Goldsone theorem for the spontaneous breakdown of spacetime symmetries

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Quarks 2018

Outline

1. Introduction: known peculiarities of the theories resulting from the spontaneous breakdown of spacetime symmetries

2. New results:

   - New massive Nambu-Goldstone bosons

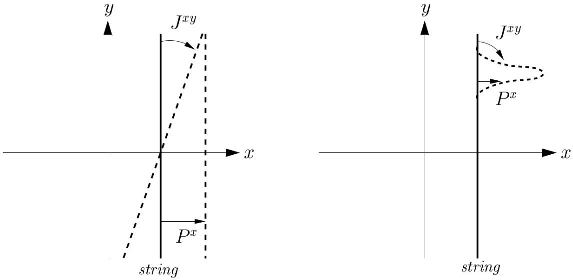
   - Understanding the inverse Higgs phenomenon

   - Goldstone's theorem

3. Conclusion

Known peculiarities

Redundant Nambu-Goldstone fields (picture from hep-th/0110285)



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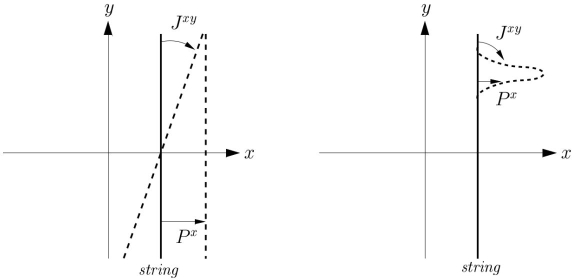

1. Introduce coset G/H : $g_H = e^{iP_\mu x^\mu} e^{iP_z \xi} e^{iM_{z\mu}\omega^\mu}$
2. Calculate Maurer-Cartan forms:

$$g_H^{-1} dg_H = iP_\mu \Omega_P^\mu + iP_z \Omega_P^z + iM_{z\mu}\Omega_M^\mu + iM_{\mu\nu}\Omega_M^{\mu\nu}$$

3. Impose inverse Higgs constraints:

$$\Omega_P^z(\partial_\mu \xi, \omega_\mu)$$

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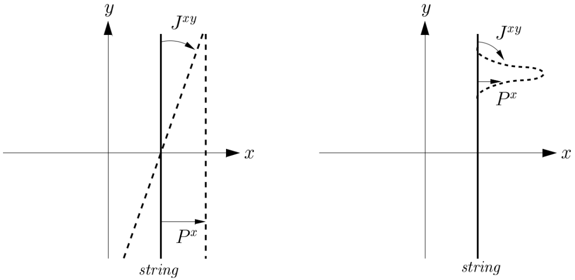

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3. Impose inverse Higgs constraints:

$$\Omega_P^z(\partial_\mu \xi, \omega_\mu) = 0 \quad \Rightarrow \quad \omega_\mu = \omega_\mu(\partial_\mu \xi)$$

Open questions

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→ *Inverse Higgs effect - a trick, an effect, a gauge choice, ... ?*

New massive Nambu-Goldstone bosons

SSB pattern: $ISO(d)_{ST} \times ISO(d)_{int} \to ISO(d)_V$

The Lagrangian of the theory:

$$\mathcal{L} = -\frac{1}{2}(\partial_i \varphi^a)^2 + \frac{1}{4}\left(\partial_{[i} V_{j]}^a\right)^2 + \varkappa V_a^i \partial_i \varphi^a + \frac{\lambda}{4d}\left(V_a^i V_i^a - dM_V^2\right)^2$$

Vacuum solution:

$$\varphi^a = \mu^2 x^a\,, \quad V_a^i = M\delta_a^i\,, \quad M = \sqrt{M_V^2 - \frac{\varkappa^2}{\lambda}}\,, \quad \mu^2 = \varkappa M$$

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Parametrizing Nambu-Goldstone modes:

$$\varphi^a(x) = \mu^2 x^a + \psi^a(x)\,, \quad V^i_a(x) = \Omega^i_a(\omega) M\,, \quad \Omega^i_a = \delta^i_a + \omega^i_a - \frac{1}{2}\omega^i_b \omega^b_a + \ldots$$

