Corrections to the Higgs mass in the MSSM: resummation of bottom quarks contributions.

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Supersymmetry and the Higgs sector

Features:

- Solves hierarchy problem.
- Can predict the SM-like Higgs mass.
- Gauge coupling unification at $\sim 10^{16}\mathrm{GeV}$.
- Possible connection to super-gravity models and string theory.
- Natural candidate for DM.

No SUSY particles have been found so far $\Rightarrow$ SUSY particles could be heavy! ($\gtrsim 1\ \mathrm{TeV}$)

Higgs potential of the MSSM

$$V_H^{MSSM} = V_{Higgs}^{MSSM} + V_{breaking}^{MSSM},$$

$$V_{Higgs}^{MSSM} = \frac{1}{8}(g_1^2 + g_2^2)(|\mathcal{H}_1|^2 - |\mathcal{H}_2|^2)^2 + \frac{1}{2}g_2^2|\mathcal{H}_1^\dagger\mathcal{H}_2|^2 + |\mu|^2(|\mathcal{H}_1|^2 + |\mathcal{H}_2|^2),$$

$$V_{Higgs}^{breaking} = \tilde{m}_1^2|\mathcal{H}_1|^2 + \tilde{m}_2^2|\mathcal{H}_2|^2 + \left(m_{12}^2\mathcal{H}_1 \cdot \mathcal{H}_2 + \text{h.c.}\right)$$

$$\mathcal{H}_1 = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}(\phi_1 - i\chi_1) \\ -\phi_1^- \end{pmatrix} \quad \mathcal{H}_2 = e^{i\xi}\begin{pmatrix} \phi_2^+ \\ v_2 + \frac{1}{\sqrt{2}}(\phi_2 + i\chi_2) \end{pmatrix}$$

$$\tan\beta \equiv t_\beta = \frac{v_2}{v_1}, \quad v = \sqrt{v_1^2 + v_2^2} = 174\ \text{GeV}$$

- Higgs sector at tree-level determined by only two variables: M_A and $\tan\beta$
- Mass of the SM-like Higgs can be predicted

Higgs potential of the MSSM

At tree level:

$$(\phi_1, \phi_2) \xrightarrow{\alpha} (h, H)\,, \quad (\chi_1, \chi_2) \xrightarrow{\beta} (A, G^0)$$

$$m^2_{h,H} = \frac{1}{2}\left(M_A^2 + M_Z^2 \mp \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta}\right)$$

If $M_A \gg M_Z$:

$$m_h^2 \approx M_Z^2 c_{2\beta}^2 < (125\ \mathrm{GeV})^2\ \text{(tree level)}$$

Quantum corrections:

$$M_h^2 = m_h^2 + \Delta m_h^2, \quad M_h^2 \lesssim (135\ \mathrm{GeV})^2$$

Uncertainties:

$$\Delta M_h^{theo} \gtrsim (1 \ldots 2)\ \mathrm{GeV}$$
$$\Delta M_h^{exp} = 0.24\ \mathrm{GeV}\ [\mathrm{PDG} - 2017]$$

Radiative corrections: Feynman Diagrammatic Approach

Higgs masses at a given order = real part of poles of propagator matrix

$$\hat{\Gamma}_{hHA}(p^2) = i \begin{pmatrix} p^2 - m_h^2 + \hat{\Sigma}_{hh}(p^2) & \hat{\Sigma}_{hH}(p^2) & \hat{\Sigma}_{hA}(p^2) \\ \hat{\Sigma}_{Hh}(p^2) & p^2 - m_H^2 + \hat{\Sigma}_{HH}(p^2) & \hat{\Sigma}_{HA}(p^2) \\ \hat{\Sigma}_{Ah}(p^2) & \hat{\Sigma}_{AH}(p^2) & p^2 - m_A^2 + \hat{\Sigma}_{AA}(p^2) \end{pmatrix}$$

CP-violating case $\Rightarrow$ 3x3 matrix

Advantages

- Precise prediction if $M_{\text{Susy}} \sim m_t$
- Includes logarithmic, non-logarithmic, suppressed terms $\mathcal{O}(v^2/M_{\text{Susy}}^2)$

Problems

Large logarithms which need to be resummed if $M_{\text{Susy}} \gg m_t$.
For $M_A \sim M_{\text{Susy}} \gg v$:

$$M_h^2 = m_h^2 - \hat{\Sigma}_{hh}^{1L}(m_h^2) + \mathcal{O}(\alpha_t^2) \approx m_h^2 + \frac{3}{\pi}\alpha_t m_t^2 \log\frac{M_S^2}{m_t^2}, \quad M_S = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$$

Radiative corrections: EFT Approach

Haber, Hempfling 1993, Carena, Quiros, Wagner 1995

- ↪ Below M_S is the SM as an effective field theory, $M_h^2 = 2\lambda(m_t)v^2$
- ↪ Effects of SUSY particles are reflected in the boundary conditions for $\lambda(M_S)$:

$$\lambda(M_S) = \frac{1}{4}(g_1^2 + g_2^2)\cos^2 2\beta + \underbrace{\Delta_{t,b}\lambda + \Delta_{\text{heavy H}}\lambda + \Delta_{\text{EWino}}\lambda + \ldots}_{\text{Threshold corrections from SUSY particles}}$$

