Dynamics of exotic branes in string theory

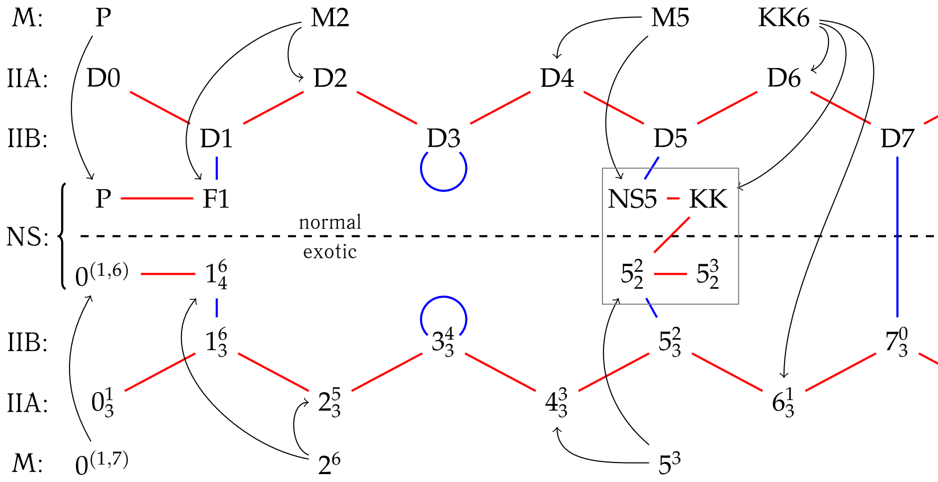
Edvard Musaev

Moscow Inst. of Physics and Technology
Kazan Federal University

based on works with
Chris Blair, Axel Kleinschmidt, Ilya Bakhmatov,
Eric Bergshoeff, Fabio Riccioni

QUARKS
Valday, 2018

Web of (some) branes



M: P M2 M5 KK6

IIA: D0 D2 D4 D6

IIB: D1 D3 D5 D7

NS: P F1 NS5 KK

normal

exotic

$0^{(1,6)}$ 1^6_4 5^2_2 5^3_2

IIB: 1^6_3 3^4_3 5^2_3 7^0_3

IIA: 0^1_3 2^5_3 4^3_3 6^1_3

M: $0^{(1,7)}$ 2^6 5^3

S-duality; T-duality; Reduction;

Why bother?

Low-energy effective action (10D IIA(B) or 11D supergravity)

$$S = \int e^{2\phi}\left(* 1 R[G] + dB \wedge dB\right) + dC \wedge dC + \ldots \\ + \text{exotic potential contributions} \qquad (1)$$

1. Cosmology Dibitetto, Grana, Blumenhagen,...
    - stabilization of scalar moduli fiels (masses = fluxes)
    - provide small positive cosmological constant (speculations around inflaton potential)
2. Black hole information Bena, Mathur, Shigemori,...
    - include exotic branes into counting of microscopic degrees of freedom inside a black hole
3. Conformal field theories Harvey, Jensen, Kimura,...
4. Non-commutative and non-associative spaces Blumenhagen, Lust, Malek,...

The results

- Action describing dynamics of exotic branes has been constructed
- This is T-duality, general coordinate and world-volume reparametrization invariant
- Coupled to a field theoretical action (SUGRA or DFT) this produces backgrounds with exotic fluxes (exotic branes)
- Depending on orientation the action reproduces those for the NS5-brane, KK5-monopole and gives actions for exotic branes
- The same is true for D-branes, which are faces of a single (9+1)-dimensional object

D-branes: geometry

10D IIA/IIB SUGRA

$$S_{\text{SUGRA}} = \int d^{10}x\sqrt{-G}e^{-2\varphi}\Big(R[G] + \boxed{H_{mnk}H^{mnk}}\Big) + \boxed{RR}\text{ gauge fields} \tag{2}$$

Fundamental string solution

$$\begin{gathered} ds^2 = H^{-3/4}\Big(-dt^2 + dx^2\Big) + H^{1/4}\Big(d\rho^2 + \rho^2 d\Omega_7^2\Big) \\ \boxed{B_{tx}} = -(H^{-1} - 1), \quad H = 1 + \frac{h}{\rho^6}, \quad e^{-2(\varphi - \varphi_0)} = H \end{gathered} \tag{3}$$

