Stationary and non-stationary solutions of the evolution equation for neutrino in matter

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Lobanov A. E., Theoretical and Mathematical Physics, 2017, 192:1, 1000–1015

- The fermions are combined in $SU(3)$-multiplets
- 1-patricle wave functions are elements of the representation space of the direct product of Poincarre group and SU(3)
- We may quantize the model
- The Fock space for the superposition of mass states can be constructed
- We obtain the pertubation series in the interaction representation
- The formulas for neutrino oscillations are in good agreement with those obtained in the phenomenological theory

An effective equation describing neutrino oscillations and its spin rotation
A. E. Lobanov, Izvestiya Vysshikh Uchebnykh Zavedenii, Fizika, **59**, No. 11, 141, (2016).[Russ. Phys. J., **59**, No. 11, 1891, (2016)].

$$\left(i\gamma^{\mu}\partial_{\mu}-\mathbb{M}-\frac{1}{2}\gamma^{\mu}f^{(e)}_{\mu}(1+\gamma^{5})\mathbb{P}^{(e)}-\frac{1}{2}\gamma^{\mu}f_{\mathrm{N}\mu}(1+\gamma^{5})\,\mathbb{I}\right)\Psi(x)=0. \quad (2.1)$$

Here, $\mathbb{I}$ is a 3×3 unit matrix , $\mathbb{M}$ — Hermitian mass matrix of the neutrino multiplet, which can be written as follows

$$\mathbb{M}=\sum_{l=1}^{3}m_{l}\mathbb{P}^{(l)}. \quad (2.2)$$

m_l are the eigenvalues of the mass matrix, which have the meaning of the masses of the multiplet components, and the matrices $\mathbb{P}^{(l)}$ are orthogonal projectors on the subspaces with these masses. Matrix $\mathbb{P}^{(e)}$ is the projector on the state of the neutrino with electron flavor.

The effective potential due to the interaction via charged currents

$$f^{\mu(e)} = \sqrt{2} G_{\mathrm{F}} \left(j^{\mu(e)} - \lambda^{\mu(e)} \right) \tag{2.3}$$

The effective potential due to the neutral current interaction

$$f_{\mathrm{N}}^{\mu} = \sqrt{2} G_{\mathrm{F}} \sum_{i=e,p,n} \left(j^{\mu(i)} \left(T^{(i)} - 2Q^{(i)} \sin^2 \theta_{\mathrm{W}} \right) - \lambda^{\mu(i)} T^{(i)} \right) \tag{2.4}$$

Here

$$j^{\mu(i)} = \{ \bar{\mathrm{n}}^{(i)} v^{0(i)}, \bar{\mathrm{n}}^{(i)} \mathbf{v}^{(i)} \} \tag{2.5}$$

are the 4-vectors of the current, and

$$\lambda^{\mu(i)} = \left\{ \bar{\mathrm{n}}^{(i)} (\zeta^{(i)} \mathbf{v}^{(i)}), \bar{\mathrm{n}}^{(i)} \left(\zeta^{(i)} + \frac{\mathbf{v}^{(i)} (\zeta^{(i)} \mathbf{v}^{(i)})}{1 + v^{0(i)}} \right) \right\} \tag{2.6}$$

are the 4-vectors of the polarizarion of the components of the medium.

In the 3-flavor model Ψ is a 12-component object. It is convenient to introduce block structure, namely to define this object as three Dirac spinors

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}. \tag{2.7}$$

In the 2-flavor model the wave function Ψ can be presented as a pair of Dirac spinors.

We will call the representation of the matrices $\mathbb{M}$, $\mathbb{P}^{(e)}$, which diagonalizes the mass matrix, **the mass representation**. We also introduce **the flavor representation** of these matrices as a representation, where the projectors $\mathbb{P}^{(e)}$ are diagonal matrices. These representations are connected by the unitary mixing matrix U_{PMNS}.