- ↪ $M_Z \ll M_\chi = M_1 = M_2 = |\mu| \leq M_s \Rightarrow$ **Split-model**
  E.Bagnaschi, G.F.Giudice, P.Slavich, A.Strumia 2014; *H.Bahl,W.Hollik 2016*
- ↪ $M_A \simeq m_t \ll M_s \Rightarrow$ **THDM**
  Haber, Hempfling 1993; *Lee, Wagner 2015*

Radiative corrections: EFT Approach

Renormalization group equation (RGE) for quartic coupling:

$$\frac{d\lambda}{d\log Q^2} = \beta_\lambda = k\beta_\lambda^{(1)} + k^2\beta_\lambda^{(2)} + \dots, \qquad k = \frac{1}{16\pi^2}$$

Solution:

$$\lambda(m_t) = \lambda(M_S) - k\beta_\lambda^{(1)}(M_S)L + k\beta_\lambda^{(1,1)}(M_S)\frac{L^2}{2} - k^2\beta_\lambda^{(2)}(M_S)L + \mathcal{O}(k^3), \quad L = \log\frac{M_S^2}{m_t^2}$$

$$\beta_g^{(1)} \to (kL)^n, n = 1, 2, \dots \to \textbf{Leading Logs (LL)}$$
$$\beta_g^{(2)} \to k(kL)^n, n = 1, 2, \dots \to \textbf{Next}-\textbf{to}-\textbf{Leading Logs (NLL)}$$
$$\text{1L threshold} \to \textbf{NLL}, \quad \text{2L threshold} \to \textbf{NNLL}$$

One-loop thresholds:

E. Bagnaschi, G. F. Giudice, P. Slavich and A. Strumia 2014

$$\Delta_t\lambda^{1L} = 6kh_t^4 s_\beta^4 \hat{X}_t^2\left(1 - \frac{\hat{X}_t^2}{12}\right), \quad \Delta_b\lambda^{1L} = 6kh_b^4 c_\beta^4 \hat{X}_b^2\left(1 - \frac{\hat{X}_b^2}{12}\right)$$

where $h_{t,b}$ are MSSM Yukawa couplings ($h_t = \frac{y_t}{s_\beta}$, $h_b = \frac{y_b}{c_\beta|1+\Delta_b|}$),
$X_t = A_t - \mu/t_\beta$, $X_b = A_b - \mu t_\beta$, $\hat{X}_{t,b} = X_{t,b}/M_{\mathrm{Susy}}$.

Radiative corrections: Hybrid Approach

- EFT calculations allow to resum large logarithms
  $\rightarrow$ should be accurate for high SUSY scale M_{Susy}
- miss however suppressed terms $\sim \frac{v}{M_{\mathrm{Susy}}}$
- diagrammatic calculation expected to be more accurate for low M_{Susy} ($\lesssim$ few TeV)

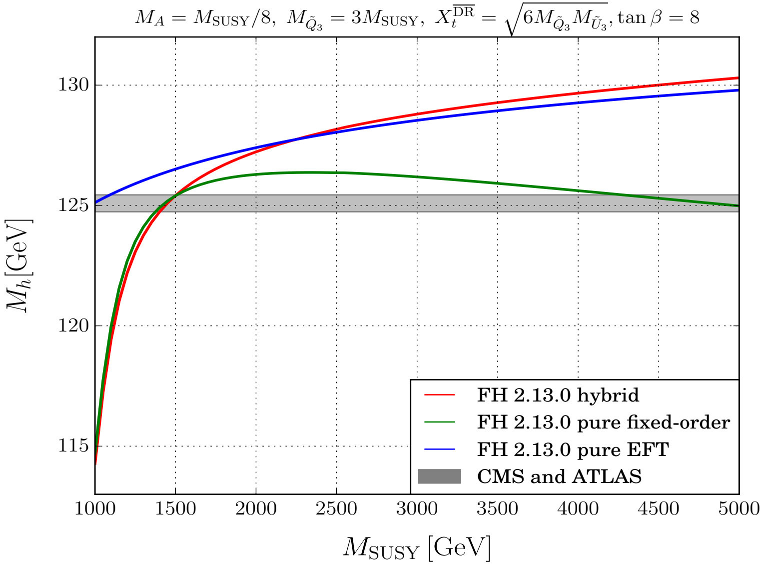
Idea: Combine both!

$$M_h^2 = (M_h^2)^{\mathrm{FO}} + \Delta M_h^2$$

$$\Delta M_h^2 = (\Delta M_h^2)^{\mathrm{EFT}}(A_i^{\overline{\mathrm{MS}}}) - (\Delta M_h^2)^{\mathrm{EFT,\ non-log}}(A_i^{\mathrm{OS}}) - (\Delta M_h^2)^{\mathrm{FO,\ logs}}(A_i^{\mathrm{OS}})$$

- Have to avoid double-counting of 1L and 2L logarithms as well as non-logarithmic terms
- EFT uses $\overline{\mathrm{DR}}$, FO uses $\overline{\mathrm{DR}}$/OS $\Rightarrow$ parameter conversion needed

Fixed order vs EFT. Example



Δ_b corrections in the MSSM.