Dp-brane solution

gauge potentials and their fluxes

$$\begin{gathered} ds^2 = H^{-\frac{7-p}{8}}\Big(-dt^2 + d\vec{x}^2_{(p)}\Big) + H^{\frac{p+1}{8}}\Big(d\rho^2 + \rho^2 d\Omega_{8-p}^2\Big) \\ \boxed{C_{tx^1\dots x^p}} = -(H^{-1} - 1), \quad H = 1 + \frac{h}{\rho^{7-p}}, \quad e^{-2(\varphi - \varphi_0)} = H^{\frac{p-3}{2}} \end{gathered} \tag{4}$$

D-branes: dynamics

Effective action for a Dp-brane

$$S_{\mathrm{DBI}} = \int_\Sigma d^{p+1}\xi \sqrt{\det\left(G+\mathcal{F}\right)} + \int_\Sigma \mathcal{C}_{p+1} \tag{5}$$

- kinetic term: dynamics of fields living on the brane $\equiv$ dynamics of the brane in space-time

$$G_{\alpha\beta} = G_{\mu\nu}\frac{\partial X^\mu}{\partial \xi^\alpha}\frac{\partial X^\nu}{\partial \xi^\beta} \tag{6}$$

- Wess-Zumino term: interaction with RR field C_{p+1} — $p+1$-form

T-duality of string on $\mathbb{T}^d$

- Mass spectrum of the string is invariant under the $O(d,d)$ group

$$\mathcal{M}^2 = \mathcal{P}^M \mathcal{H}_{MN} \mathcal{P}^N, \quad \mathcal{P}^M = \begin{bmatrix} p^m \\ w_n \end{bmatrix} \begin{matrix} \text{momentum} \\ \text{winding} \end{matrix} \tag{7}$$

- Fields $(g_{\mu\nu}, B_{\mu\nu})$ combine into linear representations

$$\mathcal{H}_{MN} = \begin{bmatrix} g - Bg^{-1}B & Bg^{-1} \\ g^{-1}B & g^{-1} \end{bmatrix}, \quad \mathcal{H} \to \mathcal{O}^T \mathcal{H} \mathcal{O}\ , \quad \mathcal{O} \in O(d,d) \tag{8}$$

- Fluxes of string theory transform as

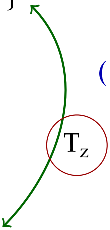
$$\begin{gathered} F_{m_1 \dots m_p x} \xleftrightarrow{T_x} F_{m_1 \dots m_p} \\ H_{xyz} \xleftrightarrow{T_x} f^x{}_{yz} \xleftrightarrow{T_y} Q^{xy}{}_z \xleftrightarrow{T_z} R^{xyz}. \end{gathered} \tag{9}$$

[Schelton, Taylor, Wecht]

T-duality orbit of NS branes

- NS5-brane, $H(R) = 1 + \frac{h}{R^2}$, $R^2 = \delta_{ij}x^i x^j + z^2$, $\{x^1, x^2, x^3\}$

$$ds^2 = H(R)\underbrace{\left(dz^2 + \delta_{ij}dx^i dx^j\right)}_{\text{transverse}} + \underbrace{ds^2_{(1,5)}}_{\text{world-volume}}, \quad B \neq 0, \tag{10}$$



- KK5 monopole, $H(r) = 1 + \frac{h}{r}$, $r^2 = \delta_{ij}x^i x^j$, z is compact

$$ds^2 = H(r)^{-1}\underbrace{\left(dz^2 + A_i dx^i\right)}_{\text{special}}{}^2 + H(r)\underbrace{\delta_{ij}dx^i dx^j}_{\text{transverse}} + ds^2_{(1,5)}, \quad B = 0 \tag{11}$$

- Nothing unusual and/or strange

[deBoer, Shigemori]

Non-geometric backgrounds

T-duality along compact x^3:

