Scattering beyond the plane-wave approximation and probing of phases of scattering amplitudes

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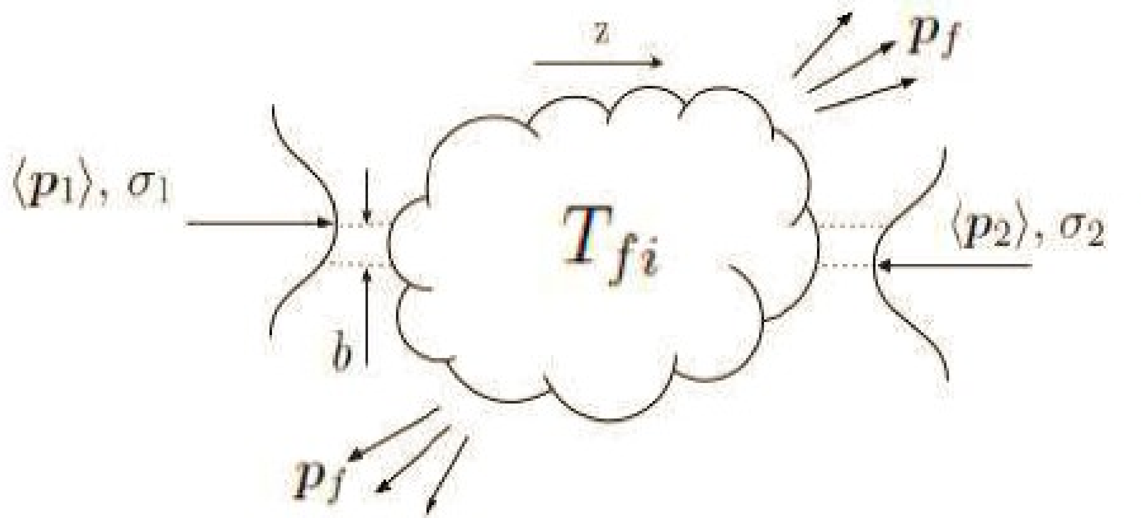
Consider a generic scattering problem 2 → N:



The plane-wave approximation is said to be applicable when:

$\sigma \ll m$ ← the packets are narrow in the momentum space

$\sigma_\perp \gg \lambda_c = \hbar/mc$ ← and wide in the coordinate space

Naively, the non-paraxial corrections to observables are

$$\sim \sigma^2/m^2 \sim \lambda_c^2/\sigma_\perp^2 \ll 1$$

Say, for the LHC beam it is less than 10^{-22}

For modern electron accelerators it is less than 10^{-14} (and some 2-3 orders larger for ILC and CLIC)

For electron microscopes it is less than 10^{-6} (!) Verbeeck, et al. 2011

Fortunately, there are also dynamical (not purely "geometrical") effects!

The plane-wave approximation in scattering

does not work if:

1. The impact parameters are large: an MD-effect (Novisibirsk)

Tikhonov 1982; Kotkin, Serbo, Schiller 1992

2. The initial particles are unstable *Ginzburg 1996; Melnikov, Serbo, 1997*

3. One describes neutrino oscillations *Akhmedov, Smirnov 2009;*

Akhmedov, Kopp, 2010

4. The in-states are not Gaussian (!) *Jentschura, Serbo, 2011; Ivanov, 2011*

5. The quantum coherence is lost (!) *Sarkadi 2016; D.K., Serbo 2017*

To be addressed in this talk

Outline

(1) Some non-Gaussian quantum states:

I. Vortex photons, electrons and neutrons with orbital angular momentum,
II. Airy photons and electrons,
III. Schrödinger cats,
IV. Their generalizations

(2) Non-paraxial wave packets and the Wigner functions

(3) Non-paraxial effects in scattering:

I. Finite momentum uncertainties and impact-parameter, “approximate” conservation laws, etc.

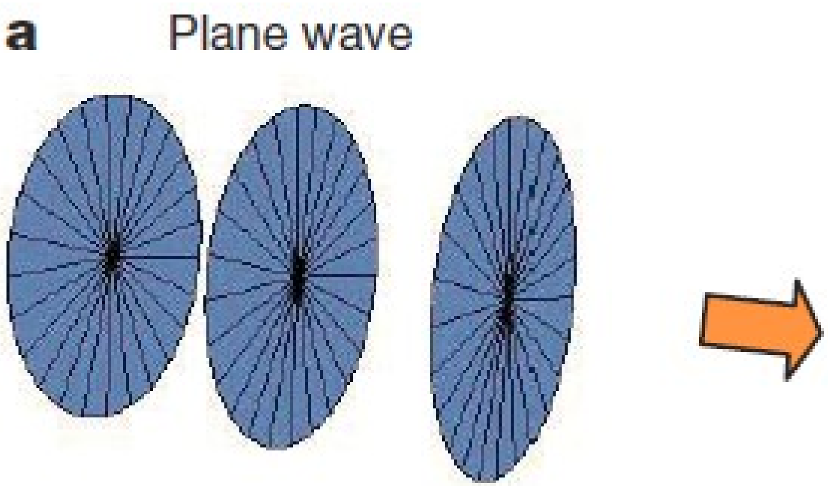
II. The cross section grows dependent upon a phase of a scattering amplitude (say, hadronic or Coulomb one)

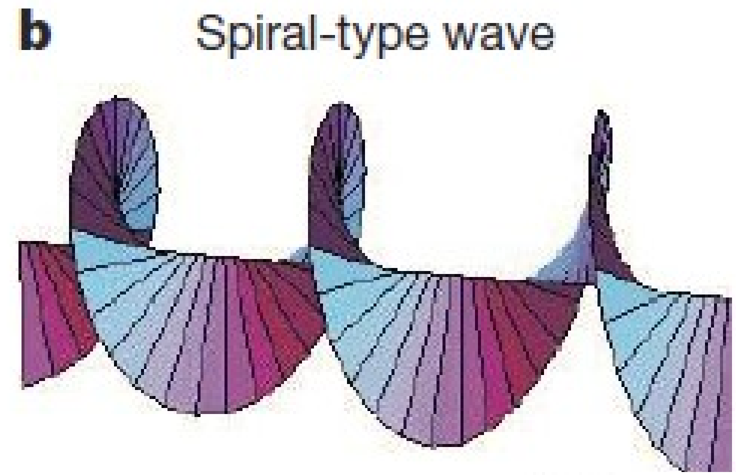
III. Quantum decoherence and the Wigner functions' negativity may affect the cross section

IV. Enhancement of the non-paraxial corrections to the plane-wave cross sections for vortex particles with large orbital angular momentum

1. Non-Gaussian states: vortex particles with orbital angular momentum (OAM)

Twisted photons: Allen, et al. 1992



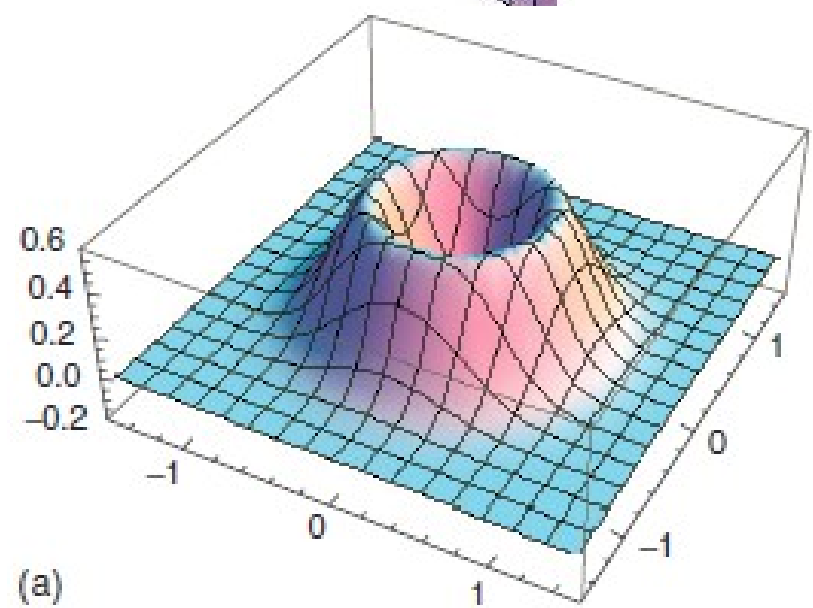


M. Uchida and A.Tonomura, Nature **464**, 737 (2010)

A Bessel-state of a free scalar particle:

