Electric current of massive fermions induced by a magnetic field in the equilibrium

Maxim Dvornikov
IZMIRAN
Troitsk, Russia

Plan of the talk

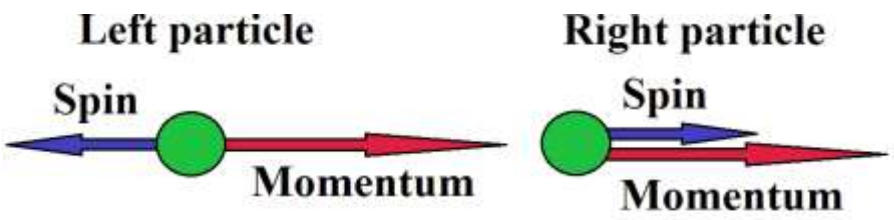
- Introduction: the chiral magnetic effect (the CME) and the magnetic field instability
- The CME in the presence of the external axial-vector field
- Influence of a particle mass and anomalous magnetic moment on the generation of the induced current

References

- **M. Dvornikov**, *Chiral magnetic effect in the presence of an external axial-vector field*, arXiv:1804.10241.
- **M. Dvornikov**, *Role of particle masses in the generation of the induced current along a magnetic field*, arXiv:1801.07788.
- **M. Dvornikov**, *Magnetic field instability driven by anomalous magnetic moments of massive fermions and electroweak interaction with background matter*, JETP Lett. **106**, 775 (2017), arXiv:1704.03403.
- **M. Dvornikov**, *Role of particle masses in the magnetic field generation driven by the parity violating interaction*, Phys. Lett. B **760**, 406 (2016), arXiv:1608.04940.

CME in a nutshell

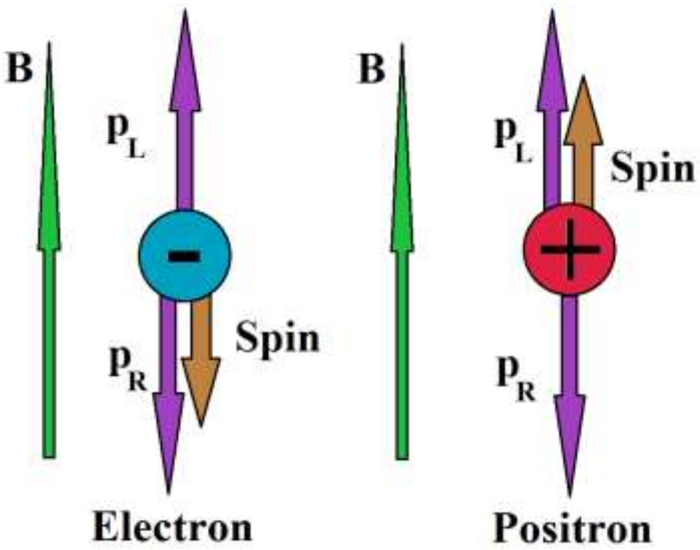
Helicity is strongly correlated with the momentum for massless particles



While interacting with a constant magnetic field **B**, electron spin is aligned opposite **B** and positron spin along **B**, at zero Landau level

Left electrons move along **B**, whereas right ones opposite **B**

Thus we can expect a flux of charged particles, i.e. electric current, along **B**



The detailed calculation by Vilenkin (1980) shows that

$$\mathbf{J} = \frac{2\alpha_{em}}{\pi}\mu_5\mathbf{B}, \quad \mu_5 = \frac{1}{2}\left(\mu_R - \mu_L\right)$$

Magnetic field instability driven by the CME

If **J**||**B** flows in the system, the Maxwell equations are modified

$$i\left(\mathbf{k}\times\mathbf{B}\right)=-i\omega\mathbf{E}+\mathbf{j}+\mathbf{j}_5 \quad i\left(\mathbf{k}\times\mathbf{E}\right)=i\omega\mathbf{B} \quad \left(\mathbf{k}\cdot\mathbf{B}\right)=0 \quad \mathbf{j}=\sigma\mathbf{E} \quad \mathbf{j}_5=\Pi\mathbf{B}$$

