Equivalent photons in proton-proton collisions at the LHC: $pp(\gamma\gamma) \to l^+l^-pp$

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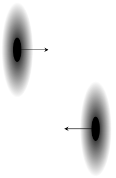
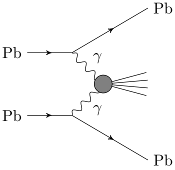
Introduction





$$\sigma \sim Z^4$$

- Integrated luminosity in pp collisions provided by the LHC to CMS in Run 2: 111 fb^{-1} at 13 TeV.
- Integrated luminosity in Pb Pb collisions provided by the LHC to CMS in HI run (2015): 0.60 nb^{-1}.
- Luminosity ratio: $1.9 \cdot 10^8$.
- For Pb, $Z = 82$; $Z^4 \approx 4.5 \cdot 10^7$.
- There could be about 4 times more events of New Physics related to electromagnetism in pp collisions than there were in Pb Pb collision.
- HI run duration is ≈ 30 days, Run 2 duration is ≈ 400 days (not counting the 2015).

A question on whether the heavy ion luminosity should be increased might open to discussion.

Introduction

Goal: derive analytical formulas to describe experimental data.

Experimental data:

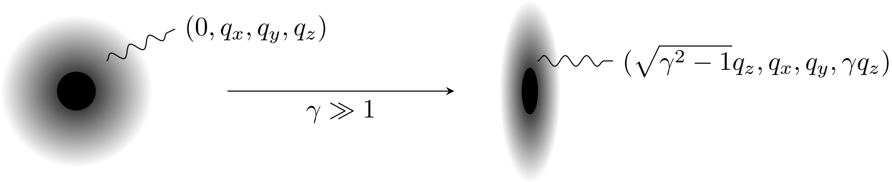
1. [1708.04053] (ATLAS): $pp \to pp\mu^+\mu^-$ at collision energy 13 TeV with integrated luminosity of 3.2 fb^{-1}.
   Fiducial cross section: 3.12 ± 0.07 (stat.) $\pm\, 0.10$ (syst.) pb.
   Cuts on the muon system parameters:

Invariant mass range	Transverse momentum	Pseudorapidity
12 GeV $< m_{\mu\mu} < 30$ GeV	> 6 GeV	< 2.4
30 GeV $< m_{\mu\mu} < 70$ GeV	> 10 GeV	< 2.4

2. ATLAS-CONF-2016-025: Pb Pb $\to$ Pb Pb $\mu^+\mu^-$ at collision energy per nucleon pair 5.02 TeV with integrated luminosity of 515 μb^{-1}.
   Fiducial cross section: 32.2 ± 0.3 (stat.)$^{+4.0}_{-3.4}$ (syst.) μb.
   Cuts on the muon system parameters:

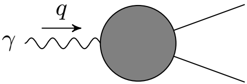
Invariant mass range	Transverse momentum	Pseudorapidity
10 GeV $< m_{\mu\mu} < 100$ GeV	> 4 GeV	< 2.4

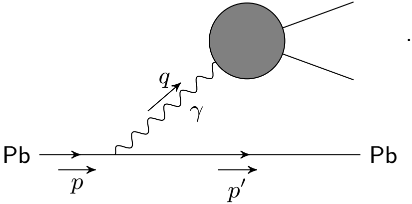
Equivalent photons approximation



Photon virtuality: $-q^2 = q_x^2 + q_y^2 + q_z^2 \ll (\gamma q_z)^2 \equiv \omega^2$

Equivalent photons approximation

Let $A_{\text{real}} =$ 

, $A_{\text{virtual}} =$ 

.

Then $A_{\text{virtual}} = A_{\text{real}} \frac{Ze}{-q^2} \frac{2E}{\omega} |\vec{q}_\perp|$.

$$\mathrm{d}\sigma_{\text{real}} = |A_{\text{real}}|^2 (2\pi)^4 \delta^{(4)}(P_f - P_i) \frac{1}{4m\omega} \mathrm{d}\rho$$

$$\mathrm{d}\sigma_{\text{virtual}} = |A_{\text{virtual}}|^2 (2\pi)^4 \delta^{(4)}(P_f - P_i) \frac{1}{4mE} \frac{\mathrm{d}^3 p'}{2E(2\pi)^3} \mathrm{d}\rho$$

