Ultra-high energy particle collisions near black holes and singularities and super-Penrose process

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Two kinds of energies as a result of collisions

1) High (unbound) energy in the centre of mass frame E_c.m.

Black holes, naked singularities, quasiblack holes, star-like configurations, wormholes

BHs: rotating or electrically charged

Proximity to horizon
Ergoregion (high angular momentum),
Extremely rapid rotation

Collisions outside and inside BH

In magnetic field

Sclar field

Particle moving towards horizon (BSW effect), Banados-Silk-Wesr PRL 2009
Head-on collisions
Fine-tuned (critical) and typical (usual) particles

2) Possibility to get high (unbounded) energies E at infinity (debris after collision) – super-Penrose process

Physical explanation and properties of BSW effect

Universal character of BSW effect near BH

Kinematic nature of the BSW effect. Role of critical trajectories

BSW effect and acceleration horizons

Geometric explanation

Kinematic explanation for collisions inside BH

Extremal versus nonextremal BHs

Kinematic censorship

Role of self-force due to gravitational radiation

BSW effect versus Penrose process: what can be seen at infinity?

Part 1

High energy processes near BHs

Key quantity: energy in centre of mass frame

1 particle $$m^2 = \left| P_\mu P^\mu \right|$$

2 particles colliding in some point

$$E^2{}_{cm} = \left| P_\mu P^\mu \right|$$

Total momentum $$P_\mu = p^{(1)}{}_\mu + p^{(2)}{}_\mu$$

$$P_a = (E_{c.m.}, 0, 0, 0) \qquad u^\mu u_\mu = -1$$

Individual E finite, energy in CM frame unbounded

Two different kinds of energy

Killing energy $E = -p_\mu \xi^\mu$ ξ^μ Killing vector

E conserved, integral of motion since metric is static or stationary

Energy in the CM frame $E_{c.m.}$

not conserved. Moreover, it is defined in one point only.
point of collision

Head-on collision

1975 - 1977 T. Piran, J. Katz and J. Shanam

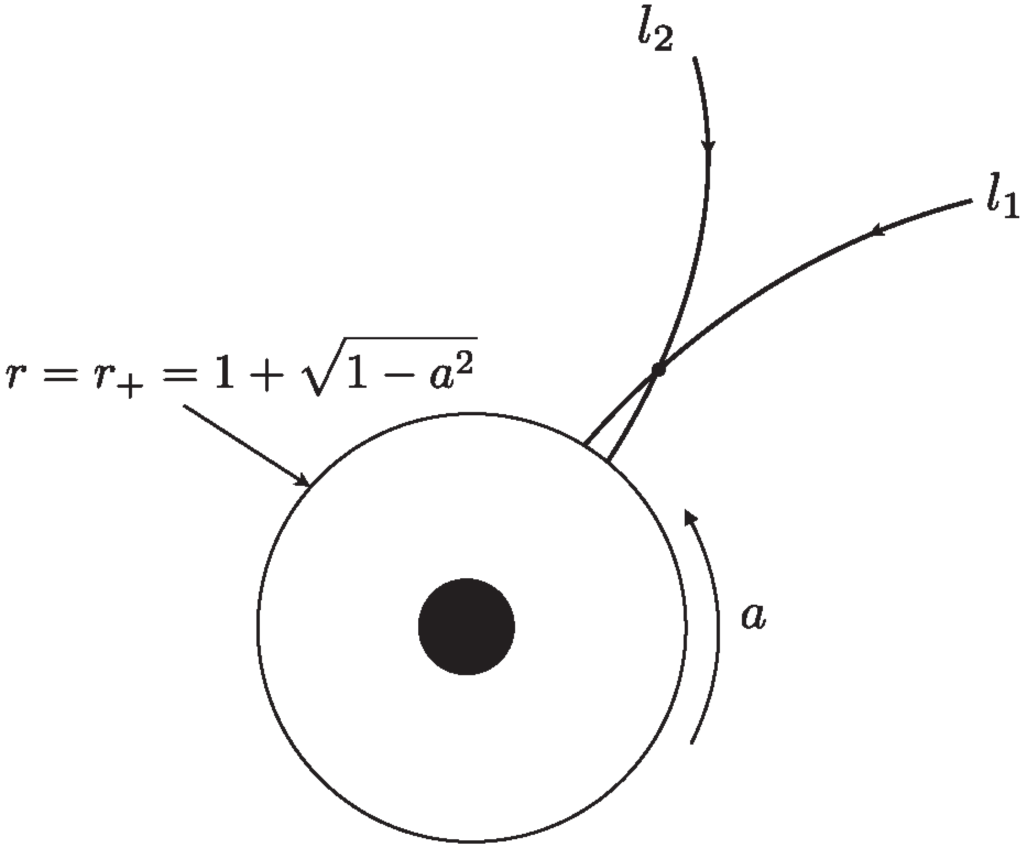
Two particles move in opposite directions near BH

Almost infinite relative blue shift

E in CM frame almost diverges

Special scenario. Particle near black (not white) hole moving away from horizon and colliding with another particle

M. Banados, J. Silk, and S. M. West PRL 2009



$$-2(1 + \sqrt{1 + a}) < l < 2(1 + \sqrt{1 - a})$$

Both particles experience blue shift, centre of mass frame is in free fall.

