975

book Mikheev Kuznetsov 2003-2014

Hollowood 2009, ...

The similar situtation if LI is violated at fundamental level.

Motivation of Lorentz Invariance violation

- Different approaches to quantum gravity
  - Discrete spacetime, loop quantum gravity, non-commutative geometry e.t.c.

    Gambini, Pullin 1999

    Douglas, Nekrasov, 2001

    ...
  - Modifications of general relativity with large space derivatives (Hořava-Lifshitz e.t.c.)

    Hořava 2009

    Blas, Pujolas, Sibiryakov 2010

    ...
- Phenomenologically in non-gravity sector
  - Special type of LV (preserving other symmetries, motivations to concrete QG approaches)

    For example, $E^2 = m^2 + p^2(1+\delta) \pm \frac{p^4}{M_{LV}^2} \pm \ldots$
  - The most general type — Standard Model Extension (SME)

    Kostelecky, Colladay 1998

Lorentz Invariance Violation: ways to constrain it

- Accurate measurements in the labs on the Earth
  Michelson-Morley-type experimiments, fine structure measurements..
- Observations in high-energy astrophysics:
  - Time-of-flight measurements (photons, neutrino, gravity waves..)
  - Modifications of some particle rections, crutial to astrophysical processes (photon decay, modification of shower formation..)
- Accumulated effects in cosmology (structure grows e.t.c.)

Summary: *Data tables: Kostelecky, Russel, 2008-2018. arXiv: 0801.0287*

Quartic dispersion relation — photon time of flight constraints

$$\mathcal{L}_{QED}^{LV} = \mathcal{L}_{QED}^{LI} \pm \frac{1}{4M_{LV,\gamma}^2} F_{ij} \Delta F^{ij}, \qquad E_\gamma^2 = p_\gamma^2 \pm \frac{p_\gamma^4}{M_{LV,\gamma}^2}$$

AGN: $M_{LV,\gamma} > 6.4 \times 10^{10}$ GeV — *Abramowski et al (H.E.S.S.) '11*

GRB: $M_{LV,\gamma} > 1.3 \times 10^{11}$ GeV — *Vasileiou et al. (Fermi-LAT) '13*

Standard Model Extension *Kostlecky, Colladay 1998*

QED sector of SME Lagrangian:

$$\mathcal{L}_{SME} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\,\overline{\psi}\,\Gamma^{\mu}D_{\mu}\,\psi - \overline{\psi}\,M\,\psi - \frac{1}{4}(k_F)_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} + (k_{AF})^{\mu}A^{\nu}\tilde{F}_{\mu\nu},$$

$(k_F)_{\mu\nu\rho\sigma}$, $(k_{AF})^{\mu}$ – LV parameters in photon sector; Γ and M:

$$\begin{aligned}\Gamma^{\mu} &= \gamma^{\mu} + c^{\mu\nu}\gamma_{\nu} + d^{\mu\nu}\gamma_5\gamma_{\nu} + if^{\mu} + \frac{1}{2}g^{\lambda\nu\mu}\sigma_{\lambda\nu} + e^{\mu}\,, \\ M &= m + a^{\mu}\gamma_{\mu} + b^{\mu}\gamma_5\gamma_{\mu} + \frac{1}{2}H^{\mu\nu}\sigma_{\mu\nu}\,.\end{aligned}$$

Lots of parameters, calculations may be very complicated. All SME coeffecients are considered perturbatively as insertions to the propagators

 $= -i(M-m),$  $= i(\Gamma^{\mu}-\gamma^{\mu})p_{\mu}\,,$

μ  $\nu = -2ip^{\alpha}p^{\beta}(k_F)_{\alpha\mu\beta\nu}\,,$ μ  $\nu = 2(k_{AF})^{\alpha}\epsilon_{\alpha\mu\beta\nu}p^{\beta}\,.$

Studies of loop processes

- Renormalizability of QED sector of SME (only infinite parts of one-loop diagrams have been calculated) *Kostelecky, Lane, Pickering, 2001*
- Finite correctons to the electron self-energy, modified electron propagator at the lowest order (simplified model, nonzero $c_{\mu\nu}$, $k_{F\,\mu\nu}$) *Cambiaso, Lehnert, Potting, 2014*