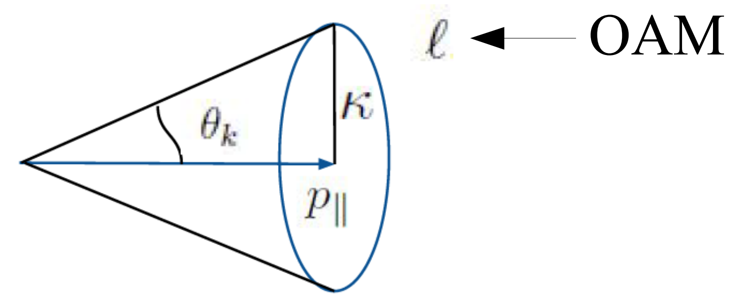
$$\psi(r) = N \, J_\ell(\kappa\rho) e^{-i\varepsilon t + i p_\parallel z + i\ell\phi_r}$$

Probability density
for a well-normalized wave packet



They form a complete and orthogonal set:

$$\langle p'_{\parallel},\kappa',\ell'|p_{\parallel},\kappa,\ell\rangle = (2\pi)^2 2\varepsilon(p)\,\delta(p_{\parallel}-p'_{\parallel})\frac{\delta(\kappa-\kappa')}{\kappa}\,\delta_{\ell\ell'}$$



$$\hat{\psi}(x) = \sum_{\ell}\int \frac{dp_{\parallel}\kappa d\kappa}{(2\pi)^2 2\varepsilon}(\langle x|p_{\parallel},\kappa,\ell\rangle\,\hat{a}_{\{p_{\parallel},\kappa,\ell\}} + \text{H.c.})$$

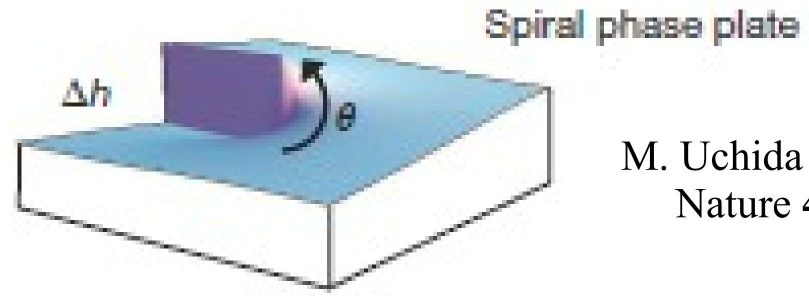
$$= \sum_{\ell}\int \frac{dp_{\parallel}\kappa d\kappa}{(2\pi)^2\sqrt{2\varepsilon}}\,(J_{\ell}(\kappa\rho)e^{-i\varepsilon t+ip_{\parallel}z+i\ell\phi_r}\hat{a}_{\{p_{\parallel},\kappa,\ell\}} + \text{H.c.})$$

$$[\hat{a}_{\{p_{\parallel},\kappa,\ell\}},\hat{a}^{\dagger}_{\{p'_{\parallel},\kappa',\ell'\}}] = (2\pi)^2\delta(p_{\parallel}-p'_{\parallel})\frac{\delta(\kappa-\kappa')}{\kappa}\,\delta_{\ell\ell'}$$

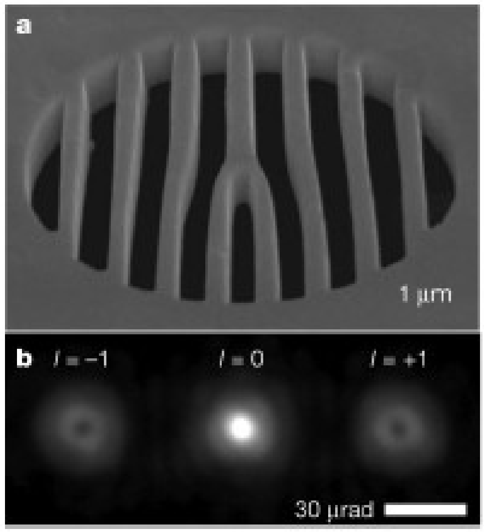
$$[\hat{\psi}(x),\hat{\psi}^{\dagger}(x')] = \sum_{\ell}\int \frac{dp_{\parallel}\kappa d\kappa}{(2\pi)^2 2\varepsilon}J_{\ell}(\kappa\rho)J_{\ell}(\kappa\rho')\,(e^{-i\varepsilon(t-t')+ip_{\parallel}(z-z')+i\ell(\phi_r-\phi'_r)} - \text{c.c.}).$$

D.K., PRA **91** (2015) 013847

Vortex electrons with E = 300 keV were generated in 2010:



M. Uchida and A. Tonomura,
Nature **464**, 737 (2010)



J. Verbeeck, et al.,
Nature ***467**, 301 (2010)*

- They can be focused to a spot of 0.1 нм

  J. Verbeeck, et al., Appl. Phys. Lett. ***99**, 203109 (2011)*
- Their OAM can be as high as 1000!

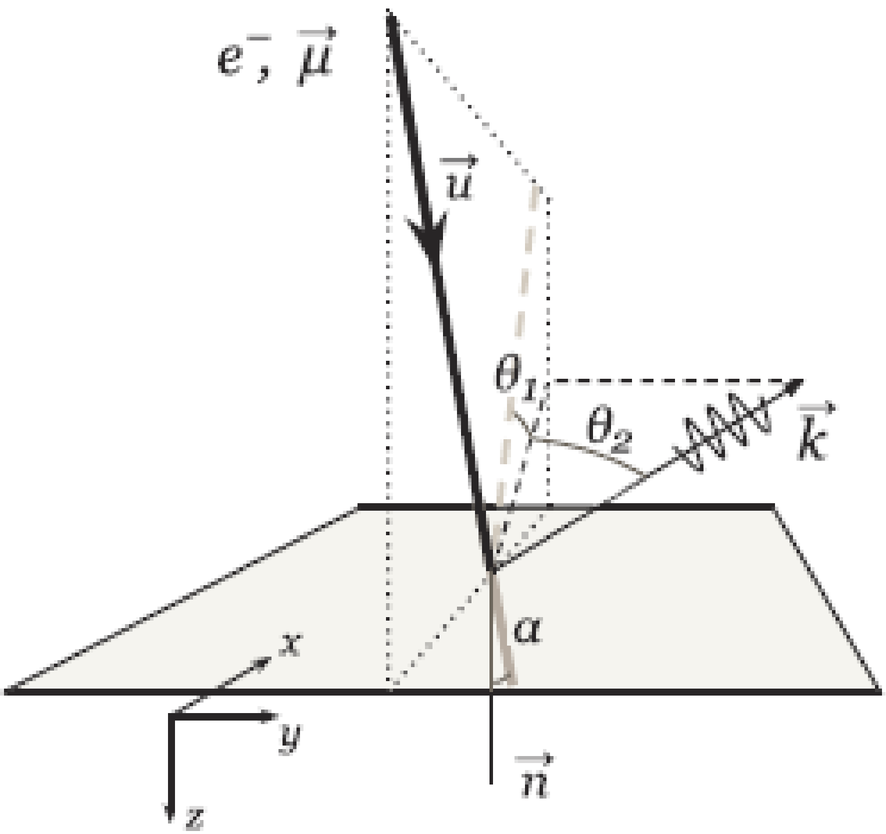
  E. Mafakheri, et al. Appl. Phys. Let. ***110**, 093113 (2017)*
- Magnetic moment of such electrons is 3 orders of magnitude larger than the Bohr magneton!

  K.Yu. Bliokh, et al., PRL ***107**, 174802 (2011)*

1. Non-Gaussian states: vortex particles with orbital angular momentum (OAM)

The huge magnetic moment → "Orbital light":

Transition radiation:



I.P. Ivanov, D.K., PRL **110** (2013) 264801

Angular asymmetry of ~ 0.1 – 1%

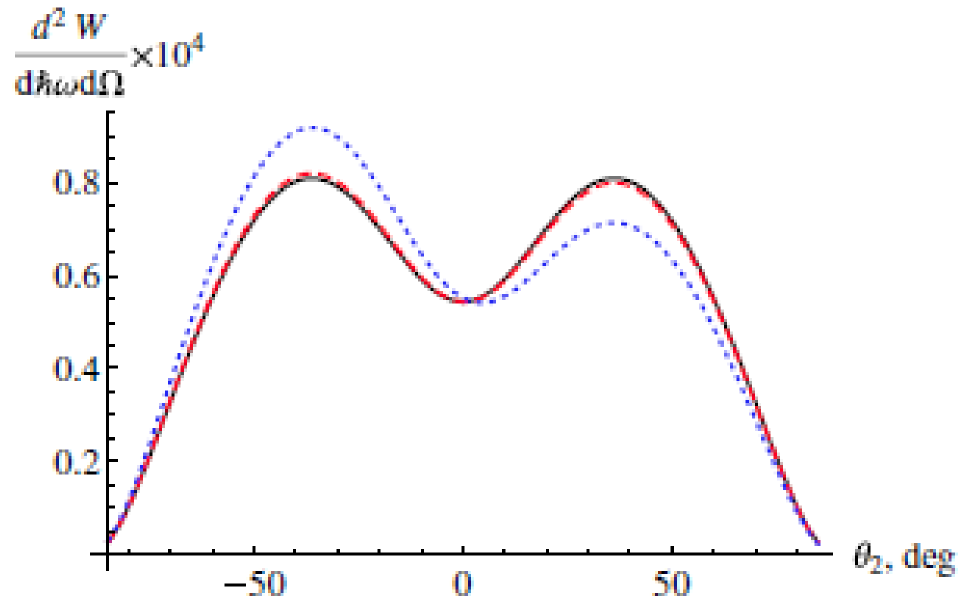


FIG. 2 (color online). Distribution of the forward TR over θ_2 for $\ell = 0$ (black solid line), $\ell = 1000$ (red dashed line), and $\ell = 10000$ (blue dotted line). Parameters are $\alpha = 70°$, $\theta_1 = -40°$, $\hbar\omega = 5$ eV.