Acceleration of particles as universal property of rotating black holes

O. Z., PRD 2010

Role of horizon

Universality of black hole physics

Unified approach to nonextremal versus extremal black holes

Energy in CM frame

$$E_{c.m.}{}^{2} = -(m_1 u^{\mu}{}_{1} + m_2 u^{\mu}{}_{2})(m_1 u_{1\mu} + m_2 u_{2\mu})$$

$$E_{c.m.}{}^{2} = m^{2}{}_{1} + m_2{}^{2} + 2m_1 m_2 \gamma$$

$$\gamma = -(u_1 u_2)$$

equatorial plane $\theta = \frac{\pi}{2} \quad (z = 0)$ Is a symmetry one

$$mu_0 = -E \qquad mu_\phi = L$$

conserved quantities

Integrals of geodesic equations

$$g_{\mu\nu} u^{\mu} u^{\nu} = -1$$

$$m\dot{t} = mu^0 = \frac{E - \omega L}{N^2} = \frac{X}{N^2}.$$

$$X = E - \omega L$$

$$2m_1 m_2 \gamma = \frac{X_1 X_2 - \varepsilon_1 \varepsilon_2 Z_1 Z_2}{N^2} - \frac{L_1 L_2}{g_\phi},$$

$$Z = \sqrt{X^2 - N^2 (m^2 + \frac{L^2}{g_\phi})}$$

$\varepsilon = -1$ for particle moving towards horizon

$\varepsilon = +1$ away from horizon

$\varepsilon_1\varepsilon_2 = -1$ head-on collision, Piran et al

$$2m_1m_2\gamma = \frac{X_1X_2 + Z_1Z_2}{N^2} - \frac{L_1L_2}{g_\phi},$$

$E^2_{c.m.}$ always unbounded near horizon

For any relationship between energies and angular momenta

$$\varepsilon_1 = \varepsilon_2 = -1$$

Energy in CM frame

$$2m_1 m_2 \gamma = \frac{X_1 X_2 - Z_1 Z_2}{N^2} - \frac{L_1 L_2}{g_\phi},$$

$$Z = \sqrt{X^2 - N^2 (m^2 + \frac{L^2}{g_\phi})}$$

Three kinds of mechanism leading to unbounded energy in CM frame

1) $N \to 0$ proximity to horizons BSW

2) $L_2 \to -\infty$ inside ergoregion, NOT near horizon Grib and Pavlov, Kerr metric

Generalization OZ

3) $\omega \to \infty$ rapid rotation (wormholes)

$$\varepsilon_1 = \varepsilon_2 = -1$$

BSW

$$2m_1 m_2 \gamma = \frac{X_1 X_2 - Z_1 Z_2}{N^2} - \frac{L_1 L_2}{g_\phi},$$

$$Z = \sqrt{X^2 - N^2\left(m^2 + \frac{L^2}{g_\phi}\right)}$$

In general case, $E^2_{c.m.}$ remains bound in horizon limit $N \to 0$

Special conditions for unbounded $E^2_{c.m.}$

Two kinds of particles (trajectories)

Usual $X_H \equiv E - \omega_H L \neq 0$

Critical $X_H \equiv E - \omega_H L = 0$

Different limiting transitions

1) point of collision approaches the horizon, and $L_1 \to L_{1(H)} = \frac{E_1}{\omega_H}$

afterwards

2) $L_1 \to L_{1(H)}$ and $N \to 0$ afterwards

In both cases

$$\lim_{L_1 \to L_{1(H)}} \lim_{N \to 0} E_{cm} = \lim_{N \to 0} \lim_{L_1 \to L_{1(H)}} E_{cm} = \infty.$$

particle 1 is critical, particle 2 is usual

Extremal versus nonextremal

Problems with attaining extremality, a=0,998 (Thorne)

$$Z=\sqrt{X^2-N^2(m^2+\frac{L^2}{g_\phi})}$$

Nonextremal

Conditions of regularity: $X_H=O(N^2)$ for critical particle

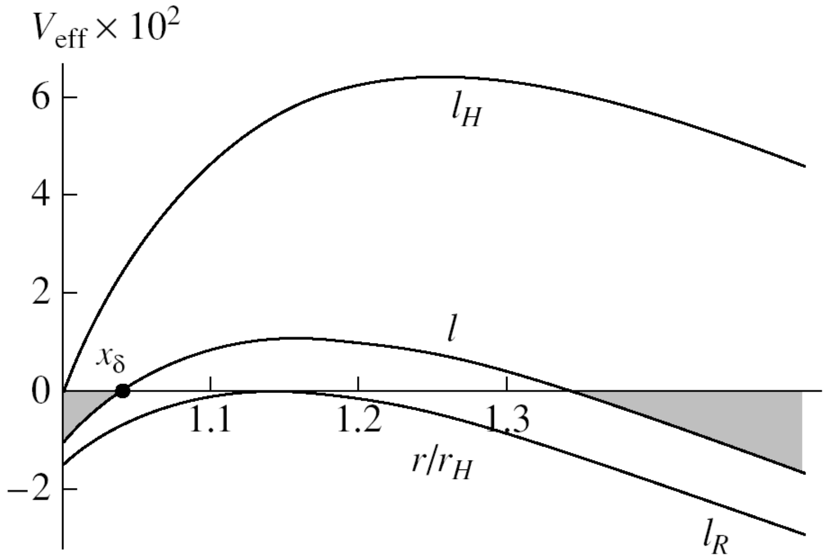
$$Z^2<0$$

For NBH, critical particle cannot reach horizon

Grib and Pavlov: nonextremal Kerr, O. Z. generalization

$$E_{c.m.}\approx\frac{m}{\sqrt{\delta}}\sqrt{\frac{2(L_H-L_2)}{1-\sqrt{1-a^2}}}$$

$L_1=L_{(H)}-\delta$ slightly noncritical



The effective potential for $A = 0.95$ and $l_R \approx 2.45$, $l = 2.5$, $l_H \approx 2.76$. Allowed zones for $l = 2.5$ are shown by the gray color.