EPA:

$$\mathrm{d}\sigma_{\text{virtual}} = \mathrm{d}\sigma_{\text{real}} \cdot n(\vec{q})\, \mathrm{d}^3 q$$

$$n(\vec{q}) = \frac{Z^2 \alpha}{\pi^2} \frac{\vec{q}_\perp^{\,2}}{\omega q^4}$$

[Berestetsky, Lifshitz, Pitaevsky, Theoretical Physics vol. 4, §99]

Equivalent photons approximation

$$
\begin{aligned}
\mathrm{d}\sigma_{\mathsf{virtual}} &= \mathrm{d}\sigma_{\mathsf{real}} \cdot n(\vec{q})\mathrm{d}^3 p' = \mathrm{d}\sigma_{\mathsf{real}} \cdot n(\omega)\mathrm{d}\omega \\
n(\vec{q}) &= \frac{Z^2\alpha}{\pi^2}\frac{\vec{q}_\perp^{\,2}}{\omega q^4} = \frac{Z^2\alpha}{\pi^2}\frac{\vec{q}_\perp^{\,2}}{\left(q_\perp^2 + (\omega/\gamma)^2\right)^2}, \\
n(\omega) &= \int n(\vec{q})\mathrm{d}^2 q_\perp \\
&= 2\pi \int\limits_0^{\hat{q}} n(\vec{q}) q_\perp \mathrm{d}q_\perp \\
&= \frac{Z^2\alpha}{\pi\omega}\left\{\ln\left[1 + \left(\frac{\hat{q}\gamma}{\omega}\right)^2\right] - \frac{1}{1 + \left(\frac{\omega}{\hat{q}\gamma}\right)^2}\right\} \\
&\approx (\omega \ll \hat{q}\gamma) \approx \frac{2Z^2\alpha}{\pi\omega}\ln\frac{\hat{q}\gamma}{\omega} \\
\hat{q} &= ?
\end{aligned}
$$

EPA spectrum cutoff

For the proton, $\hat{q} \approx \Lambda_{\text{QCD}} = 0.2\text{–}0.3$ GeV.
Dirac form factor:

$$\mathcal{J}_\mu = F(q^2)\bar{\psi}\gamma_\mu\psi,\ F(q^2) \approx \frac{1}{\left(1 - \frac{q^2}{\Lambda^2}\right)^2}\ \ (-q^2 \ll 4m_p^2),\ \Lambda^2 = 0.71\ \text{GeV}^2.$$

EPA spectrum with form factor:

$$n'(\vec{q}) = \frac{Z^2\alpha}{\pi^2}\frac{\vec{q}_\perp^{\,2}}{\omega q^4}\frac{1}{\left(1 - \frac{q^2}{\Lambda^2}\right)^2},$$

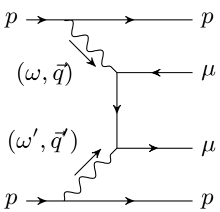
$$n'(\omega) = \int n'(\vec{q})\mathrm{d}^2q = 2\pi\int_0^\infty n'(\vec{q})q_\perp\mathrm{d}q_\perp \approx \frac{2Z^2\alpha}{\pi\omega}\left(\ln\frac{\Lambda\gamma}{\omega} - \frac{17}{12}\right)\ (\omega \ll \Lambda\gamma)$$

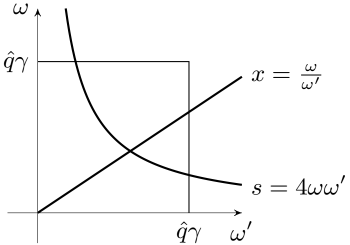
In the leading logarithmic approximation

$$n'(\omega) \approx n(\omega) = \frac{2Z^2\alpha}{\pi\omega}\ln\frac{\hat{q}\gamma}{\omega}$$

hence $\hat{q} = \Lambda \mathrm{e}^{-\frac{17}{12}} \approx 204$ MeV.
For Pb, $\Lambda \approx 80$ MeV [hep-ph/0606069] and $\hat{q} \approx 20$ MeV.

Total cross section





$$d\sigma(pp(\gamma\gamma) \to \mu^+\mu^- pp) = \sigma(\gamma\gamma \to \mu^+\mu^-) \cdot n(\omega) n(\omega') d\omega d\omega',$$

$$\sigma(\gamma\gamma \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{s}\left[\left(1 + \frac{4m_\mu^2}{s} - \frac{8m_\mu^4}{s^2}\right)\ln\frac{1+\sqrt{1-\frac{4m_\mu^2}{s}}}{1-\sqrt{1-\frac{4m_\mu^2}{s}}} - \left(1 + \frac{4m_\mu^2}{s}\right)\sqrt{1 - \frac{4m_\mu^2}{s}}\right]$$

Integration constraints: $s \equiv (q + q')^2 > 4m_\mu^2$, $\omega < \hat{q}\gamma$, $\omega' < \hat{q}\gamma$.