Berry, Balazs 1979

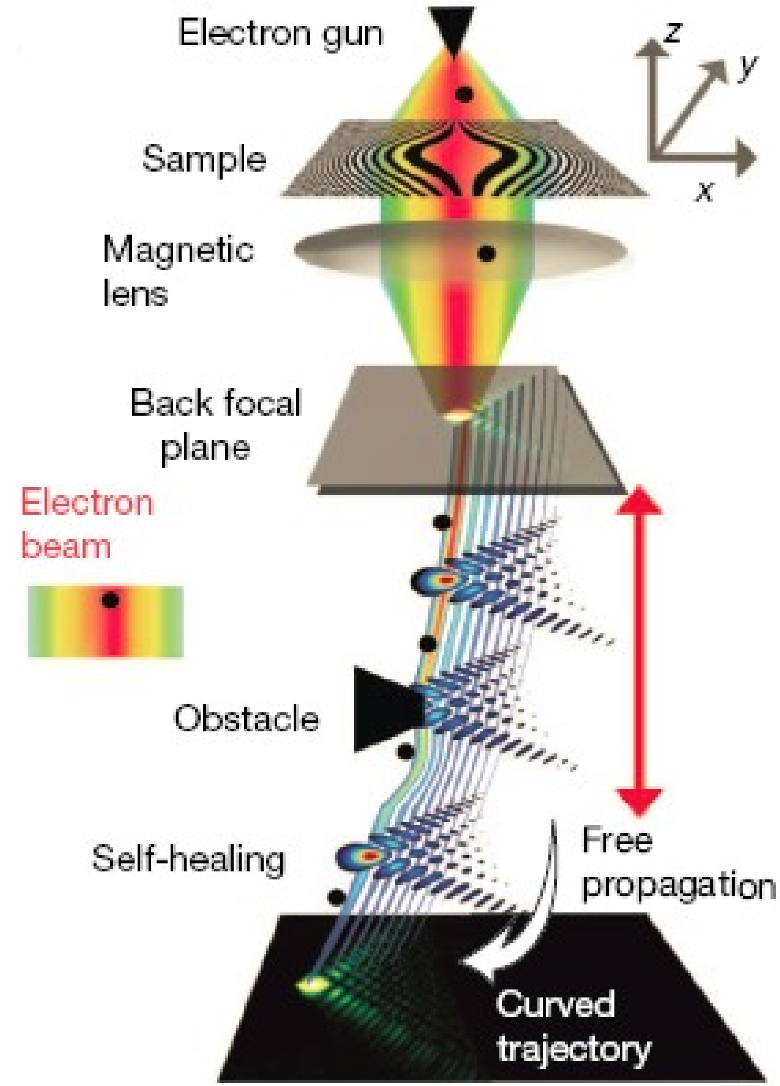
For an ideal Airy beam:

1. There is no spreading

2. Curved path in a free space

3. Self-healing after scattering

Experimental realization for 200 keV electrons →



*N. Voloch-Bloch, et al., Nature **494** (2013) 331*

We need a Lorentz-invariant description of the non-Gaussian wave packets beyond the paraxial regime!

A Gaussian packet of a massive boson:

Naumov, Naumov 2010

$$\psi(p) = \frac{2^{3/2}\pi}{\sigma} \frac{e^{-m^2/\sigma^2}}{\sqrt{K_1(2m^2/\sigma^2)}} \exp\left\{\frac{(p-\bar{p})^2}{2\sigma^2}\right\}$$

$$\int \frac{d^3p}{(2\pi)^3} \frac{1}{2\varepsilon} |\psi(p)|^2 = 1,$$

$$p^2 = \bar{p}^2 = m^2$$

In the paraxial regime this turns into a customary Gaussian packet:

$$\frac{(p-\bar{p})^2}{2\sigma^2} = -\frac{1}{2\sigma^2} \left(\delta_{ij} - \bar{u}_i \bar{u}_j\right) (\boldsymbol{p} - \bar{\boldsymbol{p}})_i (\boldsymbol{p} - \bar{\boldsymbol{p}})_j + \mathcal{O}((\boldsymbol{p} - \bar{\boldsymbol{p}})^3)$$

Mean energy:

$$\langle \varepsilon \rangle = \int d^3x\, T^{00} = \bar{\varepsilon}\, \frac{K_2\left(2m^2/\sigma^2\right)}{K_1\left(2m^2/\sigma^2\right)} = \bar{\varepsilon} \left(1 + \frac{3}{4}\frac{\sigma^2}{m^2} + \mathcal{O}(\sigma^4/m^4)\right)$$

Non-paraxial correction!

2. Non-paraxial wave packets and the Wigner functions

A relativistic generalization for a vortex boson will be:

$$\psi_\ell(p) = \frac{2^{3/2}\pi}{\sigma^{|\ell|+1}\sqrt{|\ell|!}}\, p_\perp^{|\ell|}\, \frac{e^{-m^2/\sigma^2}}{\sqrt{K_{|\ell|+1}(2m^2/\sigma^2)}}\, \exp\left\{\frac{(p-\bar{p})^2}{2\sigma^2} + i\ell\phi_p\right\}$$

They are orthogonal:

$$\int \frac{d^3p}{(2\pi)^3}\frac{1}{2\varepsilon}\, [\psi_{\ell'}(p)]^*\, \psi_\ell(p) = \delta_{\ell,\ell'}$$

An exact solution to the Klein-Gordon equation:

$$\psi_\ell(x) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2\varepsilon}\, \psi_\ell(p) e^{-ipx} = \frac{(i\rho)^{|\ell|}}{\sqrt{2|\ell|!}\,\pi}\, \frac{\sigma^{|\ell|+1}}{\varsigma^{|\ell|+1}}\, \frac{K_{|\ell|+1}(\varsigma m^2/\sigma^2)}{\sqrt{K_{|\ell|+1}(2m^2/\sigma^2)}}\, e^{i\ell\phi_r}$$

$$\varsigma = \frac{1}{m}\sqrt{\left(\bar{p}_\mu + i x_\mu \sigma^2\right)^2} = \text{inv},\ \operatorname{Re}\varsigma > 0$$

And analogously for a spinning particle...

D.K., ArXiv: 1803.09150; 1803.10166

The mean momentum of such a vortex packet is

$$\langle p_\ell^\mu \rangle = \{\langle \varepsilon_\ell \rangle, \langle \boldsymbol{p}_\ell \rangle\} = \{\bar{\varepsilon}, \bar{\boldsymbol{p}}\} \frac{K_{|\ell|+2}\left(2m^2/\sigma^2\right)}{K_{|\ell|+1}\left(2m^2/\sigma^2\right)} \simeq \{\bar{\varepsilon}, \bar{\boldsymbol{p}}\} \left(1 + \left(\frac{3}{4} + \frac{|\ell|}{2}\right) \frac{\sigma^2}{m^2}\right)$$

An invariant mass of this packet: $$m_\ell^2 = \langle p_\ell \rangle^2 \simeq m^2 \left(1 + \left(\frac{3}{2} + |\ell|\right) \frac{\sigma^2}{m^2}\right)$$

With modern technology (at el. microscopes): $$\frac{\delta m_\ell}{m_{\text{inv}}} \simeq \frac{\delta m_\ell}{m} \lesssim 10^{-3} \qquad |\ell| \sim 10^3 \quad \sigma_\perp \gtrsim 0.1 \text{ nm}$$

Analogously for the vortex electron's magnetic moment:

$$\boldsymbol{\mu}_f = \frac{1}{2} \int d^3 r \, \mathbf{r} \times \bar{\psi}_f(x) \gamma \psi_f(x) \simeq \frac{1}{2\bar{\varepsilon}} (\zeta + \hat{\boldsymbol{z}}\, \ell) \left(1 + \mathcal{O}(|\ell|\sigma^2/m^2)\right)$$

Enhancement due to the OAM!