Multiple scattering (Grib and Pavlov)

1) Particle 2 comes from infinity or is created in inner region

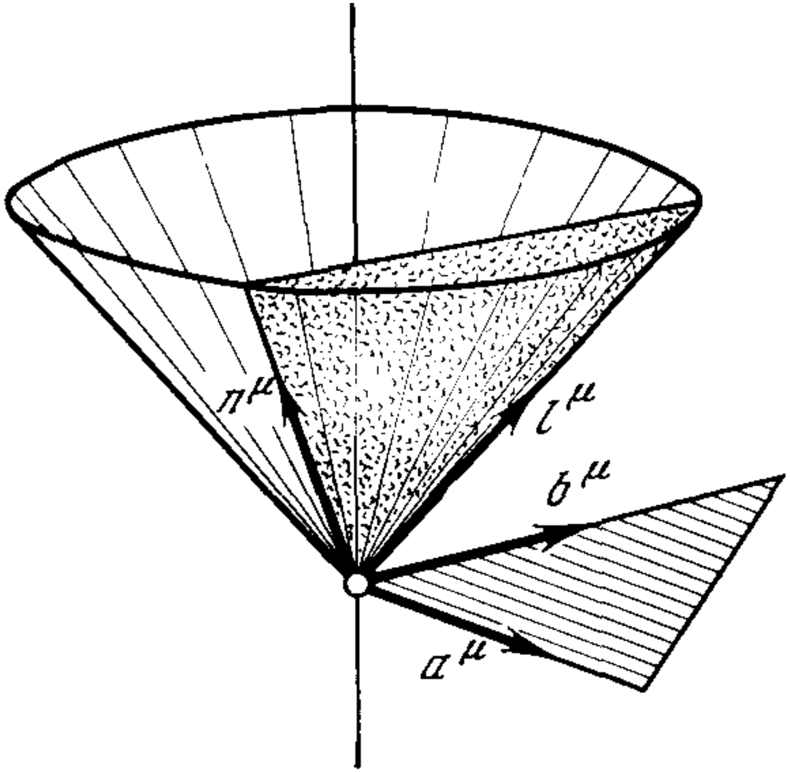
2) Collides with particle 2 there. Near-critical + usual

Geometric explanation

$\sigma_{\alpha\beta} = a_\alpha a_\beta + b_\alpha b_\beta$ lightlike vectors and N^μ

spacelike vectors a^μ, b^μ orthogonal to them

,



Four-velocity

$$u_i^\mu = \frac{l^\mu}{2\alpha_i} + \beta_i N^\mu + s_i^\mu, \; s_i^\mu = A_i a^\mu + B_i b^b$$

$$-(u_1 u_2) = \frac{1}{2}\left(\frac{\beta_1}{\alpha_2} + \frac{\beta_2}{\alpha_1}\right) - (s_1 s_2).$$

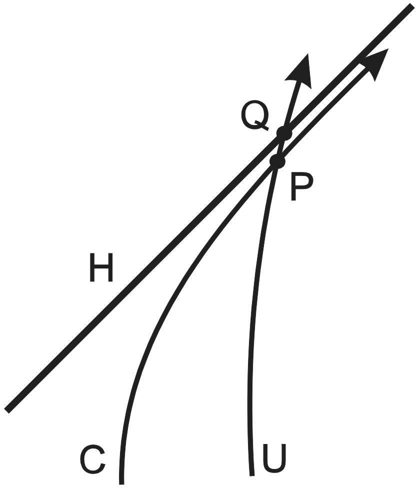
$$\alpha = 0$$

$$E_{c.m.}^2 = m_1^2 + m_2^2 - 2m_1 m_2 (u_1 u_2)$$

$$E_{c.m.}^2 = m_1^2 + m_2^2 + m_1 m_2 \left[\frac{\beta_1}{\alpha_2} + \frac{\beta_2}{\alpha_1} - 2(s_1 s_2)\right].$$

$\alpha_1 \to 0.$

Now, special condition



Kruskal-like coordinates

$$Cu^X u^Y = 1$$

$$u^X = O(\alpha) \to 0 \quad \tau = O(-\ln X) \to \infty$$

Proper time grows unbound (T. Jacobson,
Grib and Pavlov, O. Z.)