Tree-level **MSSM = type II 2HDM**:

$$\mathcal{L}_y \;=\; -y_u^{ij}\,\bar{u}_R^i Q_j^T \epsilon H_u \;+\; y_d^{ij}\,\bar{d}_R^i Q_j^T \epsilon H_d \;+\; \text{h.c.}$$

Quantum corrections when $M_{SUSY} \gg v \Rightarrow$ **MSSM = general 2HDM**

$$\mathcal{L}_{y,d}^{\text{eff}} \;=\; y_{d_i}\,\bar{d}_R^i Q_i^T \epsilon H_d \;-\; \tilde{y}_d^{ij}\,\bar{d}_R^i Q_j^T H_u^* \;+\; \text{h.c.}$$

Relation between physical mass and Yukawa coupling is modified:

$$y_{d_i} = \frac{m_{d_i}}{v_d} \Rightarrow y_{d_i} = \frac{m_{d_i}}{v_d(1+\Delta_{d_i})},$$

$$\Delta_{d_i} = -\epsilon_i t_\beta = -\frac{2\alpha_s}{3\pi} M_{\tilde{g}}\mu t_\beta\, C_0(M_{\tilde{g}}, m_{\tilde{d}_1^i}, m_{\tilde{d}_2^i}) + \ldots$$

$M_{\tilde{g}} = \mu = m_{\tilde{d}_L^i} = m_{\tilde{d}_R^i}$:

$$\Delta_{d_i} \simeq \text{sgn}(M_{\tilde{g}}\mu)\frac{\alpha_s}{3\pi} t_\beta$$

Large $\tan\beta$ & $\text{sgn}(M_{\tilde{g}}\mu) < 0$:

- $\hookrightarrow$ Bottom Yukawa is enhanced.
- $\hookrightarrow$ $\hat{X}_b^2 \sim t_\beta^2$, 2-loop next-to-leading logs are polynomials in $\hat{X}_b$.
- $\hookrightarrow$ **Corrections from bottom-type quarks can be relevant!**

FeynHiggs code

- Code for calculation of masses, mixings, decay rates, branching ratios,electroweak precision observables and flavour observables in the MSSM.
- Written in `Fortran`. Can be called from `Mathematica` or used as a library for `Fortran/C++` code. Current version `2.14.1`.
- Works with real and complex parameters.
- Mixed $\overline{\text{DR}}$/OS renormalization scheme in the FO result.

- **Fixed order**: Full 1L corrections and 2L corrections $\mathcal{O}(\alpha_s\alpha_t, \alpha_s\alpha_b, (\alpha_t+\alpha_b)^2)$ in the limit $(g_1, g_2 \to 0, p^2 \to 0)$
- **Hybrid approach**: Full LL,NLL and $\mathcal{O}(\alpha_s\alpha_t)$ NNLL resummation (assuming $M_A = M_{\text{Susy}}$). Independent intermediate electroweakino and gluino thresholds are allowed.

Parameter conversion.

- $\hat{X}_{t,b}^{\rm OS}$ is used in 1,2-loop self-energies. $\hat{X}_{t,b}^{\overline{\rm DR}}$ is used in 1-loop thresholds.

$$\Delta_t \lambda^{1L} = 6kh_t^4 s_\beta^4 \hat{X}_t^2 \left(1 - \frac{\hat{X}_t^2}{12}\right), \quad \Delta_b \lambda^{1L} = 6kh_b^4 c_\beta^4 \hat{X}_b^2 \left(1 - \frac{\hat{X}_b^2}{12}\right)$$

-

$$\hat{X}_t^{\overline{\rm DR}} = \hat{X}_t^{\rm OS}\left[1 + \left(\frac{\alpha_s}{\pi} + \frac{3}{16\pi}\left(\alpha_b\left(1+\hat{X}_b^2\right) - \alpha_t\left(1-\hat{X}_t^2\right)\right)\right)\log\frac{M_S^2}{m_t^2}\right]$$

$$\hat{X}_b^{\overline{\rm DR}} = \hat{X}_b^{\rm OS}\left[1 + \left(\frac{\alpha_s}{\pi} + \frac{3}{16\pi}\left(\alpha_t\left(1+\hat{X}_t^2\right) - \alpha_b\left(1-\hat{X}_b^2\right)\right)\right)\log\frac{M_S^2}{m_t^2}\right]$$

- $\hat{y}_b$ is used in 1,2-loop self-energies
  A. Dedes, G. Degrassi, P. Slavich 2003

$$\hat{y}_b = \frac{\sqrt{2}\overline{m}_b(m_t)}{v}\frac{1+\delta_b}{|1+\Delta_b|} - \delta y_b, \quad \delta y_b = y_b\left(\frac{\delta m_{\tilde{b}_1}^2 - \delta m_{\tilde{b}_2}^2}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2} + \frac{\delta s_{2\theta_b}}{s_{2\theta_b}} - \frac{\delta v}{v}\right)$$

  $\delta_b, \delta y_b$ **involve large logs!**

- $\frac{\sqrt{2}\overline{m}_b(M_{Susy})}{\overline{v}}\frac{1}{|1+\Delta_b|}$ is used in 1-loop thresholds.
- Conversion of y_b (see backup):

$$\frac{\sqrt{2}\overline{m}_b(M_{Susy})}{\overline{v}}\frac{1}{|1+\Delta_b|} = \hat{y}_b\left(1 + \delta_1 k + \delta_2 kL\right)$$