D.K., ArXiv: 1803.09150; 1803.10166

For scattering of wave packets instead of plane waves:

$$S_{fi} = \langle pw|\hat{S}|i\rangle = \int \prod_{i=1}^{N} \frac{d^3 p_i}{(2\pi)^3}\, \psi_i(\boldsymbol{p}_i) S_{fi}^{(pw)}$$

Is there a small parameter?

The plane-wave limit: $\sigma_i \to 0, \quad \boldsymbol{p}_i \to \boldsymbol{p}_i' \to \langle \boldsymbol{p}_i \rangle$ therefore $\frac{\boldsymbol{p}_i + \boldsymbol{p}_i'}{2} \to \langle \boldsymbol{p}_i \rangle,\ \boldsymbol{p}_i - \boldsymbol{p}_i' \to 0$

In the new variables $\frac{\boldsymbol{p}_i + \boldsymbol{p}_i'}{2} \to \boldsymbol{p}_i,\ \boldsymbol{p}_i - \boldsymbol{p}_i' \to \boldsymbol{k}_i$ we get $|\boldsymbol{k}_i| \ll |\boldsymbol{p}_i|$ when $\sigma \ll m$

A density matrix in these new variables is called a Wigner function!

Wigner 1932

The scattering probability can be expressed via the Wigner functions:

$$dW = |S_{fi}|^2 \prod_{f=3}^{N_f+2} V \frac{d^3p_f}{(2\pi)^3} = \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \frac{d^3k}{(2\pi)^3}\, d\sigma(\boldsymbol{k}, \boldsymbol{p}_{1,2})\, \mathcal{L}^{(2)}(\boldsymbol{k}, \boldsymbol{p}_{1,2}),$$

Kotkin, Serbo, Schiller, Int. J. Mod. Phys. A7 (1992) 4707

$$d\sigma(\boldsymbol{k}, \boldsymbol{p}_{1,2}) = (2\pi)^4\, \delta\Big(\varepsilon_1(\boldsymbol{p}_1 + \boldsymbol{k}/2) + \varepsilon_2(\boldsymbol{p}_2 - \boldsymbol{k}/2) - \varepsilon_f\Big)\, \delta^{(3)}(\boldsymbol{p}_1 + \boldsymbol{p}_2 - \boldsymbol{p}_f)$$
$$\times T_{fi}^{(pw)}(\boldsymbol{p}_1 + \boldsymbol{k}/2, \boldsymbol{p}_2 - \boldsymbol{k}/2) T_{fi}^{*\,(pw)}(\boldsymbol{p}_1 - \boldsymbol{k}/2, \boldsymbol{p}_2 + \boldsymbol{k}/2) \frac{1}{v(\boldsymbol{p}_1, \boldsymbol{p}_2)} \prod_{f=3}^{N_f+2} \frac{d^3p_f}{(2\pi)^3},$$

Matches the customary cross section when **k** = 0!

$$\mathcal{L}^{(2)}(\boldsymbol{k}, \boldsymbol{p}_{1,2}) = v(\boldsymbol{p}_1, \boldsymbol{p}_2) \int dt\, d^3r\, d^3R\, e^{i\boldsymbol{k}\boldsymbol{R}}\, \underline{n_1}(\boldsymbol{r}, \boldsymbol{p}_1, t) \underline{n_2}(\boldsymbol{r} + \boldsymbol{R}, \boldsymbol{p}_2, t),$$

$$v(\boldsymbol{p}_1, \boldsymbol{p}_2) = \frac{\sqrt{(p_1 p_2)^2 - m_1^2 m_2^2}}{\varepsilon_1(\boldsymbol{p}_1)\varepsilon_2(\boldsymbol{p}_2)} = \sqrt{(\boldsymbol{u}_1 - \boldsymbol{u}_2)^2 - [\boldsymbol{u}_1 \times \boldsymbol{u}_2]^2},$$

the Wigner functions

What do we lose in the paraxial regime?

For a non-relativistic Airy beam: $\psi(\boldsymbol{p}) = \pi^{3/4}\left(\frac{2}{\sigma}\right)^{3/2} \exp\left\{ -i\boldsymbol{r}_0\boldsymbol{p} - \frac{(\boldsymbol{p}-\langle\boldsymbol{p}\rangle)^2}{2\sigma^2} + \frac{i}{3}\left(\xi_x^3 p_x^3 + \xi_y^3 p_y^3\right)\right\}$

The exact Wigner function is (not everywhere positive)

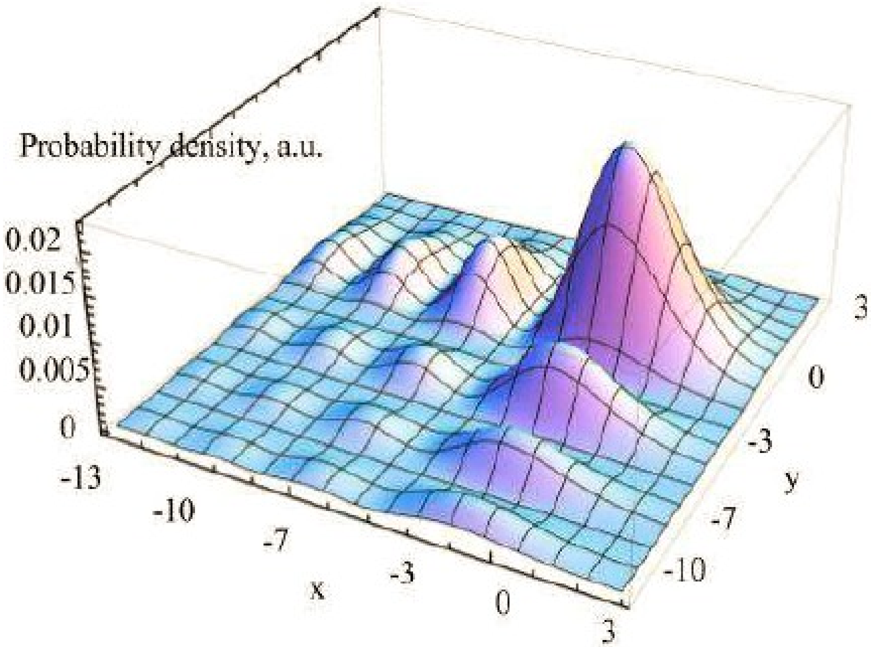
$$n(\boldsymbol{r},\boldsymbol{p},t;\boldsymbol{\xi}) = 2^{13/3}\frac{\pi}{\sigma^2\xi_x\xi_y}\exp\Big\{-\sigma^2(z-\langle z\rangle)^2 - \frac{(\boldsymbol{p}-\langle\boldsymbol{p}\rangle)^2}{\sigma^2} + \\ + \frac{1}{\sigma^2\xi_x^3}\Big(x-\langle x\rangle + \xi_x^3 p_x^2 + \frac{1}{6\sigma^4\xi_x^3}\Big) + \frac{1}{\sigma^2\xi_y^3}\Big(y-\langle y\rangle + \xi_y^3 p_y^2 + \frac{1}{6\sigma^4\xi_y^3}\Big)\Big\}, \\ \times \mathrm{Ai}\left[\frac{2^{2/3}}{\xi_x}\Big(x-\langle x\rangle + \xi_x^3 p_x^2 + \frac{1}{4\sigma^4\xi_x^3}\Big)\right]\mathrm{Ai}\left[\frac{2^{2/3}}{\xi_y}\Big(y-\langle y\rangle + \xi_y^3 p_y^2 + \frac{1}{4\sigma^4\xi_y^3}\Big)\right]$$

The approximate/paraxial one is (everywhere positive)

