Resonances in positron scattering on a supercritical nucleus and spontaneous production of e^+e^- pairs

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QUARKS-2018

Valday
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History

The Dirac equation in the Coulomb field of a nucleus:

$$E_0 = m \cdot \sqrt{1 - (Z\alpha)^2}.$$

What happens when $Z > 1/\alpha = 137$?

Pomeranchuk, I. and Smorodinsky, Ya., J. Phys. USSR (1945)

When nucleus radius is accounted for, the square root singularity is resolved, with growing Z ground state energy becomes negative.

Voronkov, V. V. and Kolesnikov, N. N., JETP (1961)

Gershtein, S. S. and Zel'dovich, Ya. B., JETP (1969)

Pieper, W. and Greiner, W., Z. Phys. (1969)

Zeldovich, Ya. B. and Popov, V. S., UFN (1971)

Greiner, W., Müller, B. and Rafelski, J. (papers, books)...

At $Z = Z_{\text{cr}} \approx 175$ ground state energy dives into lower continuum of the Dirac equation, E_0 becomes less than $-m$.

As soon as a nucleus with $Z > Z_{\text{cr}}$ is formed, spontaneous production of e^+e^- pairs occurs; positrons are radiated to spatial infinity and can be detected while electrons are captured by the nucleus. Naked supercritical nucleus is screened by electrons.

Several physical phenomena

QCD

Gribov theory of quark confinement: growing of the strong (color) charge at large distances leads to quark pair creation from vacuum. Color charge is screened and color objects do not exist as asymptotic states (confinement).

Gravity

Strong gravitational potential leads to particle creation - black hole radiates.

No experimental confirmation of the QED phenomenon up to present time!
(Some hints from graphene physics).

Kuleshov V. M., Mur V. D., Narozhny N. B., Fedotov A. M., Lozovik Yu. E. and Popov V. S., "Coulomb problem for a $Z > Z_{\rm cr}$ nucleus", Phys. Usp., V. 58, p. 785-791, 2015

Main points:

1. Resonances in positron scattering on supercritical nucleus

2. Stability of the naked nucleus with arbitrary large Z; no e^+e^- pairs creation

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Main points:

1. Resonances in positron scattering on supercritical nucleus
   WE CONFIRM IT
2. Stability of the naked nucleus with arbitrary large Z; no e^+e^- pairs creation

Recent development

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Main points:

1. Resonances in positron scattering on supercritical nucleus
   WE CONFIRM IT

2. Stability of the naked nucleus with arbitrary large Z; no e^+e^- pairs creation
   WE DO NOT AGREE

1, 2, 3, 4 volumes of Landau and Lifshitz (from Mechanics to QED)

contain considerations of the Rutherford scattering, however for $Z > 137$ new phenomena occurs.

Energy levels in the Dirac equation

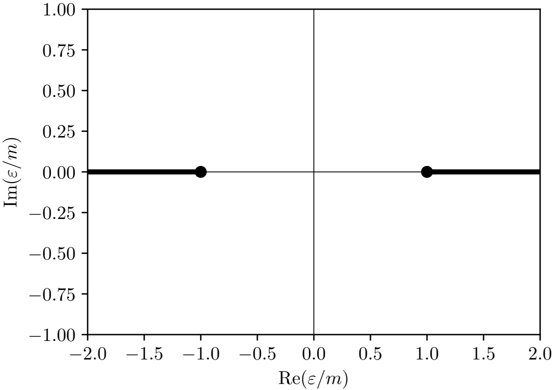
Resonances were briefly discussed in Popov-Zeldovich review (UFN, 1971) and later by

B. Müller, J. Rafelski, and W. Greiner, Z. Phys. 257, 62 (1972)

B. Müller, J. Rafelski, and W. Greiner, Z. Phys. 257, 183 (1972)

V. D. Mur and V. S. Popov, Theor. Math. Phys. 27, 429 (1976)

V. S. Popov, V. L. Eletsky, and V. D. Mur, Sov. Phys. JETP 44, 451 (1976)



Complex ε plane.

The Dirac equation

The radial functions of the Dirac equation $F(r) \equiv rf(r)$ and $G(r) \equiv rg(r)$ are determined by the following differential equations:

$$\begin{cases} \dfrac{dF}{dr} + \dfrac{\varkappa}{r} F - (\varepsilon + m - V(r))\, G = 0, \\ \dfrac{dG}{dr} - \dfrac{\varkappa}{r} G + (\varepsilon - m - V(r))\, F = 0. \end{cases}$$

Pomeranchuk, I. and Smorodinsky, Ya., J. Phys. USSR (1945)

To deal with $Z\alpha > 1$ case:

$$V(r) = \begin{cases} -\dfrac{Z\alpha}{r}, & r > R, \\ -\dfrac{Z\alpha}{R}, & r < R. \end{cases}$$