In the 2-flavor model in the mass representation

$$\mathbb{M}_0 = \frac{1}{2}(\sigma_0(m_1+m_2)-\sigma_3(m_2-m_1)), \quad \mathbb{P}_0^{(e)} = \frac{1}{2}(\sigma_0-\sigma_1 \sin 2\theta+\sigma_3 \cos 2\theta), \tag{2.8}$$

where $\sigma_i, i = 1, 2, 3$ are Pauli matrices, σ_0 is the unit matrix 2×2, θ is the mixing angle.
In the flavor representation

$$\mathbb{M} = \frac{1}{2}(\sigma_0(m_1+m_2)-(\sigma_3 \cos 2\theta-\sigma_1 \sin 2\theta)(m_2-m_1)), \quad \mathbb{P}^{(e)} = \frac{1}{2}(\sigma_0+\sigma_3). \tag{2.9}$$

U is the Pontecorvo-Maki-Nakagawa-Sakata matrix. In the 2-flavor model it takes the form

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \tag{2.10}$$

$$\mathbb{M} = U\mathbb{M}_0 U^\dagger, \qquad \mathbb{P}^{(e)} = U\mathbb{P}_0^{(e)} U^\dagger, \tag{2.11}$$

The mass states are the states, which are described by the wave functions Ψ_i $(i = \overline{1..3})$, which can be written in the mass representation as follows

$$\Psi_1 = \begin{pmatrix} \psi_1 \\ 0 \\ 0 \end{pmatrix} \qquad \Psi_2 = \begin{pmatrix} 0 \\ \psi_2 \\ 0 \end{pmatrix} \qquad \Psi_3 = \begin{pmatrix} 0 \\ 0 \\ \psi_3 \end{pmatrix}, \tag{3.1}$$

where ψ_i, $i = \overline{1..3}$ are the Dirac spinors.

We say that the neutrinos are in states with definite flavor in a given moment of time if their wave functions $\tilde{\Psi}_i$ $(i = \overline{1..3})$ in the flavor representation take the form

$$\tilde{\Psi}_1 = \begin{pmatrix} \tilde{\psi}_1 \\ 0 \\ 0 \end{pmatrix} \qquad \tilde{\Psi}_2 = \begin{pmatrix} 0 \\ \tilde{\psi}_2 \\ 0 \end{pmatrix} \qquad \tilde{\Psi}_3 = \begin{pmatrix} 0 \\ 0 \\ \tilde{\psi}_3 \end{pmatrix}, \tag{3.2}$$

where $\tilde{\psi}_i$, $i = \overline{1..3}$ are the Dirac spinors.

We will use the notation $\hat{p} = p^{\mu}\gamma_{\mu}$.
In the momentum representation the Greens function may be written as follows

$$G(p) = \left(\hat{p}\mathbb{I} - \frac{1}{2}\hat{f}(1+\gamma^5)(a\mathbb{I} + \mathbb{P}^{(e)}) - \mathbb{M}\right)^{-1}, \tag{4.1}$$

If we introduce the notations

$$H_{\mp}(p) = \left(\hat{p}\mathbb{I} - \frac{1}{2}\hat{f}(1+\gamma^5)(a\mathbb{I} + \mathbb{P}^{(e)}) \mp \mathbb{M}\right), \tag{4.2}$$

the Greens function take the form

$$G(p) = (H_{-}(p))^{-1}. \tag{4.3}$$

It can be shown that

$$G(p) = \frac{1}{2}\sum_{\zeta=\pm 1}\frac{H_+(p,\zeta)F_-(p,\zeta)}{D(p,\zeta)}(1+\zeta\mathcal{S}), \tag{4.4}$$

where

$$F_\pm(p,\zeta) = p^2 - ((pf) - R\zeta)\left(a + \frac{1}{2} \pm \frac{1}{2}\sigma_3\right) - \frac{m_1^2 + m_2^2}{2} \pm \\ \pm \frac{m_2^2 - m_1^2}{2}(\sigma_3\cos 2\theta - \sigma_1 \sin 2\theta) \pm \frac{\mathrm{i}}{4}\hat{f}(1+\gamma^5)(m_2 - m_1)\sigma_2 \sin 2\theta, \tag{4.5}$$

$$D(p,\zeta) = \left(p^2 - ((pf) - R\zeta)\left(a + \frac{1}{2}\right) - \frac{m_1^2 + m_2^2}{2}\right)^2 - \\ - \left(\frac{((pf) - R\zeta)}{2} - \frac{m_2^2 - m_1^2}{2}\cos 2\theta\right)^2 - \left(\frac{m_2^2 - m_1^2}{2}\sin 2\theta)\right)^2. \tag{4.6}$$

Here, operator $\mathcal{S}$ describes the projection of neutrino spin on its canonical momentum in the matter rest frame. In the reference frame, where the medium is moving, it takes the form

$$\mathcal{S} = \frac{1}{2R}\gamma^5(\hat{f}\hat{p} - \hat{p}\hat{f}), \quad \mathcal{S}^2 = 1, \tag{4.7}$$

where $R = \sqrt{(fp)^2 - f^2p^2}$.