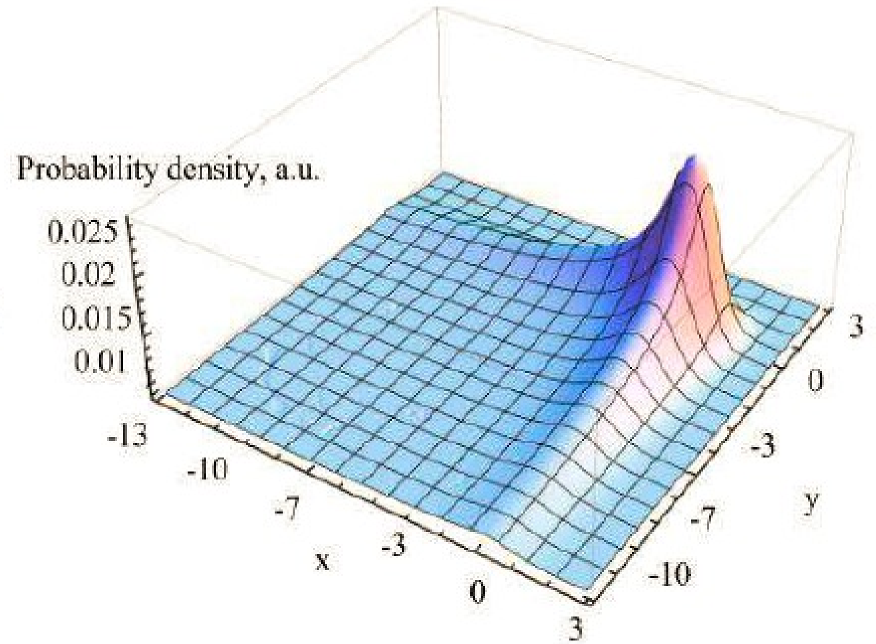
$$n(\boldsymbol{r},\boldsymbol{p},t;\boldsymbol{\xi}) = 8\,\exp\left\{-\frac{(\boldsymbol{p}-\langle\boldsymbol{p}\rangle)^2}{\sigma^2} - \sigma^2\left(\boldsymbol{r}-\langle\boldsymbol{r}\rangle+\boldsymbol{\eta}\right)^2\right\}$$

$$\boldsymbol{\eta} \equiv \boldsymbol{\eta}(\boldsymbol{p}_\perp) = \{\xi_x^3 p_x^2, \xi_y^3 p_y^2, 0\}$$

Possible quantum decoherence is lost!



The exact Airy



The approximate Airy

$$m = 1, \sigma/\langle p\rangle_z = 1/5,\ \xi_x = \xi_y = 2/\sigma,\ r_0 = z = t = \langle p\rangle_\perp = 0$$

We represent the scattering amplitude as follows: $T_{fi} = |T_{fi}| \exp\{i\zeta_{fi}\}$

$$T_{fi}(p_1 + k/2, p_2 - k/2) T^*_{fi}(p_1 - k/2, p_2 + k/2) \approx$$

$$\approx \left(|T_{fi}|^2 + \frac{1}{4} k_i k_j \mathcal{C}_{ij} + \mathcal{O}(k^4) \right) \exp\left\{ \underline{ik\partial_{\Delta p}\, \zeta_{fi}} + \mathcal{O}(k^3) \right\}$$

$$\partial_{\Delta p} = \frac{\partial}{\partial p_1} - \frac{\partial}{\partial p_2},$$

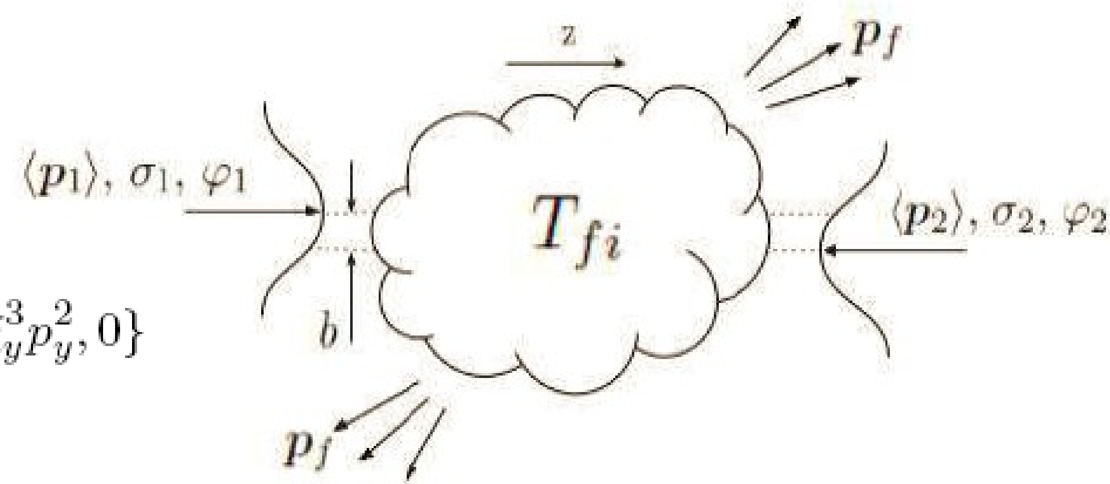
$$\mathcal{C}_{ij}(\mathbf{p}_1, \mathbf{p}_2) = |T_{fi}| \partial_{\Delta p_i} \partial_{\Delta p_j} |T_{fi}| - (\partial_{\Delta p_i} |T_{fi}|)(\partial_{\Delta p_j} |T_{fi}|)$$

$$\tilde{b}_{\varphi} = b - \frac{\partial \varphi_1(p_1)}{\partial p_1} + \frac{\partial \varphi_2(p_2)}{\partial p_2} - \left(\frac{\partial}{\partial p_1} - \frac{\partial}{\partial p_2} \right) \zeta_{fi}.$$

The amplitude's phase

Impact-parameter

Phases of the in-states



$$\frac{\partial \varphi(p)}{\partial p} = \ell \frac{\hat{z} \times p}{p_\perp^2} \quad \text{or} \quad \frac{\partial \varphi(p)}{\partial p} = \eta = \{\xi_x^3 p_x^2, \xi_y^3 p_y^2, 0\}$$

3. Non-paraxial effects in scattering

We derive the first correction to the plane-wave cross section:

$$d\sigma = dN/L \approx d\sigma^{(pw)} + \underline{d\sigma^{(1)}}$$

$$d\sigma^{(pw)} = \frac{dN^{(pw)}}{L^{(pw)}} = N_{b,1}N_{b,2}\,(2\pi)^4\delta^{(4)}(\langle p\rangle_1 + \langle p\rangle_2 - p_f)\frac{|T_{fi}|^2}{v}\prod_{f=3}^{N_f+2}\frac{d^3p_f}{(2\pi)^3}$$

provided the packets do not spread much during the collision: $t_{\text{col}} \ll t_{\text{diff}} \sim \dfrac{\sigma_b}{u_\perp} \sim \sigma_b^2\varepsilon$

$$\frac{d\sigma^{(1)}}{d\sigma^{(pw)}} = \text{“geometric” terms} + \text{dynamic terms}$$

$\sim \dfrac{\sigma_1^2}{m_1^2}$ and $\sim \dfrac{\sigma_2^2}{m_2^2}$

Also depend on the phases and on an overlap of the in-states

D.K., JHEP **03** (2017) 049

To be more precise:

$$dW = \prod_{f=3}^{N_f+2} \frac{d^3p_f}{(2\pi)^3} \frac{(2\pi)^{11}}{(\pi\,\sigma_1\sigma_2)^3} \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \frac{d^3k}{(2\pi)^3}\, \delta\left(\varepsilon_1(\boldsymbol{p}_1 - \boldsymbol{k}/2) + \varepsilon_2(\boldsymbol{p}_2 + \boldsymbol{k}/2) - \varepsilon_f\right)$$
$$\times \delta\left(\varepsilon_1(\boldsymbol{p}_1 + \boldsymbol{k}/2) + \varepsilon_2(\boldsymbol{p}_2 - \boldsymbol{k}/2) - \varepsilon_f\right) \delta(\boldsymbol{p}_1 + \boldsymbol{p}_2 - \boldsymbol{p}_f)$$
$$\times T_{fi}(\boldsymbol{p}_1 + \boldsymbol{k}/2, \boldsymbol{p}_2 - \boldsymbol{k}/2) T^*_{fi}(\boldsymbol{p}_1 - \boldsymbol{k}/2, \boldsymbol{p}_2 + \boldsymbol{k}/2)$$
$$\times \exp\Big\{ -\frac{(\boldsymbol{p}_1 - \langle \boldsymbol{p}\rangle_1)^2}{\sigma_1^2} - \frac{(\boldsymbol{p}_2 - \langle \boldsymbol{p}\rangle_2)^2}{\sigma_2^2} - \boldsymbol{k}^2 \left(\frac{1}{(2\sigma_1)^2} + \frac{1}{(2\sigma_2)^2}\right) -$$
$$-i\boldsymbol{k}\boldsymbol{b} + i\left(\varphi_1(\boldsymbol{p}_1 + \boldsymbol{k}/2) - \varphi_1(\boldsymbol{p}_1 - \boldsymbol{k}/2) + \varphi_2(\boldsymbol{p}_2 - \boldsymbol{k}/2) - \varphi_2(\boldsymbol{p}_2 + \boldsymbol{k}/2)\right) \Big\}$$