Change of variables: $s = 4\omega\omega'$, $x = \frac{\omega}{\omega'}$; $\omega = \sqrt{\frac{sx}{4}}$, $\omega' = \sqrt{\frac{s}{4x}}$

$(2m_\mu)^2 < s < (2\hat{q}\gamma)^2$, $\frac{(2\hat{q}\gamma)^2}{s} < x < \frac{s}{(2\hat{q}\gamma)^2}$.

Integration result (leading logarithmic approximation):

$$\sigma(pp(\gamma\gamma) \to pp\mu^+\mu^-) \approx 8 \cdot \frac{28}{27}\frac{\alpha^4}{\pi m_\mu^2}\ln^3\frac{\hat{q}\gamma}{m_\mu} \approx 2.2 \cdot 10^5 \text{ pb}$$

[Landau, Lifshitz, Phys.Zs.Sowjet, 6, 244 (1934)] LHC cuts?

Cut on invariant mass

$70\text{ GeV} > s > 12\text{ GeV} \gg m_\mu = 0.106\text{ GeV}.$

$\sigma(\gamma\gamma \to \mu^+\mu^-) \approx \frac{4\pi\alpha^2}{s}\left(\ln\frac{s}{m_\mu^2} - 1\right)$

$$\sigma_{\text{fid.}}(pp(\gamma\gamma) \to pp\mu^+\mu^-) = \int\limits_{\hat{s}_{\text{min}}}^{\hat{s}_{\text{max}}} \mathrm{d}s\, \sigma(\gamma\gamma \to \mu^+\mu^-) \int\limits_{s/(2\hat{q}\gamma)^2}^{(2\hat{q}\gamma)^2/s} \frac{\mathrm{d}x}{8x} n\left(\sqrt{\frac{sx}{4}}\right) n\left(\sqrt{\frac{s}{4x}}\right)$$

$$= \frac{8\alpha^4}{3\pi}\frac{1}{(2\hat{q}\gamma)^2}\frac{1}{z}\left[\ln^4 z + \left(2\ln\frac{2\hat{q}\gamma}{m_\mu} + 3\right)\left(\ln^3 z + 3\ln^2 z + 6\ln z + 6\right)\right]\Bigg|_{z=\frac{\hat{s}_{\text{min}}}{(2\hat{q}\gamma)^2}}^{\frac{\hat{s}_{\text{max}}}{(2\hat{q}\gamma)^2}}$$

$$\approx 59.6\text{ pb.}$$

Cut on transverse momentum

for 12 GeV $< s < 30$ GeV: $\quad p_T > 6$ GeV,
for 30 GeV $< s < 70$ GeV: $\quad p_T > 10$ GeV.

$$\mathrm{d}\sigma(\gamma\gamma \to \mu^+\mu^-) = \frac{8\pi\alpha^2}{s p_T} \frac{1 - \frac{2p_T^2}{s}}{\sqrt{1 - \frac{4p_T^2}{s}}} \mathrm{d}p_T \; (4m_\mu^2 \ll s)$$

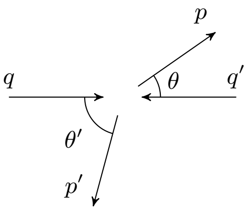
$$\sigma_{\text{fid.}}(pp(\gamma\gamma) \to pp\mu^+\mu^-) = \int\limits_{\hat{s}_{\text{min}}}^{\hat{s}_{\text{max}}} \mathrm{d}s \int\limits_{\hat{p}_T}^{\sqrt{s}/2} \mathrm{d}p_\mathrm{T} \int\limits_{s/(2\hat{q}\gamma)^2}^{(2\hat{q}\gamma)^2/s} \frac{\mathrm{d}x}{8x} \frac{\mathrm{d}\sigma(\gamma\gamma \to \mu^+\mu^-)}{\mathrm{d}p_T} n\left(\sqrt{\frac{sx}{4}}\right) n\left(\sqrt{\frac{s}{4x}}\right)$$

