Skewed Sudakov Regime

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Sudakov Form Factor

V.V. Sudakov, JETP, 1956

Vertex Parts

$\Gamma_\mu(p,q)$ in Minkowski kinematics

(The plane of external momenta $p, q, l = p - q$ contains two light-like lines crossing at the origin)

Sudakov Form Factor

$$\lim_{l^2 \to \infty} \Gamma_\mu(p,q) = -ie\gamma_\mu \exp\left(-\frac{\alpha}{2\pi} \log\left|\frac{l^2}{p^2}\right| \log\left|\frac{l^2}{q^2}\right|\right)$$

Sudakov at One Loop

Scalar

$$\lim_{l^2\to\infty} \frac{l^2}{4\pi^2} \int \frac{d^4k}{(p-k)^2 k^2 (q-k)^2} \approx \frac{i}{4} \log|\frac{l^2}{p^2}| \log|\frac{l^2}{q^2}|$$

Sudakov variables and logarithmic regions

Sudakov Results

- Coefficients by the doubly logarithmic corrections do not depend on the UV scale
- No new tensor structures
- Explicit Form Factor

Skewed Sudakov at One Loop

$$\lim_{q^2 \to 0} \frac{l^2}{4\pi^2} \int \frac{d^4k}{(p-k)^2 k^2 (q-k)^2} \approx \frac{i}{4} \log|\frac{l^2}{q^2}| C(l^2/p^2)$$

$$C(l^2/p^2) = ?$$

Relation to Sudakov:

$$\lim_{p^2 \to 0} C(l^2/p^2) \approx \log|\frac{l^2}{p^2}|$$

Tensor Structure at One Loop

Loop momentum in the denominators of the propagators can be neglected

$$\not{p}\Big(\Gamma_\mu(p,q)\Big)\not{q} \approx 2\not{p}\Big(-e^3\not{q}\not{l}\not{p}\gamma_\mu\Big)\not{q} \times \frac{\text{Scalar}}{4\pi^2 l^2}$$

$$\approx \not{p}\Big(-ie\gamma_\mu\Big)\not{q} \times \Big(-\frac{\alpha}{2\pi}\log|\frac{l^2}{p^2}|\log|\frac{l^2}{q^2}|\Big)$$

Agrees with the result

$$\Gamma_\mu(p,q) \approx -ie\gamma_\mu \exp\Big(-\frac{\alpha}{2\pi}\log|\frac{l^2}{p^2}|\log|\frac{l^2}{q^2}|\Big)$$

Definition of Leading Logs

On dimensional grounds, there are three types of logs:

$$\log|\frac{l^2}{q^2}|, \log|\frac{l^2}{p^2}|, \text{and} \log|\frac{l^2}{\mu^2}|$$

$$t \equiv \frac{\alpha}{2\pi}\log|\frac{l^2}{q^2}|$$

Leading log: $t^n F_n(l^2/p^2, l^2/\mu^2)$

Subleading logs: $\alpha^n t^m, n > 0$

Result #1: F_n depends only on l^2/p^2

No dependence on μ in the leading logarithms

Tensor Structure

Result #2:

$$\Gamma_\mu(p,q) \approx -ie\gamma_\mu F^{sk}(p,q) + ie\frac{q_\mu \not{p}}{qp}\Big(F^{sk}(p,q) - 1\Big)$$

Compare to classical Sudakov:

$$\Gamma_\mu(p,q) \approx -ie\gamma_\mu F^{S}(p,q)$$

$$F^{S}(p,q) = \exp\left(- t\log\left|\frac{l^2}{p^2}\right|\right)$$

The Skewed Form Factor

$$\lim_{p^2\to 0} F^{sk}(p,q) = F^{S}(p,q)\Phi_s(t)$$
$$s = 0, \pm$$

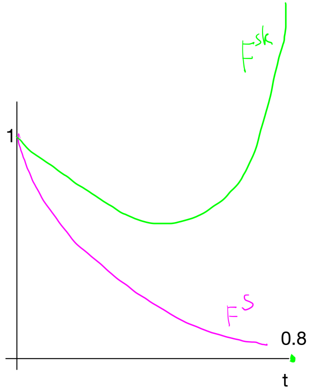
$$\frac{l^2}{p^2} > 0 \Leftrightarrow s = 0$$

$$\Phi_0(t) = \frac{1}{1-t^2-t\exp(-t)}$$

The Skewed Form Factor ...

$$\frac{l^2}{p^2} < 0 \Leftrightarrow s = \frac{l^2}{|l^2|}$$

$$\Phi_\pm(t) = \exp\big(\pm i\pi t\big)\Phi_0(t)$$



Longitudinal and Transverse Momenta of Virtual Particles

$$\Gamma_1 \equiv \frac{l^2}{4\pi^2}\int \frac{d^4k}{(p-k)^2k^2(q-k)^2} = \frac{l^2}{4\pi^2}\int \frac{d^2k_{\|}d^2k_{\perp}}{(p-k)^2k^2(q-k)^2}$$

