Photon splitting in strongly magnetized medium with taking into account positronium influence

Denis Shlenev

Yaroslavl Higher Military School of Air Defense, Russia

May 28, 2018

International Seminar *"Quarks-2018"*, Valdai, Russia

In collaboration with Roman Anikin, Michael Chistyakov and Dmitry Rumyantsev

Outline

- Introduction
- Photon dispersion with the positronium influence
- Probability of photon splitting
- Conclusion

Introduction

Magnetars (SGR) and (AXP)

$$B \sim 10^{14} - 10^{16} \text{ G} \gg B_e,$$

$B_e = m^2/e \simeq 4.41 \times 10^{13}$ G. The radio emission is observed from some magnetars (Malofeev et al. 2004, 2005, Camilo 2007 –AXP XTE J1810-197), although its characteristics differ from standard of radio pulsars.

e^+e^- – plasma is produced in two stages (Beloborodov A. M., Thompson C. 2007)

i)The hard X-ray production by Compton mechanism $\gamma e \to e\gamma$, $\omega > 2m$ (inverse Compton effect);

ii)The increase of the pitch angle, $\omega^2 \sin^2\theta > 4m^2$ and e^+e^- pair production, $\gamma \to e^+e^-$ (Klepikov N.P. 1954)

Introduction

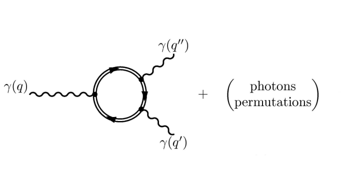
Process of photon splitting, $\gamma \to \gamma\gamma$, may soften photons below the threshold of pair creation and therefore decrease velocity of e^+e^- pair creation.

- S. Adler (1971) - photon splitting in magnetized vacuum
- M. Chistyakov, A. Kuznetsov, N. Mikheev (1999) - $\gamma \to \gamma\gamma$ is investigated in strong magnetic field (the case of non-collinear kinematics)
- M. Chistyakov, D. Rumyantsev, N. Stus (2012) - $\gamma \to \gamma\gamma$ in magnetized plasma

Introduction

- Positronium influence on dispertion properties of photon was investigated in paper of R. Anikin, N. Mikheev (2012) on example of process $\nu \to \nu\gamma$ in strong magnetic field
- Has been shown that positronium effects may significally change the probability of this process.
- Therefore, it's interesting to calculate the probability of photon splitting process in strong magnetic field with taking into account variations of dispertion properties and kinematics due to positronium influence

Feynman diagrams for the process $\gamma \to \gamma\gamma$.



Some notations

q^{μ}, q'^{μ} and q''^{μ} are the momenta of initial and final photons, $(ab)_{\perp} = a_x b_x + a_y b_y$, $(ab)_{\parallel} = a_0 b_0 - a_z b_z$, $(a\varphi b) = a_y b_x - a_x b_y$. $\varphi_{\alpha\beta} = F_{\alpha\beta}/B$ and $\tilde{\varphi}_{\alpha\beta} = \frac{1}{2}\varepsilon_{\alpha\beta\mu\nu}\varphi_{\mu\nu}$ are the dimensionless field tensor and dual field tensor correspondingly.

Photon dispersion with the positronium influence

We begin to consider the process $\gamma \to \gamma\gamma$ with investigation of the photon dispersion properties.

It is convenient to describe the propagation of the electromagnetic radiation in any active medium in terms of normal modes (eigenmodes). In turn, the polarization and dispersion properties of normal modes are connected with eigenvectors $r_\alpha^{(\lambda)}(q)$ and eigenvalues of polarization operator $\varkappa^{(\lambda)}(q)$ correspondingly.

Photon dispersion with the positronium influence

- Magnetic field without taking into account of positronium influence.

In this case the eigenvectors are $r_\mu^{(\lambda)} = b_\mu^{(\lambda)}$ (A. Shabad 1988), where

$$b_\mu^{(1)} = \frac{(\varphi q)_\mu}{\sqrt{q_\perp^2}}, \qquad b_\mu^{(2)} = \frac{(\tilde{\varphi} q)_\mu}{\sqrt{q_\parallel^2}},$$

$$b_\mu^{(3)} = \frac{q^2 \, (\Lambda q)_\mu - q_\mu \, q_\perp^2}{\sqrt{q^2 q_\parallel^2 q_\perp^2}}, \qquad b_\mu^{(4)} = \frac{q_\mu}{\sqrt{q^2}}.$$

The photon has the linear polarization.

Photon dispersion with the positronium influence

$$r_\mu^{(\lambda)} = \sum_{\lambda'=1}^{3} A_{\lambda'}^{\lambda}(q)\; b_\mu^{(\lambda')} \, .$$

Here $A_{\lambda'}^{\lambda}(q)$ are some complex coefficients, and the photon has the elliptical polarization.