Subtraction terms

$$\lambda^{2L,LL}(m_t) = \left(9(\hat{y}_b^2 - y_t^2)^2(\hat{y}_b^2 + y_t^2) - 48 g_3^2(\hat{y}_b^4 + y_t^4)\right)(kL)^2$$

$$= (-48 g_3^2 \hat{y}_b^4 + 9\hat{y}_b^6 - 9\hat{y}_b^4 y_t^2 - 48 g_3^2 y_t^4 - 9\hat{y}_b^2 y_t^4 + 9 y_t^6)(kL)^2$$

$$\lambda^{2L,NLL}(m_t) = 6 y_t^4 \left[g_3^2 \left(8\hat{X}_t^2 - \frac{4}{3}\hat{X}_t^4\right) + \left(\frac{3\hat{X}_t^2}{2} - \frac{\hat{X}_t^4}{4}\right)\left(\hat{y}_b^2\left(1+\hat{X}_b^2\right) - y_t^2\left(1-\hat{X}_t^2\right)\right) + \right.$$

$$\left. + \left(3y_t^2 + 3\hat{y}_b^2 - 16 g_3^2\right)\left(\hat{X}_t^2 - \frac{1}{12}\hat{X}_t^4\right) - \left(5y_t^2 - \hat{y}_b^2 - \frac{16}{3}g_3^2\right)\right] k^2 L +$$

$$+ 6\hat{y}_b^4 \left[g_3^2 \left(8\hat{X}_b^2 - \frac{4}{3}\hat{X}_b^4\right) + \left(\frac{3\hat{X}_b^2}{2} - \frac{\hat{X}_b^4}{4}\right)\left(y_t^2\left(1+\hat{X}_t^2\right) - \hat{y}_b^2\left(1-\hat{X}_b^2\right)\right) + \right.$$

$$\left. + \left(3y_t^2 + 3\hat{y}_b^2 - 16 g_3^2\right)\left(\hat{X}_b^2 - \frac{1}{12}\hat{X}_b^4\right) - \left(3\hat{y}_b^2 - y_t^2 - \frac{16}{3}g_3^2\right)\right] k^2 L$$

$$\delta\lambda = 24\hat{y}_b^4 k^2 L\left(\delta_1 + \delta_2 L\right) + 24\hat{y}_b^4 k^2\left(\delta_1 + \delta_2 L\right)\hat{X}_b^2\left(1 - \frac{\hat{X}_b^2}{12}\right)$$

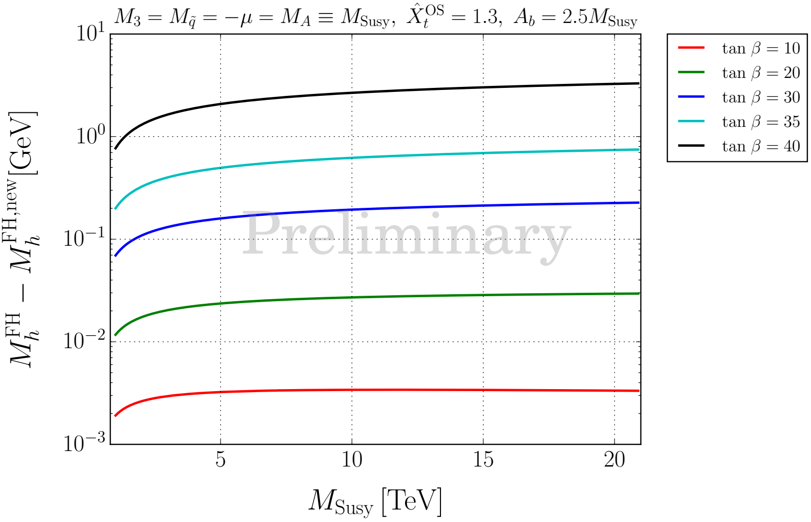
$$\hat{X}_t \equiv \frac{X_t^{OS}}{M_S}, \quad \hat{X}_b \equiv \frac{X_b^{OS}}{M_S}, \quad k = \frac{1}{16\pi^2}, \quad L \equiv \log\frac{M_S^2}{m_t^2}$$

Numerical example.

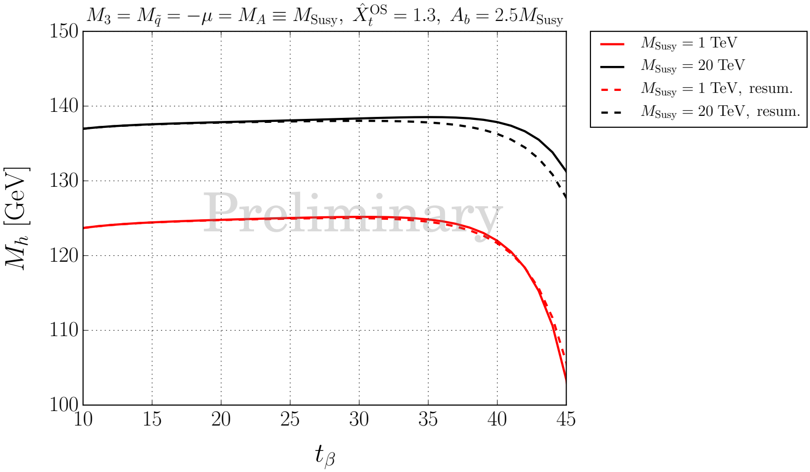
Single scale scenario:

$$M_A = M_{1,2,3} = |\mu| = M_{\text{Susy}}$$
$$A_b = 2.5\ M_{\text{Susy}}$$
$$M_{\tilde{Q}_3} = M_{\tilde{U}_3} = M_{\tilde{D}_3} = \ldots = M_{\text{Susy}}$$
$$m_t^{\text{pole}} = 173.5\ \text{GeV}\ [\text{PDG} - 2017]$$