$k_{\|}$ is a linear combination of external momenta, and

$k_{\perp}q = k_{\perp}p = 0$

Transverse plane is Euclidean

$$k_{\perp}^2 \leq 0, t \equiv -k_{\perp}^2$$

Integrating out the orientation of $k_{\perp}$ one obtains

$$\Gamma_1 = \frac{l^2}{4\pi}\int \frac{d^2k_{\|}dt\theta(t)}{(p-k)^2k^2(q-k)^2}$$

Negative Euclidean Lengths and Doubly Virtual Particles

$$\Gamma_1 = \int \frac{d^2k_{\|}dt\theta(t)}{(q-k_{\|})^2 - t + i\epsilon} G(k_{\|}, t)$$

$$G(k_{\|}, t) = \frac{l^2}{4\pi} \frac{1}{k^2\left((p-k_{\|})^2 - t\right)}$$

We will manipulate with the θ-function. As a result, integration over negative t will appear

It will be advantageous to consider doubly virtual particles with not only nonzero k^2 but also with positive $k_{\perp}^2$ (with purely imaginary components of $k_{\perp}$)

Inclination of Doubly Virtual Particles

Instead of $t \equiv -k_\perp^2$ it will be convenient to use another variable—the inclination ν

$$\nu \equiv \frac{k_\parallel^2}{k_\parallel^2 - t} = \frac{k_\parallel^2}{k^2}$$

In terms of ν

$$\theta(t) = \theta(k_\parallel^2) - \frac{k_\parallel^2}{|k_\parallel^2|}\theta\big(\nu(1-\nu)\big)$$

The Integral

$$\int \mathrm{d}^2\mathrm{k}_\parallel \mathrm{d}t\theta(\mathrm{t})\mathrm{G}(\mathrm{k}_\parallel,\mathrm{t}) = \int \mathrm{d}^2\mathrm{k}_\parallel \mathrm{d}t\theta(\mathrm{k}_\parallel^2)\mathrm{G}(\mathrm{k}_\parallel,\mathrm{t}) -$$

$$-\int_0^1 \frac{\mathrm{d}\nu}{\nu^2}\int \mathrm{d}^2\mathrm{k}_\parallel \mathrm{k}_\parallel^2 \mathrm{G}\big(\mathrm{k}_\parallel,\mathrm{k}_\parallel^2(1-1/\mathrm{nu})\big)$$

The part with the integration over time-like longitudinal momentum vanishes because of Feynman's $i\epsilon$

Inclination Representation

As a result

$$\Gamma_1 = -\int_0^1 \mathrm{d}\nu \int \frac{\mathrm{d}^2 \mathrm{k}_\parallel}{(\mathrm{k}_\parallel - \mathrm{q}\nu)^2 + \mathrm{q}^2\nu(1-\nu) + \mathrm{i}\epsilon} \mathrm{G}(\mathrm{k}_\parallel, \nu)$$

$$\mathrm{G}(\mathrm{k}_\parallel, \nu) = \left(\frac{\mathrm{l}^2}{4\pi}\right) \frac{1}{(\mathrm{p} - \mathrm{k}_\parallel)^2 - \mathrm{k}_\parallel^2(1 - 1/\nu)}$$

Leading Logarithm

Leading logarithm comes from a vicinity of $k_{\|} = q\nu$ To extract it, one uses the formula

$$\int_A \frac{d^2k}{k^2 + q^2 + i\epsilon} \approx -i\pi \log |\frac{A^2}{q^2}|$$

$A_{\pm}$ are limits of integration around zero $k_{\pm} = k_0 \pm k_1$

The Answer

$$\Gamma_1 \approx \frac{i}{4} \log|\frac{l^2}{q^2}| \int_0^1 \frac{l^2 d\nu}{p^2(1-\nu) + l^2\nu + i\epsilon}$$

$$C(l^2/p^2) = \int_0^1 \frac{l^2 d\nu}{p^2(1-\nu) + l^2\nu + i\epsilon}$$

As expected

$$C(l^2/p^2) \approx \log|\frac{l^2}{p^2}|$$

In all Orders...

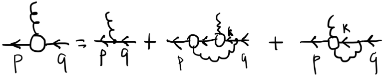


Figure: Schwinger-Dyson equation for 1PI vertex part

$$G_\mu(p, q) = \int_0^{2\pi} d\phi \int_0^1 d\nu \int d^2k_\parallel \; \frac{I_{\mu\lambda}(p,k;q-k)\not{k}\gamma^\lambda}{(k_\parallel - \nu q)^2 + q^2\nu(1-\nu) + i\epsilon}$$

$$k_\perp = \sqrt{-k_\perp^2}\left(\cos(\phi), \sin(\phi)\right), \; k_\perp^2 = k_\parallel^2\left(\frac{1}{\nu} - 1\right)$$

after radiating the photon the incoming fermion is “inclined” to carry away the fraction of the initial momentum ν

Conclusions

- Skewed Sudakov regime was introduced
- Doubly virtual particles and their inclination were defined
- Consideration of the Skewed Sudakov using incination gives simultaneously simplification and generalization of the classical Sudakov treatment
- Skewed Sudakov may have phenomenological applications