An effective equation describing neutrino oscillations and its spin rotation

$$\left(i\gamma^{\mu}\partial_{\mu} - \mathbb{M} - \frac{1}{2}\gamma^{\mu}f^{(e)}_{\mu}(1+\gamma^5)\mathbb{P}^{(e)} - \frac{1}{2}\gamma^{\mu}f_{\mathrm{N}\mu}(1+\gamma^5)\,\mathbb{I}\right)\Psi(x) = 0. \quad (5.1)$$

Lobanov A. E. "Particle quantum states with indefinite mass and neutrino oscillations."2015.
arXiv:1507.01256[hep-ph].

We will search for the stationary solutions in the form

$$\Psi(x) = e^{-i(px)}\Psi_0, \tag{5.2}$$

where the spinor Ψ_0 is an arbitrary 8-component spinor, which does not depend on the coordinates of the event space. Here the component of the 4-momentum p^0 is the neutrino energy, and $\mathbf{p}$ is the canonical momentum of the neutrino multiplet.

It can be shown that for the moving medium the solutions may be found as the eigenfunctions of the operator which describes the polarization state of the neutrino

$$\gamma^5 \frac{\hat{f}\hat{p} - \hat{p}\hat{f}}{2\sqrt{(fp)^2 - f^2p^2}} \to \frac{(\boldsymbol{\Sigma}\mathbf{p})}{|\mathbf{p}|} \tag{5.3}$$

For the matter at rest $f^{\mu} = \{f^0, 0, 0, 0\}$ the neutrino helicity is conserved, and the 8-component spinor (5.2) can be chosen as the eigenfunction of the helicity operator.

We will use the standard representation of the γ-matrices.
8-component spinors, which describe the neutrino with definite helicity, take the form

$$\Psi^T = \left(A_1^\zeta \chi_1, A_1^\zeta \chi_2, A_2^\zeta \chi_1, A_2^\zeta \chi_2, B_1^\zeta \chi_1, B_1^\zeta \chi_2, B_2^\zeta \chi_1, B_2^\zeta \chi_2 \right) e^{-i(px)}, \quad (5.4)$$

where χ_1, χ_2 are complex values such that

$$\frac{(\sigma \mathbf{p})}{|\mathbf{p}|} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \zeta \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}. \quad (5.5)$$

The matrix equation on the coefficients A_n^ζ, B_n^ζ $(n = 1, 2)$ follows from the neutrino evolution equation.

If the matter is unpolarized, then the dispersion law takes the form

$$(p^0 - \zeta|\mathbf{p}|)^2 + 2(\zeta|\mathbf{p}| - \frac{f_0}{2}(a + 1/2))(p^0 - \zeta|\mathbf{p}|) - \frac{m_1^2 + m_2^2}{2} =$$
$$= \frac{\xi}{2}\sqrt{(f_0(p^0 - \zeta|\mathbf{p}|) - (m_2^2 - m_1^2)\cos 2\theta)^2 + (m_2^2 - m_1^2)^2 \sin^2 2\theta}, \quad (5.6)$$

where ξ is either 1 or -1.

Coefficients A_n^ζ, B_n^ζ $n = 1, 2$ are defined up to a multiplicative constant N

$$NA_1^\zeta = \frac{f_0 \sin 2\theta}{4}(p^0 - \zeta|\mathbf{p}| + m_1)(p^0 - \zeta|\mathbf{p}| - m_2), \tag{5.7}$$

$$NA_2^\zeta = -\frac{f_0 \sin 2\theta}{4}(p^0 - \zeta|\mathbf{p}| - m_1)(p^0 - \zeta|\mathbf{p}| - m_2), \tag{5.8}$$

$$NB_1^\zeta = \frac{f_0^2 \sin^2 2\theta}{8}(p^0 - \zeta|\mathbf{p}|) + \frac{1}{2}\left(\zeta|\mathbf{p}| - \frac{f_0}{2}(a + \frac{1}{2} - \frac{\cos 2\theta}{2})\right) \cdot \cdot \left(m_1^2 - m_2^2 - \xi\Delta + f_0(p^0 - \zeta p)\cos 2\theta\right), \tag{5.9}$$