Kinematic explanation

$$E_{c.m.}^2 = -(p_1^\mu + p_2^\mu)(p_{1\mu} + p_{2\mu}) = m_1^2 + m_2^2 - 2m_1 m_2 u_1^\mu u_{2\mu}.$$

$$\gamma = -u_1^\mu u_{2\mu} = \frac{1}{\sqrt{1-w^2}}$$

BSW effect occurs if $w \to 1$ w is relative velocity

$$w^2 = 1 - \frac{(1-v_1^2)(1-v_2^2)}{[1 - v_1 v_2 (\vec{n}_1 \vec{n}_2)]^2}$$

The most interesting case: $\boldsymbol{v_1 < 1,}$ $v_2 \to 1$

Collision of rapid particle with target

Relative velocity close to c

$$ds^2 = -N^2 dt^2 + g_{\phi\phi}(d\phi - \omega dt)^2 + dl^2 + g_{zz} dz^2$$

Attached to observer

$$h_{(0)\mu} = -N(1,0,0,0),$$

$$h_{(1)\mu} = (0,1,0,0),$$

$$h_{(2)\mu} = \sqrt{g_{zz}}(0,0,0,1),$$

$$h_{(3)\mu} = \sqrt{g_{\varphi\varphi}}(-\omega,0,0,1)$$

$$-u_\mu h^\mu_{(0)} = \frac{E - \omega L}{N},$$

$$u_\mu h^\mu_{(3)} = \frac{L}{\sqrt{g_{\varphi\varphi}}}.$$

$$E - \omega L = \frac{mN}{\sqrt{1 - v^2}},$$

If

$$V_\mu = h_{\mu(0)}$$

then

$$V_\mu \xi^{(3)\mu} = 0$$

ZAMO

Horizon limit

1) Usual particle, $E \neq \omega_+ L$ $\quad v \to 1$

2) Critical particle $E = \omega_+ L$ $\quad v \to v_0 < 1$

Acceleration versus decceleration

Naïve expectation: to achieve large $E_{c.m.}$

we must have large velocities and individual energies.

No! The condition of criticality selects slow particle among all possible ones

$$E - \omega L = \frac{mN}{\sqrt{1-v^2}},$$

“Almost” any particle is rapid (usual one)

Special subset of slow particles is responsible for large energy in CM frame

Strong gravity ensures BSW effect since it almost “halts” this kind of particles.

Role of gravitational radiation

Naively: it bounds the growth of E in CM, restricts BSW effect

More careful inspection: under rather general assumptions (radial acceleration is finite in OZAMO frame, asimuthal force tends to zero not too slowly) the critical trajectories do exist. As a consequence, the BSW effect persists.

Details: I. V. Tanatarov and O. Z., **PRD 2013**

BSW effect survives!

Acceleration of particles by nonrotating charged black holes

O. Z. JETP Letters 2010

Role of rotation $L_1 = \frac{E_1}{\omega_H}$ If $\omega_H \to 0$ $L_1 \to \infty$