In the MHD approximation $\sigma \gg \omega$, one gets the Faraday equation for the large scale magnetic field evolution

$$\frac{\partial\mathbf{B}}{\partial t}=\alpha\left(\nabla\times\mathbf{B}\right)+\eta\nabla^2\mathbf{B} \quad \alpha=\frac{\Pi}{\sigma} \quad \eta=\frac{1}{\sigma}$$

The Faraday equation has the unstable solution

$$B(k,t)=B_0\exp\left[\int_{t_0}^{t}\left(|\alpha|-\eta k\right)k\,dt'\right]$$

If $k < |\alpha|/\eta$, this solution describes the exponential growth of a seed magnetic field B_0

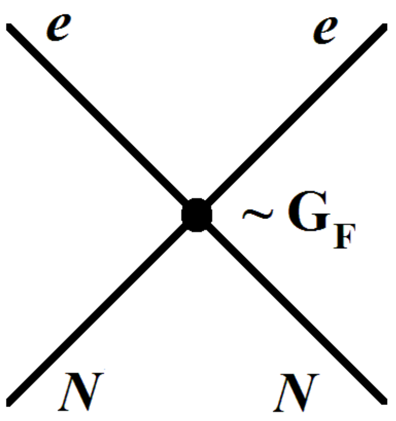
CME under the influence of an external axial-vector field

External axial-vector field

We discuss the interaction of charged fermions with an axial-vector field

$$L_{\text{int}} = -\bar{\psi}\gamma^{\mu}\gamma^{5}\psi V_{\mu}$$

Example: electroweak interaction with background matter



If matter is unpolarized and at rest

$$L_{\text{int}} = -\bar{\psi}\gamma^{0}\left[V_{L}\frac{1}{2}(1-\gamma^{5}) + V_{R}\frac{1}{2}(1+\gamma^{5})\right]\psi$$

The effective potentials $V_{\mathrm{L,R}} \sim G_{\mathrm{F}}\, n_{\mathrm{background}}$

Chiral matter with nonzero chemical potentials and axial-vector field

$$L = L_0 + L_B + L_5 =$$

$$\bar{\psi}_L[\gamma^\mu(i\partial_\mu + eA_\mu) - \gamma^0 V_L]\psi_L + \mu_L \psi_L^+ \psi_L + (L \to R)$$

$$\xrightarrow{???}$$

$$\bar{\psi}_L[\gamma^\mu(i\partial_\mu + eA_\mu)]\psi_L + (\mu_L - V_L)\psi_L^\dagger \psi_L + (L \to R)$$

Effective chemical potentials $\mu_{R,L}^{(eff)} = \mu_{R,L} - V_{R,L}$

The CME should be modified (Dvornikov & Semikoz, 2015)

$$\mathbf{J} = \frac{2\alpha_{em}}{\pi}(\mu_5 + V_5)\mathbf{B}, \; V_5 = \frac{1}{2}(V_L - V_R) \sim G_F n_{background}$$

Criticism

- Kaplan et al. (2017), on the basis of the Nielsen & Ninomiya (1983) method for the derivation of the CME, claimed that the electroweak background matter cannot contribute to the CME.
- Sadofyev & Isachenkov (2011), using the chiral hydrodynamics approach (Son & Surowka, 2009), demonstrated that an external axial-vector field does not contribute explicitly to the CME.

Spectrum of a massive fermion interacting with background matter under the influence of a magnetic field

$$\left[\gamma^{\mu}\left(i\partial_{\mu}+eA_{\mu}\right)-m-\gamma^{0}\left(V_{L}P_{L}+V_{R}P_{R}\right)\right]\psi=0$$

Energy spectrum at the lowest Landau level with $n = 0$

$$E_{n=0}=\bar{V}+\sqrt{(p_{z}+V_{5})^{2}+m^{2}} \quad \bar{V}=\frac{1}{2}(V_{R}+V_{L})$$