The photon polarization operator in this case can be presented in the following form

$$\mathcal{P}_{\alpha\beta} = \sum_{\lambda} \varkappa^{(\lambda)} \frac{r_\alpha^{(\lambda)} (r_\beta^{(\lambda)})^*}{(r^{(\lambda)})^2}$$

Photon dispersion with the positronium influence

The physical polarization vectors of the photons

$$\varepsilon_\alpha^{(1)}(q) = -b_\alpha^{(1)} = \frac{(q\varphi)_\alpha}{\sqrt{q_\perp^2}}, \qquad \varepsilon_\alpha^{(2)}(q) = -b_\alpha^{(2)} = \frac{(q\tilde{\varphi})_\alpha}{\sqrt{q_\parallel^2}}$$

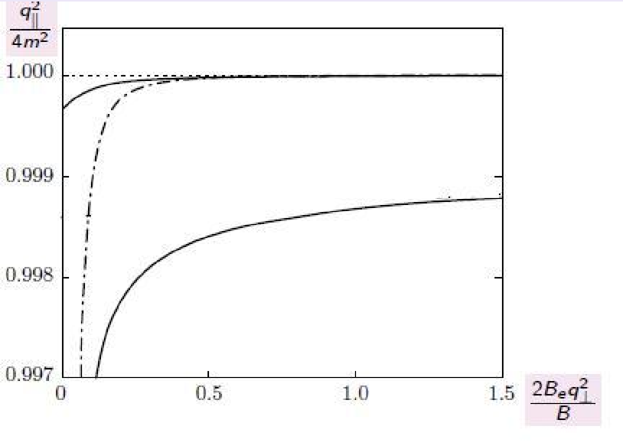
are just as in the pure magnetic field.
The corresponding eigenvalues are

$$\varkappa^{(1)} \simeq -\frac{\alpha}{3\pi}\, q_\perp^2\,, \quad \varkappa^{(2)} \simeq -\frac{2\alpha}{\pi}\, eB\, \left[\mathcal{J}(q_\parallel) + H\left(\frac{q_\parallel^2}{4m^2}\right)\right]$$

$H(z)$ is the field contributions in the polarization operator.
$\mathcal{J}(q_\parallel)$ is the positronium contributions in the polarization operator.

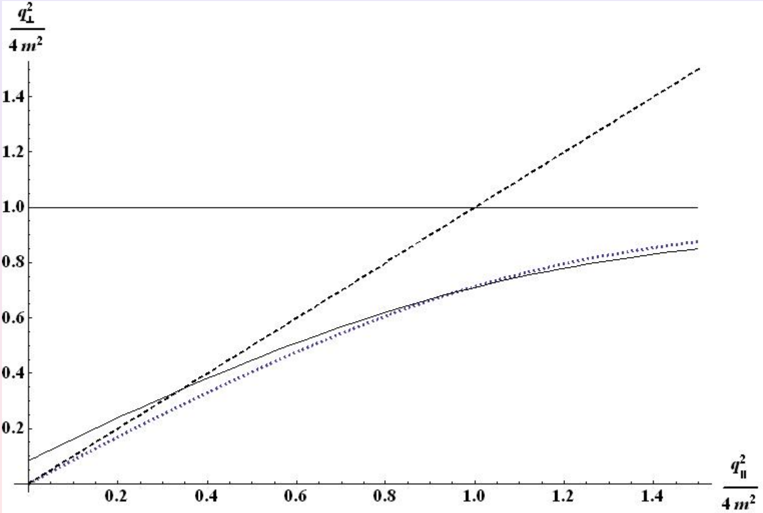
Photon dispersion with the positronium influence

The dispersion laws of the mode-2 photon.



Photon dispersion with the positronium influence

The dispersion laws of the mode-2 photon for $B = 200 B_e$.



Amplitude of photon splitting

$$\mathcal{M}_{1\to 12} = -i\frac{2\pi^2}{\alpha eB}\left(\frac{\alpha}{\pi}\right)^{3/2}\frac{(q'\varphi q'')(q'\tilde{\varphi}q'')}{[q_\perp^2 q_\perp'^2 q_\parallel''^2]^{1/2}}H(q_\parallel''^2),$$

$$\mathcal{M}_{1\to 22} = -i\frac{2\pi^2}{\alpha eB}\left(\frac{\alpha}{\pi}\right)^{3/2}\frac{(q'q'')_\parallel}{[q_\perp^2 q_\parallel'^2 q_\parallel''^2]^{1/2}}((qq'')_\perp H(q_\parallel'^2) + (qq')_\perp H(q_\parallel''^2)),$$