$$NB_2^\zeta = \frac{f_0^2 \sin^2 2\theta}{8}(p^0 - \zeta|\mathbf{p}|) + \frac{1}{2}\left(p^0 - m_2 - \frac{f_0}{2}(a + \frac{1}{2} - \frac{\cos 2\theta}{2})\right) \cdot \cdot \left(m_1^2 - m_2^2 - \xi\Delta + f_0(p^0 - \zeta|\mathbf{p}|)\cos 2\theta\right). \tag{5.10}$$

where $\Delta = \sqrt{(f_0(p^0 - \zeta|\mathbf{p}|) - (m_1^2 - m_2^2)^2 \cos 2\theta)^2 + (m_1^2 - m_2^2)^2 \sin^2 2\theta}.$

For the ultrarelativistic left-handed neutrinos the solutions corresponding to $\xi = 1$ and $\xi = -1$ can be written as follows

$$\Psi_{+} = \begin{pmatrix} \sin\phi \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \\ -\sin\phi \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \\ \cos\phi \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \\ -\cos\phi \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \end{pmatrix} \frac{e^{-i(\varepsilon_{+} t - \mathbf{px})}}{\sqrt{2}}, \quad \Psi_{-} = \begin{pmatrix} \cos\phi \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \\ -\cos\phi \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \\ -\sin\phi \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \\ \sin\phi \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \end{pmatrix} \frac{e^{-i(\varepsilon_{-} t - \mathbf{px})}}{\sqrt{2}} \tag{5.11}$$

where ε_{+}, ε_{-} are different solutions of the equation, which represents the dispersion law.

Here we use the notations

$$\sin\phi = \frac{2f_0p_0\sin 2\theta}{\sqrt{(2f_0p_0\sin 2\theta)^2 + (2f_0p_0\cos 2\theta - (m_2^2 - m_1^2))^2}}, \quad (5.12)$$

$$\cos\phi = \frac{(m_2^2 - m_1^2) - 2f_0p_0\cos 2\theta}{\sqrt{(2f_0p_0\sin 2\theta)^2 + (2f_0p_0\cos 2\theta - (m_2^2 - m_1^2))^2}}, \quad (5.13)$$

$$\Delta = \sqrt{(2f_0p^0 - (m_2^2 - m_1^2)^2\cos 2\theta)^2 + (m_2^2 - m_1^2)^2\sin^2 2\theta} \quad (5.14)$$

The stationary states for the neutrino in medium are not the mass states. In the limit $f^0 \to 0$ in the mass representation the solutions of the evolution equation are the wave functions of the mass states

$$\Psi_+ = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \chi_1 \\ \chi_2 \\ -\chi_1 \\ -\chi_2 \end{pmatrix} e^{-i(\varepsilon_+ t - \mathbf{px})}, \quad \Psi_- = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_1 \\ \chi_2 \\ -\chi_1 \\ -\chi_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{-i(\varepsilon_- t - \mathbf{px})}, \quad (5.15)$$

As we have expected, the stationary states in vacuum are the mass states.

In the flavor representation the stationary solutions with the momentum **p** for $\xi = 1$ and $\xi = -1$ take the form

$$\tilde{\Psi}_+ = \begin{pmatrix} \sin\theta_m \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \\ -\sin\theta_m \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \\ \cos\theta_m \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \\ -\cos\theta_m \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \end{pmatrix} \frac{e^{-i(\varepsilon_+ t - \mathbf{px})}}{\sqrt{2}}, \quad \tilde{\Psi}_- = \begin{pmatrix} \cos\theta_m \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \\ -\cos\theta_m \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \\ -\sin\theta_m \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \\ \sin\theta_m \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \end{pmatrix} \frac{e^{-i(\varepsilon_- t - \mathbf{px})}}{\sqrt{2}}, \tag{5.16}$$

where $\theta_m = \theta + \phi$ is the effective mixing angle in matter, which is defined as follows

$$\cos 2\theta_m = \frac{(m_1^2 - m_2^2)\cos 2\theta - 2f^0 p^0}{\Delta}, \qquad \sin 2\theta_m = \frac{(m_1^2 - m_2^2)\sin 2\theta}{\Delta}. \tag{5.17}$$