The simplest model of QED without LI

Two different maximal velocities: $(1 + c_\gamma)$ — for photon,
$(1 + c_f)$ — for electron (or other fermion)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi}\gamma^\mu D_\mu \psi - m_f \bar{\psi}\psi -$$
$$-\frac{c_\gamma}{2} F_{ij} F^{ij} - i c_f \bar{\psi}\gamma^i D_i \psi.$$

f — an arbitrary charged fermion (may be e, μ, τ, top-quark or others..)
Dispersion relations:

$$\gamma : \qquad E_\gamma^2 = (1 + c_\gamma)^2 \, p_\gamma^2 \simeq (1 + 2c_\gamma) \, p_\gamma^2,$$
$$\text{fermion} : \qquad E_f^2 = (1 + c_f)^2 \, p_f^2 + m_f^2 \simeq (1 + 2c_f) \, p_f^2 + m_f^2.$$

Tree-level propagators are calculated exactly on c_f, c_γ (simple rescaling of LI ones) $\rightarrow$ non-perturbative treatment of LV parameters.

Photon polarization operator

LI case (dimensional regularisation)

$$\Pi^{LI}_{\mu\nu}(p_\gamma) = \left(\eta_{\mu\nu} - \frac{(p_\gamma)_\mu (p_\gamma)_\nu}{p_\gamma^2}\right) p_\gamma^2 \Pi(p_\gamma^2),$$

$$\Pi(p_\gamma^2) = -\frac{e^2}{2\pi^2}\int_0^1 dx\, x(1-x)\left[\frac{1}{\epsilon} + \ln 4\pi - \gamma_E - \ln\frac{m_f^2 - x(1-x)p_\gamma^2}{\mu^2}\right].$$

LV case

$$\Pi_{\mu\nu}(p_\gamma) = \left[(1-c_e)p_\gamma^2(P_1)_{\mu\nu} - 2c_e\vec{p}_\gamma^{\,2}(P_2)_{\mu\nu}\right]\Pi(\hat{p}_\gamma^2),$$

$\hat{p}_\gamma = (E_\gamma, (1+c_f)\vec{p}_\gamma)$. Projectors: $P_1^{\mu\nu} = \eta^{\mu\nu} - \frac{p_\gamma^\mu p_\gamma^\nu}{p_\gamma^2}$, $P_2^{\mu\nu} = -\delta_i^\mu \delta_j^\nu \left(\delta^{ij} - \frac{p_\gamma^i p_\gamma^j}{\vec{p}_\gamma^{\,2}}\right)$

$$\Pi(\hat{p}_\gamma^2) = \frac{e^2}{2\pi^2}\int_0^1 dx\, x(1-x)\ln\left(1 - x(1-x)\hat{p}_\gamma^2/m_f^2\right) + \Pi_0,$$

On-shell renormalization scheme: $\Pi_0 = 0$ (no corrections in IR).

Modified photon propagator (Coulomb gauge)

Sum over one-particle reduced diagrams

$$D^{00}_{1-loop}(p_\gamma) = -\frac{1}{(1-c_f)\vec{p}_\gamma^{\,2}}, \qquad D^{0i}_{1-loop}(p_\gamma) = 0,$$

$$D^{ij}_{1-loop}(p_\gamma) = -\frac{1}{1-\Pi(\hat{p}_\gamma^2)(1-c_f)} \cdot \frac{\delta^{ij} - \frac{p_\gamma^i p_\gamma^j}{\vec{p}_\gamma^{\,2}}}{p_0^2 - \vec{p}_\gamma^{\,2}(1+2c_\gamma+2(c_\gamma-c_f)\Pi(\hat{p}_\gamma^2))}.$$

$\frac{1}{1-\Pi(\hat{p}_\gamma^2)(1-c_f)}$ gives the Landau pole. The pole of the 2nd denominator gives the modified photon dispersion relation,

$$E_\gamma^2 = \vec{p}_\gamma^{\,2}\left(1+2c_\gamma+2(c_\gamma-c_f)\Pi\left(2(c_f-c_\gamma)\vec{p}_\gamma^{\,2}\right)\right),$$

which may be computed exactly in a several limits.