The eigenvalue of the polarization operator $\varkappa^{(2)}$ becomes large near the electron-positron pair production threshold. This suggests that the renormalization of the wave function for a photon of this polarization should be taken into account:

$$\varepsilon_\alpha^{(2)}(q) \to \varepsilon_\alpha^{(2)}(q)\sqrt{Z_2}, \quad Z_2^{-1} = 1 - \frac{\partial \varkappa^{(2)}(q)}{\partial \omega^2}.$$

Probability of the photon splitting

The expression for the photon splitting probability can be defined as follows:

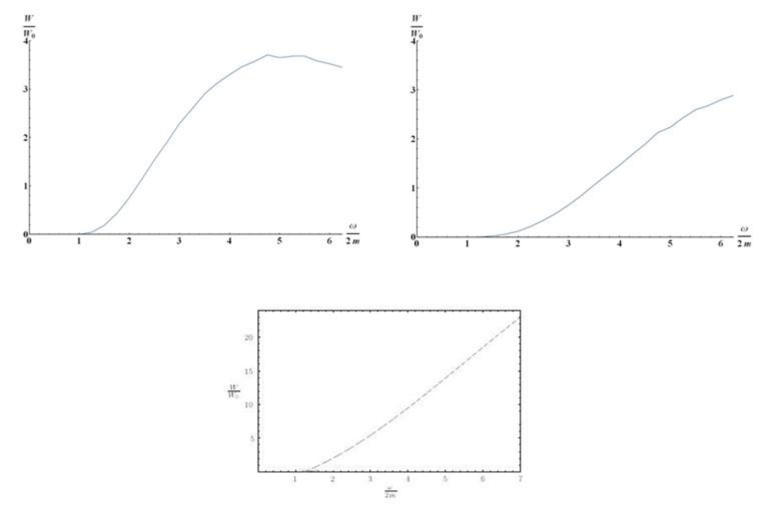
$$W_{\lambda\to\lambda'\lambda''} = \frac{g_{\lambda'\lambda''}}{32\pi^2\omega_\lambda}\int \left|\mathcal{M}_{\lambda\to\lambda'\lambda''}\right|^2 \times$$

$$\times Z_\lambda Z_{\lambda'} Z_{\lambda''}\delta(\omega - \omega' - \omega'')\frac{d^3k''}{\omega'\omega''},$$

where the factor $g_{\lambda'\lambda''} = 1 - (1/2)\,\delta_{\lambda'\lambda''}$ is inserted to account for the possible identity of the final photons.

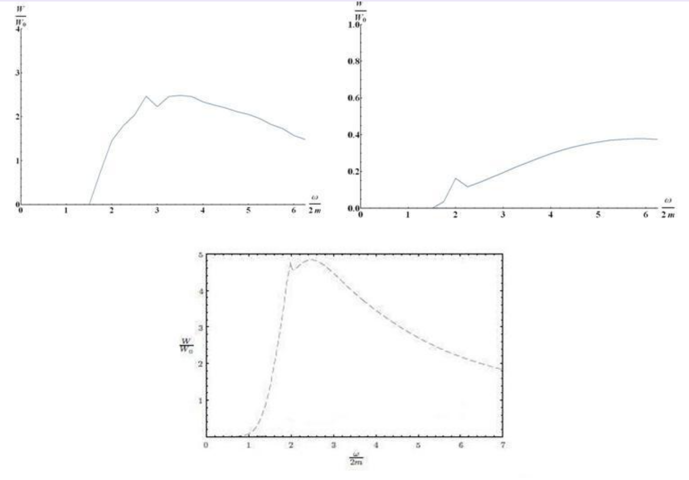
Probability of the photon splitting

Probability of the channel $\gamma_1 \to \gamma_1 \gamma_2$ in a strong magnetic field $(B/B_e = 200)$, $W_0 = (\alpha/\pi)^3\, m \simeq 3.25 \cdot 10^2 cm^{-1}$



Probability of the photon splitting

Probability of the channel $\gamma_1 \to \gamma_1\gamma_2$ in a strong magnetic field $(B/B_e = 200)$, $W_0 = (\alpha/\pi)^3\, m \simeq 3.25 \cdot 10^2 cm^{-1}$



Conclusion

- We have considered the process of photon splitting, $\gamma \to \gamma\gamma$ in strong magnetic field with taking into account positronium influence.
- The changes of the photon dispersion properties in this case are investigated. It has been shown that the photons polarization vectors are just as in the pure magnetic field.
- The probability of photon splitting is calculated. Positronium influence suppresses the probabilities of channels $\gamma_1 \to \gamma_1\gamma_2$ and $\gamma_1 \to \gamma_2\gamma_2$ in comparison with pure magnetic field.

Thank you!!!