Effective Lagrangian(s):

$$\mathcal{L}_{\psi,A} = -\frac{1}{2}(\partial_i \psi^a)^2 + \frac{1}{4}(\partial_{[i} A^a_{j]})^2 - \frac{1}{2}\varkappa^2 A^i_j A^j_i + \varkappa A^i_a \partial_i \psi^a$$

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A^i_j integrated out: $\mathcal{L}_\psi = -\frac{1}{4}\left((\partial_i \psi^a)^2 + (\partial_a \psi^a)^2\right)$

Applying the coset space construction

The corresponding coset space: $g_H = e^{i\tilde{P}_\mu x^\mu} e^{i\bar{P}_a \psi^a} e^{\frac{i}{2}\bar{M}_{ab}\omega^{ab}}$

Covariant derivatives: $D_\mu \psi^a = \partial_\mu \psi^a - \mu^2 \omega^a_\mu\,, \quad D_\mu \omega^{\lambda\sigma} \simeq \partial_\mu \omega^{\lambda\sigma}$

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Imposing inverse Higgs constraints:

$$\mathcal{L}_\psi = -\tfrac{1}{8}(D_{\{i}\psi_{a\}})^2 = -\tfrac{1}{4}\Big((\partial_i\psi^a)^2 + (\partial_a\psi^a)^2\Big)$$

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How to obtain a theory including fields charged only under SO_V?

Understanding the inverse Higgs phenomenon

Which coset should be used within the coset space technique?

Polar decomposition: $\chi(x) = \gamma(x)\tilde{\chi}(x)\,, \quad \tilde{\chi}^T(x)(\hat{Z}_a\chi_{vac}(x)) = 0$

Introduce $\chi(x), \ \tilde{\chi}(x)$ as:

$$\chi(x) = (\phi^1, ..., \phi^d, V_1^1, ..., V_d^d, \theta), \quad \tilde{\chi}(x) = (\tilde{\phi}^1, ..., \tilde{\phi}^d, \tilde{V}_1^1, ..., \tilde{V}_d^d, \tilde{\theta})$$

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- Hence, $\quad \tilde{\chi}(x) = (0, ..., 0, V_1^1, ..., V_d^d, \theta), \quad \gamma(x) = e^{i\bar{P}_a\xi^a}$

Since homogeneously transforming quantities are obtained from $\gamma^{-1}d\gamma$,

one should not introduce ω^{ab} at all!

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Does suitable $\Omega_b^a(\psi)$ exist?

Yes, if one can find any suitable coset:

consider $g_H = e^{i\tilde{P}_\mu x^\mu} e^{i\bar{P}_a \psi^a} e^{i\bar{M}_{ab}\omega^{ab}}$ and find the searched for expression.

Via polar decomposition:

$$\gamma(x) = e^{i\bar{P}_a\psi^a} e^{\frac{i}{2}\bar{M}_{ab}\omega^{ab}}\,, \qquad \omega^{ab} = \omega^{ab}(\psi^a)$$

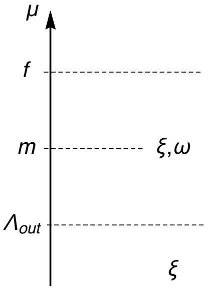
Goldstone's theorem

Let one be given an SSB pattern:

$$G \to H,$$

and let Z_a be broken generators and $B_n \in Z_a : \hat{B}_n \Phi|_0 \neq 0$, then:

- n_{NG} = nuber of B_n
- Nambu-Goldstone fields corresponding to B_α such that $[P_\mu, B_\alpha] \sim B_n$ are massive



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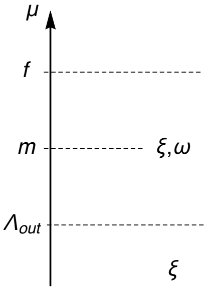

If some of the generators always act trivially at the origin, they never give rise to Nambu-Goldstone fields.

The conformal group: $\forall \Phi \; \hat{K}_n \Phi = 0$

Conclusion

- The action of the generators on the vacuum at the origin uniquely fixes the number of Nambu-Goldstone fields
- Some of the Nambu-Goldstone fields are necessarily gapped
- Inverse Higgs constraints is a trick used to uncharge fields under the action of broken but acting trivially at the origin generators