One-loop correction to the photon dispersion relation

$$\mathbf{y \equiv (c_f - c_\gamma)\frac{E_\gamma^2}{m_f^2}}.$$

- $y < -2$: the polarization operator acquires imaginary part. Following the Optical theorem, photon decays to f.-antif. pair. $y = -2$ — the threshold.
- $y \gg 1$ — logarithmic correction

$$E_\gamma^2 = p_\gamma^2 \left[1 + 2c_\gamma + \frac{e_f^2}{6\pi^2}(c_\gamma - c_f)\cdot\left[\ln\left(2\,(c_f - c_\gamma)\,\frac{p_\gamma^2}{m_f^2}\right) - \frac{5}{3}\right]\right].$$

- $|y| \ll 1$ — quartic correction. The sign minus before the quartic term appears for both positive and negative y.

$$E_\gamma^2 = p_\gamma^2\,(1 + 2c_\gamma) - \frac{p_\gamma^4}{M_{LV}^2},$$

where the effective LV scale M_{LV} is:

$$M_{LV} = \frac{\sqrt{15}\pi}{e_f}\cdot\frac{m_f}{|c_f - c_\gamma|}.$$

One-loop correction to photon velocity, $y \gg 1$ case

Group photon velocity $c_{ph} = \frac{\partial E_\gamma}{\partial p_\gamma}$:

$$c_\gamma^{ph} = c_\gamma - \frac{e^2}{6\pi^2} \cdot (c_f - c_\gamma) \cdot \ln\left(2(c_f - c_\gamma)\frac{\vec{p}_\gamma^2}{m^2}\right).$$

Coincides with renormgroup calculations

(based on infinite parts of loop diagrams) *Kostelecky, Lane, Pickering 2001*

if the renormgroup scale μ is taken as $\mu = \sqrt{c_\gamma - c_f}\, E_\gamma$

Interpretation: set $c_e = 0$ (redefinition)

The LV photon polarization operator, considered on-shell may be considered as off-shell polarization operator, calculated in LI theory with the squared photon momentum

$$q^2 \equiv E_\gamma^2 - \vec{k}^2 = 2(c_\gamma - c_f)\, E_\gamma^2.$$

The case of the logarithmic correction $y \gg 1$ corresponds to $q^2 \gg m^2$.

Constraints on LV for fermions, $y \ll 1$ case

The dispersion relation $E_\gamma^2 = p_\gamma^2 (1 + 2c_\gamma) - p_\gamma^4 / M_{LV}^2$ have been tested by photon time-of-flight observations. The best constraint: $M_{LV} < M_{LV}^{GRB} \equiv 1.3 \times 10^{11}$ GeV. In terms of c_f and c_γ, the constraint is (if the condition $|y| \ll 1$ is valid)

$$|c_f - c_\gamma| < \frac{\sqrt{15}\pi}{e_f} \cdot \frac{m_f}{M_{LV}} \simeq 40 \cdot \frac{e}{e_f} \cdot \frac{m_f}{M_{LV}}$$

Bounds on $|c_f - c_\gamma|$ for different fermions:

fermion	photon timing bounds (this work)	current bounds	
electron	$1.5 \cdot 10^{-13}$	10^{-15}	*Altschul 2010*
muon	$3 \cdot 10^{-11}$	$\sim 10^{-11}$	*Altschul 2006*
tau-lepton	$1.2 \cdot 10^{-9}$	$\sim 10^{-8}$	*Altschul 2006*
t-quark	$1.6 \cdot 10^{-7}$	$\sim 10^{-2}$	*D0 collab. 2012*

The best constraints on c_f for tau-lepton and top-quark!

Conlusions

- In the absence of LI for charged fermions photon dispersion relation (and physical velocity) acquires non-trivial one-loop correction.
- Experimental non-observation of photon dispersion leads to constraints on LV for all charged fermions. These constraints are the best in the literature for heavy fermions (tau-lepton, top-quark)
- Effective Lagrangian for photons acquires high space derivative terms like $\delta\mathcal{L} \sim -\frac{1}{M_{LV}^2} F_{\mu\nu} \Delta F^{\mu\nu}$.
- Is it possible to obtain sign $+$? Contribution from W^+W^- loop?
- arXiv:1705.07796[hep-th], accepted to PRD

Thank you for your attention!