$$\mathcal{L}(\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{k}) = \frac{(2\pi)^7 v}{(\pi\,\sigma_1\sigma_2)^3}\, \delta\left(\varepsilon_1(\boldsymbol{p}_1 + \boldsymbol{k}/2) - \varepsilon_1(\boldsymbol{p}_1 - \boldsymbol{k}/2) + \varepsilon_2(\boldsymbol{p}_2 - \boldsymbol{k}/2) - \varepsilon_2(\boldsymbol{p}_2 + \boldsymbol{k}/2)\right)$$
$$\times \exp\Big\{ -\frac{(\boldsymbol{p}_1 - \langle \boldsymbol{p}\rangle_1)^2}{\sigma_1^2} - \frac{(\boldsymbol{p}_2 - \langle \boldsymbol{p}\rangle_2)^2}{\sigma_2^2} - \boldsymbol{k}^2 \left(\frac{1}{(2\sigma_1)^2} + \frac{1}{(2\sigma_2)^2}\right) -$$
$$-i\boldsymbol{k}\boldsymbol{b} + i\left(\varphi_1(\boldsymbol{p}_1 + \boldsymbol{k}/2) - \varphi_1(\boldsymbol{p}_1 - \boldsymbol{k}/2) + \varphi_2(\boldsymbol{p}_2 - \boldsymbol{k}/2) - \varphi_2(\boldsymbol{p}_2 + \boldsymbol{k}/2)\right) \Big\}.$$

$$\frac{d\sigma^{(1)}}{d\sigma^{(pw)}} = -\frac{1}{4}\left(\frac{\mathrm{Tr}\sigma_1^2 - 3\boldsymbol{u}_1\sigma_1^2\boldsymbol{u}_1}{\varepsilon_1^2} + \frac{\sqrt{\Delta u \alpha^{-1} \Delta u}}{v}\,\partial_{p_1}\sigma_1^2\partial_{p_1}\frac{v}{\sqrt{\Delta u \alpha^{-1} \Delta u}} + (1\to 2)\right) -$$
$$-\frac{1}{2}\Bigg(\frac{1}{8}\,\frac{\sqrt{\Delta u \alpha^{-1} \Delta u}}{|T_{fi}|^2}\,(\partial_{\varepsilon_1} + \partial_{\varepsilon_2})\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2}\right)\left(\mathrm{Tr}\alpha_\perp^{-1} - \frac{1}{2}\boldsymbol{u}_1\alpha_\perp^{-1}\boldsymbol{u}_1 - \frac{1}{2}\boldsymbol{u}_2\alpha_\perp^{-1}\boldsymbol{u}_2\right)$$
$$\times \frac{|T_{fi}|^2}{\sqrt{\Delta u \alpha^{-1} \Delta u}} - \frac{1}{2|T_{fi}|^2}\,\mathcal{C}_{ij}\,\alpha_{\perp}{}^{-1}_{ij} - 2\langle \boldsymbol{b}_\varphi\rangle\alpha_\perp^{-1}\partial_{\Delta p}\zeta_{fi} + \partial_{\Delta p}\zeta_{fi}\alpha_\perp^{-1}\partial_{\Delta p}\zeta_{fi}\Bigg)$$

3. Non-paraxial effects in scattering

Interference of the incoming packets is governed by

$$\left(\frac{1}{2\sigma_1^2}+\frac{1}{2\sigma_2^2}\right)^{-1}=\frac{2\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}$$

← Due to the finite overlap of the two non-orthogonal packets!

A corresponding term in the cross section is:

$$\frac{d\sigma^{(1)}}{d\sigma^{(pw)}}\propto\ \frac{2\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}\left[\frac{\Delta\boldsymbol{u}}{|\Delta\boldsymbol{u}|}\times\left[\frac{\Delta\boldsymbol{u}}{|\Delta\boldsymbol{u}|}\times\langle\boldsymbol{b}_\varphi\rangle\right]\right]\cdot\left(\frac{\partial}{\partial\boldsymbol{p}_2}-\frac{\partial}{\partial\boldsymbol{p}_1}\right)\zeta_{fi}\Big|_{\boldsymbol{p}_{1,2}=\langle\boldsymbol{p}\rangle_{1,2}}$$

$$\Delta\boldsymbol{u}=\boldsymbol{u}_1-\boldsymbol{u}_2 \qquad \boldsymbol{b}_\varphi=\boldsymbol{b}-\frac{\partial\varphi_1(\boldsymbol{p}_1)}{\partial\boldsymbol{p}_1}+\frac{\partial\varphi_2(\boldsymbol{p}_2)}{\partial\boldsymbol{p}_2}$$

This contribution defines an azimuthal asymmetry of the scattering:

$$\mathcal{A}[\boldsymbol{b}_\varphi]=\frac{dW[\boldsymbol{b}_\varphi]-dW[-\boldsymbol{b}_\varphi]}{dW[\boldsymbol{b}_\varphi]+dW[-\boldsymbol{b}_\varphi]}=\frac{d\sigma[\boldsymbol{b}_\varphi]-d\sigma[-\boldsymbol{b}_\varphi]}{d\sigma[\boldsymbol{b}_\varphi]+d\sigma[-\boldsymbol{b}_\varphi]}=\frac{d\sigma^{(1)}[\boldsymbol{b}_\varphi]-d\sigma^{(1)}[-\boldsymbol{b}_\varphi]}{2d\sigma^{(pw)}}+\mathcal{O}(\sigma^4)$$

D.K., JHEP **03** (2017) 049

3. Non-paraxial effects in scattering

$$\mathcal{A} = \frac{2\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}\left[\frac{\Delta \boldsymbol{u}}{|\Delta \boldsymbol{u}|}\times\left[\frac{\Delta \boldsymbol{u}}{|\Delta \boldsymbol{u}|}\times\langle \boldsymbol{b}_\varphi\rangle\right]\right]\cdot\left(\frac{\partial}{\partial \boldsymbol{p}_2}-\frac{\partial}{\partial \boldsymbol{p}_1}\right)\zeta_{fi}\Big|_{p_{1,2}=\langle p\rangle_{1,2}}$$

There are two scenarios:

1. Off-center collision of the Gaussian beams
2. Central collision of non-Gaussian beams (vortex particles, Airy beams, etc.)

For a 1 + 2→ 3 + 4 process in the collider frame:

$$\langle \boldsymbol{p}\rangle_1 = -\langle \boldsymbol{p}\rangle_2 \equiv \boldsymbol{p} = \boldsymbol{u}\varepsilon = \{0,0,p\},\ \ \Delta\boldsymbol{u} = 2\boldsymbol{u},\ \ v = |\Delta\boldsymbol{u}|$$

$$\frac{\partial}{\partial \boldsymbol{p}_1}-\frac{\partial}{\partial \boldsymbol{p}_2} = 8\boldsymbol{p}\frac{\partial}{\partial s}+4\left(\boldsymbol{p}_3-\boldsymbol{p}\right)\frac{\partial}{\partial t} \qquad t=(p_1-p_3)^2,\ \ s=(p_1+p_2)^2$$

and the asymmetry simplifies:

$$\mathcal{A} = 4\frac{2\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}\langle \boldsymbol{b}_\varphi\rangle \boldsymbol{p}_3\,\frac{\partial\zeta_{fi}(s,t)}{\partial t}$$

← Shows how the phase changes with the transferred momentum!

It is odd with respect to $\phi_3 \to \phi_3 \pm \pi$

An up-down asymmetry!

D.K., JHEP **03** (2017) 049

3. Non-paraxial effects in scattering

The 1st scenario: non-central collision of Gaussian beams with $b \lesssim \sigma_b$
(identical beams, relativistic energies, small scattering angles) – say, ee → X, pp → X, etc.

$$\mathcal{A} \approx -2\,\frac{\lambda_c}{\sigma_b}\,\cos\phi_{sc}\,\sqrt{\tau_0}\,\frac{\partial\zeta_{fi}}{\partial\tau_0},\quad \tau_0 = \frac{-t}{4m^2}$$

or, alternatively:

$$\mathcal{A} \approx -2\,\frac{\lambda_c}{\sigma_b}\,\cos\phi_{sc}\,\frac{1}{\gamma}\frac{\partial\zeta_{fi}}{\partial\theta_{sc}}$$

Just a linear "geometric" suppression!

In other words: $d\sigma^{(1)} \propto f(s,t)\,\lambda_c^2/\sigma_b^2$ and there is a region where *f(s,t)* is very large!

In QED (West, Yennie, 1968): $\frac{1}{\gamma}\frac{\partial\zeta_{fi}}{\partial\theta_{sc}} \sim \frac{\alpha_{em}}{\gamma\theta_{sc}}$ ⟹ $\mathcal{A} = \mathcal{O}\left(\frac{\lambda_c}{\sigma_b}\frac{\alpha_{em}}{\gamma\theta_{sc}}\right)$ ← Lorentz invariant!