Angular momentum versus charge

Reissner-Nordstrom

Pure radial motion

$\omega_H = 0$ and $L_1 = L_2 = 0$

particles charged, nongeodesic motion

$$ds^2 = -dt^2 f + \frac{dr^2}{f} + r^2 d\omega^2.$$

$$f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$$

$$m^2 \dot{r}^2 = (E - \frac{qQ}{r})^2 - m^2 f.$$

$$mu^0 = m\dot{t} = \frac{1}{f}(E - \frac{qQ}{r}),$$

$$X_i = E_i - \frac{q_i Q}{r}, Z_i = \sqrt{X_i^2 - m^2 f}.$$

$$\frac{E_{cm}^2}{2m^2} = 1 + \frac{X_1 X_2 - Z_1 Z_2}{fm^2}$$

Rotating BH

1 critical + 1 usual

Static charged BH

Q

$$E^2_{c.m.} \approx \frac{const}{f}$$

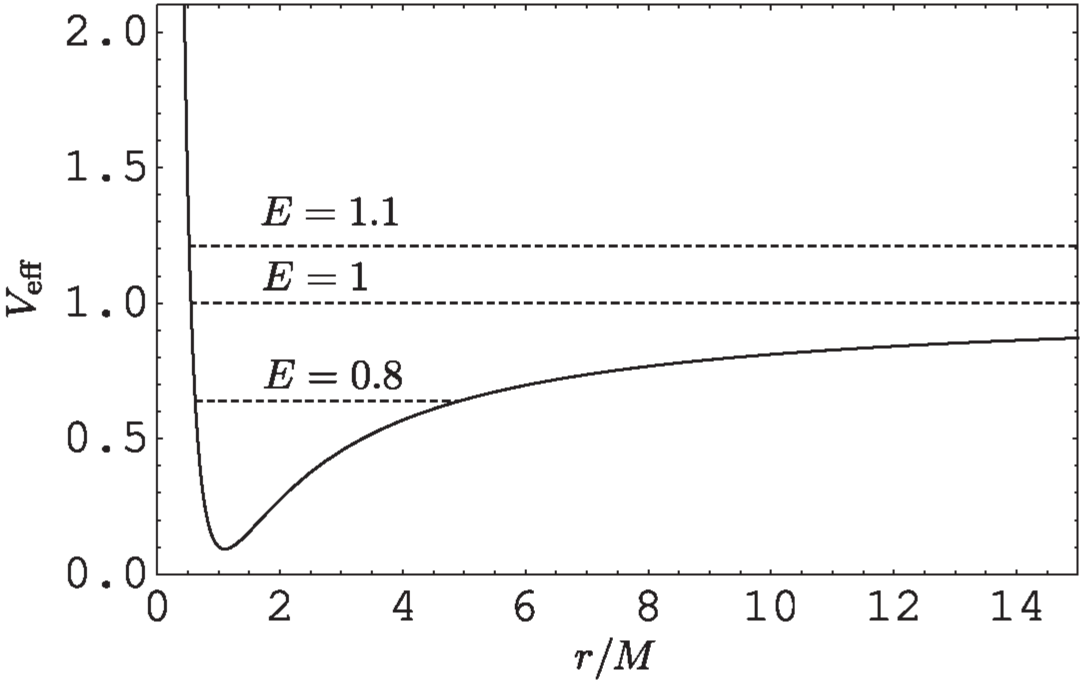
L

q

Alternative mechanisms of getting unbounded energies in CM frame

Patil, Joshi, Kimura, Nakao

RN metric, naked singularity



$$Q \approx M$$

$$Q > M$$

Black hole

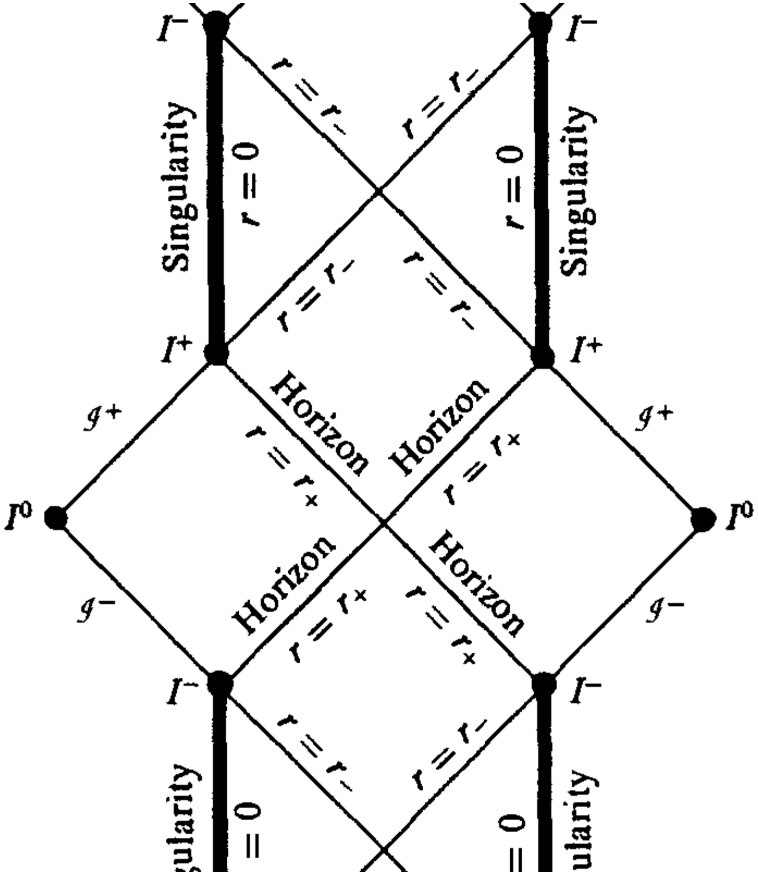
$$Q < M$$

Naked singularity

Small N

Small f in point of collision

Collisions near inner horizon



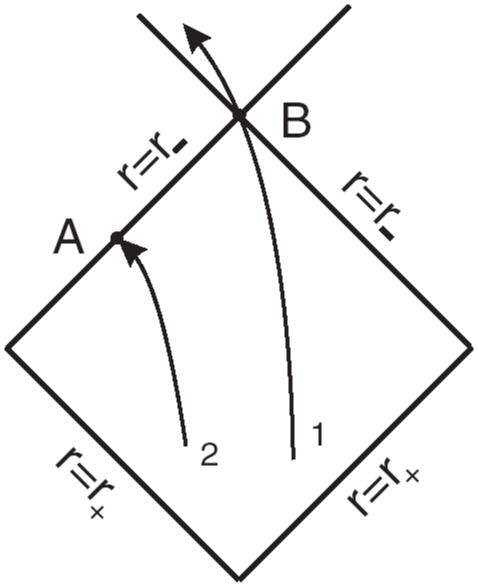
Two particles
(r,t)

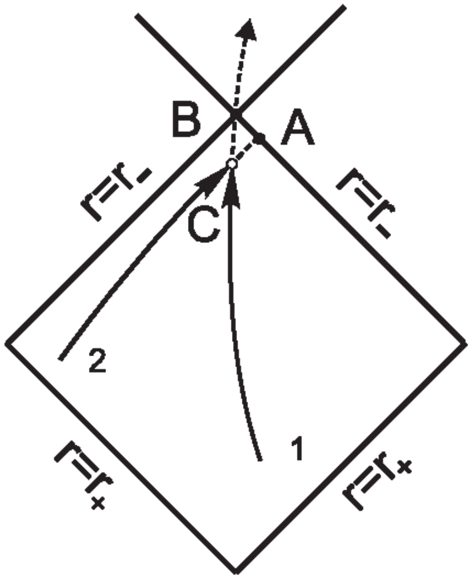
$$\lim_{r \to r_H} E_{c.m.}(r) = \infty$$

Inside:
Two different points with same r
(U,V) Kruskal coordinates.