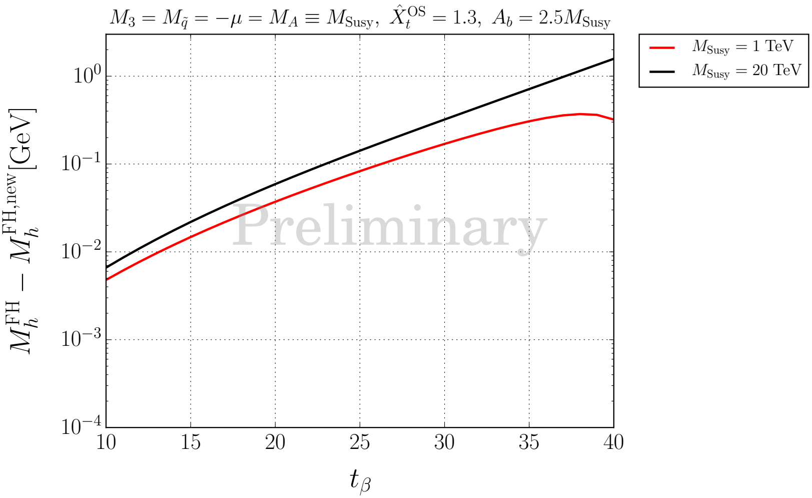
Numerical example.



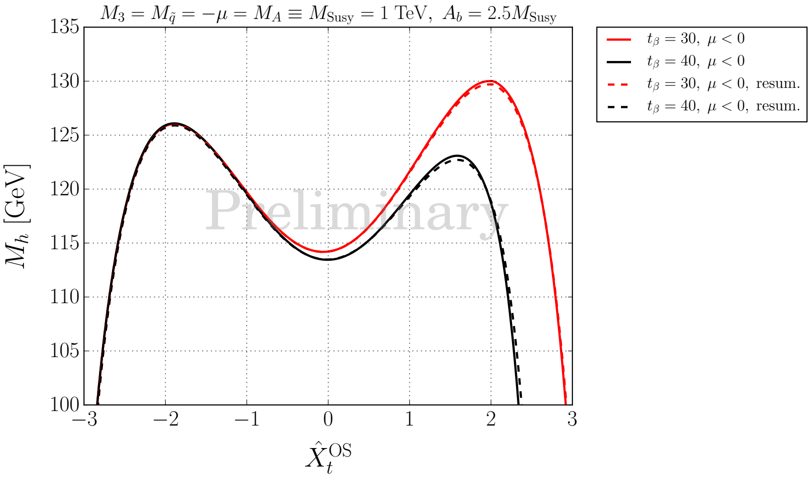
Numerical example.



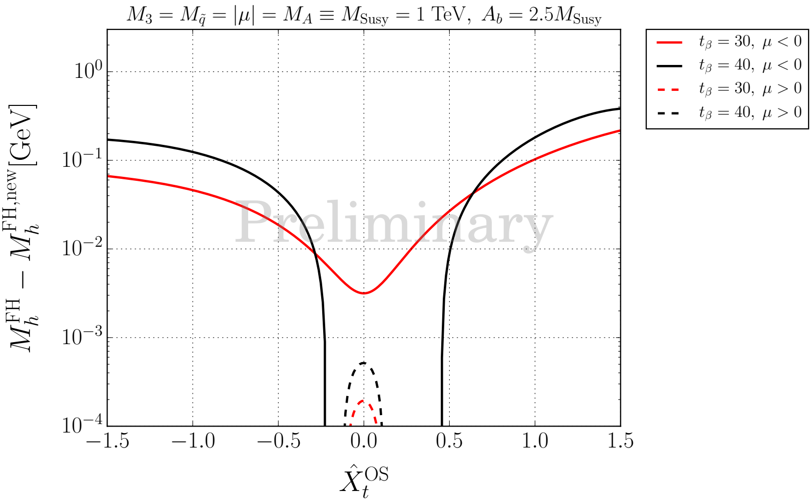
Numerical example.