When the 4-vectors of current and polarization do not depend on the coordinates of the event space, the solution can be written with the help of the matrix exponential using the method developed in the following papers

Lobanov A. E. Phys. Lett. B. 2005. **619**, N 1-2. P. 136

Arbuzova E. V., Lobanov A. E., Murchikova E. M. Phys. Rev. D. 2010. **81**, N 4. 045001

$$\Psi(x) = U(x)\Psi_0 \qquad (6.1)$$

where

$$\Psi_0 = \frac{1}{2}(1 - \gamma^5\gamma_\mu s_0^\mu)\left(\psi^0 \otimes e_j\right), \quad \bar{\Psi}_0\Psi_0 = 2. \qquad (6.2)$$

Here ψ^0 is a constant bispinor, e_j is an arbitrary unit vector in the three dimensional vector space over the field of complex numbers, s_0^μ being the 4-vector of neutrino polarization, which satisfies the condition $(us_0) = 0$.

In the flavor representation the solution of this type takes the form

$$\Psi(x) = \frac{1}{\sqrt{2q_0}} U(\gamma_\mu q^\mu + m)\Psi_0,$$
$$U = \sum_{\zeta=\pm 1} \exp -i \left(\frac{(qx)}{2m}(m_1 + m_2) + \frac{1}{2}\left((fx) - \zeta F\right)(a + 1/2) \right)$$
$$\times \left(\cos\frac{Z_\zeta}{2} - i \sin\frac{Z_\zeta}{2} \left(X_\zeta \sigma_1 - Y_\zeta \sigma_3\right) \right),$$
$$X_\zeta = \frac{1}{Z_\zeta}\left(\frac{(qx)}{m}(m_2 - m_1)\sin 2\theta \right),$$
$$Y_\zeta = \frac{1}{Z_\zeta}\left(\frac{(qx)}{m}(m_2 - m_1)\cos 2\theta - \frac{1}{2}\left((fx) - \zeta F\right) \right),$$
$$Z_\zeta = \sqrt{\left(\frac{(qx)(m_2-m_1)}{m}\sin 2\theta\right)^2 + \left(\frac{(qx)(m_2-m_1)}{m}\cos 2\theta - \frac{(fx)-\zeta F}{2}\right)^2}. \tag{6.3}$$

Here, the following notations are used

$$F = \frac{(fx)(fq) - (qx)f^2}{\sqrt{(fq)^2 - f^2 q^2}},$$
$$s^\mu = \frac{q^\mu (fq) - f^\mu q^2}{m\sqrt{(fq)^2 - f^2 q^2}}. \tag{6.4}$$

q^μ is the kinetic momentum of the neutrino, $q^2 = m^2$.

In the ultrarelativistic limit the 4-velocity of the neutrino $u^\mu = \{u^0, \mathbf{u}\}$, which is connected to the kinetic momentum as follows $u^\mu = q^\mu/m$, can be related to the coordinates of the particle as follows

$$x^\mu = u^\mu \tau. \tag{7.1}$$

Therefore, we obtain the solution of the quasi-classical neutrino evolution equation *Lobanov A.E., Chukhnova A.V.* Moscow University Physics Bulletin, 2017, Vol. 72, No. 5.

The evolution of the neutrino is characterized by the only parameter τ, which is the proper time

$$\tau = L/|\mathbf{u}|. \tag{7.2}$$

Let us consider the spin-flavor transitions between the states with definite flavors. The projectors on these states are given by the following matrices

$$\mathbb{P}_0^{(\alpha)} = \frac{1}{2}\left(1 + \xi_0\sigma_3\right), \quad \mathbb{P}_0^{(\beta)} = \frac{1}{2}\left(1 + \xi'_0\sigma_3\right), \quad \xi_0, \xi'_0 = \pm 1. \qquad (7.3)$$

Let's assume that in each these states the neutrino has definite helicity, i. e.

$$s_0^{(\alpha)\mu} = \zeta_0 s_{sp}^{\mu}, \quad s_0^{(\beta)\mu} = \zeta'_0 s_{sp}^{\mu}, \quad s_{sp}^{\mu} = \{|\mathbf{u}|, u^0\mathbf{u}/|\mathbf{u}|\}. \qquad (7.4)$$