Solution of the Dirac equation

Solution at $r < R$:

$$\begin{pmatrix} F \\ G \end{pmatrix} = \text{const} \cdot \sqrt{\beta r} \cdot \begin{pmatrix} \mp J_{\mp(1/2+\varkappa)}(\beta r) \\ J_{\pm(1/2-\varkappa)}(\beta r) \frac{\beta}{\varepsilon + m + \frac{Z\alpha}{R}} \end{pmatrix},$$

where $\varkappa = \pm 1, \pm 2, \ldots$, $\beta = \sqrt{\left(\varepsilon + \frac{Z\alpha}{R}\right)^2 - m^2}$. Upper (lower) signs correspond to $\varkappa \lessgtr 0$.

Solution at $r > R$:

$$\begin{pmatrix} F \\ G \end{pmatrix} = \begin{pmatrix} \sqrt{m+\varepsilon} \\ -\sqrt{m-\varepsilon} \end{pmatrix} \exp(-\rho/2)\rho^{i\tau} \begin{pmatrix} Q_1 + Q_2 \\ Q_1 - Q_2 \end{pmatrix},$$

where $\tau = \sqrt{(Z\alpha)^2 - \varkappa^2}$, $\rho = 2\lambda r = -2ikr$, $\lambda = \sqrt{(m-\varepsilon)(m+\varepsilon)}$, and

$$\begin{cases} Q_1 = C \cdot \dfrac{-\frac{iZ\alpha m}{k} + \varkappa}{-i\tau + \frac{iZ\alpha\varepsilon}{k}} \cdot {}_1F_1\left(i\tau - \dfrac{iZ\alpha\varepsilon}{k}, 2i\tau + 1, \rho\right) + \\ \qquad + D \cdot \dfrac{-\frac{iZ\alpha m}{k} + \varkappa}{i\tau + \frac{iZ\alpha\varepsilon}{k}} \rho^{-2i\tau} {}_1F_1\left(-i\tau - \dfrac{iZ\alpha\varepsilon}{k}, -2i\tau + 1, \rho\right), \\ Q_2 = C \cdot {}_1F_1\left(1 + i\tau - \dfrac{iZ\alpha\varepsilon}{k}, 2i\tau + 1, \rho\right) + D \cdot \rho^{-2i\tau} {}_1F_1\left(1 - i\tau - \dfrac{iZ\alpha\varepsilon}{k}, -2i\tau + 1, \rho\right), \end{cases}$$

where C and D are arbitrary coefficients. Unlike textbooks case both regular and singular at $r = 0$ solutions are taken into account.

The ratio C/D should be determined by matching with the solution at $r < R$.

Scattering phase and the position of the poles

The scattering phase $\delta_\varkappa(\varepsilon, Z)$ is determined by investigating the behavior of the wave function at large r:

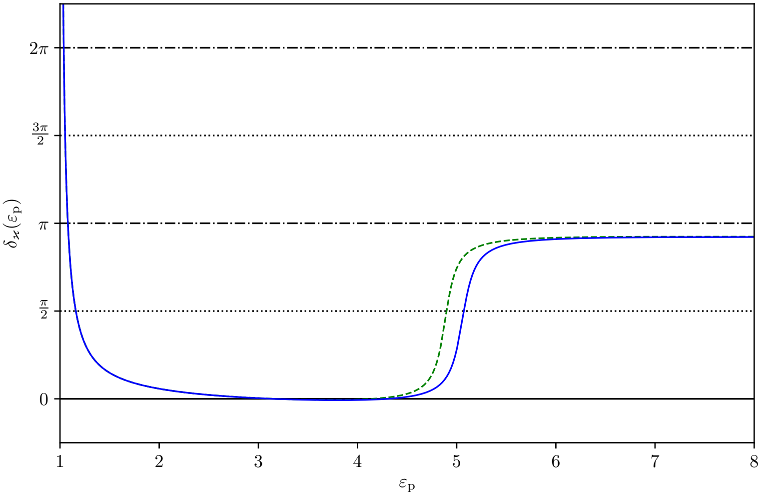
$$\left(\begin{array}{c} F \\ G \end{array}\right)\Bigg|_{r\to\infty} \propto \left(\begin{array}{c} \sqrt{m+\varepsilon} \\ -\sqrt{m-\varepsilon} \end{array}\right)\left\{e^{i(kr+\frac{Z\alpha\varepsilon}{k}\ln(2kr))}e^{2i\delta} \pm e^{-i(kr+\frac{Z\alpha\varepsilon}{k}\ln(2kr))}\right\},$$