For electrons of E = 300 keV focused to 0.1 nm
and $\theta_{sc} \sim 10^{-2} - 10^{-1}$ we have the following conservative estimate:

$$|A| \sim 10^{-4} - 10^{-3}\quad (!)$$

← And the same estimate within the 2nd scenario

One can detect a contribution of the Coulomb phase!

Similar estimates were also obtained by Ivanov 2012 and Ivanov, et al. 2016

3. Non-paraxial effects in scattering

A parameter which is usually employed: $\rho = \mathrm{Re}T_{fi}/\mathrm{Im}T_{fi} = 1/\tan\zeta_{fi}$

Once the Coulomb phase is known, one can retrieve also the hadronic phase!

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THE EUROPEAN PHYSICAL JOURNAL C

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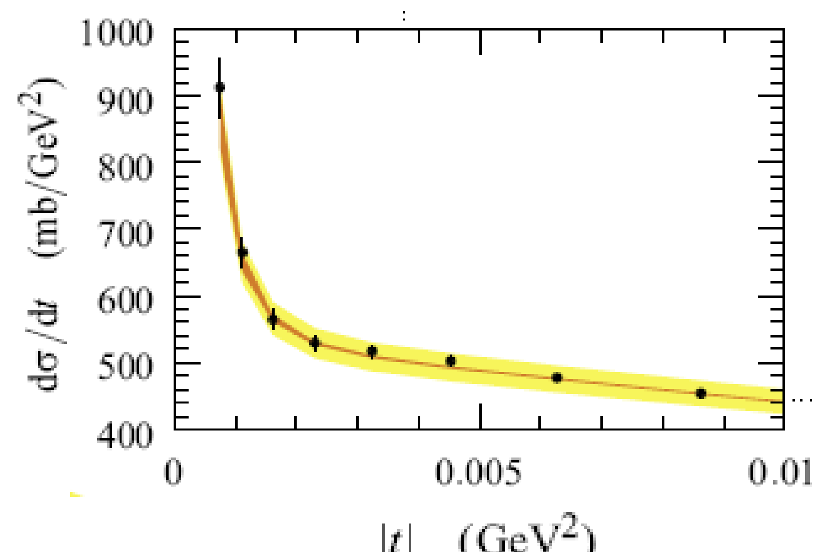
Regular Article - Experimental Physics

Measurement of elastic pp scattering at $\sqrt{s} = 8$ TeV in the Coulomb–nuclear interference region: determination of the ρ-parameter and the total cross-section

TOTEM Collaboration

$|t|$, from 6×10^{-4} to $0.2\,\mathrm{GeV}^2$

The region of a Coulomb-hadronic interference



3. Non-paraxial effects in scattering

Taking the same models as TOTEM, one can give more precise estimates of the asymmetry induced by the hadronic phase:

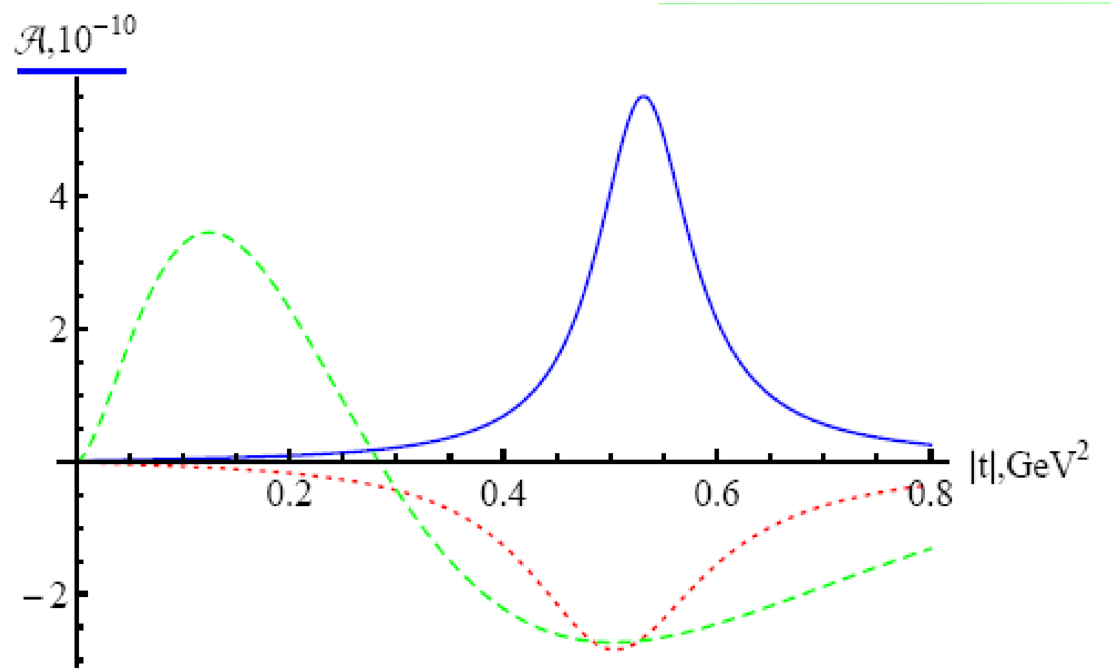
$$\frac{\partial \zeta_{fi}}{\partial t} = -\frac{\tau}{\tau^2 + (t + |t_0|)^2}$$ – the so-called standard parametrization, red dotted line

$$\frac{\partial \zeta_{fi}}{\partial t} = -\frac{\rho t_d}{(\rho t_d)^2 + (t - t_d)^2}$$ – the one by Bailly et al. (EHS-RCBC Collaboration), Z. Phys. C **37**, 7 (1987) blue solid line

$$\frac{\partial \zeta_{fi}}{\partial t} = \zeta_1(\kappa + \nu t)\left(\frac{-t}{1\,\text{GeV}^2}\right)^{\kappa-1} e^{\nu t}$$ – the so-called peripheral parametrization V. Kundrát and M. Lokajíček, Z. Phys. C **63**, 619 (1994) the green dashed line

For pp-collisions
the beams are too wide...

$$\frac{\lambda_c}{\sigma_b} \sim 10^{-11}.$$



3. Non-paraxial effects in scattering

When the parameter λ_c/σ_b is small, the quantum decoherence does not reveal itself in scattering and the Wigner functions stay everywhere positive (the WKB approximation).

Is there a chance to probe negative values of a Wigner function in scattering?

Beam-beam collision $\rightarrow$ beam + target

For scattering of an electron off an atom, an analogous small parameter is

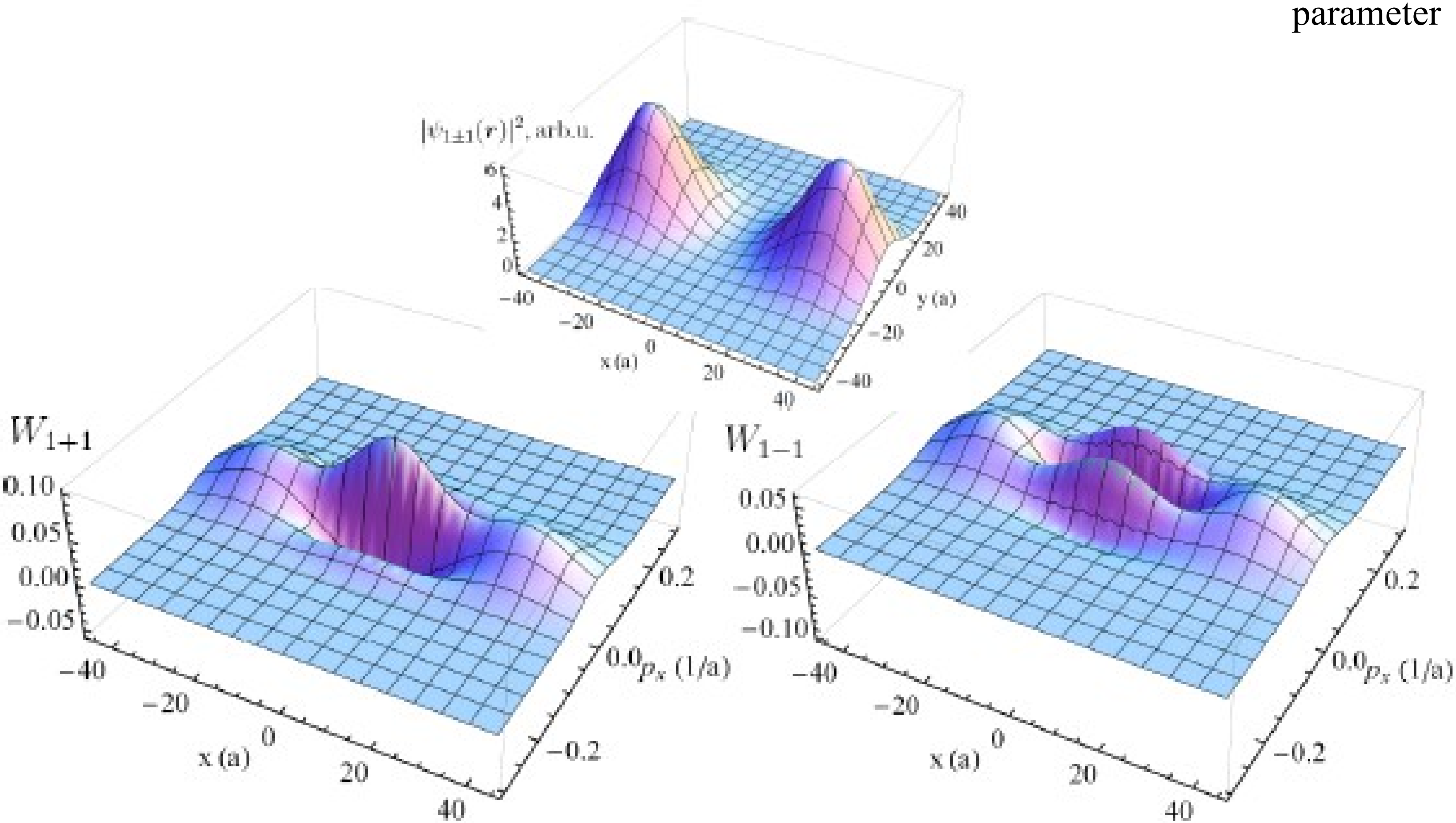
$$a/\sigma_b \qquad a \approx 0.053 \text{ nm is a Bohr radius}$$