The probability in this case is given by the following expression

$$W_{\alpha\to\beta} = \frac{1+\xi_0\xi'_0}{2}\frac{1+\zeta_0\zeta'_0}{2}W_1 + \frac{1+\xi_0\xi'_0}{2}\frac{1-\zeta_0\zeta'_0}{2}W_2 + \\ + \frac{1-\xi_0\xi'_0}{2}\frac{1+\zeta_0\zeta'_0}{2}W_3 + \frac{1-\xi_0\xi'_0}{2}\frac{1-\zeta_0\zeta'_0}{2}W_4. \qquad (7.5)$$

$$W_1 = \frac{1}{2}\Bigg(\frac{1}{2}(1-\zeta_0(ss_{sp}))^2(1-S^2_{+1}X^2_{+1}) + \frac{1}{2}(1+\zeta_0(ss_{sp}))^2(1-S^2_{-1}X^2_{-1})+$$
$$+(1-(ss_{sp})^2)(C_{+1}C_{-1}+S_{+1}S_{-1}Y_{+1}Y_{-1})\cos(\omega\tau)+$$
$$+\xi_0(1-(ss_{sp})^2)(S_{+1}Y_{+1}C_{-1}-C_{+1}S_{-1}Y_{-1})\sin(\omega\tau)\Bigg),$$

$$W_2 = \frac{1}{2}\Bigg(\frac{1}{2}(1-(ss_{sp})^2)(2-S^2_{+1}X^2_{+1}-S^2_{-1}X^2_{-1})-$$
$$-(1-(ss_{sp})^2)(C_{+1}C_{-1}+S_{+1}S_{-1}Y_{+1}Y_{-1})\cos(\omega\tau)-$$
$$-\xi_0(1-(ss_{sp})^2)(S_{+1}Y_{+1}C_{-1}-C_{+1}S_{-1}Y_{-1})\sin(\omega\tau)\Bigg),$$

$$W_3 = \frac{1}{2}\Bigg(\frac{1}{2}(1-\zeta_0(ss_{sp}))^2S^2_{+1}X^2_{+1} + \frac{1}{2}(1+\zeta_0(ss_{sp}))^2S^2_{-1}X^2_{-1}+$$
$$+(1-(ss_{sp})^2)S_{+1}S_{-1}X_{+1}X_{-1}\cos(\omega\tau)\Bigg),$$

$$W_4 = \frac{1}{2}\Bigg(\frac{1}{2}(1-(ss_{sp})^2)(S^2_{+1}X^2_{+1}+S^2_{-1}X^2_{-1})-$$
$$-(1-(ss_{sp})^2)S_{+1}S_{-1}X_{+1}X_{-1}\cos(\omega\tau)\Bigg).$$

$$
\begin{aligned}
Y_\zeta &= \frac{1}{Z_\zeta}\Big(\big((fu) - \zeta R\big)/2 - \big(m_2 - m_1\big)\cos 2\theta\Big),\\
X_\zeta &= \frac{1}{Z_\zeta}\Big(\big(m_2 - m_1\big)\sin 2\theta\Big),\\
Z_\zeta &= \sqrt{\Big(\big((fu) - \zeta R\big)/2 - \big(m_2 - m_1\big)\cos 2\theta\Big)^2 + \Big(\big(m_2 - m_1\big)\sin 2\theta\Big)^2}.
\end{aligned} \tag{7.8}
$$

If we assume $u^0 \approx |\mathbf{u}|$, then for the medium at rest

$$
\begin{aligned}
Y_+ &= -\cos 2\theta, & X_+ &= \sin 2\theta,\\
Y_- &= -\cos 2\theta_{eff}\,, & X_- &= \sin 2\theta_{eff}\,,
\end{aligned} \tag{7.9}
$$

where θ_{eff} is the effective mixing angle in matter.

$$
C_{\pm 1} = \cos\big(\tau Z_{\pm 1}/2\big), \quad S_{\pm 1} = \sin\big(\tau Z_{\pm 1}/2\big), \quad \omega = R(1/2 + a). \tag{7.10}
$$

Generally, these spin-flavor transition probabilities are characterized by six frequences. If $u^0 \approx |\mathbf{u}|$ then

$$L^0_{osc} = \frac{2\pi|\mathbf{u}|}{Z_{+1}} \tag{7.11}$$

is the flavor oscillation length in vacuum, and the parameter

$$L^f_{osc} = \frac{2\pi|\mathbf{u}|}{Z_{-1}} \tag{7.12}$$

is the flavor oscillation length in matter. Four combinational oscillation lengths

$$L^c_{osc} = \left|\frac{4\pi|\mathbf{u}|}{Z_{-1} \pm Z_{+1} \pm 2\omega}\right| \tag{7.13}$$

arise due to correlations between flavor transitions and the spin rotation. The number of such combinational lengths under certain conditions can be equal to two.