Collisions near inner horizon

Again, one of two particle should be critical. Then, the following cases are possible.

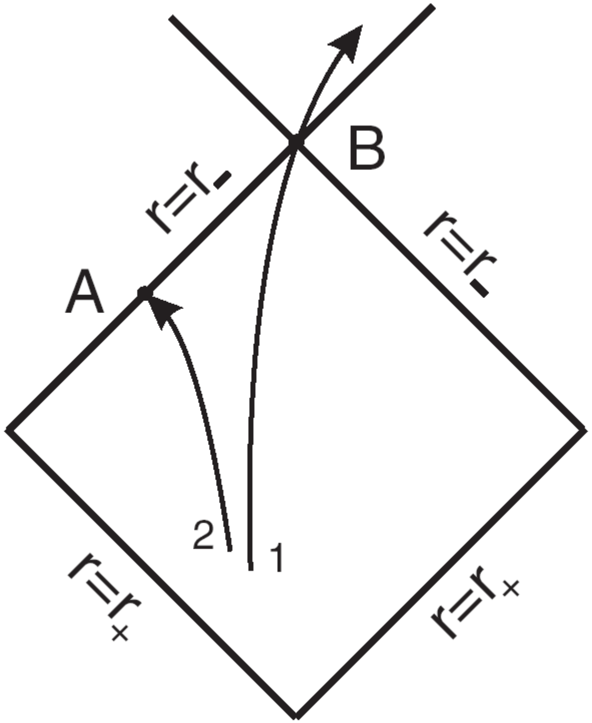




Kinematic censorship preserved

Fig. 1. Impossibility of strong version of BSW effect. Critical particle 1 passes through bifurcation point whereas usual one 2 hits left horizon

Fig. 2. The weak version of BSW effect. Near-horizon collision between Critical particle 1 and usual one 2.



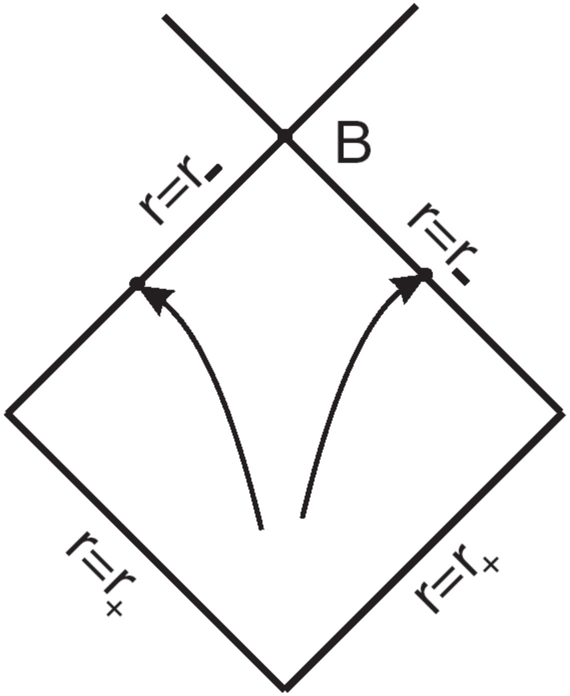


Fig. 4. Impossibility of strong version of PS effect. Two usual particles hit different branches of horizon.

Fig. 3. Impossibility of strong version. Critical particle 1 passes through bifurcation point, whereas
a usual one 2 hits left horizon.

Kinematic censorship

Kinematic censorship as general principle (Yu. Pavlov, O.Z.)