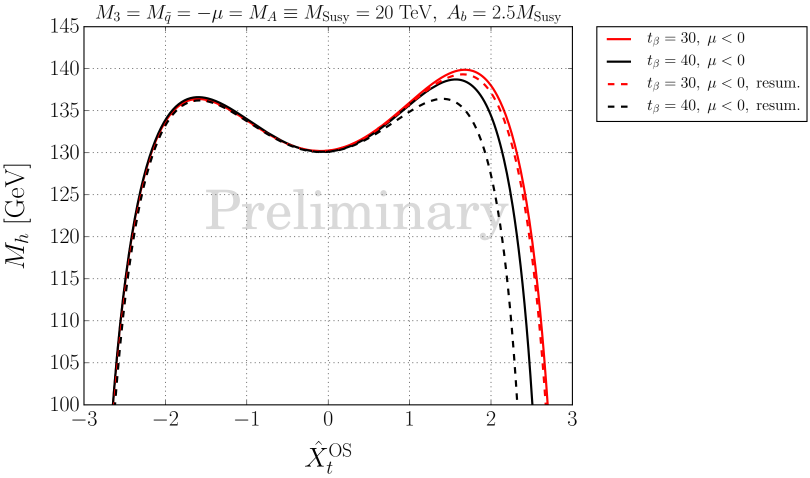
Numerical example.



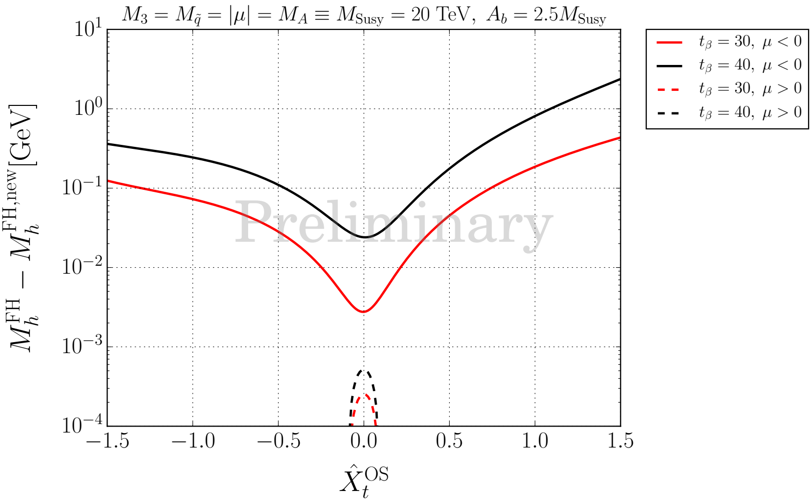
Numerical example.



Numerical example.



Numerical example.



Summary and Outlook

What is done so far?

- LL,NLL resummation of $\mathcal{O}(\alpha_b)$, $\mathcal{O}(\alpha_b\alpha_s, \alpha_b\alpha_t)$ contributions is implemented into `FeynHiggs` code.
- Preliminary results seem to indicate that effect can be fairly large $\sim \mathcal{O}(1\ \mathrm{GeV})$ in the large $\tan\beta$ regime

What comes next?

↪ Implementation of resummation of $\sim m_b^2 \alpha_{\mathrm{EW}} \log \frac{M_S^2}{m_t^2}$.

↪ Implementation of τ-lepton contributions in the hybrid approach.

↪ Implementation of NNLL resummation of $\mathcal{O}(\alpha_b)$, $\mathcal{O}(\alpha_b\alpha_s, \alpha_b\alpha_t)$ contributions.

Backup slides

Δ_b corrections in the MSSM. t_β resummation.

Bottom mass counter-term in the OS scheme:
L.Hofer, U.Nierste, D.Scherer 2009.

$$\delta m_b = \frac{m_b}{2}[\Sigma_b^{LL}(m_b^2) + \Sigma_b^{RR}(m_b^2)] + \Sigma_b^{RL}(m_b^2),$$
$$\Sigma_b(p) = \not{p}\left[\Sigma_b^{LL}(p^2)P_L + \Sigma_b^{RR}(p^2)P_R\right] + \Sigma_b^{RL}(p^2)P_L + \Sigma_b^{LR}(p^2)P_R$$

When $t_\beta \to \infty$: $\delta m_b = \Sigma_b^{RL}(m_b^2) \simeq \Sigma_b^{RL}(0) := m_b\Delta_b = m_b\epsilon_i t_\beta \Rightarrow \delta y_b = y_b\epsilon_i t_\beta$.

1. δy_b **is UV finite** *when* $t_\beta \to \infty$
2. *There are no* t_β*-enhanced contributions to* δy_b *from genuine multiloop diagrams M.Carena, D.Garcia, U.Nierste, C.Wagner 2000.*

$$\delta y_b^{(1)} = y_b\epsilon_i t_\beta,\ \delta y_b^{(2)} = \delta y_b^{(1)}\epsilon_i t_\beta, \ldots, \delta y_b^{(n+1)} = \delta y_b^{(n)}\epsilon_i t_\beta,$$
$$y_b^{(0)} = y_b + \delta y_b^{(1)} + \delta y_b^{(2)} + \ldots = y_b\left(1 + \epsilon_i t_\beta + (\epsilon_i t_\beta)^2 + \ldots\right) = \frac{y_b}{1-\epsilon_i t_\beta}$$