$$= \frac{8\alpha^4}{3\pi} \int\limits_{\hat{s}_{\text{min}}}^{\hat{s}_{\text{max}}} \mathrm{d}s \, \frac{1}{s^2} \ln^3 \frac{(2\hat{q}\gamma)^2}{s} \left(\ln \frac{1 + \sqrt{1 - \frac{4\hat{p}_T^2}{s}}}{1 - \sqrt{1 - \frac{4\hat{p}_T^2}{s}}} - \sqrt{1 - \frac{4\hat{p}_T^2}{s}} \right)$$

$$\approx 5.37 \text{ pb} + 0.91 \text{ pb} = 6.28 \text{ pb}.$$

Cut on pseudorapidity

Cut: $|\eta| < \hat\eta < 2.4$.
$\eta = -\ln\tan(\theta/2)$, so $10^\circ \lesssim \theta \lesssim 170^\circ$.
In terms of phase space variables:



$$\frac{\mathrm{e}^{-\hat\eta}}{1-\sqrt{1-\frac{4p_T^2}{s}}} < \frac{\omega}{p_T} < \frac{\mathrm{e}^{\hat\eta}}{1+\sqrt{1-\frac{4p_T^2}{s}}}$$

$$\sigma_{\text{fid.}}(pp(\gamma\gamma)\to pp\mu^+\mu^-) = \int\limits_{\hat s_{\text{min}}}^{\hat s_{\text{max}}} \mathrm{d}s \int\limits_{\hat p_T}^{\sqrt{s}/2} \mathrm{d}p_T \int\limits_{\mathrm{e}^{-2\hat\eta}\cdot\frac{1+\sqrt{1-\frac{4p_T^2}{s}}}{1-\sqrt{1-\frac{4p_T^2}{s}}}}^{\mathrm{e}^{2\hat\eta}\cdot\frac{1-\sqrt{1-\frac{4p_T^2}{s}}}{1+\sqrt{1-\frac{4p_T^2}{s}}}} \frac{\mathrm{d}x}{8x}\frac{\mathrm{d}\sigma(\gamma\gamma\to\mu^+\mu^-)}{\mathrm{d}p_T} n\left(\sqrt{\frac{sx}{4}}\right) n\left(\sqrt{\frac{s}{4x}}\right)$$

$$= \frac{4\alpha^4}{\pi}\int\limits_{\hat s_{\text{min}}}^{\hat s_{\text{max}}} \frac{\mathrm{d}s}{s^2} \int\limits_{\hat p_T}^{\sqrt{s}/2} \frac{\mathrm{d}p_T}{p_T} \frac{1-\frac{2p_T^2}{s}}{\sqrt{1-\frac{4p_T^2}{s}}} \int\limits_{\mathrm{e}^{-2\hat\eta}\cdot\frac{1+\sqrt{1-\frac{4p_T^2}{s}}}{1-\sqrt{1-\frac{4p_T^2}{s}}}}^{\mathrm{e}^{2\hat\eta}\cdot\frac{1-\sqrt{1-\frac{4p_T^2}{s}}}{1+\sqrt{1-\frac{4p_T^2}{s}}}} \frac{\mathrm{d}x}{x} \ln\frac{(2\hat q\gamma)^2}{sx} \ln\left(\frac{(2\hat q\gamma)^2}{s}\cdot x\right)$$

≈ 2.85 pb $+ 0.50$ pb $= 3.35$ pb

Cuts summary

$pp(\gamma\gamma) \to \mu^+\mu^- pp$

No cuts		$2.2 \cdot 10^5$ pb
12 GeV $< \sqrt{s} <$ 30 GeV	54.0 pb	59.6 pb
30 GeV $< \sqrt{s} <$ 70 GeV	5.65 pb	
12 GeV $< \sqrt{s} <$ 30 GeV, $p_T >$ 6 GeV	5.37 pb	6.28 pb
30 GeV $< \sqrt{s} <$ 70 GeV, $p_T >$ 10 GeV	0.91 pb	
12 GeV $< \sqrt{s} <$ 30 GeV, $p_T >$ 6 GeV, $\|\eta\| < 2.4$	2.85 pb	3.35 pb
30 GeV $< \sqrt{s} <$ 70 GeV, $p_T >$ 10 GeV, $\|\eta\| < 2.4$	0.50 pb	