The spin-flip probability W is the sum of W_2 and W_4

$$W = \frac{1}{2}\,\mathcal{A}\,(A_1(1-\cos\omega_1\tau)+A_2(1-\cos\omega_2\tau)+A_3(1-\cos\omega_3\tau)+A_4(1-\cos\omega_4\tau)) \tag{8.1}$$

where

$$\mathcal{A} = 1-(ss_{sp})^2. \tag{8.2}$$

$$1-(ss_{sp})^2 = \frac{\mathbf{v}^2\sin^2\vartheta}{(vu)^2-1} \tag{8.3}$$

does not depend on the medium density. It depends on the 4-velocities of the medium v^μ and the neutrino u^μ. Here, ϑ is the angle between the vectors of the velocities of the medium and the neutrino.

If $\mathbf{u} > \mathbf{v}$, then the maximum value of this amplitude is obtained when

$$\cos\vartheta_{max} = \frac{\sqrt{v_0^2 - 1}/v_0}{\sqrt{u_0^2 - 1}/u_0}, \tag{8.4}$$

It is equal to

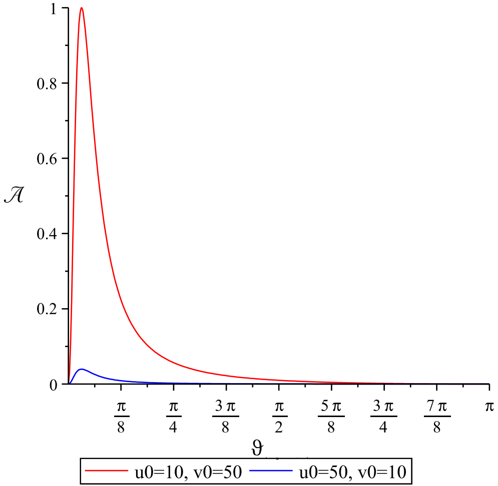
$$(1 - (ss_{sp})^2)_{max} = \frac{v_0^2 - 1}{u_0^2 - 1}. \tag{8.5}$$

If $\mathbf{u} < \mathbf{v}$, the maximum value of this amplitude is obtained when

$$\cos\vartheta_{max} = \frac{\sqrt{u_0^2 - 1}/u_0}{\sqrt{v_0^2 - 1}/v_0}, \tag{8.6}$$

It is equal to

$$(1 - (ss_{sp})^2)_{max} = 1. \tag{8.7}$$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0.8
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7π/8
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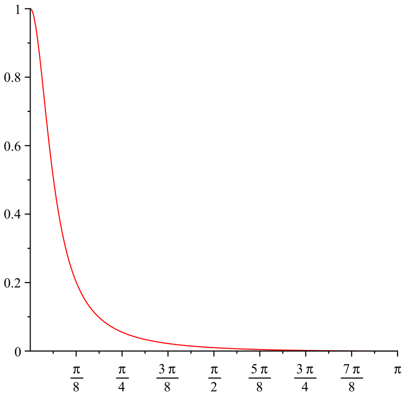
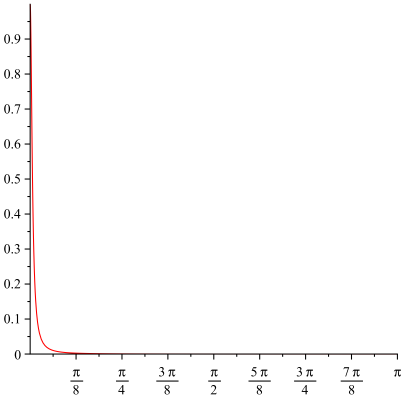
u0=10, v0=50
u0=50, v0=10