$$e^{2i\delta_\varkappa} = -\frac{1}{\varkappa + \frac{iZ\alpha m}{k}} \cdot \frac{\frac{C}{D}\cdot\frac{\Gamma(2i\tau)}{\Gamma\left(i\tau+\frac{iZ\alpha\varepsilon}{k}\right)}\rho^{i\tau}\left(-\rho\right)^{-i\tau} - \frac{\Gamma(-2i\tau)}{\Gamma\left(-i\tau+\frac{iZ\alpha\varepsilon}{k}\right)}\rho^{-i\tau}\left(-\rho\right)^{i\tau}}{\frac{C}{D}\cdot\frac{\Gamma(2i\tau)}{\Gamma\left(1+i\tau-\frac{iZ\alpha\varepsilon}{k}\right)} - \frac{\Gamma(-2i\tau)}{\Gamma\left(1-i\tau-\frac{iZ\alpha\varepsilon}{k}\right)}}.$$

The resonance of the scattering amplitude corresponds to the pole of the S-matrix element $S \equiv e^{2i\delta}$, so we immediately get an equation for the position of this pole in the ε-plane:

$$\frac{C}{D}\cdot\frac{\Gamma\left(2i\tau\right)}{\Gamma\left(1+i\tau-\frac{iZ\alpha\varepsilon}{k}\right)} - \frac{\Gamma\left(-2i\tau\right)}{\Gamma\left(1-i\tau-\frac{iZ\alpha\varepsilon}{k}\right)} = 0.$$

Scattering phase and the position of the poles



Dependence on $\varepsilon_{\rm p}$ of the scattering phase $\delta_{-1}(\varepsilon_{\rm p}, 232)$ ($Z = 232$ and $\varkappa = -1$) for a nucleus with radius $R = 0.031/m$. The blue solid line corresponds to the exact phase, the green dashed line corresponds to the approximation $1/m \gg R$ in matching, a'la *Kuleshov et al.*

Resonance (Gamov) states

Where do resonances come from?

- At $Z < Z_{\rm cr}$ (ground state energy $\varepsilon > -m$) they are bound states, spectrum of which are given by zeroes of denominator of S.

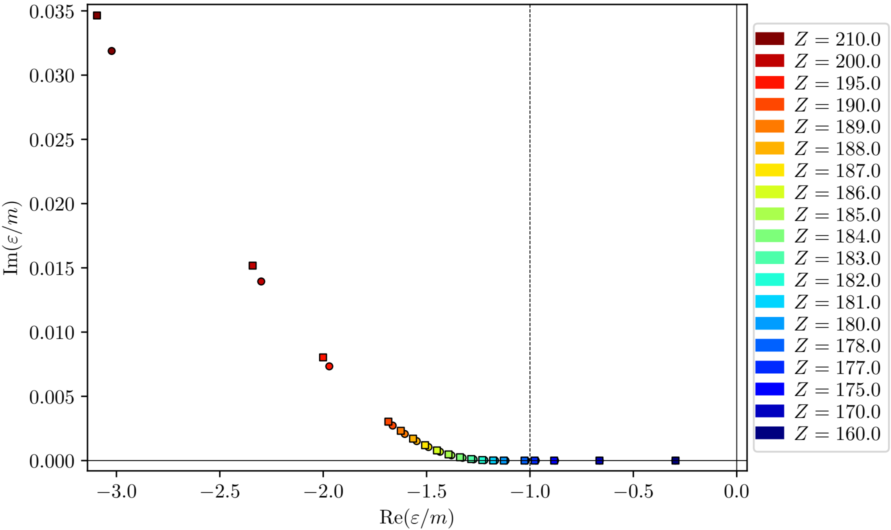
- At $Z > Z_{\rm cr}$ pole goes to the second sheet of energy plane ABOVE left cut,

$$\varepsilon = -\varepsilon_0 + \boxed{\frac{i}{2}\gamma}$$

Unusual sign of γ

Kuleshov et al.:
At $Z > Z_{\rm cr}$ state is at lower continuum — sign of energy should be reversed (negative electron energies at lower continuum describe positive positron energies).

BUT: resonance state originates from upper continuum ...



The dependence of the ground state energy on Z. The square markers are for the exact values of the energy and the round markers are for the approximate ones. At $Z = Z_{\text{cr}}$ the bound states become resonances with positive $\text{Im}[\varepsilon]$.

The Dirac equation for positron in the field of (positively charged) nucleus

$$Z\alpha \longrightarrow -Z\alpha, \quad \tilde{V}(r) = -V(r)$$

Solving this equation we found scattering phase δ.

What about resonances?

At first glance — no bound states, so no resonances.

However we should not forget, that the Dirac equation describes *BOTH* electrons and positrons. Thus "the Dirac equation for positron" describes *BOTH* positrons and electrons. It describes bound states, which this time originates from lower continuum, where electrons are situated.