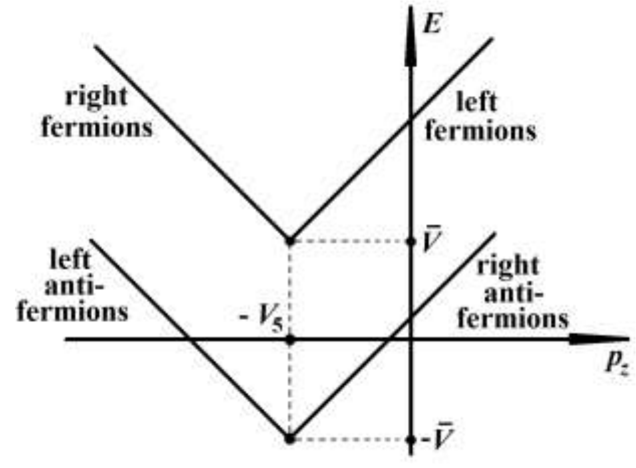
In the chiral limit $m \to 0$, the energy spectrum reads

$$E_{eL}=p_{z}+V_{L},\ -V_{5}<p_{z}<+\infty$$

$$E_{eR}=-p_{z}+V_{R},\ -\infty<p_{z}<-V_{5}$$

$$E_{\bar{e}R}=p_{z}-V_{R},\ -V_{5}<p_{z}<+\infty$$

$$E_{\bar{e}L}=-p_{z}-V_{L},\ -\infty<p_{z}<-V_{5}$$



Calculation of anomalous current

Higher Landau levels with $n > 0$ do not contribute to the current **J||B**

Lowest energy level with $n = 0$ contribution

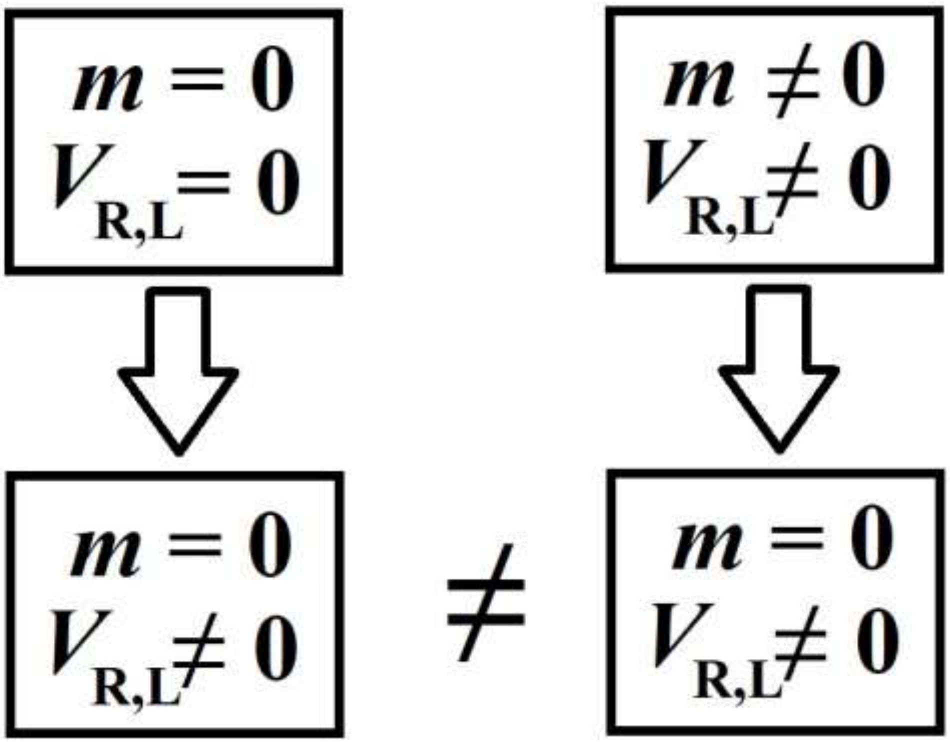
$$\vec{J}_{e,\bar{e};R,L} = \mp e \int dp_y dp_z \bar{\psi}_{e,\bar{e};R,L} \vec{\gamma} \psi_{e,\bar{e};R,L} f(E^{(n=0)}_{e,\bar{e};R,L} \mp \mu_{R,L})$$