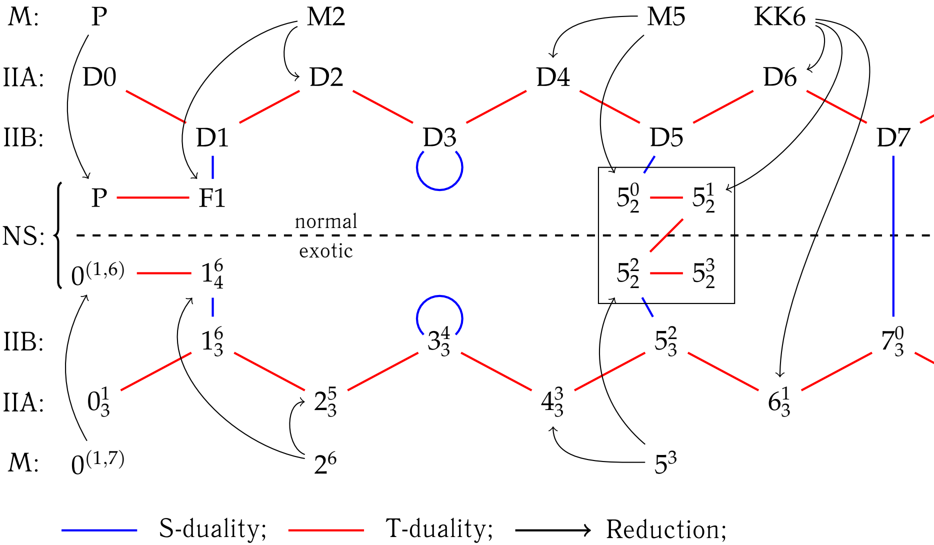
- Q-brane, $H(\rho) = 1 + h \log\frac{\mu}{\rho}, \quad \rho^2 = (x^1)^2 + (x^2)^2$

$$
\begin{aligned}
ds^2 &= \frac{H}{H^2 + h^2\theta^2}\Big(\underbrace{(dx^4)^2 + (dx^3)^2}_{\text{special}}\Big) + H\Big(\underbrace{(dx^1)^2 + (dx^2)^2}_{\substack{\text{transverse}\\ (\rho,\theta)}}\Big) + ds^2_{(1,5)},\\
B &= \frac{h\theta}{H^2 + h^2\theta^2} dx^4 \wedge dx^3
\end{aligned}
\tag{12}
$$

- Non-trivial monodromy around the brane $\theta \to \theta + 2\pi$
- Non-commutativity of open string coordinates
- This is supposed to have Q-flux $Q_\theta{}^{43} = h$

[deBoer, Shigemori]

Web of (some) branes



Challenge for exotic branes

Want to

- describe dynamics of exotic branes as fundamental objects of string theory
- do this in T-duality and gauge invariant way

For that need to understand

- What are symmetries of these objects? E.g. NS5-brane does not have Killing directions, while its T-dual, KK5-monopole, has one.
- How the world-volume fields fit into T-duality multiplets? Linear T-duality multiplets contain more fields, than available degrees of freedom.
- What potentials these interact with? E.g.: NS5-brane, KK5-monopole and exotic Q- and R-branes have very different fluxes and hence potentials

Doubled geometry

- Momentum and winding modes of a string unify into a doubled momentum;

$$\mathcal{P}^M = \begin{bmatrix} p^m \\ w_n \end{bmatrix} \Longrightarrow \mathbb{X}^M = \begin{bmatrix} x^m \\ \tilde{x}_m \end{bmatrix} = \begin{bmatrix} x_R^m + x_L^m \\ x_R^m - x_L^m \end{bmatrix} \tag{13}$$

- section constraint for consistency of the theory, kills half of the coordinates
- Choosing the T-duality frame $T_x : x \longleftrightarrow \tilde{x}$
- NS5-brane and NS exotic branes interact with $G_{\mu\nu}, B_{\mu\nu}, \varphi$
- Generalized metric is a T-duality covariant way of dealing with such backgrounds

$$\mathcal{H}_{MN} = \begin{bmatrix} g - Bg^{-1}B & Bg^{-1} \\ g^{-1}B & g^{-1} \end{bmatrix}, \quad \text{in analogy with} \quad F_{\mu\nu} = \begin{bmatrix} 0 & \vec{E} \\ -\vec{E} & *_3 \vec{B} \end{bmatrix} \tag{14}$$

DFT monopole

Kaluza-Klein understanding of the theory

$$\begin{aligned} ds^2_{\rm DFT} &= \mathcal{H}_{MN} d\mathbb{X}^M d\mathbb{X}^N \\ &= (g_{\mu\nu} - B_\mu{}^\rho B_{\rho\nu}) dx^\mu dx^\nu + 2B_\mu{}^\nu dx^\mu d\tilde{x}_\nu + g^{\mu\nu} d\tilde{x}_\mu d\tilde{x}_\nu . \end{aligned} \tag{15}$$

The Taub-NUT-like solution in coordinates $(z, y^i, x^a, \tilde{z}, \tilde{y}_i, \tilde{x}_a)$

$$\begin{aligned} ds^2_{\rm DFT} &= H(1 + H^{-2}A^2) dz^2 + H^{-1} d\tilde{z}^2 + 2H^{-1} A_i (dy^i d\tilde{z} - \delta^{ij} d\tilde{y}_j dz) \\ &\quad + H(\delta_{ij} + H^{-2} A_i A_j) dy^i dy^j + H^{-1} \delta^{ij} d\tilde{y}_i d\tilde{y}_j \\ &\quad + \eta_{ab} dx^a dx^b + \eta^{ab} d\tilde{x}_a d\tilde{x}_b , \\ e^{-2d} &= He^{-2\phi_0} . \end{aligned} \tag{16}$$