At small Z the ground state energy equals

$$\varepsilon = -m(1 - (Z\alpha)^2/2).$$

With growing Z energy crosses zero and at $Z > Z_{\rm cr}$ first resonance (Gamov) state emerges. It is situated on the second sheet of energy plane below right cut:

$$\varepsilon_{\rm ps} = \varepsilon_0 - \frac{i}{2}\gamma.$$

Proper place for the resonance state!

Qualitative explanation of the resonance phenomena in the e^+N^+ system; M.I.Eides question

A Schrödinger-like equation from the Dirac equation for positron:

$$\chi'' + k^2\chi = 0,$$

where $k^2 = 2m\,(E - U)$, $E = \frac{\varepsilon^2 - m^2}{2m}$. The effective potential is seen to be made of two terms: $U = U_1 + U_2$, where:

$$U_1 = \frac{\varepsilon}{m}\tilde{V} - \frac{1}{2m}\tilde{V}^2 - \frac{\varkappa\,(1-\varkappa)}{2mr^2},$$

and

$$U_2 = \frac{\tilde{V}''}{4m\left(\varepsilon - m - \tilde{V}\right)} + \frac{3}{8m}\frac{\left(\tilde{V}'\right)^2}{\left(\varepsilon - m - \tilde{V}\right)^2} + \frac{\varkappa\tilde{V}'}{2mr\left(\varepsilon - m - \tilde{V}\right)}.$$

for $\varepsilon = m$ and $\varkappa = 1$, we get

$$U = \frac{Z\alpha}{r} - \frac{4(Z\alpha)^2 - 3}{8mr^2}.$$

At short distances the terms $\propto 1/r^2$ dominates and, for a supercritical nucleus, they lead to attraction, while the Coulomb term dominates at $r \geq 1/m$. This attractive force explains the existence of resonances in the e^+N^+ system. For $Z\alpha > 1$ fall to the center occurs.

Naked nucleus with arbitrary large Z?

Kuleshov V. M., Mur V. D., Narozhny N. B., Fedotov A. M., Lozovik Yu. E. and Popov V. S., "Coulomb problem for a $Z > Z_{\mathrm{cr}}$ nucleus", Phys. Usp., V. 58, p. 785-791, 2015

The scattering phase is real

$\Downarrow$

e^+e^- pairs are not produced

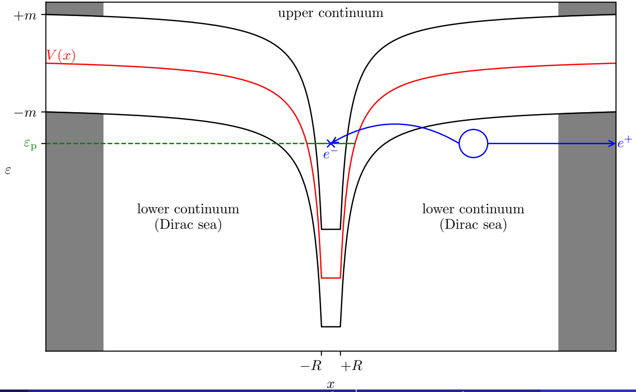
$\Downarrow$

Naked supercritical nucleus is stable

No naked nucleus with $Z > Z_{\rm cr}$

<u>OUR OPINION:</u>

- Conservation of energy allows pairs production.
- Resonance in the heavy nucleus - positron system just signal pair production.
- What happens:
  - Without nucleus empty level in lower continuum is positron.
  - With nucleus the energy of empty level gets imaginary part, so it decays with the lifetime $1/\gamma$.
  - In this time electron which occupies the state with the same energy in the lower continuum "jumps" to this empty level creating "charged vacuum", while positron (hole in the lower continuum) is radiated to "infinity".



Conclusions

1. Rutherford scattering of positron on supercritical nucleus do exhibit resonance behavior; width of this resonance equals time in which e^+e^- pair is produced by supercritical nucleus.
2. *Kuleshov et al.* wrote in Conclusions of their paper, that stable state of the supercritical nucleus consists of empty states of upper continuum, empty discrete levels and occupied states of lower continuum. It is just the picture I described. Occupied by electrons states of lower continuum which come from the upper continuum when Z grows and becomes larger that $Z_{\rm cr}$ form "charged vacuum" — its charge equals $-n$, where n is the number of these levels. Compensating charge $+n$ was carried out by produced positrons.
3. Conclusive evidence: Let us consider positron scattering on a nucleus with, say, $Z = Z_{\rm cr} + 10$. Two electrons from charged vacuum are spread at $r \approx 1/m$. Resonance in this case will be approximately at the same place which we found before having approximately the same width. This width gives the lifetime of positron in vicinity of supercritical nucleus; during this time positron flies away leaving nuclei screened by vacuum electrons.

Supercritical nucleus is not naked - its electric charge is partially screened by these electrons.