Pb Pb $(\gamma\gamma) \to \mu^+\mu^-$Pb Pb

No cuts	$7.19 \cdot 10^9$ μb
10 GeV $< \sqrt{s} <$ 100 GeV	119 μb
also $p_T >$ 4 GeV	34.2 μb
also $\|\eta\| < 2.4$	30.9 μb

1708.04053 (ATLAS)

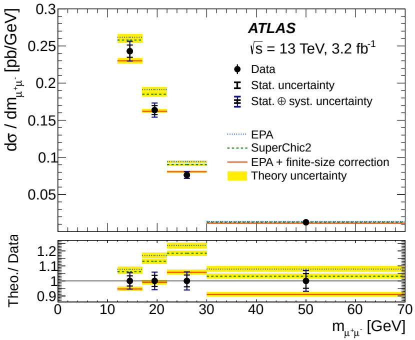
$pp \to pp\ \mu^+\mu^-$ with $\sqrt{s} = 13$ TeV and integrated luminosity of 3.2 fb^{-1}.

Cuts:

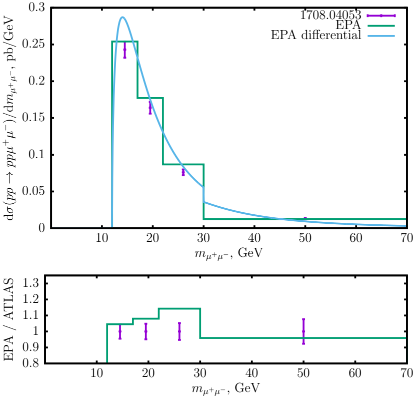
for 12 GeV $< m_{\mu\mu} < 30$ GeV: $p_T(\mu) > 6$ GeV, $|\eta(\mu)| < 2.4$.

for 30 GeV $< m_{\mu\mu} < 70$ GeV: $p_T(\mu) > 10$ GeV, $|\eta(\mu)| < 2.4$.

Experiment



EPA



Fiducial cross section:

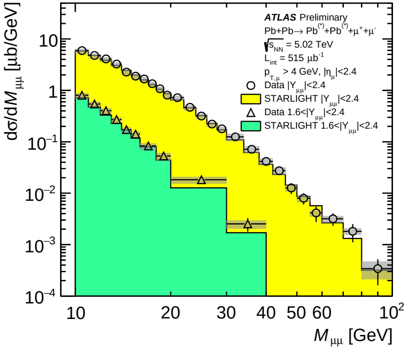
3.12 ± 0.07 (stat.) $\pm\ 0.10$ (syst.) pb — 3.35 pb

ATLAS-CONF-2016-025

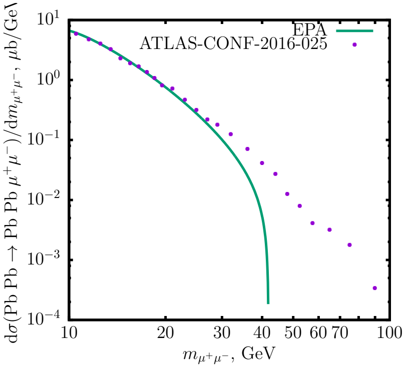
Pb Pb $\to$ Pb Pb $\mu^+\mu^-$ with $\sqrt{s_{\text{NN}}} = 5.02$ TeV and integrated luminosity of $515\ \mu\text{b}^{-1}$.
Cuts:

- 10 GeV $< m_{\mu\mu} < 100$ GeV.
- $p_T(\mu) > 4$ GeV.
- $|\eta(\mu)| < 2.4$.

Experiment



EPA



Fiducial cross section:

32.2 ± 0.3 (stat.)$^{+4.0}_{-3.4}$ (syst.) μb — $30.9\ \mu$b

Conclusion

- The LHC can be used to search for New Physics in photon-photon collisions.
- Photon invariant mass can reach $2\hat{q}\gamma \approx 2.8$ TeV in pp collisions with $\sqrt{s_{pp}} = 13$ TeV and 100 GeV in Pb Pb collisions with $\sqrt{s_{NN}} = 5.03$ TeV.
- We have developed analytical formulas for the fiducial cross section of lepton pair production in peripheral collisions of charged particles. Fiducial cross sections calculated for the reactions $pp(\gamma\gamma) \to pp\mu^+\mu^-$ and Pb Pb $(\gamma\gamma) \to$ Pb Pb $\mu^+\mu^-$ were found to be in agreement with the experimental data.