With the harmonic function $H(z, y) = 1 + \dfrac{h}{z^2 + \delta_{ij} y^i y^j}$

[Berman, Rudolph]

Physical subspace

- The doubled coordinates $\mathbb{X}^M = (x^z, x^i, x^a, \tilde{x}_z, \tilde{x}_i, \tilde{x}_a)$ are identified with the parameters $z, y^i, \tilde{z}, \tilde{y}_i$ with a high level of ambiguity
- The choice matters

$$\begin{aligned}
(x^z, x^1, x^2, x^3) &= (z, y^1, y^2, y^3), && \text{NS5-brane },\\
(x^z, x^1, x^2, x^3) &= (\tilde{z}, y^1, y^2, y^3), && \text{KK5-brane},\\
(x^z, x^1, x^2, x^3) &= (\tilde{z}, \tilde{y}_1, y^2, y^3), && \text{Q-brane},\\
(x^z, x^1, x^2, x^3) &= (\tilde{z}, \tilde{y}_1, \tilde{y}_2, y^3), && \text{R-brane},\\
(x^z, x^1, x^2, x^3) &= (\tilde{z}, \tilde{y}_1, \tilde{y}_2, \tilde{y}_3), && \text{R'-brane},
\end{aligned} \tag{17}$$

- The harmonic function is always

$$H(z, y^1, y^2, y^3) = 1 + \frac{h}{z^2 + \delta_{ij} y^i y^j} \tag{18}$$

- Non-geometric backgrounds depend on dual coordinates

Embedding of the brane

$$\begin{array}{cccccc|cccccccc} x^0 & x^1 & x^2 & x^3 & x^4 & x^5 & y^1 & y^2 & y^3 & y^4 & \tilde{y}_1 & \tilde{y}_2 & \tilde{y}_3 & \tilde{y}_4 \\ \hline \times & \times & \times & \times & \times & \times & & & & & & & & \\ \multicolumn{6}{c}{\underbrace{\hphantom{xxxxxxxxxxxxxxxxxxxxxxxx}}_{\text{world-volume}}} & \multicolumn{8}{c}{\underbrace{\hphantom{xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}}_{\text{transverse directions}}} \end{array} \tag{19}$$

Coordinates of the space-time = scalar fields on the brane

$$\Downarrow$$

no issues with doubled geometry, section condition etc.

- doubled number of scalar fields on the brane
- projection condition which leaves only half of them (to keep SUSY)

The action

$$
\begin{aligned}
S_{\mathrm{DBI}} = \int d^6\xi\, e^{-2d}\sqrt{|h_{ab}|}\sqrt{1+e^{2d}|h_{ab}|^{-1/2}(\bar{\lambda}_{\mathrm{brane}}\mathcal{C})^2}\,\times \\
\times\sqrt{-\left|g_{\mu\nu}\partial_\alpha X^\mu\partial_\beta X^\nu+\mathcal{H}_{MN}\hat{D}_\alpha Y^M\hat{D}_\beta Y^N-\frac{e^d(\det h)^{-1/4}\bar{\lambda}_{\mathrm{brane}}\mathcal{G}_{\alpha\beta}}{\sqrt{1+e^{2d}(\det h)^{-1/2}(\bar{\lambda}_{\mathrm{brane}}\mathcal{C})^2}}\right|}\,,
\end{aligned}
\tag{20}
$$

where

$$
\mathcal{G}_{\alpha\beta}=2\partial_{[\alpha}\tilde{c}_{\beta]}+\widetilde{\mathcal{C}}_{\alpha\beta}
\tag{21}
$$

is a worldvolume field strength with the following pullback of RR fields:

$$
\begin{aligned}
\widetilde{\mathcal{C}}_{\alpha\beta}=&\left(\mathcal{C}_{\mu\nu}-(B_{\mu\nu}+\frac{1}{2}A_\mu{}^M A_\nu{}^N\Gamma_{MN})\mathcal{C}+\sqrt{2}A_\mu{}^M\Gamma_M\mathcal{C}_\nu\right)\partial_{[\alpha}X^\mu\partial_{\beta]}X^\nu \\
&+\sqrt{2}\Gamma_M\left(\mathcal{C}_\mu-\frac{1}{\sqrt{2}}A_\mu{}^N\Gamma_N\mathcal{C}\right)\partial_{[\alpha}X^\mu\hat{D}_{\beta]}Y^M-\frac{1}{2}\Gamma_{MN}\mathcal{C}\hat{D}_{[\alpha}Y^M\hat{D}_{\beta]}Y^N.
\end{aligned}
\tag{22}
$$