2L RGEs in the gaugeless limit

D.Buttazzo, G.Degrassi, P.P.Giardino, G.F.Giudice, F.Sala, A.Salvio, A.Strumia 2013

$$\frac{dg_3}{d\log Q^2} = \frac{1}{2} g_3^3 k \left[-7 + k\left(-26 g_3^2 - 2y_t^2 - 2y_b^2\right)\right]$$

$$\frac{dy_t}{d\log Q^2} = \frac{1}{2} y_t k \left[\frac{9}{2} y_t^2 + \frac{3}{2} y_b^2 - 8g_3^2 + k\left(6y_t^2\left(6g_3^2 - 2y_t^2 - \frac{11}{24} y_b^2 - \lambda\right) + \right.\right.$$
$$\left.\left. + y_b^2\left(-\frac{1}{4} y_b^2 + 4g_3^2\right) + \frac{3}{2}\lambda^2 - 108 g_3^4\right)\right]$$

$$\frac{dy_b}{d\log Q^2} = \frac{1}{2} y_b k \left[\frac{9}{2} y_b^2 + \frac{3}{2} y_t^2 - 8g_3^2 + k\left(6y_b^2\left(6g_3^2 - 2y_b^2 - \frac{11}{24} y_t^2 - \lambda\right) + \right.\right.$$
$$\left.\left. + y_t^2\left(-\frac{1}{4} y_t^2 + 4g_3^2\right) + \frac{3}{2}\lambda^2 - 108 g_3^4\right)\right]$$

$$\frac{d\lambda}{d\log Q^2} = k\left[6\left(\lambda^2 + \lambda y_t^2 + \lambda y_b^2 - y_t^4 - y_b^4\right) + \frac{k}{2}\left(y_t^2(60 y_t^4 - 3y_t^2\lambda + 80 g_3^2\lambda \right.\right.$$
$$- 64 g_3^2 y_t^2 - 72\lambda^2 - 42 y_b^2\lambda - 12 y_t^2 y_b^2) + y_b^2(-72\lambda^2 - 3y_b^2\lambda + 80 g_3^2\lambda + 60 y_b^4 -$$
$$\left.\left. - 12 y_b^2 y_t^2 - 64 y_b^2 g_3^2) - 78\lambda^3\right)\right]$$

Analytical solutions to 1L RGEs

$\sim y_b^2$ terms are neglected!

$$\frac{g_3^2(M_{\rm Susy})}{g_3^2(m_t)} = \frac{1}{1+7k\log\frac{M_{\rm Susy}^2}{m_t^2}}$$

$$\frac{y_t^2(M_{\rm Susy})}{y_t^2(m_t)} = \frac{2g_3^2(m_t)}{\left(2g_3^2(m_t)-9y_t^2(m_t)\right)\left(1+7k\log\frac{M_{\rm Susy}^2}{m_t^2}\right)^{\frac{8}{7}}+9y_t^2(m_t)\left(1+7k\log\frac{M_{\rm Susy}^2}{m_t^2}\right)}$$

$$\frac{y_b^2(M_{\rm Susy})}{y_b^2(m_t)} = \frac{\left(1+7k\log\frac{M_{\rm Susy}^2}{m_t^2}\right)^{-\frac{8}{7}}}{\left(1-\frac{9y_t^2(m_t)}{2g_3^2(m_t)}\left(1-\left(1+7k\log\frac{M_{\rm Susy}^2}{m_t^2}\right)^{-\frac{1}{7}}\right)\right)^{\frac{1}{3}}}$$

$v_{\text{OS}} \longrightarrow v_{\overline{\text{MS}}}$ conversion

D.Buttazzo, G.Degrassi, P.P.Giardino, G.F.Giudice, F.Sala, A.Salvio, A.Strumia 2013

$$v_{\overline{\text{MS}}}^2 = v_{\text{OS}}^2 + \frac{3}{(4\pi)^2}\Big[m_t^2 - 2A_0(m_t^2) - \frac{1}{6}\left(2m_w^2 + m_z^2 + M_h^2\right) + \\ + \frac{m_w^2}{M_h^2 - m_w^2}A_0(M_h^2) + \left(\frac{4}{3} - \frac{c_w^2}{s_w^2}\right)A_0(m_z^2) + \\ + \left(\frac{11}{3} + \frac{c_w^2}{s_w^2} - \frac{M_h^2}{M_h^2 - m_w^2}A_0(m_w^2)\right)\Big]$$

Two-loop $\hat{y}_b$ at large $M_{\rm Susy}$

$$\delta_b = \frac{4}{3} g_s^2 k \left(\frac{A_b}{M_{\rm Susy}} - \log \frac{M_{\rm Susy}^2}{m_t^2} \right) + \frac{y_t^2 k}{2} \left(\frac{7}{4} - \frac{5}{2} \log \frac{M_{\rm Susy}^2}{m_t^2} \right) +$$

$$+ \frac{y_b^2 k}{2 c_\beta^2} \left(\frac{3}{4} - 3 \log \frac{M_{\rm Susy}^2}{m_t^2} \right)$$

$$\frac{\delta v}{v} = -\frac{k}{4} \left[y_t^2 \left(3 + \left(\frac{A_t}{M_{\rm Susy}} \right)^2 \right) + \frac{y_b^2}{c_\beta^2} \right]$$