Accounting for the correct energy spectrum, the current reads

$$\vec{J} = \frac{2\alpha_{em}}{\pi} \mu_5 \vec{B}$$

External axial-vector field (electroweak interaction with background matter) does not directly contribute to the CME

Non-equivalence of different approaches for the CME description



$$\vec{J} = \frac{2\alpha_{em}}{\pi}\left(\mu_5 + V_5\right)\vec{B} \qquad \vec{J} = \frac{2\alpha_{em}}{\pi}\mu_5\vec{B}$$

Anomalous current of massive particles induced by electroweak interaction and anomalous magnetic moments

Motivation

- The CME appears only if charged fermions are massless, i.e. $\boldsymbol{\mu}_L \neq \boldsymbol{\mu}_R$.
- The chiral symmetry should be restored.
- A first order chiral phase transition is possible only if physics beyond the standard model exists (Cline et al., 2017).
- QCD first order chiral phase transitions are discussed (Hands, 2001).
- Thus the issue of the generation of a current **J||B** for massive particles is important for the astrophysical applications since such a current leads to the magnetic field instability and can be used for the generation of strong magnetic fields.

Charged fermion interacting with a magnetic field, accounting for its anomalous magnetic moment, and with electroweak matter

$$\left[\gamma^{\mu}\left(i\partial_{\mu}+eA_{\mu}\right)-m-\frac{\mu}{2}\sigma_{\mu\nu}F^{\mu\nu}-\gamma^{0}\left(V_{L}P_{L}+V_{R}P_{R}\right)\right]\psi=0$$

Anomalous magnetic moment $\mu=\frac{e}{2m}\left(\frac{\alpha_{em}}{2\pi}+\ldots\right)$

Exact solution of the Dirac equation was found by Studenikin et al. (2012)

Energy spectrum

At higher energy levels with $n > 0$

$$E = V + \varepsilon$$

$$\varepsilon = \sqrt{p_z^2 + 2eBn + m^2 + (\mu B)^2 + V_5^2 + 2sR^2}$$

$$R^2 = \sqrt{(p_z V_5 - \mu B m)^2 + 2eBn\left[(\mu B)^2 + V_5^2\right]}$$

$$s = \pm 1$$

This spectrum is not symmetric with respect to change $p_z \rightarrow -p_z$

The asymmetry ~ $\boldsymbol{\mu} B\, m\, V_5$

Is there a current **J||B** in this system?

- Particles moving along and opposite magnetic field, i.e. having different signs of p_z, will have different velocities $v_z = p_z / \boldsymbol{\varepsilon}$.
- Bubnov et al. (2017); Dvornikov (2017) claimed that there is an anomalous current **J||B** in the system ~ $\boldsymbol{\mu} B\, m\, V_5$.
- Only higher energy levels with $n > 0$ contribute to this current.

Careful calculation of the current

The general expression for the current

$$\vec{J} = \Pi \vec{B}$$

$$\Pi = -\frac{\alpha_{em}}{\pi} \sum_{n=1}^{\infty} \sum_{s=\pm 1} \int_{-\infty}^{+\infty} dp_z \left(f_e - f_{\bar{e}} \right)$$

$$\times \left[p_z \left(1 + s \frac{V_5^2}{R^2} \right) - s \frac{\mu B m V_5}{R^2} \right]$$

After quite lengthy but straightforward calculations (including the integration by parts) one can show that Π= 0, i.e. the current **J||B** is not induced

Results

- The CME is quite robust.
- An external axial-vector field does not contribute to the current **J||B**.
- We established that nonzero mass of charged fermions destroy the CME in the extended system which includes the electroweak interaction with matter and anomalous magnetic moments.
- The second result generalizes our previous finding (Dvornikov, 2016), where only electroweak matter was taken into account.

Acknowledgements

- RFBR (Russia)
- The Competitiveness Improvement Program at the Tomsk State University
- Organizers of Quarks-2018 for the invitation