In any act of collision energy remains finite

Extremal black holes: infinite proper time

Nonextremal black hole, outside: interval shrinks to point

Nonextremal black hole, inside: two different branches of horizon

Part 2

High energy collisions near black holes and super-Penrose process

“Standard” Penrose process

Decay of particle $0 \longrightarrow 1 + 2$

$E_0 = E_1 + E_2$ $E_2 < 0$ $E_1 > E_0$

Efficiency $\eta = \dfrac{E_1 - E_0}{E_0}$ ergoregion

Collisional Penrose process

$$1 + 2 \longrightarrow 3 + 4$$

BSW process

Unbounded energy in the centre of mass (CM) frame

$E_{c.m.}$ versus Killiing energy measured at infinity

Even in spite of unbounded $E_{c.m.}$

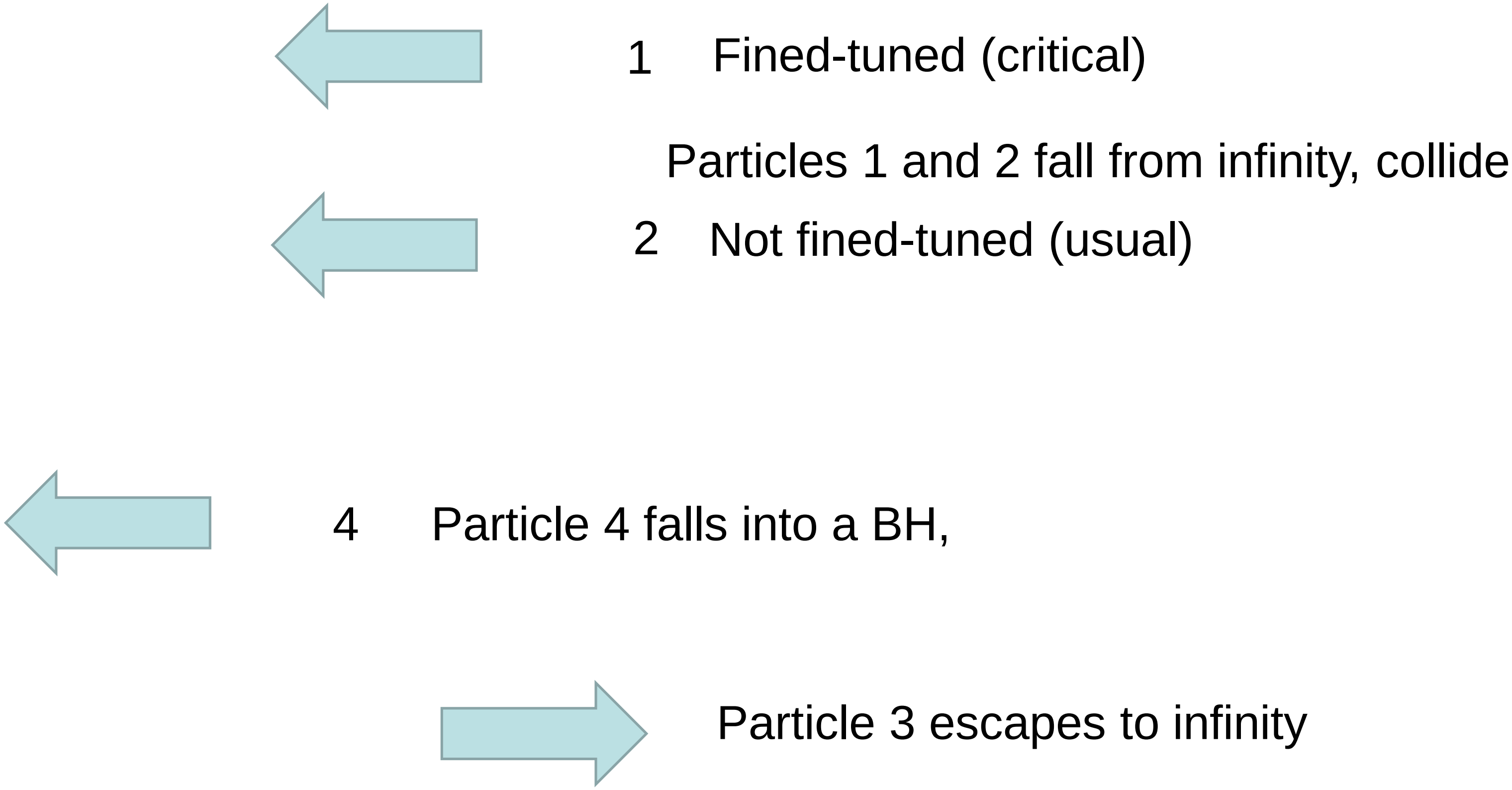
typically quite modest

Equatorial plane

Kerr	Excess less than 50 %	Mejer et al, 2012 Harada et al 2012
Dirty black holes		OZ 2012

Dirty = surrounded ny matter, NOT Kerr BH

Standard scenario.



1 Fined-tuned (critical)

Particles 1 and 2 fall from infinity, collide

2 Not fined-tuned (usual)

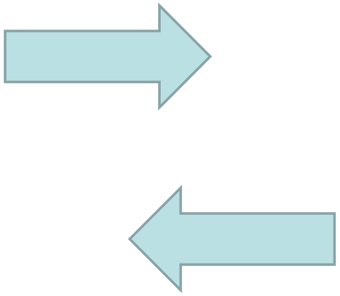
4 Particle 4 falls into a BH,

Particle 3 escapes to infinity

Particle 3 moves immediately after colliison
towards BH and bounces
or moves to inifinity at once

From analysis of conservation laws:
Particle 3 is critical or near-critical, particle 4 is usual

J. Schnittman (2014)



1 Near-critical moves from BH

2 Not fined-tuned (usual)

head-on collision

Amplification, factor about 14 Kerr, numerics

Harada et al 2015 Analytical derivation for Kerr

O.Z. dirty black holes, analytically

Unbounded efficiency (super-Penrose process)

Is it possible? Test particles approximation

E. Berti, R. Brito and V. Cardoso, 2015 Kerr, numerics

O. Z. 2015 Dirty BH, analytically

Head-on collision of usual particles

Near horizon, particle should move towards BH

White holes (Grib and Pavlov 2014)

or special scenario of multiple collisions in case of BH

Particle 1 (moves from BH) is usual

Unbounded efficiency (super-Penrose process)

E. Berti, R. Brito and V. Cardoso, 2015 Kerr, numerics

O. Z. 2015 Dirty BH, analytically

Near horizon, particle should move towards BH

White holes (Grib and Pavlov 2014)

or special scenario n case of BH

We can try to prepare required state for SPP (usual particle moving from BH)

Is it possible ot obtain it as a result of previous collision?