Two-loop $\hat{y}_b$ at large $M_{\rm Susy}$

$$\delta y_b = \frac{8}{3} g_s^2 k \log\frac{M_{\rm Susy}^2}{m_t^2} + y_t^2 k \Bigg[\frac{5}{4} + \frac{4\pi}{3\sqrt{3}} - \left(\frac{1}{2} - \frac{3}{4}\frac{A_t^2}{M_{\rm Susy}^2} \right) \log\frac{M_{\rm Susy}^2}{m_t^2} +$$

$$+ \frac{5}{4}\frac{A_t^2}{M_{\rm Susy}^2} + \frac{\pi}{3\sqrt{3}}\frac{A_t A_b}{M_{\rm Susy}^2} -$$

$$- \frac{1}{4}\frac{A_t}{M_{\rm Susy}} \left(\frac{A_t}{M_{\rm Susy}} + \frac{2\overline{m}_b}{\overline{m}_t c_\beta} \right) \left(1 + \frac{A_t}{M_{\rm Susy}}\frac{\overline{m}_t c_\beta}{\overline{m}_b} \right) \log\left| \frac{A_t}{M_{\rm Susy}} + \frac{\overline{m}_b}{\overline{m}_t c_\beta} \right| -$$

$$- \frac{1}{4}\frac{A_t}{M_{\rm Susy}} \left(\frac{A_t}{M_{\rm Susy}} - \frac{2\overline{m}_b}{\overline{m}_t c_\beta} \right) \left(1 - \frac{A_t}{M_{\rm Susy}}\frac{\overline{m}_t c_\beta}{\overline{m}_b} \right) \log\left| \frac{A_t}{M_{\rm Susy}} - \frac{\overline{m}_b}{\overline{m}_t c_\beta} \right| \Bigg] +$$

$$+ \frac{y_b^2}{c_\beta^2} \Bigg[\frac{7}{4} - \frac{A_b^2}{M_{\rm Susy}^2} \left(\frac{3}{2} - \frac{\pi}{\sqrt{3}} \right) - \log\frac{2\overline{m}_b}{\overline{m}_t c_\beta} - \frac{11}{4}\log\frac{M_{\rm Susy}^2}{m_t^2} -$$

$$- \frac{1}{4}\left(1 + \frac{A_t}{M_{\rm Susy}}\frac{\overline{m}_t c_\beta}{\overline{m}_b} \right) \log\left| \frac{A_t}{M_{\rm Susy}} + \frac{\overline{m}_b}{\overline{m}_t c_\beta} \right| -$$

$$- \frac{1}{4}\left(1 - \frac{A_t}{M_{\rm Susy}}\frac{\overline{m}_t c_\beta}{\overline{m}_b} \right) \log\left| \frac{A_t}{M_{\rm Susy}} - \frac{\overline{m}_b}{\overline{m}_t c_\beta} \right| \Bigg]$$

δ_1 and δ_2 coefficients.

$$\delta_1 = \frac{4}{3} g_s^2 \left(\frac{A_b}{M_{\mathrm{Susy}}} - 1 \right) + y_t^2 \left[\frac{3}{8} - \frac{4\pi}{3\sqrt{3}} - \frac{A_t^2}{M_{\mathrm{Susy}}^2} - \frac{\pi}{3\sqrt{3}} \frac{A_t A_b}{M_{\mathrm{Susy}}^2} + \right.$$

$$+ \frac{1}{4} \frac{A_t}{M_{\mathrm{Susy}}} \left(\frac{A_t}{M_{\mathrm{Susy}}} + \frac{2\overline{m}_b}{\overline{m}_t c_\beta} \right) \left(1 + \frac{A_t}{M_{\mathrm{Susy}}} \frac{\overline{m}_t c_\beta}{\overline{m}_b} \right) \log \left| \frac{A_t}{M_{\mathrm{Susy}}} + \frac{\overline{m}_b}{\overline{m}_t c_\beta} \right| +$$

$$\left. + \frac{1}{4} \frac{A_t}{M_{\mathrm{Susy}}} \left(\frac{A_t}{M_{\mathrm{Susy}}} - \frac{2\overline{m}_b}{\overline{m}_t c_\beta} \right) \left(1 - \frac{A_t}{M_{\mathrm{Susy}}} \frac{\overline{m}_t c_\beta}{\overline{m}_b} \right) \log \left| \frac{A_t}{M_{\mathrm{Susy}}} - \frac{\overline{m}_b}{\overline{m}_t c_\beta} \right| \right] +$$

$$+ \frac{y_b^2}{c_\beta^2} \left[-1 + \frac{A_b^2}{M_{\mathrm{Susy}}^2} \left(\frac{3}{2} - \frac{\pi}{\sqrt{3}} \right) + \log \frac{2\overline{m}_b}{\overline{m}_t c_\beta} + \right.$$

$$\left. + \frac{1}{4} \left(1 + \frac{A_t}{M_{\mathrm{Susy}}} \frac{\overline{m}_t c_\beta}{\overline{m}_b} \right) \log \left| \frac{A_t}{M_{\mathrm{Susy}}} + \frac{\overline{m}_b}{\overline{m}_t c_\beta} \right| + \frac{1}{4} \left(1 - \frac{A_t}{M_{\mathrm{Susy}}} \frac{\overline{m}_t c_\beta}{\overline{m}_b} \right) \log \left| \frac{A_t}{M_{\mathrm{Susy}}} - \frac{\overline{m}_b}{\overline{m}_t c_\beta} \right| \right]$$

$$\delta_2 = -4 g_s^2 - \frac{3}{4} y_t^2 \left(1 + \frac{A_t^2}{M_{\mathrm{Susy}}^2} \right) + \frac{5 y_b^2}{4 c_\beta^2}$$