The action

Keeping only NS-NS fields $\mathbb{X}^M = (x^m, \tilde{x}_m)$:

$$S^{(NS)}_{DBI} = \int d^6\xi e^{-2d}\sqrt{\det(h_{ab})} \times \sqrt{-\det\left(g_{\mu\nu}\partial_\alpha X^\mu\partial_\beta X^\nu + \Pi_{MN}\, D_\alpha\mathbb{X}^M D_\beta\mathbb{X}^N\right)}, \tag{23}$$

Kaluza-Klein-like projections

$$\begin{aligned} D_\alpha\mathbb{X}^M &= \partial_\alpha\mathbb{X}^M + \partial_\alpha X^\mu A_\mu{}^M \\ \Pi_{MN} &= \mathcal{H}_{MN} - h^{ab}\mathcal{H}_{MP}\mathcal{H}_{NQ}k_a^P k_b^Q, \\ h_{ab} &= \mathcal{H}_{MN}k_a^M k_b^M. \end{aligned} \tag{24}$$

The section constraint (algebraic):

$$\eta_{MN}k_a^M k_b^N = 0, \tag{25}$$

Allowed number of non-zero Killing vectors is no less than half of the total.

T-duality frames

$$\eta_{MN}k_a^M k_b^N = 0, \quad \eta_{MN} = \begin{bmatrix} 0 & 1_{4\times4} \\ 1_{4\times4} & 0 \end{bmatrix} \tag{26}$$

Killing vectors $k_a^M = (k_a^m; \tilde{k}_{am})$ satisfying the constraint and the embedding

	world-volume						transverse directions							
	x^0	x^1	x^2	x^3	x^4	x^5	y^1	y^2	y^3	y^4	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{y}_4$
NS5	×	×	×	×	×	×	●	●	●	●	k	k	k	k
KK5	×	×	×	×	×	×	●	●	●	k	k	k	k	●
Q	×	×	×	×	×	×	●	●	k	k	k	k	●	●
R	×	×	×	×	×	×	●	k	k	k	k	●	●	●
R′	×	×	×	×	×	×	k	k	k	k	●	●	●	●

(27)

● — localization direction
k — Killing direction

T-duality frames: D-branes

The same is true for Dp-branes

	0	1	2	3	4	5	6	7	8	9	$\tilde{1}$	$\tilde{2}$	$\tilde{3}$	$\tilde{4}$	$\tilde{5}$	$\tilde{6}$	$\tilde{7}$	$\tilde{8}$	$\tilde{9}$
D0	×	●	●	●	●	●	●	●	●	●	×	×	×	×	×	×	×	×	×
D1	×	×	●	●	●	●	●	●	●	●	●	×	×	×	×	×	×	×	×
D2	×	×	×	●	●	●	●	●	●	●	●	●	×	×	×	×	×	×	×
D3	×	×	×	×	●	●	●	●	●	●	●	●	●	×	×	×	×	×	×
D4	×	×	×	×	×	●	●	●	●	●	●	●	●	●	×	×	×	×	×
D5	×	×	×	×	×	×	●	●	●	●	●	●	●	●	●	×	×	×	×
D6	×	×	×	×	×	×	×	●	●	●	●	●	●	●	●	●	×	×	×
D7	×	×	×	×	×	×	×	×	●	●	●	●	●	●	●	●	●	×	×
D8	×	×	×	×	×	×	×	×	×	●	●	●	●	●	●	●	●	●	×
D9	×	×	×	×	×	×	×	×	×	×	●	●	●	●	●	●	●	●	●

The results

- Action for exotic branes
- which is invariant
- which produces backgrounds for the normal and exotic NS branes
- the choice depends on orientation in the doubled space
- D-branes are fancy

Discussion

Future directions

- Consider U-duality orbits and construct effective actions for that
- Field theories on worldvolume (especially for D-branes)
- Construct gauge and T-invariant Wess-Zumino term in the same frame as the kinetic action
- Microscopic meaning of the non-geometric D-branes (if any)

What's the use of all that?

- Tadpole cancellation conditions for flux compactifications (Bianchi identities), support for internal space
- String behavior on such backgrounds: non-commutativity and non-associativity
- Little string theories from NS five-branes

Thank you!