Full scenario

Step 1. Particles 1 and 2 ingoing: fall from infinity and collide near BH

Step 2. They produce usual outgoing particle 3

Step 3. Particle 3 collides with particle 4 falling from inifinity (head-on collision)

Result: particle 5 with unbounded energy moving to infinity

One of particles (say, 2) falling from infinity has to have mass (N is lapse funcit

$m_2 = O(N^{-2})$ Kerr metric, E. Leiderschneider and T. Piran 2015

General approach (O.Z., 2015)

$$ds^2 = -N^2dt^2 + g_\varphi(d\varphi - \omega dt)^2 + \frac{dr^2}{A} + g_\theta d\theta^2$$

Equatorial plane, redefine radial coordinate

Effective metric

$$ds^2 = -N^2dt^2 + g_\phi(d\phi - \omega dt)^2 + \frac{dr^2}{N^2}$$

Conservation laws

$E_{in} = E_{fin}$ $L_{in} = L_{fin}$ Consequence: $X_{in} = X_{fin}$

Let p particles collide and produce q new particles.

$$\sum_{i=1}^{p} \sigma_i Z_i = \sum_{k=1}^{q} \sigma_k Z_k .$$ radial momentum

Conservation laws + forward-in-time condition X>0

Near-horizon limit, $N_c \to 0$

Statement. If in the initial configuration usual outgoing particles are absent, they cannot appear after collision.

Previous statement applis to case with finite masses, etc.

For finite masses and angular momenta,
We cannot obtain a usual outgoing particle as a result of previous collision

If we relax this condition, it is possible to obtain a usual outgoing Particle, provided

$$m_2 = O(N^{-2})$$

Attempt to find loophole

Fractional degrees allow $X = O(N^s)$ $0 < s < 1$

Inconsistent with conservaiton laws

Generalizes observation of
E. Leiderschneider and T. Piran

Collision with a supermassive particle

Collision near past horizons (white holes)

BH is unsuitable for SP P

Super-Penrose process (naked singularity)

Both particles are ingoing and usual, come from inifnity. Particle 1 bounces back from potential barrier. Collides with particle 2.
Ingoing-→outgoing
Head-on collisions

Debris from head-on collision. Significant enhancement

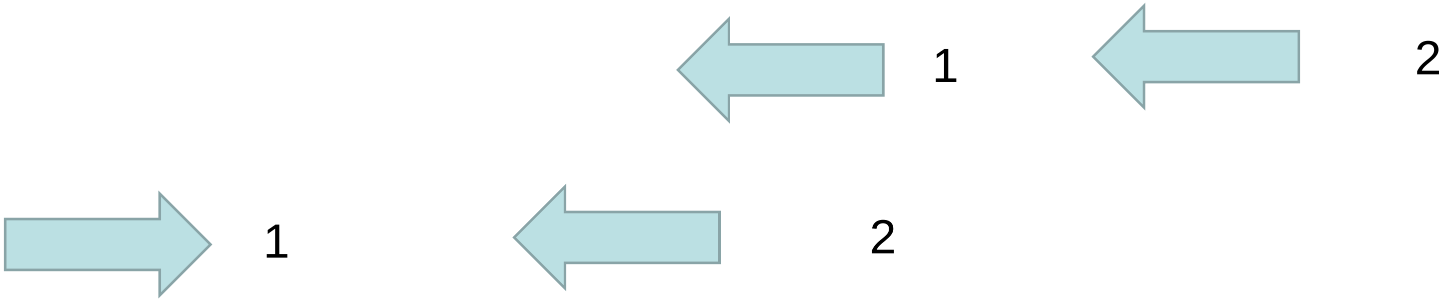
Critical particle moves away from black hole (outgoing)

Usual particle moves towards black hole

Outgoing usual particle
O. Z. (2014) analytically
V. Cardoso et al (2014) numeric findings

Super-Penrose process (naked singularity)

Both particles 1 and 2 are ingoing and usual, come from infinity.



Particle 1 bounces
back from potential barrier. Ingoing → outgoing

Head-on collision, SPP (OZ 2014)

Detailed description for Kerr. Harada et al, 2016

General approach, Tanatarov and O. Z. 2017

General approach Tanatarov and O. Z. (2017)

Wald inequalities

Particle with mass μ and Killing energy E decays into two massless fragments. Fragment's frequency measured at infinity ν_∞

its emitted frequency measured in the rest frame of the decaying particle ν

$$\frac{E}{\mu}-\sqrt{\frac{E^2}{\mu^2}+g_{tt}}\le\frac{\nu_\infty}{\nu}\le\frac{E}{\mu}+\sqrt{\frac{E^2}{\mu^2}+g_{tt}}$$

Wald 1974

Wald inequalities for collisional Penrose process

1+2 =compound particle, decays to massless 3 and 4

$$\mu = E_{c.m.} = 2\hbar\nu$$

$$E - \sqrt{E^2 + \mu^2 g_{tt}} \leq 2\hbar\nu_\infty \leq E + \sqrt{E^2 + \mu^2 g_{tt}}$$

Thus $\hbar\nu_\infty$ can be large (diverge) only if μ is large (diverging)

$\mu \to \infty$ $$\hbar\nu_\infty \approx \frac{\mu}{2}\sqrt{g_{tt}}$$

Concluisons

High energy collisions due to horizon

Role of critical trajectories

Force does not spoil effect

Rotating or charged BH

Universality

Energy of debris at infinity

Modest extraction in standard scenarios

Enhancement in head-on collision

SPP near BH is impossible

Near naked sing. possible

Alternative scenarios (far from horizon – large L or rapid rotation)

Thank you!