$u_0 = 10,\ v_0 = 10$

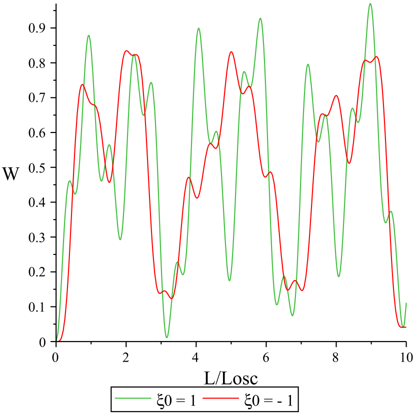


$u_0 = 100,\ v_0 = 100$

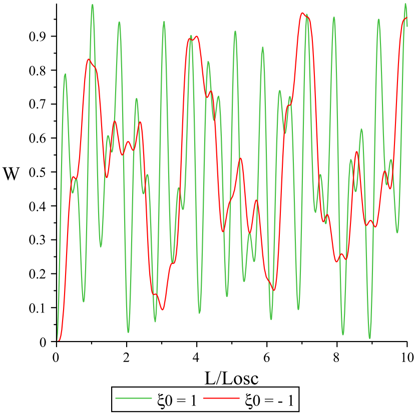
We demonstrate the figures for the dependence of spin-flip on the distance from the detector. The scale on the horisontal axis is L/L_{osc}, where L_{osc} is the flavor oscillations length in vacuum. The k parameter characterizes the degree of medium impact on the neutrino propagation

$$k = \frac{\sqrt{2} G_{\mathrm{F}} n}{|m_1 - m_2|}, \tag{8.8}$$

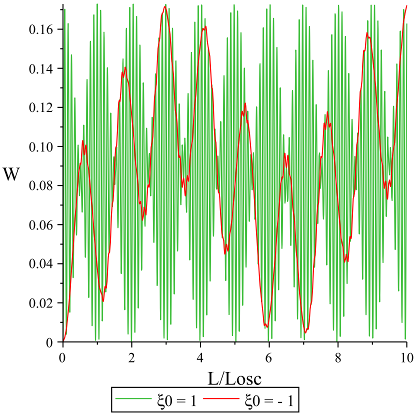
where n is the medium density in the laboratory frame.



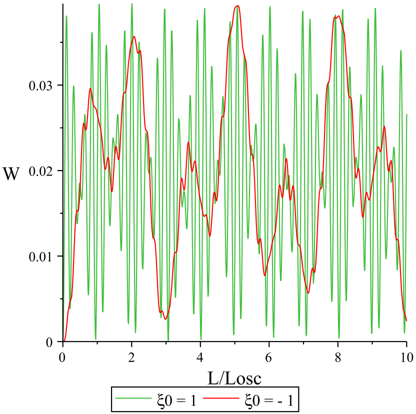
$\xi_0 = \pm 1,\ u_0 = 10,\ v_0 = 50,$
$k = 10,\ \cos\vartheta = \cos\vartheta_{max}$



$\xi_0 = \pm 1,\ u_0 = 10,\ v_0 = 50,$
$k = 20,\ \cos\vartheta = \cos\vartheta_{max}$



$\xi_0 = \pm 1,\ u_0 = 10,\ v_0 = 50,$
$k = 10,\ \cos\vartheta = 0.9$



$\xi_0 = \pm 1,\ u_0 = 50,\ v_0 = 10,$
$k = 10,\ \cos\vartheta = \cos\vartheta_{max}$

Conclusion

- We find the Greens function of neutrino in dense medium.
- We show that the stationary states in medium differ from the mass states.
- The wave function, describing the state with a definite flavor in matter can be constructed as a linear combination of the stationary states. The coefficients in this linear combination depend on the mixing angle in matter.
- We obtain the wave functions of the spin-flavor coherent states, which in quasi-classical limit become similar to the solutions of the quasi-classical evolution equation. These solutions describe the neutrino with definite velocity.

- We obtain the spin-flip probability for ultrarelativistic neutrinos. The pattern of such transitions depends on the initial flavor state of the neutrino, as well as on the density of the medium and the velocities of neutrino and the medium.
- There is no spin rotation for the medium at rest.
- The spin-flip probability is non-zero only if the directions of neutrino propagation and the movement of the medium are close (but not equal) to each other.

Thank you for your attention!