which is 137 times larger than λ_c/σ_b !

For electron beams focused to 0.1 nm one can enter the non-paraxial regime!

The so-called Schrödinger's cat state $|\boldsymbol{r}_0\rangle \pm |-\boldsymbol{r}_0\rangle$
has a not-everywhere positive Wigner function

$\boldsymbol{r}_0$ is an impact parameter



In the Born approximation, the number of scattering events is:

$$\frac{d\nu}{d\Omega} = N_e \int d^2b\, d^2p\; n(\boldsymbol{b})\, W(\boldsymbol{b},\boldsymbol{p}) (f(\boldsymbol{Q}-\boldsymbol{p}))^2$$

The target's transverse profile

The projectile's Wigner function

The Born amplitude

For a wide Gaussian target of hydrogen in the ground 1s state:

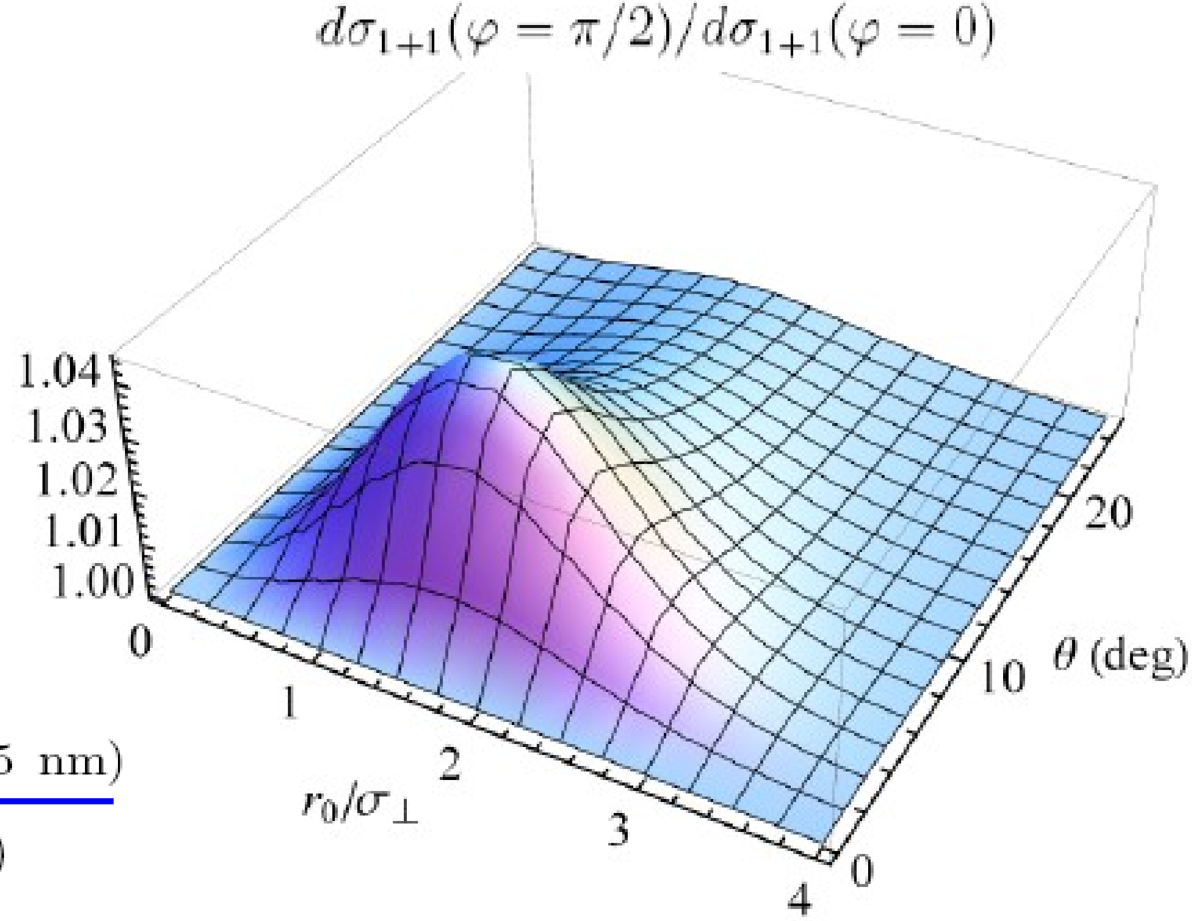
$$\frac{d\nu_{1\pm1}}{d\Omega} = \mathcal{N}_{1\pm1} \int\limits_0^\infty dx\, e^{-xg}\, \frac{x + x^2 + x^3/6}{1 + xa^2/(8\sigma_\perp^2)} \left(\cosh\left(\frac{\boldsymbol{b}_0\cdot\boldsymbol{r}_0}{\Sigma^2}\right) e^{-r_0^2/(2\Sigma^2)} \pm \right.$$

$$\left. \pm \cos\left(2\boldsymbol{r}_0\cdot\boldsymbol{p}_f \frac{xa^2/(8\sigma_\perp^2)}{1 + xa^2/(8\sigma_\perp^2)}\right) \exp\left\{-\frac{r_0^2}{2\sigma_\perp^2(1 + xa^2/(8\sigma_\perp^2))}\right\} \right)$$

Quantum interference does contribute to the cross section already in the Born approximation!

D.K., V.G. Serbo, PRL **119** (2017) 173601

The quantum interference also results in an angular asymmetry:



$$\sigma_\perp = 2a \approx \underline{0.1 \text{ nm } (\text{FWHM} \approx 0.25 \text{ nm})}$$

$$p_i = p_f = 20/a \ (\varepsilon_{\text{kin}} = 5.6 \text{ keV})$$

$$\theta = 10^\circ$$

Several per cent!

D.K., V.G. Serbo, PRL **119** (2017) 173601

Conclusion

- The non-paraxial effects are effectively attenuated by λ_c/σ_b, not always by λ_c^2/σ_b^2, and originate thanks to a finite overlap of the incoming wave packets
- For instance, for vortex electrons with high OAM, $d\sigma^{(1)}/d\sigma_{\text{pw}} \sim |\ell|\,\lambda_c^2/\sigma_b^2$.
- In beam-beam collisions, the Wigner functions of the non-Gaussian in-states turn out to be everywhere positive
- For well-focused electron beams of different spatial profiles these effects can already reach $\sim 0.1 - 1\%$.
- For QED, they can compete with the NNLO- or even with the NLO corrections
- A contribution of the Coulomb phase to the cross section in elastic *ee* scattering can reach ~ 0.1% and can already be measured
- The quantum decoherence (connected with the Wigner functions' negativity) may reveal itself in scattering off an atomic target already in the Born approximation; The corresponding effects can also reach 1 – 10% for the Schrödinger's cat states.

The steps further and issues:

- Proton spin puzzle: vortex beams can be more sensitive to the partons' angular momenta.

  Does a deep inelastic $e^{(tw)} + p \to X$ scattering bring some new information?

- Are there effects that are enhanced when ultrarelativistic massive particles with a non-Gaussian profile collide or are scattered off a target?

  How can one use ultrarelativistic vortex electrons?

- Can we employ scattering/annihilation as a means for quantum tomography?

- ….